Duration Time Analysis of Spouse Employment in the U.S. Army

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August 1990

United States Army
Research Institute for the Behavioral and Social Sciences

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### Title
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### Type of Report
Final

### Date of Report
1990, August

### Page Count
10

### Abstract
The Army Family Research Program (AFRP) is a 5-year integrated research program that supports the Chief of Staff of the Army (CSA) White Paper 1983: The Army Family and The Army Family Action Plans (1984-1990) by developing databases, models, program evaluation technologies, and policy options that assist the Army to retain quality soldiers, improve soldier and unit readiness, and increase family adaptation to Army life. This report presents a conceptual model of analysis of duration of spouse employment in general, and of Army spouses in particular. The utility of the model is presented in the context analysis of data from the 1985 Department of Defense (DoD) Survey of Spouses.
DURATION TIME ANALYSIS OF SPOUSE EMPLOYMENT IN THE U.S. ARMY

CONTENTS

INTRODUCTION ........................................ 1
DEFINITION, CENSORING, AND EXPLANATORY VARIABLES ............ 1
ALTERNATIVE FUNCTIONAL FORMS .................................. 2
REFERENCES ....................................................... 7

LIST OF TABLES

Table 1. Characteristics of special distributions ............... 3
DURATION TIME ANALYSIS OF SPOUSE EMPLOYMENT IN THE U.S. ARMY

1.0 Introduction

Recently, the subject of duration analysis has found a footing not only in the actuarial, physical, and biomedical sciences but also in social sciences such as economics (Baldwin, 1983; Kiefer, 1988; Heckman & Singer, 1982, 1985), psychology (Fellman, Goldberg, & May, 1987), and sociology (Allison, 1985; Koo, Suchindran, & Griffith, 1984; Tuma, 1983; Tuma & Hannan, 1984). For example, econometricians employ it to analyze duration of spells of unemployment and employment. Psychometricians use it to analyze time taken to complete a task and similar activities while sociometrists use it to evaluate duration of marriage, divorce, and time span between births.

An objective of this paper is to present a conceptual model of analysis of duration of employment of spouses in general, and that of Army spouses, in particular. The second section deals with a brief discussion of alternative functional forms used in the literature to analyze duration data. The third section outlines reasons for selection by us of a specific functional form suitable for analysis of economic data on employment duration.

2.0 Definition, Censoring and Explanatory Variables

Duration is objectively defined with respect to a time origin and the end of the time period for analysis. Ideally, all individuals or observations need to be comparable at the beginning of the period of analysis. The periods of duration should also be homogeneous. For analysis of employment or unemployment, one should separate periods of boom from that of recessions. The data on duration of employment with current employer in the 1985 Department of Defense (DoD) Survey of Spouses (hereafter referred to as the Survey of Spouses) conform with this definition because the period of three years prior to date of the survey comprised an economic boom period in the United States. Also, most of the spouses did not exceed a duration of three years because of the general institutional practice of having soldiers undergo a Permanent Change of Stations (PCS) every three years. For analysis of the Survey of Spouses data, it was planned to use the origin of time as January 1982 and the period of three years ending with December 1984.

Duration analysis often uses survey data so that the period of duration is not likely to be completed for several observations on the date of the survey. This is called right-censoring of the data. For example, for the Survey of Spouses, the data for spouses who were working with their current employers were right-censored at the time of their responses. To account for right-censoring, duration analysis assumes that the censored individuals are representative of individuals who survive during the period. The measurement of the period of duration is given by $X_i$ where:

$$X_i = \min (T_i, C_i)$$

where $T_i = $ duration time of individual $i$; and

$C_i = $ censoring time of individual $i$. 

1
In short, the duration period is the smaller of the time to termination or the time to censure. In the proposed analysis of the Survey of Spouses data, the duration of employment of spouses was to be calculated in months, for a sample of spouses employed between January 1982 and the month of the survey.

3.0 Alternative Functional Forms

Duration (or hazard, survival/failure) is defined as the probability of occurrence of an event per unit of time (e.g., duration of employment or unemployment of an individual during a period of time). The functional form of such a probability dependent variable is non-linear. The methodology that is generally used is that of maximum likelihood because Ordinary Least Squares regressions yield biased estimates (Flinn & Heckman, 1982). The information on duration is often more useful that the mere incidence of finding a job or that of being laid off from work because the socioeconomic and psychological problems associated with, say, unemployment, may accentuate with an increase in duration. Analysis of duration of employment is also crucial for discussion of such phenomena as career commitment and career advancement which generally result from an increase in duration of employment with a specific employer.

Since the functional form of duration is likely to vary with the theoretical structure employed by an analyst, it is not surprising that there are at least thirteen special mathematical distributional functions employed for analysis of duration data. These distributions and their properties are outlined in Table 1. Of these, the most frequently used functional form in the literature is the exponential, perhaps because it is the only distribution with only one parameter, with mean = standard deviation = 1/rho (see Table 1). There are, however, several limitations of this distribution for analysis of economic data on employment duration. For example, the exponential distribution requires that if and only if \( f_T(t) = \text{rho} \text{ (constant}), \)

\[ F_T(t) = \exp.(-\text{rho}(t)) \text{, so that the rate of increase in duration is monotonic or constant.} \]

Lancaster (1979) rightly questioned the assumption of such a constant duration rate for his analysis of unemployment spells. In theory, he expected an increasing (instead of constant) duration of unemployment. In the context of economic theory, however, there is no a priori reason to expect a monotonic or an increasing duration function. In order to accommodate an increasing, constant (monotonic) or a decreasing function, Lancaster initially used a Weibull distribution which is defined by:

\[ F(t) = 1 - \exp.(-\lambda(t)\exp.\alpha) \]

If alpha = 1, equation (1) is reduced to the exponential distribution noted above. It's density is given by:

\[ f(t) = \lambda(\alpha.t)\exp.(\alpha - 1)\exp.(-\lambda(t)\exp.\alpha) \]

and its duration rate is given by:
Table 1

Characteristics of Special Distributions

<table>
<thead>
<tr>
<th></th>
<th>Survivor function</th>
<th>Density function</th>
<th>Hazard</th>
<th>No of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Exponential</td>
<td>$e^{-\rho t}$</td>
<td>$pe^{-\rho t}$</td>
<td>$\rho$</td>
<td>1</td>
</tr>
<tr>
<td>(ii) Gamma</td>
<td>incomplete gamma function</td>
<td>$\frac{\rho(\rho t)^{\alpha-1} e^{-\rho t}}{\Gamma(\alpha)}$</td>
<td>$\rho$</td>
<td>2</td>
</tr>
<tr>
<td>(iii) Weibull</td>
<td>$\exp[-(\rho t)^\alpha]$</td>
<td>$\kappa \rho(\rho t)^{\alpha-1} \exp[-(\rho t)^\alpha]$</td>
<td>$\kappa \rho(\rho t)^{\alpha-1}$</td>
<td>2</td>
</tr>
<tr>
<td>(iv) Gompertz-Makeham</td>
<td>$- \alpha \rho_0 t e^{-\rho_0 t}$</td>
<td>$\frac{\kappa(\kappa \rho_0)^\alpha}{(1 - \kappa \rho_0)^\alpha}$</td>
<td>$\frac{\kappa}{1 + \kappa \rho_0}$</td>
<td>2</td>
</tr>
<tr>
<td>(v) Compound exponential</td>
<td>$\frac{(\kappa \rho_0)^\alpha}{(l - \kappa \rho_0)^\alpha}$</td>
<td>$\frac{\kappa(\kappa \rho_0)^\alpha}{(l - \kappa \rho_0)^\alpha}$</td>
<td>$\frac{\kappa}{l + \kappa \rho_0}$</td>
<td>3</td>
</tr>
<tr>
<td>(vi) Orthogonal polynomial</td>
<td>$e^{-\rho t}[1 + \kappa_1 \rho t - \kappa_2 \rho t(\rho t - 2)]$</td>
<td>$\rho e^{-\rho t}[1 + \kappa_1 L_1(\rho t) + \kappa_2 L_2(\rho t)]$</td>
<td>nonmonotonic</td>
<td>2</td>
</tr>
<tr>
<td>(vii) Log normal</td>
<td>$- \sqrt{2 \pi \rho}$</td>
<td>$\kappa e^{-1}[1 + (\rho t)^\alpha]^{-1}$</td>
<td>$\frac{\kappa e^{-1} \rho^\alpha}{[1 + (\rho t)^\alpha]}$</td>
<td>2</td>
</tr>
<tr>
<td>(viii) Log logistic</td>
<td>$[1 + (\rho t)^\alpha]^{-1}$</td>
<td>$\kappa e^{-1}[1 + (\rho t)^\alpha]^{-1}$</td>
<td>$[1 + (\rho t)^\alpha]$</td>
<td>4</td>
</tr>
<tr>
<td>(ix) Generalized F</td>
<td>$- \sqrt{2 \pi \rho}$</td>
<td>$\kappa e^{-1}[1 + (\rho t)^\alpha]^{-1}$</td>
<td>$[1 + (\rho t)^\alpha]$</td>
<td>2</td>
</tr>
<tr>
<td>(x) Inverse Gaussian</td>
<td>$- \sqrt{2 \pi \rho}$</td>
<td>$\kappa e^{-1}[1 + (\rho t)^\alpha]^{-1}$</td>
<td>$[1 + (\rho t)^\alpha]$</td>
<td>2</td>
</tr>
<tr>
<td>(xi) Translation</td>
<td>$- \sqrt{2 \pi \rho}$</td>
<td>$\kappa e^{-1}[1 + (\rho t)^\alpha]^{-1}$</td>
<td>$[1 + (\rho t)^\alpha]$</td>
<td>2</td>
</tr>
<tr>
<td>(xii) Scale family</td>
<td>$\Phi(\rho t)$</td>
<td>$\rho g(\rho t)$</td>
<td>$\rho h''(\rho t)$</td>
<td>1 extra for origin</td>
</tr>
<tr>
<td>(xiii) Proportional hazard family</td>
<td>$[\mathcal{L}(t)]^\alpha$</td>
<td>$\Psi[\mathcal{L}(t)]^{\alpha-1} l(t)$</td>
<td>$\Psi h''(t)$</td>
<td>1 extra for proportionality</td>
</tr>
</tbody>
</table>

\[ \lambda(t) = \frac{f(t)}{1 - F(t)} = \lambda \alpha t \exp(\alpha - 1) \]

An empirical value of \( \alpha \) in equation (3) determines whether the duration rate is increasing or not, as is shown below:

\[ \alpha > 1, \ \frac{d\lambda}{dt} > 0 \quad \text{(increasing rate)} \]

\[ \alpha = 1, \ \frac{d\lambda}{dt} = 0 \quad \text{(constant rate)} \]

\[ \alpha < 1, \ \frac{d\lambda}{dt} < 0 \quad \text{(decreasing rate)}. \]

Lancaster (1979) specified an \( i \)'th person's duration rate as:

\[ \lambda_i^*(t) = \alpha t \exp(\alpha - 1) \exp(\beta'x_i) \]

where \( x_i \) is a vector of \( i \)'th person's characteristics.

Empirically, Lancaster's maximum likelihood estimate of \( \alpha \) was 0.77, a result indicating decreasing duration rate of unemployment. Conversely, for employment, we would expect an increase in the probability or rate of continuation of an individual, the longer the individual or the spouse stays with the employer.

Lancaster also reported an interesting finding which is useful for selection of specific functional form for analysis of Survey of Spouses data. Lancaster's estimate of \( \alpha \) increased as he added more explanatory variables to the model. This result indicates that the decreasing hazard rate implied by his first estimate was at least partly due to the heterogeneity caused by the initially omitted explanatory variables rather than true duration dependence.

Since it is virtually impossible to include all of the relevant variables, Lancaster used an alternative specification to account for such exclusion from the duration function:

\[ \mu_i^*(t) = v_i \lambda_i(t) \]

where \( \lambda_i(t) \) is the same as in equation (7) and \( v_i \) is an unobserved random variable assumed to be independently and identically distributed as Gamma (1, \( \sigma^2 \)). The random variable \( v_i \) is a proxy for all the unobservable explanatory variables. Amemiya (1985) obtains the following decreasing duration function from equation (8) because of the addition of heterogeneity denoted by \( \sigma^2 \):

\[ \lambda^*(t) = \lambda(t)(1 - F^*(t))^\sigma \]

where \( (1 - F^*(t))^\sigma \) is a decreasing function of \( t \). Under this new model, Lancaster found the Maximum Likelihood Estimate of \( \alpha \)
to be 0.9. Hence he argues that a decreasing duration rate in his model is caused more by heterogeneity rather than by true duration dependence.

In the proposed analysis of the data from the Survey of Spouses, the use of the Gamma distribution was planned to account for heterogeneity because not all of the relevant explanatory variables for inclusion in the model were available.
4.0 REFERENCES


