ELEVATED TEMPERATURE CRACK GROWTH IN TITANIUM ALUMINIDES

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Crack growth of Titanium intermetallics at elevated temperatures has been studied. Based on limited amount of test data available in the literature, it is concluded that, for Titanium aluminides at temperatures over 700°C, $C_t$ correlates better with $da/dt$ than the LEFM parameter $K$. An alternative definition of $C_t$ has also been derived in this study. The new $C_t$ definition is based on a line integral along the outer boundary of the whole structure. Such an alternative definition has made it easier to calculate $C_t$ for structural components.

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 INTRODUCTION</td>
<td>1-1</td>
</tr>
<tr>
<td>2.0 CREEP CRACK GROWTH DATA COLLECTION</td>
<td>2-1</td>
</tr>
<tr>
<td>3.0 CREEP CRACK GROWTH PARAMETERS</td>
<td>3-1</td>
</tr>
<tr>
<td>3.1 Crack Tip Parameters in TDFM</td>
<td>3-2</td>
</tr>
<tr>
<td>3.2 Creep Crack Growth Behavior in Ti-24Al-11Nb</td>
<td>3-8</td>
</tr>
<tr>
<td>3.2.1 Data Analyses Methods</td>
<td>3-9</td>
</tr>
<tr>
<td>3.2.2 Results and Discussion</td>
<td>3-11</td>
</tr>
<tr>
<td>3.3 Creep-Fatigue Crack Growth Behavior</td>
<td>3-13</td>
</tr>
<tr>
<td>4.0 RE-EXAMINATION AND APPLICATION OF $C_t$</td>
<td>4-1</td>
</tr>
<tr>
<td>4.1 Pseudo-Potential $P^*$ and $C_t$</td>
<td>4-1</td>
</tr>
<tr>
<td>4.2 Steady-State Creep</td>
<td>4-3</td>
</tr>
<tr>
<td>4.3 Small Scale Creep</td>
<td>4-3</td>
</tr>
<tr>
<td>4.4 Creep in Transition Period</td>
<td>4-8</td>
</tr>
<tr>
<td>4.5 $C_0(t)$ versus $C_t$</td>
<td>4-9</td>
</tr>
<tr>
<td>5.0 CONCLUSION AND RECOMMENDATIONS</td>
<td>5-1</td>
</tr>
<tr>
<td>6.0 REFERENCES</td>
<td>6-1</td>
</tr>
</tbody>
</table>

APPENDIX A LISTING OF THE COMPUTER CODE | A-1 |
APPENDIX B EQUATIONS FOR CALCULATING $\dot{V}_C$ AND $\dot{V}$ [69] | B-1 |
APPENDIX C RESULTS OF THE ANALYSIS OF CREEP CRACK GROWTH DATA OF TITANIUM ALUMINIDES | C-1 |
APPENDIX D DERIVATION OF $\frac{d}{da} \int_{A_C} f dA$ | D-1 |
APPENDIX E RELATIONSHIP BETWEEN $C_{os}$ and $C_t$ | E-1 |
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1</td>
<td>UDRI Test Matrix for Creep Crack Growth Tests on Ti-24Al-11Nb</td>
<td>3-16</td>
</tr>
<tr>
<td>3-2</td>
<td>Values of the Creep Constant &quot;A&quot; at Various Test Temperatures</td>
<td>3-17</td>
</tr>
<tr>
<td>3-3</td>
<td>Tensile Yield Strength and Elastic Modulus at Various Temperatures</td>
<td>3-17</td>
</tr>
<tr>
<td>4-1</td>
<td>Creep Fracture Constants</td>
<td>4-11</td>
</tr>
<tr>
<td>4-2</td>
<td>Effects of Creep Zone Shape (n=5)</td>
<td>4-11</td>
</tr>
<tr>
<td>4-3</td>
<td>Example Material Properties</td>
<td>4-12</td>
</tr>
<tr>
<td>4-4</td>
<td>$C^*$ and $t_T$ for the Example Problem</td>
<td>4-12</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-2</td>
<td>Creep Behavior of Ti-24Al-11Nb at Stress Levels of 137.8 MPa, 172.3 MPa and Temperature of 750°C [11]</td>
<td>2-6</td>
</tr>
<tr>
<td>2-4</td>
<td>Steady Strain Rate vs. Stress at 700°C and 750°C for Creep of Ti-24Al-11Nb [11]</td>
<td>2-8</td>
</tr>
<tr>
<td>2-5</td>
<td>Crack Growth Per Cycle Curves for 750°C [13]</td>
<td>2-9</td>
</tr>
<tr>
<td>2-6</td>
<td>Crack Growth Rate Curves for 750°C [13]</td>
<td>2-10</td>
</tr>
<tr>
<td>2-7</td>
<td>Sustained Load Crack Growth Rate for 750°C [13]</td>
<td>2-11</td>
</tr>
<tr>
<td>2-8</td>
<td>Effect of Temperature on Creep Behavior of Ti-24Al-11Nb [12]</td>
<td>2-12</td>
</tr>
<tr>
<td>2-10</td>
<td>Creep Crack Growth Rate Behavior of Ti-24Al-11Nb [12]</td>
<td>2-14</td>
</tr>
<tr>
<td>2-12</td>
<td>Effect of Frequency on Fatigue Crack Growth of Ti-24Al-11Nb at 650°C [27]</td>
<td>2-16</td>
</tr>
<tr>
<td>3-1</td>
<td>Creep Deformation Behavior of Ti-24Al-11Nb</td>
<td>3-18</td>
</tr>
<tr>
<td>3-2</td>
<td>Schematic Representation of the Levels of Creep Deformation Under Which Creep Crack Growth Can Occur</td>
<td>3-19</td>
</tr>
<tr>
<td>3-3</td>
<td>(a) Load Line Deflection, ( V_c ), as a Function of Time for Bodies of Crack Lengths ( a ) and ( a+\Delta a ) at Various Load Levels, and (b) the Definition of the ( C_t ) Parameter</td>
<td>3-20</td>
</tr>
</tbody>
</table>
LIST OF FIGURES (continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4</td>
<td>da/dt Versus $C_t$ Data for Ti-24Al-11Nb Obtained from Specimens of Two Sizes</td>
<td>3-21</td>
</tr>
<tr>
<td>3-5</td>
<td>da/dt Versus $K$ Data for Ti-24Al-11Nb Obtained From Specimens of Two Different Sizes</td>
<td>3-22</td>
</tr>
<tr>
<td>3-6</td>
<td>da/dt Versus $C_t$ Data for Ti-24Al-11Nb Obtained at 650°C in Air and Vacuum Environments</td>
<td>3-23</td>
</tr>
<tr>
<td>3-7</td>
<td>da/dt Versus $K$ Data for Ti-24Al-11Nb Obtained at 650°C in Air and Vacuum Environments</td>
<td>3-24</td>
</tr>
<tr>
<td>3-8</td>
<td>Creep Crack Growth Behavior of Ti-24Al-11Nb in Air at Various Temperatures</td>
<td>3-25</td>
</tr>
<tr>
<td>3-9</td>
<td>Creep Crack Growth Behavior of Ti-24Al-11Nb in Vacuum at Various Temperatures</td>
<td>3-26</td>
</tr>
<tr>
<td>3-10</td>
<td>Representation of Creep-Fatigue Crack Growth Behavior in Terms of the $(C_t)_{avg}$ Parameter for New Cr-Mo-V Steels Tested at 1000°F (538°C) in Air and Argon</td>
<td>3-27</td>
</tr>
<tr>
<td>3-11</td>
<td>Superimposition of Creep-Fatigue Crack Growth Data According to the $(C_t)_{avg}$ Parameter Concept for New and Ex-Service Cr-Mo-V Steels Tested at 1000°F (538°C) and 800°F (427°C)</td>
<td>3-28</td>
</tr>
<tr>
<td>3-12</td>
<td>$C_t$ as a Function of Time for Cyclic Operation</td>
<td>3-29</td>
</tr>
<tr>
<td>3-13</td>
<td>Normalized Crack Tip Stress as a Function of Normalized Time at a Fixed Distance Ahead of the Crack Tip for Fast Loading Followed by a Sustained Load</td>
<td>3-30</td>
</tr>
<tr>
<td>4-1</td>
<td>Schematic of Small Scale Creep</td>
<td>4-13</td>
</tr>
<tr>
<td>4-2</td>
<td>Comparison of $C_t$ for $n=5$</td>
<td>4-14</td>
</tr>
<tr>
<td>4-3</td>
<td>Comparison of $C_t$ for $n=7$</td>
<td>4-15</td>
</tr>
<tr>
<td>4-4</td>
<td>Comparison of $C_t$ for $n=10$</td>
<td>4-16</td>
</tr>
<tr>
<td>4-5</td>
<td>Comparison of $C_t$ for $n=15$</td>
<td>4-17</td>
</tr>
<tr>
<td>4-6</td>
<td>Comparison of $C_t$ for $n=20$</td>
<td>4-18</td>
</tr>
</tbody>
</table>
LIST OF FIGURES (concluded)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-1</td>
<td>Creep Zone at Two Different Crack Depths Under the Same Loading History</td>
<td>D-2</td>
</tr>
<tr>
<td>E-1</td>
<td>Relationship Between $C_t$ and $C_o$ for Small Scale Creep</td>
<td>E-3</td>
</tr>
</tbody>
</table>
1.0 INTRODUCTION

For aircraft structures under constant amplitude cyclic loads, the relationship between fatigue crack growth rate, da/dN, and cyclic stress intensity factor, ΔK, can usually be characterized by one of the laws proposed by Paris, Walker [1], Collipriest [2], Hop-Rau [3], or Forman [4,5]. However, in many aerospace problems, the stress can vary significantly from cycle to cycle. Such variation in loading, or spectrum loading can have significant effects on fatigue crack growth rate [6]. The overloads generally retard crack growth below the levels predicted from constant amplitude conditions using the crack growth laws referenced above. If load interaction effects are ignored, the predicted life is generally conservative, but more realistic crack growth prediction can be achieved by the use of retardation models.

For pure mechanical loads at relatively low temperatures, the retardation models proposed by Wheeler [7] or Willenborg [8] have been widely used. However, the load spectra in aircraft structures often consist not only of sustained mechanical stresses but may also include high amplitude transient thermal stresses. The transient thermal stresses, at elevated temperatures, can cause increased plastic deformation and crack surface closure at crack tips, and thus may significantly influence the subsequent fatigue crack growth behavior. During the sustained load period which often follows the transient thermal stresses, time-dependent creep may also occur locally in the crack tip region, which will also influence the subsequent crack growth behavior.

Recent developments in processing technology has led to the emergence of a series of high temperature titanium alloys and intermetallics. Some of these materials are being considered for structural components of advanced aircraft and hypervelocity vehicles. Intelligent use of these materials requires the
ability to predict the life of the component under service conditions. The current push to use these materials at elevated temperature over a long period of time has drawn attention to the time-dependent (creep) elevated temperature response of high temperature titanium alloys and intermetallics.

A technique for fatigue crack growth modeling with retardation and acceleration effects in a thermal-mechanical environment is needed but does not exist. The objective of this proposed research (Phases I and II) is to develop and demonstrate a new technique for modeling fatigue crack growth with retardation/acceleration effects in combined mechanical thermal stress applications at elevated temperatures.

In this Phase I study, experimental data on crack growth of titanium alloys and intermetallics were first compiled. In all of the data collected in this literature survey, the crack growth rates, da/dt, were correlated with the stress intensity factor K, which is a valid parameter in lower temperature range but may not be appropriate for the high temperature applications when creep deformation occurs. Several crack tip parameters, including K, C*, C(t), and C_t, were then evaluated for their applicability and practicality for titanium alloys and intermetallics in the high temperature environments. It is identified under this task that C_t may be the best parameter for characterizing creep crack growth of titanium alloys. Application of C_t to creep-fatigue crack growth modeling of titanium intermetallics is also discussed briefly in this report even though there is not enough data to substantiate it. In the last task of this phase I study, C_t was reexamined and alternative ways of calculating C_t were derived. C_t can be calculated more easily for components using this method.

It is concluded after this Phase I study that research in both the analytical and the experimental areas, on creep crack growth
of titanium alloys and intermetallics in high temperature environments is far from complete. Recommendations for future research work in these areas are made.
2.0 CREEP CRACK GROWTH DATA COLLECTION

Extensive studies have been conducted in the past for predicting the crack growth behavior of Ni-base superalloys and stainless steels at elevated temperatures. The crack growth in a number of nickel base superalloys has been successfully characterized with Linear Elastic Fracture Mechanics (LEFM) parameters. However, since different materials can behave differently in high temperature environments, the use of LEFM parameters, cannot be universally justified. For example the preliminary studies on the high temperature crack growth behaviors of titanium aluminides have shown that \( \frac{da}{dt} \) may not uniquely correlate with \( K \). The crack growth at elevated temperatures has been shown to be very complex and depends on load cycles, time and environmental conditions. Further, special attention is required to deal with the situation of complex loading at elevated temperatures such as a flight spectrum involving hold time, overload, etc.

The amount of elevated temperature crack growth data in titanium alloys is limited. Specifically, creep crack growth data on titanium alloys are scarce. The crack growth data, available in the literature, under different hold times and overloads are limited to relatively lower temperatures. Since these alloys are designed for applications at elevated temperatures with long hold periods, information regarding the time-dependent response as well as the means of implementing this information into a life prediction model is essential.

The technical community currently has no standard for evaluating creep crack growth behavior at elevated temperatures. A number of parameters (other than \( K \)) such as the nonlinear elastic parameter, \( J^* \)-integral, or the creep crack growth parameters, the \( C^* \)-integral, and reference stress, etc., have been identified which could be used to correlate the crack growth behavior at
elevated temperatures. Saxena, et al, [9] have successfully demonstrated the advantages of C_t over K for creeping solids.

The titanium alloys for the most part have not attracted much application in industry at temperatures greater than 500°C due to their limited oxidation resistance above this temperature. At temperatures less than 500°C, the creep process in these alloys is not important. Therefore, it is not surprising that hardly any data are available on creep crack growth of titanium alloys. It is anticipated that the new elevated temperature titanium alloys such as Ti-1100, IMI 834, etc., will see application at temperatures above 500°C in aerospace components; also titanium aluminides are expected to sustain complex loading at temperatures up to 700°C. As indicated earlier, these alloys might sustain measurable creep damage at operating temperatures under flight spectrum loadings, involving overload, hold time, etc. Hence, a strong need exists for a reasonable parameter which could uniquely account for damage caused by complex load excursions at high temperatures.

In an attempt to demonstrate the feasibility of use of C^+, C(t), C_t, and K to describe the damage evolution during crack growth in high temperature titanium alloys and intermetallics, an extensive literature survey was conducted to obtain elevated temperature crack growth data. During this survey, only limited data on time dependent crack growth were available on titanium alloys; therefore, the study focused on intermetallics. The limited data on the creep deformation [10,11], creep crack growth [10-12], hold time, creep fatigue interaction [10], and thermal fatigue [13] of titanium aluminide which were available were used to demonstrate a better correlation of crack growth rates with time-dependent fracture mechanics parameters. The original data were correlated with the stress intensity factor K. These results are summarized in Figures 2-1 through 2-12. In Figures 2-1 and 2-2, creep behavior of Ti-24Al-11Nb at several different
load levels are illustrated for temperatures at 700°C and 750°C, respectively. Rupture time and final failure strain for the above two tests are shown in Figures 2-3 and 2-4. Crack growth curves for Titanium aluminides at 750°C by [13] are shown in Figures 2-5 to 2-7 in terms of K. It is noted from these three figures that crack growth rate of Titanium aluminides at 750°C does not correlate well with K. Recent test data by Khobaib [12] for Ti-24Al-11Nb are plotted in Figures 2-8 to 2-11. Again, it is concluded from these figures that crack growth rate of Ti-24Al-11Nb does not correlate well with K. These data will be replotted in terms of $C_t$ in section 3 of this report. Balsone et al [27] have studied the frequency effects on crack growth and their data is shown in Figure 2-12.

The fatigue crack growth rate (FCGR) behavior of titanium alloys at elevated temperature has been extensively studied [14-19]. Most of the studies, however, describe either the correlation of microstructures with FCGR or the effect of composition on FCGR. The effect of environment has also been investigated [20,21] and an inert environment seems to improve FCGR resistance at higher temperatures. The effect of frequency and stress ratio has been mainly studied at room temperature, where the main interests have been the role of hydrogen. The issue of hold time effect also have been extensively studied [22-26]. However, these studies are limited in scope, because the temperatures of interest are relatively low, and the main interest in these studies also centers around the effect of hydrogen. Currently, Balsone, et al [27], are investigating the effect of frequency on the elevated temperature FCGR of Ti-24Al-11Nb. Reportedly, the crack growth increases at lower frequencies, but a clear time dependent model as in the case of Ni-base superalloy [28] has not been reported. The inert environment, of course, slows down the FCGR, but more work is needed to define the creep contribution on the crack growth behavior at these elevated temperatures.
The crack growth rate retardation due to single or multiple overload cycles on FCGR have been studied by numerous workers [29-49]. Almost all investigations were conducted at room temperature where no creep effect is involved. The retardation effects have been explained by (1) compressive residual stresses ahead of the crack tip, (2) crack closure, and (3) the strain hardening effect in the overloaded plastic zone.
Figure 2-1. Creep Behavior of Ti-24Al-11Nb at Stress Levels of 275.6 MPa, 241.2 MPa and 206.7 MPa and Temp. of 700°C (Rev. 11)
Figure 2-2. Creep Behavior of Ti-24Al-11Nb at Stress Levels of 137.8 MPa, 172.3 MPa and 206.7 MPa and Temp. of 750°C (Ref. 11)
Figure 2-3. Plot of Stress vs. Rupture Life for Creep Rupture of Ti-24Al-11Nb (Ref. 11)
Figure 2-4. Steady Strain Rate vs Stresses at 700°C for Ti-24Al-11Nb (Ref. 11)
Figure 2-5. Crack Growth Per Cycle Curves for 750°C (Ref. 13)
Figure 2-6. Crack Growth Rate Curves for 750°C (Rev. 13)
Figure 2-7. Sustained Load Crack Growth Rate for 750°C (Ref. 13)
Effect of Temperature on Creep Behavior of Ti-24Al-11Nb (Ref. 12)

Figure 2-8.

(N/IN)
Figure 2-9. Creep Crack Growth Behavior of Ti-24Al-11Nb (Ref. 12)

2-13
Figure 2-10. Creep Crack Growth Rate Behavior of Ti-24Al-11Nb (Ref. 12)
Figure 2-11. Creep Crack Growth Behavior of Ti-24Al-11ND (Ref. 12)
Figure 2-12. Effect of Frequency on Fatigue Crack Growth of Ti-24Al-11Nb at 650°C (Ref. 27)
3.0 CREEP CRACK GROWTH PARAMETERS

Titanium aluminides are an important class of candidate materials for application in the manned hypersonic vehicle program. In the temperature regime of 600 to 900°C, these materials have high specific strength properties and possess good environmental and creep resistance [12]. However, before they can be used in the aerospace structures, it is necessary to develop accurate models for predicting their crack growth behavior in the intended service environment.

Recently, some studies have been performed for characterizing the creep and creep-fatigue crack growth behavior in a titanium aluminide intermetallic [12,13]. These studies were planned with the objective of correlating the crack growth data using the linear elastic fracture mechanics (LEFM) parameters. However, after completion of these tests it was recognized that the crack growth was accompanied by considerable creep deformation. Thus, the crack growth rate correlations presented in terms of the LEFM parameters are not as good as those in the low temperature applications.

In this section, the results from the creep crack growth tests for which all raw data were available, are reanalyzed using time-dependent fracture mechanics (TDFM) concepts and crack tip parameters. Analytically estimated values of load-line deflection rates were obtained by considering the creep deformation rates of these materials and were subsequently used to estimate the TDFM parameters. In the subsequent discussion, the TDFM concepts are first briefly reviewed and subsequently, the results of the analysis of the creep crack growth data are presented and discussed. Unfortunately, the raw data for the creep-fatigue tests were not available and could not be reanalyzed in the same way as the creep crack growth data. However, some discussion on the applicability of TDFM concepts to creep-fatigue crack growth is presented in this section.
3.1 Crack Tip Parameters in TDFM

Figures 2-1 and 2-2 show the creep deformation behavior of Ti-24Al-11Nb at 700 and 750°C, respectively. At both temperatures, the strain versus time behavior shows the classical three region behavior consisting of primary, secondary and tertiary creep. At a constant temperature, such creep behavior can be represented by an equation of the type given below:

\[ \dot{\varepsilon}_c = A_1 \varepsilon_c^{p} \sigma^{n_1 (1+p)} + A_2 \varepsilon_s^{n_2} + A_3 (\varepsilon_t)^p \sigma^{n_3 (1-p)} \]  

where, \( \varepsilon_c = \varepsilon_p + \varepsilon_s + \varepsilon_t \)

\( \varepsilon_p, \varepsilon_s, \varepsilon_t \) are primary, secondary and tertiary components, respectively, of the total creep strain, \( \varepsilon_c \), \( \sigma \) is applied stress, and \( A_1, p, n_1, A_2, n_2, A_3, p_3, n_3 \) are regression constants which describe the creep deformation trend in their various regimes.

In Figures 2-1 and 2-2 it is observed that the creep ductilities are in excess of twenty percent at all stress levels and at both temperatures. Also, the primary creep behavior is limited to short periods following loading. In cracked bodies of ductile materials, tertiary creep is expected to be restricted to the process zone near the crack tip and is not anticipated to contribute significantly to the remote load-point deflection. Thus, the most significant contribution to the load-line deflection rate in titanium aluminide specimens will be from the second term (secondary creep rate) on the right hand side of Equation (3-1). However, initially upon loading, the crack tip stress field will redistribute and an elastic strain rate term must be added to the creep strain rate to determine the overall strain rate, \( \dot{\varepsilon} \) which can be written as:

\[ \dot{\varepsilon} = \dot{\sigma}/E + A_2 \sigma^n \]  

(3-2)
where, $\dot{\sigma}$ is the rate of change of stress with time. Figure 3-1 shows the plot of $\dot{\varepsilon}_s$ versus $\sigma$ at both test temperatures for Ti-24Al-11Nb. The following equation represents the steady-state creep rate at both temperatures.

$$\dot{\varepsilon}_s = 9.5 \times 10^{-17} \exp \left[ - \frac{87,700}{R} \left( \frac{1}{T} - 1.0277 \times 10^{-3} \right) \right] \sigma^{5.5}$$

(3-3)

where, $R$ is the universal gas constant and $T$ is the absolute temperature in Kelvin.

The crack tip parameters discussed in this section will be formulated for elastic, power-law creep materials which deform according to Equation (3-2). However, if necessary, these parameters can be generalized to include the contributions from the primary and tertiary creep [51,52]. It may be necessary to include contributions from primary creep at low stresses and low temperatures. However, sufficient data are not available at the present time to proceed with such an analysis.

There are three crack tip parameters in TDFM. These are the $C^*$-Integral [54-56], $C(t)$-Integral [57] and the $C_t$ parameter [57]. In order to completely understand the regions of validity of these parameters, it is necessary to consider the size of the region of creep deformation as shown in Figure 3-2. During the small-scale-creep regime, the creep deformation is localized and the creep zone size is small in comparison to the remaining ligament size. This is analogous to the small-scale-yielding (SSY) regime in linear elastic fracture mechanics [57,58]. During the transition regime, the creep zone is no longer small compared to crack size or the remaining ligament size. This is analogous to the elastic-plastic regime. The third region is the extensive creep regime in which the creep deformation has spread through the entire ligament. This regime is analogous to the
fully-plastic regime and is also referred to the steady-state regime because the creep deformation behavior is dominated by steady-state creep.

The $C^*$-Integral was proposed as a creep analog to the $J$-Integral and is defined as follows [53]:

$$C^* = \int_{\Gamma} W^* dy - T_i \frac{\partial \dot{u}_i}{\partial x} ds \quad (3-4)$$

where, $W^*$ is determined by the following equation:

$$W^* = \int_0^{l_{ij}} \sigma_{kl} d\epsilon_{kl} \quad (3-5)$$

$T_i$ is the traction vector along a path $\Gamma$ (considering the region enclosed by the path as a free body) which originates at a point along the lower crack surface, and goes around counter-clockwise, and ends at a point on the upper crack surface. Thus, the contour encloses the crack tip. $\sigma_{ij}$ and $\dot{\epsilon}_{ij}$ are the stress and strain rate tensors and $\dot{u}_i$ is the deflection rate vector along the direction of the traction. $ds$ is an arc length element along $\Gamma$. $C^*$ has also been interpreted as the difference between the energy rates (or power) provided to two bodies with incrementally differing crack lengths which are otherwise identical.

Thus, $C^*$ is given by:

$$C^* = - \frac{1}{B} \frac{dU^*}{d\bar{a}} \quad (3-6)$$

where, $B$ is thickness and $U^*$ is the stress-power $(\sigma_{ij} \dot{\epsilon}_{ij})$ times $[n/(n+1)]$ and is obtained by calculating the area under the load versus load-point deflection rate plot. $C^*$ also uniquely
characterizes the crack tip stress and strain rate fields [59,60]:

\[ \sigma_{ij} = \left( \frac{c^*}{AI_n r} \right)^{1/n+1} \bar{\sigma}_{ij} \theta \] \quad (3-7)

\[ \dot{\epsilon}_{ij} = A \left( \frac{c^*}{AI_n r} \right)^{n/n+1} \bar{\epsilon}_{ij} \theta \] \quad (3-8)

where, \( I_n \) is a constant dependent on the \( n \) value, \( r \) is the radial direction from the crack tip and \( \bar{\sigma}_{ij} \theta \) and \( \bar{\epsilon}_{ij} \theta \) are dimensionless angular functions.

In the small-scale and the transition creep regimes in which components often lie, the elastic strain rate term in Equation (3-2) cannot be ignored. Thus, \( C^* \) is no longer path-independent (because strain rate is not just a function of stress alone due to the \( \dot{\sigma} \) term) and it also does not uniquely characterize the crack tip stress fields. Thus, additional crack tip parameters need to be defined to characterize creep crack growth behavior. The \( C(t) \)-Integral and the \( C_t \) parameter have been introduced to generalize \( C^* \) to include the small-scale and the transition creep regions.

The \( C(t) \)-Integral of Bassani and McClintock [56] is the same as \( C^* \) in eq. (3-4) except it is determined along a path very near the crack tip \( (r \to 0) \). \( C(t) \) uniquely characterizes the amplitude of the crack tip stress fields at all times ranging from small-scale creep to extensive creep conditions. Under extensive creep conditions, it reduces to \( C^* \). The value of the \( C(t) \)-Integral can be approximated by the following equation [61]:

\[ C(t) = \left( \frac{t_T}{t} + 1 \right) C^* \] \quad (3-9)
where, \( t \) is the elapsed time and \( t_T \) is the transition-time for extensive creep conditions to develop. Under constant load conditions, \( t_T \) has been given by \([58,62]\):

\[
t_T = \frac{K^2(1-\nu^2)}{E(n+1)C^*}
\]  

(3-10)

where, \( K \) is the stress intensity parameter, \( \nu \) is Poisson's ratio and \( E \) is elastic modulus. As can be seen clearly from Equation (3-4) that, \( C(t) \rightarrow C^* \) as \( t \rightarrow \infty \). The major drawback of using \( C(t) \) is that in the small-scale and transition creep regions it can not be measured at the load-point. It can only be calculated.

The \( C_t \) parameter defined by Saxena \([57]\) is also a generalization of \( C^* \) but is different from the \( C(t) \)-Integral. \( C_t \) generalizes upon the energy rate interpretation of \( C^* \) as follows. Consider several identical pairs of cracked specimens. Within each pair, one specimen has a crack length \( a \) and the other has an incrementally larger crack length \( a + \Delta a \). The specimen of each pair are loaded to various constant load levels \( P_1', P_2', P_3', \ldots, P_i', \ldots \) at elevated temperatures, and the load-line deflection as a function of time is recorded (Figure 3-3a). The load-line deflection due to creep is \( V_c \). It is assumed that no crack extension occurs in any of the specimens and the instantaneous response is linear elastic. At a fixed time, the load versus deflection rate, \( \dot{V}_c \), behavior is plotted for all specimens. A schematic of the expected behavior is shown in Figure 15b. Several such plots can be generated from these tests by varying time.

The area between the \( P-\dot{V}_c \) curves for specimens with crack length \( a \) and \( a + \Delta a \), is called \( \Delta U_t^* \) (the subscript denotes that this
value is at a fixed time, t). The $C_t$ parameter is given by the following equation,

$$ C_t = -\frac{1}{B} \frac{\partial U_t^*}{\partial a} \quad (3-11) $$

where, $B$ is specimen thickness. As $t \to \infty$, $C_t = C^*$ by definition because $\partial U_t^*/\partial a = dU^*/da$ under steady-state conditions.

$C_t$ can be measured at the loading pins and can also be calculated as will be discussed later. It has also been shown to be directly related to the rate of expansion of the creep zone size in the small scale creep region [63] and it may then be argued that the rate of crack tip damage accumulation must scale with the rate of creep zone expansion. Thus, $C_t$ can be expected to characterize the creep crack growth rate in the small-scale creep and the transition creep region. This has been borne out by a considerable amount of data [64].

In summary, there are three crack tip parameters in TDFM; $C^*$, $C(t)$ and $C_t$. $C^*$ is a subset of both $C(t)$ and $C_t$ because the latter parameters reduce to $C^*$ under extensive creep conditions. In the small-scale and the transition creep regions, $C^*$ loses its significance but $C(t)$ and $C_t$ continue to retain their significance. Also, in these regimes, $C(t) \neq C_t$. Since $C_t$ can also be measured always at the load point, it has an advantage over $C(t)$ which can only be calculated. Both parameters can be generalized to include primary and tertiary creep [51,65]. Recent studies have shown that $C_t$ can successfully characterize creep crack growth rate data over a wide range of creep deformation conditions. Therefore, we have used $C_t$ exclusively for correlating creep crack growth data in Ti-24Al-11Nb.
Since C(t), C_t, and C^* are defined only for stationary cracks, their applicability for growing cracks becomes increasingly more important in the lower temperature ranges of the creep regime. Several analytical and experimental studies have addressed this issue [66-70] and it is now well established that these parameters may be used for slowly growing cracks. One approach to assure the validity of C_t is to compare the overall deflection rate at the load-point, $\dot{V}$, (which includes the deflection rate associated with the growing crack) with the deflection rate due to creep deformation only ($\dot{V}_c$). For constant load conditions used during creep crack growth testing, the two are related in the following way [69]:

$$\dot{V}_c = \dot{V} - \frac{B(da/dt)}{P} \left[ \frac{2K^2}{E} + (m+1)J_p \right] \quad (3-12)$$

where, P is load, B is specimen thickness, $J_p$ is plastic contribution to J-Integral and m is the Ramberg-Osgood exponent relating plastic strain to applied stress. For the tests conducted in the dominantly elastic regime, $J_p=0$. When $\dot{V}_c/\dot{V} < 0.8$, the validity of C_t as a crack tip parameter becomes suspect. As can be seen from Equation (3-12), this condition is more likely to be violated at high values of da/dt. Low temperatures will cause $\dot{V}_c$ to be smaller. Thus, in an extreme case of the subcreep temperature regime, $\dot{V}_c=0$ and C_t can no longer be expected to be a viable crack tip parameter.

3.2 Creep Crack Growth Behavior in Ti-24Al-11Nb

All the data reported in this section is from the work of Khobaib, et al, [12] generated as a part of an Air Force sponsored program at the University of Dayton Research Institute (UDRI). Table 3-1 shows the summary of all tests conducted and
the loading conditions used. All specimens were compact tension (CT) type. The test temperatures were 650, 700, 750 and 800°C. Tests were conducted in an air environment as well as in vacuum at each temperature. The nominal width of the specimens tested in air was 40 mm (1.58 in.) with one exception. Specimen 88-258 was tested in air but was only 20 mm (.79 in.) wide. These tests were conducted under constant load conditions and the crack length was continuously monitored. No measurement of the load-line deflection was made during these tests. The test durations ranged from less than 2 hours to 70 hours which yielded data in the crack growth rate range of $2.54 \times 10^{-2}$ to $2.54 \text{ mm/hr (.001 to .1 in/hr)}$. These data were analyzed as a part of this project and the $da/dt$ were correlated with $C_t$. For comparison, plots of $da/dt$ versus $K$ were also generated. The data analysis method is described next and the results are discussed subsequently.

3.2.1 Data Analysis Methods

Since the load-line displacements were not available, $C_t$ could only be calculated. Equation (3-3) was used to estimate the value of $A$ in Equation (3-2) at the four test temperatures (see Table 3-2). The value of $A$ for 650 and 800°C are extrapolations of the actual data at 700 and 750°C. Estimated values of the .2% yield strength and the elastic modulus [71] at these temperatures are listed in Table 3-3.

The crack size versus time ($a$ vs. $t$) data were used to calculate $da/dt$ by the secant method. The corresponding values of the average crack size and time for the same interval were also calculated. The minimum crack extension between successive data points was taken to be 0.25 mm (0.01 in.). A computer program was developed to reduce this data into the $da/dt$ versus $C_t$ format. Various other parameters such as $K$, transition time, predicted load-line deflection rate, $\dot{V}$, and the predicted
load-line deflection rate due to creep, $\dot{V}_C$, were also calculated for each data point. The listing of the computer code is given in Appendix A and the equations used to calculate the various parameters are given below.

$C_t$ was calculated using the following equation [63,72]:

$$C_t = \frac{4a(1-\nu^2)\beta \bar{r}_c(\theta)}{E(n-1)} \frac{K^4}{W} \frac{(EA)^{2/n-1}(F'/F)t-(n-3)(n-1)}{+ C^*} \quad (3-13)$$

where, $a = (1/2\pi) [(n+1)^2/1.38n]^{2/(n-1)}$ for $3 < n < 13$, $\beta = .33$ and $\bar{r}_c(\theta) = .4$ which were estimated from finite element analysis [63]. $F = K$-calibration factor = $(K/P)BW^{1/2}$ and $F' = dF/d(a/w)$. $C^*$ can be calculated from the following equation for CT specimens [73]:

$$C^* = \frac{h_1WA}{(1-a/W)^n} \left[ \frac{P}{1.455BW\eta} \right]^{n+1} \quad (3-14)$$

where, $h_1$ is a function of $a/W$ and $n$ given in Reference 73. The values of $h_1$ for $n = 5$ and 7 were linearly interpolated to obtain the values for 5.5. $\eta$ is given as follows:

$$\eta = (\phi^2 + 2\phi + 2)^{1/2} - (\phi + 1) \quad (3-15)$$

where, $\phi = \frac{2a}{W-a}$.

The stress intensity parameter, $K$, is calculated using the standard expression for CT specimens [74]. The transition time ($t_T$) is then calculated from Equation (3-10). The load-line deflection rate, $\dot{V}$, and the load-line deflection rate due to creep deformation, $\dot{V}_C$, are also calculated using equations given in Appendix B. The ratio $\dot{V}_C/\dot{V}$ is important in assuring that

3-10
creep deformation dominates the behavior of the specimen. If ductile tearing or environmental attack causes the crack to grow out of the creep deformation zone, the above ratio will approach a value of zero. If the above ratio exceeds 0.8, the crack growth rate data are expected to correlate well with $C_t$ [69].

3.2.2 Results and Discussion

The tabular results from the analyses of all specimens are given in Appendix C. In the following discussion, the primary goal will be to examine the applicability of TDFM concepts to titanium aluminides. The influence of temperature and environment will also be considered.

One of the ways to experimentally validate a crack tip parameter is to test specimens of different size, geometry, starting crack lengths and applied load levels and to determine whether the data fall on a unique trend. At 750°C in air, the test matrix (see Table 3-1) includes one test each on specimens with widths of 40 and 20 mm (specimens 88-094 and 88-258). Since all other test conditions were identical, the results of these tests are plotted in Figure 3-4 on a $da/dt$ versus $C_t$ plot and in Figure 3-5 on a $da/dt$ versus $K$ plot. The data for specimen 88-094 show a dip which may have occurred due to transient conditions during testing. The ratio $\dot{V}_c/\dot{V}$ was also somewhat less than 0.8 for specimen 88-258 and the test lasted for slightly more than six hours. It appears that the load level for this test was on the high side for generating slow subcritical crack growth data. Thus, these data are not sufficient to make any definite conclusions about the applicability of TDFM crack tip parameters. Nevertheless, there appears to be a better correlation between $da/dt$ than that between $da/dt$ and $K$ in the data from these tests.
Figure 3-6 shows the da/dt versus $C_t$ behavior for crack growth tests conducted at 650°C in air and in vacuum. At equal $C_t$ values, the crack growth rates in air appear to be much higher than those in vacuum. Comparing the values of $\dot{V}$ and $\dot{V}_C$ for these tests (see Appendix C), it appears that for the test conducted in air (88-111), $\dot{V}_C << \dot{V}$ throughout the test. Hence, a unique correlation between da/dt and $C_t$ is not expected. Even for the specimen tested in vacuum (88-269), $\dot{V}_C / \dot{V} < 0.8$ except for the first two data points. Thus, even though the data trend for air and vacuum are in the right direction, no definite conclusions can be made from these data. In Figure 3-7, the crack growth data from the same two specimens are plotted as a function of the stress intensity parameter, $K$. At equal $K$ values, the da/dt is higher for tests conducted in vacuum. This result cannot be rationalized except by assuming that da/dt does not correlate with $K$ under these conditions. It appears that these data are in the transition regime between $C_t$ and the $K$ controlled regimes for which no suitable parameters are yet available.

Figure 3-8 shows the crack growth data from all tests conducted in air and Figure 3-9 shows all the data from the tests conducted in vacuum. The air data show temperature effects. In other words, da/dt is dependent on temperature. However, the crack growth rates are the highest for 650°C and the lowest for 800°C which is opposite to the expected trend. This is attributed to the problems in correlating the low temperature data with $C_t$ because of the high applied loads resulting in very high crack growth rates causing problems discussed earlier. This is further confirmed by comparing the data in vacuum generated at different temperatures (Figure 3-9). With the exception of the test at 650°C for which the condition $\dot{V}_C / \dot{V} < 0.8$ was not met for most of the test, the temperature effects are normalized when da/dt is
expressed as a function of $C_t$. This result is consistent with other similar data in stainless steels [70] and in Cr-Mo-V steels [57].

In summary, creep crack growth data in a titanium aluminide appears to correlate well with $C_t$ in the temperature range of 700 to 800°C. There are strong indications that TDFM concepts are necessary for characterizing the high temperature crack growth rates in this material. At 650°C, these correlations are suspect. It is recommended that a systematic investigation involving specimens of different geometries, sizes and load-levels be undertaken to examine this issue in much more detail. It is also recommended that load levels be selected such that the resultant test durations are at least one week and some tests last for several weeks. Load-line deflection should be monitored continuously during these tests. These data can then be used for definitely establishing the role of TDFM in predicting the elevated temperature crack growth behavior of these materials.

3.3 Creep-Fatigue Crack Growth Behavior

It has been recently shown [75] that the average crack growth rate, $(da/dt)_{avg}$, during a hold time can be characterized by the average value of $C_t$ parameter $(C_t)_{avg}$ during the hold time. $(da/dt)_{avg}$ and $(C_t)_{avg}$ are defined as follows:

$$(da/dt)_{avg} = \frac{1}{t_h} \left[(da/dN) - (da/dN)_o\right]^{(3-16)}$$

where, $t_h$ is hold time, $da/dN$ is crack growth rate per cycle and $(da/dN)_o$ is the crack growth rate for a loading waveform with zero hold time but with loading and unloading times which are the
same as for the waveform with hold time of $t_h$.

$$(C_t)_{avg} = \frac{1}{t_h} \int_0^{t_h} C_t \, dt \quad (3-17)$$

If $t_h << t_T$, $(C_t)_{avg}$ can be estimated by integrating the small-scale-creep term in Equation (3-13), which is the first term on the right hand side of the equation.

Thus, $(C_t)_{avg}$ can be analytically estimated by the following equation:

$$\frac{2a(1-\nu^2)\beta\bar{R}_C(\theta) K^4}{E(n-1)} \frac{(EA)^2/n-1(F'/F)t^{-(n-3)}(n-1)}{(3-18)\ W}$$

If load-line deflections are measured during the hold time, $(C_t)_{avg}$ can also be measured. Figure 3-10 shows the $(da/dt)_{avg}$ versus $(C_t)_{avg}$ behavior for a 1Cr-1Mo-.25V steel for hold times ranging from 50 seconds to 24 hours at 538°C. Figure 3-11 shows the same data plus the additional creep-fatigue crack growth data at 427°C and creep crack growth data at various temperatures. Several observations can be made from these data.

The $(da/dt)_{avg}$ appears to uniquely correlate with $(C_t)_{avg}$ for different hold times and also for two quite different temperatures. It also shows that the creep-fatigue crack growth rates are somewhat lower than the creep crack growth rates at equal $C_t$. However, this does not imply that the component lives will be enhanced by periodic unloading. In fact, the result is quite the opposite. Periodic unloading does decrease the crack growth rates at the same $C_t$ but it also increases the magnitude of $C_t$ itself for the same applied load and crack size. Figures
3-12 and 3-13 schematically show $C_t$ as a function of time for wave forms involving periodic unloading. The unloading interrupts the crack tip stress relaxation and during reloading a stress peak is regenerated every cycle [76] followed by stress relaxation during hold time. This can also be envisioned as resharpening of the crack tip due to periodic loading and unloading.

The behavior of Cr-Mo-V steels is similar to that of titanium aluminide under elevated temperature creep conditions. Therefore, the TDFM framework described above is expected to apply to titanium aluminides. This should, of course, be established by conducting an appropriate experimental program.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>88-11</td>
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<td>0.804</td>
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<td>Vac</td>
<td>0.310</td>
<td>0.805</td>
<td>0.1796</td>
</tr>
</tbody>
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B = thickness 1 in. = 25.4 mm
W = width 1 kip = 4.45 kilo-Newton
P = load
Table 3-2

Values of the Creep Constant "A" at Various Test Temperatures

<table>
<thead>
<tr>
<th>Test Temp (°C)</th>
<th>A (ksi) $^{-n} \text{hr}^{-1}$</th>
<th>A (MPa) $^{-n} \text{hr}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>3.29 x 10^{-13}</td>
<td>8.05 x 10^{-18}</td>
</tr>
<tr>
<td>700</td>
<td>3.88 x 10^{-12}</td>
<td>9.5 x 10^{-17}</td>
</tr>
<tr>
<td>750</td>
<td>3.61 x 10^{-11}</td>
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</tr>
<tr>
<td>800</td>
<td>2.70 x 10^{-10}</td>
<td>6.60 x 10^{-15}</td>
</tr>
</tbody>
</table>

Note: $n = 5.5$

Table 3-3

Tensile and Elastic Modulus at Various Temperatures

<table>
<thead>
<tr>
<th>Test Temp (°C)</th>
<th>$\sigma_{YS}$</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ksi</td>
<td>MPa</td>
</tr>
<tr>
<td>650</td>
<td>41</td>
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</tr>
<tr>
<td>800</td>
<td>35</td>
<td>241.3</td>
</tr>
</tbody>
</table>
Figure 3-1. Creep Deformation Behavior of Ti-24Al-11Nb
Figure 3-2. Schematic Representation of the Levels of Creep Deformation Under Which Creep Crack Growth Can Occur.
Figure 3-3. (a) Load Line Deflection, $V_c$, as a Function of Time for Bodies of Crack Lengths $a$ and $a+\Delta a$ at Various Load Levels, and (b) the Definition of the $C_t$ Parameter
Figure 3-4. da/dt Versus Ct Data for Ti-24Al-11Nb Obtained From Specimens of Two Sizes
Figure 3-5. da/dt Versus K Data for the Ti-24Al-11Nb Obtained from Specimens of Two Sizes
Figure 3-6. da/dt Versus C_t Data for Ti-24Al-11Nb Obtained at 650°C in Air and Vacuum Environments
Figure 3-7. da/dt Versus K Data for Ti-24Al-11Nb Obtained at 650°C in Air and Vacuum Environments
Figure 3-8. Creep Crack Growth Behavior of Ti-24Al-11Nb in Air at Various Temperatures
Figure 3-9. Creep Crack Growth Behavior of Ti-24Al-11Nb in Vacuum at Various Temperatures
Cr-Mo-V Rotor Steels
1000°F

- = Air, 0.014 hr
- = Air, 0.47 hr
- = Air, 24.0 hr
- = Argon, 0.5 hr

Scatter Band

Pure Creep Data (1000°F)

Figure 3-10. Representation of Creep-Fatigue Crack Growth Behavior in Terms of the \((C_t)_{avg}\) Parameter for New Cr-Mo-V Steels Tested at 1000°F (538°C) in Air and Argon (Ref. 75)
Figure 3-11. Superimposition of Creep-Fatigue Crack Growth Data According to the $(C_t)_{avg}$ Parameter Concept for New and Ex-Service Cr-Mo-V Steels Tested at 1000°F (538°C) and 800°F (427°C) (Ref. 75)
Figure 3-12. Schematic of $C_t$ as a Function of Time for Cyclic Operation
Figure 3-13. Normalized Crack Tip Stress as a Function of Normalized Time at a Fixed Distance Ahead of the Crack Tip for Fast Loading Followed by a Sustained Load
4.0 Reexamination and Application of $C_t$

As discussed in section 3 of this report, $C_t$ appears to be the best parameter, among $C_t$, $C(t)$, $K$ and $C^*$, for characterizing creep crack growth of titanium intermetallics under transient as well as steady-state creep. The same behavior for steel has been reported by Saxena and his co-workers (see e.g. [63]) in their experimental results. The good correlation between $C_t$ and creep crack growth can be attributed to the fact that $C_t$ is proportional to the rate of expansion of the creep zone [63], which characterizes the damage zone at the crack tip region under creep conditions.

Although both $C_t$ and $C(t)$ are related to the crack tip damage (or stress) under creep conditions, $C_t$ can be readily measured from load line deflection curves in a test while $C(t)$ can only be computed. On the other hand, $C(t)$ is easier to compute than $C_t$ for structural components. This section reexamines the definition of $C_t$ and provides alternative ways of computing $C_t$.

4.1 Pseudo-Potential $P^*$ and $C_0$

For a two-dimensional structure, we define pseudo-potential $P^*$ as

$$ P^* = \int_{\partial A_t} T_i \dot{u}_i \, ds - \int_A W^* \, dA \quad (4-1) $$

where $A$ is the entire body, $\partial A_t$ is the boundary of $A$ on which traction is specified, and $W^*$ is defined by

$$ W^* = \int_0^{\epsilon_{ij}} \sigma_{kl} \, d\epsilon_{kl} \quad (4-2) $$
It is worth noting that, even though the expression of $W^*$ in eq. (4-2) looks simple and somewhat familiar, $W^*$ does not bear any physical meaning. The first term in eq. (4-1), which is the total power which flows into the body at a given instant, depends only on the current state (stresses, strains, and strain rates) of the body. On the contrary, the second term in eq. (4-1) is history dependent. In other words, for a structure body $A$, the first part of $P^*$ is a function of its current state while the second part of $P^*$ depends on the loading path from time zero.

For a stationary crack, we then define another quantity $C_0(t)$ as

$$C_0(t) = \frac{dP^*}{da} = \frac{d}{da} \left[ \int_{\partial A_t} T_i \dot{u}_i ds - \int_A W^* dA \right]$$

(4-3)

where $a$ is the crack length. It will be shown later in this report that $C_0$ and $C_t$ are uniquely related in small scale creep and steady-state creep conditions. It should be noted that $A$ in eq. (4-3) is the entire body and eq. (4-3) is defined for two stationary cracks at length of $a$ and $a+\Delta a$ under the same loading conditions. The subscript "o" of $C_0$ stands for the outer boundary. By following the same procedure as that used by Rice [77] in deriving the well-known $J$-integral, it can be shown that eq. (4-3) can be reduced to

$$C_0(t) = \int_{\partial A} (W^* n_i - T_i \dot{u}_i, l) ds - \int_A (W^* a - \sigma_{ij} \dot{\epsilon}_{ij}, a) dA$$

$$= C(t)|_{\partial A} - \int_A (W^* a - \sigma_{ij} \dot{\epsilon}_{ij}, a) dA$$

(4-4)

where $A$ is the entire structure body and $\partial A$ is the entire outer boundary of the body. Thus, the new parameter $C_0$ defined in eq. (4-3) is nothing but an area integral plus the $C(t)$ integral [56] computed along the entire outer boundary of the body. The area
integral in eq. (4-4), however, is not always zero since, in general, $W^*$ is a function of $\dot{\epsilon}_{ij}$, $\dot{\sigma}_{ij}$, ..., implying $W^*_a \neq \sigma_{ij}\dot{\epsilon}_{ij}$. The inequality of $C_0$ and $C(t)$ has also been observed numerically by Bassani, et al, [63] and Leung, et al, [78]. Further, like $C_t$ and $C(t)$, $C_0(t)$, in general, is not path independent.

4.2 Steady-State Creep

From eq. (4-4), it can be easily deduced that, at long times $t \to \infty$,

$$C_0(t) = C_t(t) = C(t) = C^* = \int_\Gamma (W^* n_1 - T_i \dot{u}_i, 1) ds$$

(4-5)

where $\Gamma$ is any integration contour which encompasses the crack tip counterclockwise, because

$$W^* = W^*(\dot{\epsilon}_{ij})$$

(4-6)

$$W^*_a = (\partial W^*/\partial \dot{\epsilon}_{ij})\dot{\epsilon}_{ij}, a = \sigma_{ij}\dot{\epsilon}_{ij}, a$$

(4-7)

is true under steady-state creep conditions. Therefore, for steady-state creep, $C_0$ and $C_t$ represent the same physical quantity.

4.3 Small Scale Creep

We next examine the behavior of $C_0$ at the other end of the time spectrum, very small times $t \to 0$. As illustrated in Figure 4-1, when the elapsed time $t$ is much less than the transition-time $t_T$ defined by eq. (3-10), the creep dominant zone $A'_C$ is much smaller than the surrounding elastic zone $A_e$. The creep dominant zone $A'_C$ has the same shape as the creep zone $A_C$ defined by Riedel and
Rice [58] but is larger in size. The latter is the locus of points where the equivalent creep strain equals to the equivalent elastic strain. As will be shown later in this report in an example problem, it is found that good agreements can be achieved consistently for a wide range of creep exponents n when $A'_C$ is set to be six times the size of $A_C$ defined by Riedel and Rice [58]. For small scale creep, it can be argued that

$$\int_A W^* dA = \int_{A'_C} W^* dA + \int_{A_e} W^* dA \approx \int_{A'_C} W^* dA$$

(4-8)

since integration of $W^*$ over the elastic region $A_e$ is negligible compared with the contribution from the creep dominant region $A'_C$. From eq. (4-8), it can be seen that $A'_C$ has to be larger than $A_C$ because at the boundary of $A_C$, total creep strains equal to total elastic strains and the $W^*$ values at the boundary of $A_C$ are still not negligible. Also, the net power dissipated in the elastic region is near zero, i.e.,

$$\int_{\partial A} T_i \dot{u}_i ds \approx \int_{\partial A'_C} T_i \dot{u}_i ds$$

(4-9)

With eqs. (4-8) and (4-9), $C(t)$ defined in eq. (4-3) now reduces to

$$C(t) = \frac{d}{da} \left[ z \int_{\partial A'_C} T_i \dot{u}_i ds - \int_{A'_C} W^* dA \right]$$

(4-10)

where $z=1$ under load-control conditions and $z=0$ under displacement-control conditions. It is worth noting that $C^*$ was defined [53] originally under displacement-control conditions while $C_t$ was defined [57] originally under load-control conditions. That is, the P-◊ curves should be constructed by running constant displacement rate tests when measuring $C^*$ but
constant load tests when measuring $C_t$. However, at steady-state creep conditions, both constant load tests and constant displacement rates will lead to the same P-ÂV curve due to the fact that $P$ and $ÂV$ at steady-state creep conditions are uniquely related. The same conclusion can not be drawn for $C_t$ at small scale creep or transition creep conditions, because, under these conditions, the P-ÂV relation [72]

$$
\dot{V}_c = \frac{4a(1-\nu^2)}{E(n-1)} \frac{P^3}{B} \frac{F^4}{W^2} \frac{2/(n-1)}{(n-3)/(n-1)} \tau
$$

(4-11)

is true only for constant loads.

Inside the creep dominant zone $A_c'$ the overall strain rate is determined by the power law relationship given in eq. (3-1). Thus, in $A_c'$

$$
W^* = \frac{n}{(n+1)} \sigma_{ij} \dot{\epsilon}_{ij}
$$

(4-12)

and the stresses, strain rates, and displacements behave asymptotically like the HRR field [58-60]:

$$
\sigma_{ij} = \sigma_o \left[ \frac{C(t)}{\sigma_o I_n r} \right]^{1/(n+1)} \tilde{\sigma}_{ij}(\theta)
$$

(4-13)

$$
\dot{\epsilon}_{ij} = \dot{\epsilon}_o \left[ \frac{C(t)}{\sigma_o I_n r} \right]^{n/(n+1)} \tilde{\epsilon}_{ij}(\theta)
$$

(4-14)

$$
\dot{u}_i = r \dot{\epsilon}_o \left[ \frac{C(t)}{\sigma_o I_n r} \right]^{n/(n+1)} \tilde{u}_i(\theta)
$$

(4-15)

4-5
where dimensionless functions $\tilde{I}_{ij}$, $\tilde{\sigma}_{ij}$, $\tilde{\varepsilon}_{ij}$, and $\tilde{u}_i$ have been tabulated by Shih [79] and Symington and Shih [80] for various $n$ values, $\sigma_0$ is yield strength, and $\dot{\varepsilon}_0$ is strain rate at $\sigma_0$.

In eq. (4-10), the creep dominant zone $A'_c$ is not fixed in space but is moving and changing in size with different crack lengths, implying that the differentiation procedure used by Rice [77] in deriving $J$ is not applicable here.

We further assumed that the creep zone for the stationary crack of lengths $a$ and $a+\Delta a$ are self similar. That is, it is assumed that

$$r_c(\theta, t) = F(t) F_{cr}(\theta)$$  \hspace{1cm} (4-16)

where $r_c$ is radial distance measured from the crack tip of the boundary $\partial A_c$ of the creep zone $A_c$. This assumption is valid as long as the body is under small scale creep. In fact, Riedel and Rice [58] have shown that, under constant loads,

$$r_c(\theta, t) = r_c^0(t) F_{cr}(\theta)$$  \hspace{1cm} (4-17a)

$$r_c^0(t) = \frac{1}{2\pi} \left( \frac{K}{\tilde{E}} \right)^2 \frac{(n+1)\dot{\varepsilon}_0 n E^n t^{2(n-1)}}{2\pi(1-\nu^2)\sigma_0^n}$$  \hspace{1cm} (4-17b)

where $K$ is stress intensity factor and $F_{cr}(\theta)$ is a non-dimensional function.

Substituting eqs. (4-12) to (4-15) and applying the differentiation technique described in Appendix D, equation (4-10) can be reduced to

$$C_{os}(t) = \frac{6}{n+1} F_{cr}(\frac{\pi}{2}) \left[ L_n r_c^o \frac{\partial C(t)}{\partial a} + M_n C(t) \frac{\partial}{\partial a} r_c^o \right]$$  \hspace{1cm} (4-18)
where the additional subscript "s" on C stands for small scale creep, the factor 6 comes from the assumption that $A'_c$ is six times the size of $A_c$, and

$$L_n = \frac{1}{R} \int_{\partial A_c} \tilde{\sigma}_{ij} \tilde{u}_{ij} d[s/r_c(\frac{\pi}{2}, t)]$$

$$M_n = \frac{1}{R} \int_{\partial A_c} \tilde{\sigma}_{ij} \tilde{\varepsilon}_{ij} d[s/r_c(\frac{\pi}{2}, t)]$$

(4-19)

(4-20)

are dimensionless constants which depend on the creep constant $n$ and shape function $F_{cr}(\theta)$ of the creep zone $A_c$. Creep zone shapes under mode I loading for different $n$ values have been computed analytically by Riedel and Rice [58]. For the creep zone shapes computed by Riedel and Rice [58], $L_n$ and $M_n$ for $n=5, 7, 10, 15,$ and 20 have been calculated and are tabulated along with $F_{cr}(\frac{\pi}{2})$ in Table 4-1. $L_n$ and $M_n$ values listed in Table 4-1 were calculated based on the $\tilde{\sigma}_{ij}$, $\tilde{\varepsilon}_{ij}$, and $\tilde{u}_{i}$ functions tabulated in References [79, 80] by Shih and his co-workers. To study the sensitivity of $L_n$ and $M_n$ on the shape of the creep zones, a creep zone shape computed per finite element method by Riedel [81] and an artificially assumed double circle creep zone for $n=5$ were also used to calculate $L_n$ and $M_n$, and their results are shown in Table 4-2. It is seen in this table that $(L_n + M_n)$, which determines $C_0$, is not sensitive to the creep zone shapes tested here.

Under constant load condition, Eq. (4-18) can be further simplified as

$$C_{os}(t) = \frac{6}{n+1} F_{cr}(\frac{\pi}{2}) (L_n + M_n) \frac{K^4(1-\nu^2)}{E^3(n+1)\pi t} K'[\frac{(n+1) I_n E^n At}{2\pi(1-\nu^2)}]$$

or

(4-21)
where $K' = dK/da$. Eqs. (4-21) and (4-22) are very similar to the asymptotic expressions derived by Saxena [63, 72], i.e. very similar to the first term of eq. (3-13). Like $C_t$, at small scale creep, $C_{os}(t)$ is also proportional to the rate of creep zone size growth. As discussed in Appendix E, it can be shown that, under small scale creep and constant load conditions,

$$C_t = \frac{n+1}{4} C_{os}$$

(4-23)

### 4.4 Creep in Transition Period

For the transition period between small scale creep and steady-state creep, it is proposed that $C_t$ is the sum of its asymptotic values at $t<<t_T$ and at $t\rightarrow\infty$, i.e.,

$$C_t(t) = \frac{n+1}{4} C_{os}(t) + C^*$$

(4-24)

where $C_{os}$ is calculated per eq. (4-18). Under constant load conditions, $C_{os}$ can also be calculated by eqs. (4-21) or (4-22). To check the validity of eq. (4-23), the problem of a compact tension specimen under a constant force $P$ analyzed by Bassani, et al, [63] was chosen as an example. Dimensions and material properties of this example problem are listed in Table 4-3. $C^*$ and $t_T$ values for this example for various $n$ values were then calculated and summarized in Table 4-4. The resulting $C_o(t)$ values are plotted in Figures 4-2 to 4-6 for $n=5, 7, 10, 15, \text{and } 20$, respectively. In these figures, three curves are shown:
\((n+1)C_{os}(t)/4, [(n+1)C_{os}(t)/4 + C^*]\), and a reference curve \\
\(t \frac{(n-3)}{(n-1)} C^*[1+(\frac{t}{t^*})^{\frac{n-3}{n-1}}]\), which has been shown [63] to be a very good approximation of \(C_t\) for the entire time range. From these figures, it is seen that the agreement between \([(n+1)C_{os}/4 - C^*]\) and \(C_t\) are excellent for the entire time range for all \(n\) values tested.

4.5 \(C_o(t)\) versus \(C_t\)

As indicated in section 4.1 and 4.2, \(C_o(t)\) and \(C_t\) are similar in the following aspects:

(a) Both \(C_o\) and \(C_t\) become \(C^*\) at steady-state creep.
(b) Under small scale creep conditions, both \(C_o\) and \(C_t\) are proportional to the rate of creep zone growth.
(c) Under a constant load, both \(C_o\) and \(C_t\) behave asymptotically like \(t^{-(n-3)/(n-1)}\) at very small \(t\), which is much less than the asymptotic value of \(t^{-1}\) for \(C(t)\).
(d) \(C_t\) and \(C_o\) are related by \(C_t=(n+1)C_o/4\) for small scale creep. Thus, \(C_o\) can correlate with creep crack growth data as well as \(C_t\) does.
(e) Both parameters are defined for stationary cracks.

However, there is a basic difference between \(C_o\) and \(C_t\): \(C_o\)'s definition, eqs. (4-3) and (4-4), is based on a single loading (constant load or variable load) on a single structure, while \(C_t\)'s definition, eq. (3-11), is based on multiple loading (load remain constant in each single one) on multiple but identical structure bodies. However, it has been shown that, like \(C_o\), \(C_t\) can also be computed based on a single loading for small scale creep and steady-state creep [57].
Furthermore, both $C_t$ and $C_o$ can be used to handle variable loading even though the existing $C_t$ definition does not give a very clear indication on how the $C_t$ should be calculated under variable loading conditions. To calculate $C_t$ for a crack under variable loading, either $C_o$ formulation in Eq. (4-3) or the $C_t$-$r_c$ relationship similar to that in Eq. (4-22) should be used.

To sum up this section, an alternative way of calculating $C_t$ has been derived. While both the existing and the newly proposed definitions of $C_t$ are as effective and accurate in evaluating test specimens, the new $C_t$ definition provides a more convenient way of computing $C_t$ for analyzing structural components.
### Table 4-1

**Creep Fracture Constants**

<table>
<thead>
<tr>
<th>$n$</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{cr}(\pi/2)$</td>
<td>0.3156</td>
<td>0.3214</td>
<td>0.3262</td>
<td>0.3292</td>
<td>0.3300</td>
</tr>
<tr>
<td>$L_n$</td>
<td>0.3944</td>
<td>0.3980</td>
<td>0.4030</td>
<td>0.4101</td>
<td>0.4131</td>
</tr>
<tr>
<td>$M_n$</td>
<td>0.6177</td>
<td>0.6373</td>
<td>0.6623</td>
<td>0.6919</td>
<td>0.7131</td>
</tr>
</tbody>
</table>

### Table 4-2

**Effects of Creep Zone Shape ($n=5$)**

<table>
<thead>
<tr>
<th></th>
<th>Analytical&lt;sup&gt;(a)&lt;/sup&gt;</th>
<th>FEM&lt;sup&gt;(b)&lt;/sup&gt;</th>
<th>Double Circles&lt;sup&gt;(c)&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{cr}(\pi/2)$</td>
<td>0.3156</td>
<td>0.2703</td>
<td>0.3156</td>
</tr>
<tr>
<td>$L_n$</td>
<td>0.3944</td>
<td>0.4095</td>
<td>0.4556</td>
</tr>
<tr>
<td>$M_n$</td>
<td>0.6177</td>
<td>0.5521</td>
<td>0.5281</td>
</tr>
<tr>
<td>$L_m + M_n$</td>
<td>1.0121</td>
<td>0.9616</td>
<td>0.9837</td>
</tr>
</tbody>
</table>

<sup>(a)</sup> Reference 58  
<sup>(b)</sup> Reference 81  
<sup>(c)</sup> $F_{cr} = 0.3156 \sin \theta$
Table 4-3
Example Material Properties

\begin{align*}
E &= 20.4 \times 10^3 \text{ ksi} \\
\nu &= 0.3 \\
\sigma_o &= 19.0 \text{ ksi} \\
\dot{\varepsilon}_o &= 0.003444 \text{ hr}^{-1} \\
F'/F &= 2.912 \\
w &= 2 \text{ in.} \\
a &= 0.941 \text{ in.} \\
p &= 3450 \text{ lbf} \\
K &= 21.52 \text{ ksi}\sqrt{\text{in}}
\end{align*}

Table 4-4
C* and t_T for the Example Problem

<table>
<thead>
<tr>
<th>n</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>C* (ksi-in/hr)</td>
<td>6.1081</td>
<td>2.0540</td>
<td>0.4203</td>
<td>0.0309</td>
<td>0.002243</td>
</tr>
<tr>
<td>t_T (hrs)</td>
<td>0.5637</td>
<td>1.2572</td>
<td>4.4684</td>
<td>41.745</td>
<td>438.50</td>
</tr>
</tbody>
</table>
Figure 4-1. Schematic of Small Scale Creep
Figure 4-2. Comparison of $C_0$ and $C_t$ for $n=5$
Figure 4-3. Comparison of $C_0$ and $C_*$, for $n=7$
Figure 4-4. Comparison of $C_0$ and $C_t$ for $n=10$
Figure 4-5. Comparison of $C_0$ and $C_t$ for $n=15$.
Figure 4-6. Comparison of $C_0$ and $C_t$ for $n=20$
5.0 CONCLUSION AND RECOMMENDATIONS

An extensive literature survey for creep crack growth data for titanium alloys in the high temperature environments has been conducted in this Phase I study. Creep crack growth data is found to be scarce if not non-existent in the open literature. The limited amount of data obtained in this study on Ti-24Al-11Nb alloys is expressed in terms of stress intensity factor $K$, which may not be an adequate parameter to characterize crack growth in the creep region. Among $K$, $C^*$, $C(t)$, and $C_t$, an evaluation was then conducted in this Phase I research to see which of the four parameters is best suited for characterizing creep crack growth of titanium alloys. It is concluded, after the evaluation, that $C_t$ may be the most promising candidate for describing creep crack growth of titanium alloys in the high temperature region. $C_t$ can be easily measured in a test through its load line deflection history and has been found to correlate well with crack growth test data for steels in the high temperature environments. To be able to fully utilize $C_t$ for the life prediction of a titanium alloy structure, it is important that $C_t$ should also be computed easily and accurately for geometries and loading conditions other than those of laboratory tests. As a first step of achieving this, the definition of $C_t$ was reexamined and another parameter $C_o(t)$ was proposed in this report. The new parameter, $C_o(t)$, can be calculated by integrating either along the outer boundary and over the entire area of the structure or along the boundary of the creep dominant zone. $C_o$ is found to be proportional to the growth rate of the creep dominant zone at the crack tip region. $C_t$ and $C_o$ are closely related under constant load condition. They both approach $C^*$ at steady-state creep. Under small scale creep conditions, they are proportional to each other, with a proportionality constant $[(n+1)/4]$. It is also found that $[(n+1)C_o(t)/4+C^*]$ behaves almost exactly like $C_t$ over the entire time spectrum. However, $C_o(t)$ can be calculated for both constant and variable load conditions. Therefore, $C_o(t)$ can be
viewed as an alternative form of $C_t$ but is easier to compute under the more general loading conditions.

It is apparent that more studies, both analytical, numerical, and experimental, are needed to fully understand creep crack growth of titanium aluminides in the high temperature environments. To name a few, it is recommended that the following studies be performed:

(a) Extensive laboratory tests should be conducted to understand the creep crack growth behavior of titanium alloys. More specifically, it is necessary to evaluate $C_t$ as the controlling parameter for creep crack growth, to find the application range for $K$ and $C_t$, to study the effects of hold time on crack growth, to obtain data on overload effects, to monitor creep zone growth, to study size effects and to have test data on both slow and fast propagating cracks.

(b) Analytically, there is a need to study the limitations on $C_0(t)$, to explore the theoretical ground of crack growth retardation and crack growth through the use of micro-mechanics models, to find a parameter for running cracks (cracks propagating in a higher speed), and to develop a way of characterizing crack growth in a temperature range where both LEFM and TDFM are important.

(c) Numerically, it is necessary to verify the concept of $C_0(t)$ proposed in this report, to study the effect of running crack, to establish a systematic way of calculating $C_0$ and/or $C_t$ for use in actual structural life prediction, to build a solution base (or handbook) for $C_0$ for the geometries and loading conditions encountered the most by the Air Force, and to develop a crack growth software package such that it can cover both LEFM, TDFM, and transition between LEFM and TDFM.
6.0 REFERENCES

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   Shuttle Orbiting Division, Space System Group, Sept. 1979.

3. Yuen, A., Hopkins, S. W., Leverant, G. R., and Rau, C. A., 
   "Correlation Between Fracture Surface Appearance and Fracture Mechanics Parameters for Stage II Fatigue Crack 


   1986.


    the 1988 Titanium Aluminides Meeting, 1-3 November 1988, Stratford, CT.

    Indianapolis, IN.


6-5


APPENDIX A

Listing of the Computer Code
100 KEY OFF
110 REM this program is stored under the name udri.bas
111 CLS
115 LOCATE 5,10: INPUT "MATERIAL TYPE";M$
116 LOCATE 7,10: INPUT "SPECIMEN ID";X$
117 LOCATE 9,10: INPUT "TEST TEMPERATURE (C)";TP
120 CLS
130 INPUT "Do you want to use previously stored spec. size & material data";YS
140 IF Y$ = "Y" GOTO 150 ELSE 200
150 OPEN "FCT" FOR INPUT AS #1
160 INPUT #1, YS, N, AA, E, BN, B, W, P
170 CLOSE #1
180 GOTO 310
200 CLS
210 LOCATE 2,10: INPUT "Yield Strength (ksi)";YS
220 LOCATE 4,10: INPUT "creep Exponent (n)";N
230 LOCATE 6,10: INPUT "Creep Coeff. (ksi^-n.hr^-1)";AA
240 LOCATE 8,10: INPUT "Elastic Modulus (ksi)";E
250 LOCATE 10,10: INPUT "Nominal Thickness (in)";BN
260 LOCATE 12,10: INPUT "Crack Plane Thickness (in)";B
270 LOCATE 14,10: INPUT "Specimen Width (in)";W
280 LOCATE 16,10: INPUT "Test Load (kips)";P
290 LOCATE 20,10: INPUT "DO YOU WANT TO EDIT THE DATA (Y/N)";Y$
300 IF Y$ = "Y" GOTO 200 ELSE 301
301 OPEN "FCT" FOR OUTPUT AS #1
302 WRITE #1, YS, N, AA, E, BN, B, W, P
303 CLOSE #1
310 CLS
320 DIM AB(50), T(50), DADT(50), CT(50), CSTAR(50), K(50), TT(50), V(50), VC(50)
321 LOCATE 5,10: INPUT "DO YOU WANT TO USE CRACK GROWTH DATA FROM FILE(Y/N)";YS
322 IF Y$ = "Y" GOTO 323 ELSE 338
323 LOCATE 10,10: INPUT "FILE NAME";FS
330 OPEN FS FOR INPUT AS #1
331 INPUT #1, NN
332 FOR I = 1 TO NN
333 INPUT #1, AB(I), T(I), DADT(I)
334 NEXT I
335 CLOSE #1
336 GOTO 410
338 CLS
339 LOCATE 5,10: INPUT "NUMBER OF DATA POINTS";NN
340 CLS
350 FOR I = 1 TO NN
355 LOCATE 3,10: PRINT "DATA SET NO. ";I
360 LOCATE 5,10: INPUT "CRACK SIZE(IN)";AB(I)
370 LOCATE 7,10: INPUT "TIME (HRS)";T(I)
380 LOCATE 9,10: INPUT "CRACK GROWTH RATE (IN/HR)";DADT(I)
390 CLS
400 NEXT I
410 CLS
420 PRINT "DATA SET ",", CRACK SIZE"," TIME"," DADT"
430 FOR I = 1 TO NN
440 PRINT USING "##.##";I;
450 PRINT USING "####.####",";AB(I);
460 PRINT USING "####.####",";T(I);
470 PRINT USING "#####.#####",";DADT(I)
480 NEXT I
490 LOCATE 20,10: INPUT "DO YOU WANT TO EDIT ANY OF THE DATA (Y/N)";YS
500 IF Y$ = "Y" GOTO 510 ELSE 600
510 CLS

A-2
520 LOCATE 10,10: INPUT "WHICH POINT DO YOU WANT TO EDIT";
530 LOCATE 12,10: INPUT "NEW CRACK SIZE";AB(II)
540 LOCATE 14,10: INPUT "NEW TIME";T(II)
550 LOCATE 16,10: INPUT "NEW DADT";DADT(II)
560 LOCATE 20,3: INPUT "NEXT DATA POINT TO BE EDITED (HIT ENTER IF EDITING COMPLETE)";
570 IF II > 0 GOTO 580 ELSE 410
580 CLS
590 GOTO 530
600 CLS
610 LOCATE 10,10: INPUT "DO YOU WANT TO STORE THE DATA IN A FILE(Y/N)";YS
620 IF YS = "Y" GOTO 630 ELSE 1000
630 LOCATE 12,10: INPUT "WHAT DO YOU WANT TO CALL THIS DATA FILE";F$
640 CLS
650 OPEN FS FOR OUTPUT AS #1
655 WRITE #1, NN
660 FOR I = 1 TO NN
670 WRITE #1, AB(I), T(I), DADT(I)
680 NEXT I
690 CLOSE #1
1000 CLS
1010 REM BEGIN CALCULATIONS
1020 PI = 3.141592
1030 ALPHA2 = (.5/PI)*((N+1)^2/(2*N*.69))^(2/(N-1))
1035 AA = AA*YS^N
1040 REM SUBROUTINE FOR CALCULATING H1 VALUES
1050 GOSUB 4000
1060 FOR I = 1 TO NN
1070 AOW = AB(I)/W
1080 F = .866+.4*AA-13.2*AA^2+14.72*AA^3-5.6*AA^4
1090 KOP = (((2*AA)/((1-AOW)^1.5))*F/(B*W^.5)
1100 K(I) = KOP*P
1110 REM SUBROUTINE FOR INTERPOLATING H1 VALUES
1111 GOSUB 4200
1120 TT(I) = 2*.91*(((K(I))^2)/(W*E))*DDF*.33*RDTC
1121 CT(I) = CT + CSTAR(I)
1122 VDOTE = ((2*(K(I))^2)/E)*GADT(I)*B/P
1130 GAMMA = 2+.9717*AA-.9756*AA^2-.6371*AA^3+.6352*AA^4
1140 ETA = .2932-.7261*AA+.4889*AA^2-.369*AA^3-.09324*AA^4
1150 V(I) = VDOTE + VDOTSSC
1300 NEXT I
1310 GOTO 5000
4000 REM SUBROUTINE TO CALCULATE H1 VALUES
4010 H250 = 2.58064-.3554543*N+3.498268*E-02*N^2-1.574719*E-03*N^3+2.938609*E-05*N^4
4020 H375 = 2.355219-.5008945*N+5.437075*E-02*N^2-2.870103*E-03*N^3+5.632168*E-05*N^4
4030 H500 = 2.355219-.5008945*N+5.437075*E-02*N^2-2.870103*E-03*N^3+5.632168*E-05*N^4
4040 H625 = 2.089532 - 3824516*N + 4.028168*N^2 - 2.102551*E-03*N^3 + 4.072901*E-05*N^4
4050 H750 = 1.993401 - 3399606*N + 0.0384689*N^2 - 2.612296*E-03*N^3 + 4.14364*E-05*N^4
4060 H100 = 1.677875 - 1254355*N + 6.621381*E-03*N^2 - 1.493825*E-04*N^3
4070 REM
4080 REM SUBROUTINE TO INTERPOLATE HI VALUES
4090 IF AOW < .375 GOTO 4220 ELSE 4240
4100 H1 = H250 + ((H375 - H250)/.125)*(AOW -.25)
4110 RETURN
4120 IF AOW < .5 GOTO 4250 ELSE 4270
4130 H1 = H375 + ((H500 - H375)/.125)*(AOW -.375)
4140 RETURN
4150 IF AOW < .625 GOTO 4280 ELSE 4300
4160 H1 = H500 + ((H625 - H500)/.125)*(AOW -.5)
4170 RETURN
4180 IF AOW < .75 GOTO 4310 ELSE 4330
4190 H1 = H625 + ((H750 - H625)/.125)*(AOW -.625)
4200 RETURN
4210 IF AOW < .875 GOTO 4340 ELSE 4360
4220 H1 = H750 + ((H100 - H750)/.125)*(AOW -.75)
4230 RETURN
4240 PRINT "CRACK SIZE","TIME","da/dt","K";
4250 PRINT "in","hr","in/hr","ksi.in^5"
4260 FOR I = 1 TO NN
4270 PRINT USING "###.###";AB(I);  
4280 PRINT USING "####.#";T(I);  
4290 PRINT USING "###.###";K(I);  
4300 PRINT USING "#####.###";CT(I)*1000;
4310 PRINT USING "##.##","TT")
4320 PRINT USING "#####.###";(CSTAR(I))*1000  
4330 NEXT I
5000 REM OUTPUT CODE
5010 CLS
5020 PRINT "CRACK SIZE","TIME","da/dt","K";
5030 PRINT "in","hr","in/hr","ksi.in^5"
5040 FOR I = 1 TO NN
5050 PRINT USING "###.###";AB(I);  
5060 PRINT USING "####.#";T(I);  
5070 PRINT USING "###.###";VC(I);  
5080 PRINT USING "#####.###";V(I);  
5090 PRINT USING "#####.###";V(I);  
5100 PRINT USING "#####.###";V(I);  
5110 NEXT I
5120 LOCATE 25,10:INPUT "CONTINUE";Y$ 
5130 CLS
5140 PRINT "CRACK SIZE"," VDOT"," VDOT"," TT"," CSTAR"
5150 PRINT " (in)"," (in/hr)"," (in/hr)"," (hr)","(in.lbs/in^2.hr)"
5160 PRINT "(in)"," (in/hr)"," (in/hr)"," (hr)","(in.lbs/in^2.hr)"
5170 FOR I = 1 TO NN
5180 PRINT USING "###.###";AB(I);  
5190 PRINT USING "#####.###";VC(I);  
5200 PRINT USING "#####.###";V(I);  
5210 PRINT USING "#####.###";(CSTAR(I))*1000 
5220 NEXT I
5230 LOCATE 25,10:INPUT "WOULD YOU LIKE A HARD COPY OF THE RESULTS (Y/N)";Y$ 
5240 IF Y$ = "Y" GOTO 5240 ELSE 5580
5250 LPRINT " MATERIAL TYPE:";MS$;
5260 LPRINT " SPECIMEN ID:";X$;
5270 LPRINT " TEST TEMP (C):";TP;
5280 LPRINT " TEST LOAD (Kips):";P;
5290 LPRINT " WIDTH (in):";W;
5300 LPRINT " NOMINAL THICKNESS(in):";BN
5310 LPRINT " SIDEgroove (%):";((BN-B)/BN)*100 
5320 LPRINT A-4
LPRINT " CREEP EXP (n):";N;
LPRINT " CREEP COEF. (Ksi^-n.hr^-1):";AA*YS^-N
LPRINT " ELAS MOD (Ksi):";E;
LPRINT " YIELD STR (Ksi):";YS

LPRINT "CRACK SIZE","TIME","da/dt";
LPRINT "Ct";
LPRINT " in ","hr","in/hr";
LPRINT " in.lbs./in^2.hr";
LPRINT " ksi.in^5"

FOR I = 1 TO NN
LPRINT USING " #.###";AB(I);
LPRINT USING " ######";T(I);
LPRINT USING " #.##### ";DADT(I);
LPRINT USING "#####.###";(CT(I))*1000;
LPRINT USING "###.####";K(I)
NEXT I

LPRINT "CRACK SIZE"," VCDOT"," VDOT"," TT"," CSTAR"
LPRINT " (in )"," (in/hr)""," (in/hr)"," (hr)","(in.lbs/in^2.hr)"

FOR I = 1 TO NN
LPRINT USING " #.###";AB(I);
LPRINT USING " #.#####";VC(I);
LPRINT USING " #.#####";V(I);
LPRINT USING "#####.###";TT(I);
LPRINT USING "#####.###";(CSTAR(I))*1000
NEXT I

CLS
LOCATE 5,10:INPUT "DO YOU WANT TO STORE DATA FOR PLOTTING (Y/N)";YS
IF YS = "Y" GOTO 5610 ELSE END

LOCATE 10,10: INPUT "FILE NAME";FS
OPEN FS FOR OUTPUT AS #1
FOR I = 1 TO NN
WRITE #1, (1000)*CT(I),DADT(I), K(I)
NEXT I
CLOSE #1
END
APPENDIX B

Equations for Calculating $\dot{V}_c$ and $\dot{V}$ [69]

\[
\dot{V}_c = \frac{2B(1-\nu^2)}{E\rho} \beta K^4 a(E\Delta t)^{2/n+1} + \dot{V}_{ss} \tag{B-1}
\]

where, $\dot{V}_{ss} = \frac{BWC^*}{P\eta_1} \tag{B-2}$

\[
\eta_1 = \frac{1}{(1-a/w)} - \frac{n}{n+1} (\gamma - \alpha_1/n) \tag{B-3}
\]

\[
\gamma = 2 + 0.9717 \frac{a}{w} - 0.9756 \left(\frac{a}{w}\right)^2 - 0.6371 \left(\frac{a}{w}\right)^3 + 0.6352 \left(\frac{a}{w}\right)^4 \tag{B-4}
\]

\[
\alpha_1 = 0.2932 - 0.7261 \left(\frac{a}{w}\right) + 0.4889 \left(\frac{a}{w}\right)^2 + 0.369 \left(\frac{a}{w}\right)^3 - 0.09324 \left(\frac{a}{w}\right)^4 \tag{B-5}
\]

All symbols have been defined previously.

\[
\dot{V} = \dot{V}_c + \frac{B}{P} \frac{2K^2}{E} \tag{B-6}
\]

Since the true stress-strain data for these materials was not available, it was assumed that the instantaneous behavior of these specimens was linear elastic.
APPENDIX C

Results of the Analysis of Creep Crack Growth Data of Titanium Aluminides
MATERIAL TYPE: Titanium Aluminide  
SPECIMEN ID: 88-111  
TEST TEMP (C): 650

TEST LOAD (Kips): 2.142  
WIDTH (in): 1.5785  
NOMINAL THICKNESS (in): .3942

SIDE GROOVE (%): 0

CREEP EXP (n): 5.5  
CREEP COEF. (Ksi^-n.hr^-1): 3.29E-13

ELAS MOD (Ksi): 12700  
YIELD STR (Ksi): 41

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<tr>
<th>CRACK SIZE (in)</th>
<th>TIME (hr)</th>
<th>da/dt (in/hr)</th>
<th>Ct (in.lbs./in^2.hr)</th>
<th>K (ksi.in^.5)</th>
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<th>VDOT (in/hr)</th>
<th>TT (hr)</th>
<th>CSTAR (in.lbs/in^2.hr)</th>
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MATERIAL TYPE: Titanium Aluminide  
SPECIMEN ID: 88-269  
TEST TEMP (C): 650

TEST LOAD (Kips): .558  
WIDTH (in): .C  
NOMINAL THICKNESS (in): .181  
SIDEGROOVE (%): 0

CREEP EXP (n): 5.5  
CREEP COEF. (Ksi^-n.hr^-1): 3.29E-13  
ELAS MOD (Ksi): 12700  
YIELD STR (Ksi): 41

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<th>K (ksi.in^0.5)</th>
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MATERIAL TYPE: Titanium Aluminide   SPECIMEN ID: 88-268   TEST TEMP (C): 700
SIDEGROOVE (%): 0

CREEP EXP (n): 5.5   CREEP COEF. (Ksi^-n.hr^-1): 3.88E-12
ELAS MOD (Ksi): 12400   YIELD STR (Ksi): 39

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CREEP EXP (n): 5.5  CREEP COEF. (Ksi^n.hr^-1): 3.61E-11  ELAS MOD (Ksi): 12000  YIELD STR (Ksi): 37

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TEST LOAD (Kips): .418  WIDTH (in): .804  NOMINAL THICKNESS (in): .19  SIDE GROOVE (%): 0

CREEP EXP (n): 5.5  CREEP COEF. (Ksi^-n.hr^-1): 3.61E-11  ELAS MOD (Ksi): 12000  YIELD STR (Ksi): 37

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SPECIMEN ID: 88-270  
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TEST LOAD (Kips): .321  
WIDTH (in): .801  
NOMINAL THICKNESS (in): .1758
SIDEGROOVE (%): 0

CREEP EXP (n): 5.5  
CREEP COEF. (Ksi^-n.hr^-1): 3.61E-11
ELAS MOD (Ksi): 12000  
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MATERIAL TYPE: Titanium Aluminide  
SPECIMEN ID: 88-266  
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TEST LOAD (Kips): .323  
WIDTH (in): .803  
NOMINAL THICKNESS (in): .1758
SIDEGROOVE (%): 0

CREEP EXP (n): 5.5  
CREEP COEF. (Ksi^-n.hr^-1): 3.61E-11
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C-9
MATERIAL TYPE: Titanium Aluminide  SPECIMEN ID: 88-260  TEST TEMP (C): 750

TEST LOAD (Kips): .359  WIDTH (in): .802  NOMINAL THICKNESS(in): .18
SIDEGROOVE (%): 0

CREEP EXP (n): 5.5  CREEP COEF. (Ksi^-n.hr^-1): 3.61E-11
ELAS MOD (Ksi): 12000  YIELD STR (Ksi): 37

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SPECIMEN ID: 88-254  
TEST TEMP (°C): 800

TEST LOAD (Kips): 0.252  
WIDTH (in): 0.7978  
NOMINAL THICKNESS (in): 0.178

SIDE GROOVE (%): 0

CREEP EXP (n): 5.5  
CREEP COEF. (Ksi^-n.hr^-1): 2.7E-10

ELAS MOD (Ksi): 11700  
YIELD STR (Ksi): 35

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<th>TT (hr)</th>
<th>CSTAR (in lbs/in^2/hr)</th>
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C-11
MATERIAL TYPE: Titanium Aluminide  SPECIMEN ID: 88-267  TEST TEMP (C): 800

TEST LOAD (Kips): .31  WIDTH (in): .805  NOMINAL THICKNESS (in): .1796  SIDEGROOVE (%): 0

CREEP EXP (n): 5.5  CREEP COEF. (Ksi^-n.hr^-1): 2.7E-10  ELAS MOD (Ksi): 11700  YIELD STR (Ksi): 35

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C-12
APPENDIX D

Derivation of \( \frac{d}{da} \int_{A_c} f dA \)

\[
\frac{\Delta}{\Delta a} \int_{A_c} f dA = \frac{1}{\Delta A} \left[ \int_{A_c+\Delta A_c} (f+\Delta f) dA - \int_{A_c} f dA \right]
\]

\[
= \int_{A_c} \frac{\Delta f}{\Delta a} dA + \frac{1}{\Delta a} \int_{\Delta A_c} (f+\Delta f) dA
\]

\[
= \int_{A_c} \frac{\Delta f}{\Delta a} dA + \frac{1}{\Delta a} \int_{\partial A_c} (f+\Delta f) \Delta r_c ds
\]

\[
\frac{d}{da} \int_{A_c} f dA = \int_{A_c} f, a dA + \int_{\partial A_c} f \cdot r_c, a ds \quad (D-1)
\]
Figure D-1. Creep Zone at Two Different Crack Depths Under the Same Loading History
Appendix E
Relationship Between $C_{os}$ and $C_t$

As shown in Figure E-1, $C_t$ has been defined as [57] the area $A$ between two $P-V$ curves for crack lengths of $a$ and $a+\Delta a$ divided by $\Delta a$. The $P-V$ curves are, as illustrated in Figure 15, constructed by running a series of constant load tests on specimens with crack lengths $a$ and $a+\Delta a$. However, under small scale creep conditions, it has been shown [72] that, under constant loads, $\dot{V}_C$ is related to $P$ by

$$\dot{V}_C = \frac{4a(1-\nu^2)}{E(n-1)} \frac{P^3}{B} \frac{t^{n-1}}{W^2} \frac{F_{t(n-1)}}{n-1}$$  \hspace{1cm} (E-1)$$

where $a=0.159155[(n+1)^2/1.38n]^{2/(n-1)}$, $E$ is Young's modulus, $\nu$ is Poisson's ratio, $A$ and $n$ are creep constants defined in eq. (3-2), $B$ is plate thickness, $W$ is plate width, $P$ is the applied load, and $F$ the proportional constant in calculating $K$ (see eq. (3-13)). With eq. (E-1), it has been shown [72] that

$$C_t = \frac{P\dot{V} F'}{BW F} = \frac{[\text{Area } A+B]}{\Delta a} = \frac{1}{4} \frac{[\text{Area } B+C]}{\Delta a} = \frac{1}{4} \frac{P}{\Delta a} \frac{d\dot{V}}{da}$$  \hspace{1cm} (E-2)$$

Eq. (E-2) implies that, under small scale creep conditions, $C_t$ can be obtained based on a single constant load test at load level $P$ instead of a series of constant load tests at several increasing load levels from zero to $P$.

Under small scale creep, $C_o$ is calculated by eq. (4-10). From eq. (4-9), it can be seen that, under constant load $P$,

$$P\dot{V}_C = \int_{\partial A_t} T_i \dot{u}_i \, ds = \int_{\partial A_C} T_i \dot{u}_i \, ds$$  \hspace{1cm} (E-3)$$

Furthermore, by virtual work theory,
\[
\int_{\partial A_C} T_i \dot{u}_i \, ds = \int_{A_C} \dot{W} \, dA = \int_{A_C} \sigma_{ij} \dot{e}_{ij} \, dA \tag{E-4}
\]

Therefore, under constant load conditions, the asymptotic solution of \( C_0 \) defined in eq. (4-10) can be reduced to

\[
C_{os} = \frac{1}{n+1} p \frac{q^*}{a} \tag{E-5}
\]

Eqs. (E-2) and (E-5) lead to a relationship between \( C_t \) and \( C_0 \) at small scale creep conditions as

\[
C_t = \frac{n+1}{4} C_{os}, \quad (t/t_T) << 1 \tag{E-6}
\]
Figure E-1. Relationship between $C_1$ and $C_0$ at Small Scale Creep