Perspectives on Modeling Electromagnetic Sea Scatter

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The purpose of this report is to provide a brief perspective on the status of various theories used to predict the scattering of electromagnetic waves from the sea surface. The three theoretical approaches discussed are the Global Boundary Value Problem (GBVP), the Composite Surface Model (CSM), and the Surface Feature Scattering Model (SFSM). It is concluded, among other things, that (1) the GBVP approaches are inadequate without further consideration of surface "sharpness" in the scattering integral; (2) the basic conceptual difficulties that prevent the CSM from being a satisfactory physical theory should be confronted; (3) while the SFSMs can account for scattering behavior inexplicable by the other methods, there is still no way to relate the generation and distribution of the scattering features to measurable environmental variables. The present state of understanding of sea scatter is judged to be inadequate as a basis for reliable predictions.
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I. INTRODUCTION

Although the sea is a special case of the general class of rough surfaces, it is a particularly attractive laboratory for rough surface scattering because it is a unified physical system governed by known laws of hydrodynamics. Empirically, its height distribution seems fairly gaussian, it is well-behaved (unlike, say, the land surface in the vicinity of the Grand Canyon, or downtown Manhattan), and it possesses a measurable surface height spectrum - so it can be easily characterized. This has attracted intense interest from the oceanographic community, since signals scattered from such a surface must contain information useful in the remote sensing of its spectrum. As a result, there has been an uncritical preoccupation with those scattering models that relate the scattered field to the surface spectrum.

In the development of models of sea scatter based on physical theory, there are essentially two basic, and distinct, approaches. Historically, the first approach assumed the surface return to have its origin in scattering features, or obstacles, actually present on or near the sea surface. Initially, the choice of scattering obstacles related more to the pre-existence of convenient scattering solutions than to insights gained from observing the sea, and the results had limited credibility. More recently, feature models have become more realistic by focussing on wedge shapes, as suggested by the Stokes waves and sharp crests observed on most natural water surfaces, and the sloshes and plumes suggested by the properties of wave groups and the hydrodynamics of breaking waves.

The other approach to theoretical modeling derives the scattered field from a global boundary-value problem (GBVP) in which the sea as a whole is considered a boundary surface whose corrugations are described by some kind of statistical process. There is an enormous literature devoted to the theory of surface scatter from this point of view, stemming from the importance not only of radar sea scatter, but radar ground scatter and sonar reverberation (the acoustic equivalent of radar clutter) from both the surface and bottom of the sea. It is the GBVP approach that leads to the analytical expression of Bragg resonance scattering that has dominated the theory of sea backscatter since the late 1960's. However, it has been found that this approach (and its variants, such as the Composite Surface model) fails to explain a variety of scattering behaviors that are encountered at low grazing angles particularly, and it is this failure that has led to the renewed interest in feature scattering mentioned above.

The purpose of this report is to provide a perspective on the status of the theories used to predict the scattering of electromagnetic waves from the sea surface. It is argued that there is at present no satisfactory theory of sea scatter and that considerable research is still needed.

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II. REVIEW OF MODELING THEORIES AND THEIR STATUS

A. The Global Boundary Value Problems

The approach to sea scatter as a global boundary-value problem derives essentially from work in the early 1950's associated with such names as Rice (small-perturbation solution using Rayleigh's hypothesis to produce a scattering cross section proportional to the surface wavenumber spectrum), Isakovitch, and Eckart (scalar integral formulations using the assumptions of physical optics). Most of the work on GBVP's since that time has been in the elaboration and reformulation of these basic ideas, with the vector nature of electromagnetic scattering usually brought into the integral formulation by using the familiar Stratton-Chu formulation of the boundary-value problem.

A most useful expression of this point of view is found in a recent paper by Fung and Pan, where a first-order iteration of the physical optics (Kirchhoff) approximation is shown to provide the conventional small-perturbation (Bragg scatter) solution when \( kh << 1 \) (\( h \) is rms surface displacement), and the expected (polarization-independent) physical optics result when \( kh >> 1 \). Transition between these asymptotic regimes is afforded by series expansions of integrals over powers of the surface correlation function. In the case of large \( kh \), these expressions resemble those found in Beckmann and Spizzichino. The transition from small to large \( kh \) is illustrated in Fig. 1, where Fung and Pan compare their second-order theory (1st-order iteration), shown by the solid lines, with the asymptotic small perturbation (SP) and Kirchhoff approximations for three values of \( kh \). It would appear that this higher-order theory provides a smooth transition between the two extreme approximations. If it can be assumed that the iteration scheme is convergent, in the sense that higher-order terms produce increasingly less important corrections, then Fig. 1c suggests that the physical optics (or Kirchhoff) approximation to the GBVP holds down to values of \( kh \) as small as \( kh = 2 \) (provided also that the correlation length \( l \) is not too small, i.e., \( kl > 2 \)). Although these results were derived for perfectly conducting boundary conditions, they should apply reasonably well to the actual sea surface for incident angles less than about 60 degrees.

For radar scattering by the sea, \( kh = 2 \) and \( kl = 2 \) correspond to rms sea waveheights and correlation lengths of about 1/3 of a radar wavelength, so under most sea conditions (waveheights greater than a foot) the integral formulation of the GBVP in the physical optics approximation should describe scattering from the sea at microwave wavelengths (less than a foot) down to rather small scales. This conclusion has certain important implications.

It is convenient to examine these implications by noting that most of the important behavior of the scattering cross section in this approximation is expressed in the following one-dimensional schematic equation, where \( F_p(\theta) \) is an angular factor (for polarization \( p \)), \( S \) denotes the sea surface, \( k_1 = k \sin \theta, k_2 = k \cos \theta \) are the horizontal/vertical components of the incident propagation vector, and \( C(x) \) is the surface correlation coefficient:

\[
\sigma^o(\theta) = k^2 F_p(\theta) \int_S dx e^{-i2k_1 x} [e^{-4k_2^2 h^2} [1 - C(x)] - e^{-4k_2^2 h^2}]
\]

The statistical properties of the sea surface enter through the correlation coefficient \( C(x) \), and in Fung and Pan the exponential containing \( C(x) \) is expanded as an infinite series in powers of its argument. In other applications by Holliday, et al., and Thompson, \( C(x) \) is left in the exponent and expressed as the Fourier transform of an assumed sea spectrum.
Most of the existing theoretical approaches to sea scatter can be discussed by referring to Eqn.(1). First, the small-perturbation Bragg scatter solution is obtained by letting $k h \ll 1$, so that the exponential containing $C(x)$ is replaced by its argument, and the integral becomes the Fourier transform of the correlation function, which is just the surface wavenumber spectrum evaluated at the Bragg wavenumber $2k_B = 2k \sin \theta$

\begin{equation}
(2) \quad \sigma_B^0(\theta) = 4k^4 \cos^2 \theta F_p(\theta) \int_{-\infty}^{\infty} dx e^{-i2k_1x} [h^2 C(x)] = 8\pi k^4 \cos^2 \theta F_p(\theta) W(2k_1)
\end{equation}

The form of the angular factor $F_p(\theta)$ depends on how carefully slope terms are accounted for in the original integral expression (see, e.g., Leader\textsuperscript{9}). It should be noted that this expression could apply globally only for (HF) radar wavelengths of tens of meters at sea states up to about $SS=3$.

The classic perturbation theory for a rough surface, embodied in the so-called Rayleigh-Rice solution referred to at the beginning of this section, leads to an expression identical to (2). In this case the surface perturbations are assumed small \textit{ab initio}, and the scattered field is expressed as an expansion of outgoing plane waves above the surface. There have been some attempts recently to develop perturbation theories within the integral-equation formulation of the scattering problem by invoking the extinction theorem (vanishing of the total field underneath the surface) to determine the perturbation coefficients (see Nieto-Vesperinas and Garcia\textsuperscript{10}, Rodriguez\textsuperscript{11}, and also, Winebrenner and Ishimaru\textsuperscript{12}). However, Jackson, Winebrenner and Ishimaru\textsuperscript{13} have found that the results obtained by the two different perturbation methods (Rayleigh-Rice and Extinction Theorem) are identical through fifth order; but while there is therefore nothing to be gained by the newer approach, a related phase perturbation technique described in reference 11 is claimed to afford more rapid convergence under certain conditions. Since these methods originate in a general integral formulation of the scattering problem, they must, and do, reproduce the usual Kirchhoff approximation as one option. For this reason they provide alternative ways of looking at the results obtained by the more conventional approach adopted in this report, although it is not clear that they provide any really new insights.

Under almost all real sea conditions at microwave frequencies, $kh$ will globally be much greater than 1, so only values of $C(x)$ close to unity, or $x$ close to 0, will contribute, and the form of the correlation function at the origin becomes very important. This is an old problem in rough-surface scattering, and the conventional approach is to expand $C(x)$ about $x=0$ and retain the first two terms. For a well behaved $C(x)$, one writes:

\begin{equation}
(3) \quad h^2 C(x) = h^2 + (1/2) h^2 C(0) x^2 + (1/24) h^2 C(0) x^4 +
\end{equation}

where the first two coefficients are the surface height variance and the mean square surface slope, respectively. The third coefficient contains the mean square surface curvature, and has traditionally been ignored.

It should be noted that in this approximation regime, the concept of Bragg scatter does not enter. In fact, by defining $C(x)$ as an integral over the sea wavenumber spectrum, Holliday\textsuperscript{7} has shown that, at least at the smaller incident angles, the cross section is relatively insensitive to truncation of the wave spectrum at values of $K$ well below the Bragg wavenumber $2k$; i.e., the predictions of the theory at the...
smaller incidence angles (e.g. \( \theta < 20^\circ \)) do not require the existence of Bragg resonant surface waves. In fact, taking Eqn.(3) to contain the entire dependence of radar cross section on surface properties, we see that the cross section depends ultimately only on surface height \((h)\), surface roughness (slopes), and surface sharpness (curvatures). Moreover, if the integral in (1) is performed using the first two terms in (3), the result has only a weak dependence on radar frequency, which was one of the experimental results both Holliday, et al, and Thompson were trying to explain. Let us illustrate this by substituting the first two terms of (3) into (1), introducing the surface correlation length \(L\) by setting \((1/2)C(0) = -i/L^2\) so that the second term may be expressed in terms of a mean-square surface slope \(<s^2> - h^2/L^2\):

\[
\sigma^0(\theta) \sim k^2 F_p(\theta) \int_{-\infty}^{\infty} dx e^{-2ik_1 x} \exp[-2k_2^2 <s^2>]x^2
\]

Looking at the integrand, it is clear that the major contribution to the scattered field emerges from the very small portion of the scattering surface defined by

\[
dx \sim [1/(2k_2^2<s^2>)]^{1/2} - \lambda
\]

(for an incident angle of \(45^\circ\) and a nominal rms sea slope of 0.15.) This measure may be viewed as a scattering correlation length and is seen to be only of the order of a radar wavelength, so any resonance would be strongly damped and the concept of Bragg resonant scatter becomes suspect. The integral in (4) can be done exactly, yielding the conventional expression for specular scatter:

\[
\sigma^0(\theta) = [\sec^4 \theta/<s^2>] \exp[-\tan^2 \theta/<s^2>]
\]

It should be noted that in this approximation, which is a direct consequence of (1) and (3) without curvature terms, the cross section depends only on incident angle and slope variance and is independent of both frequency and polarization. When compared to measurements, however, it falls off much too rapidly with increasing incident angle; obviously, something is missing that should supply the scattered field at medium to high angles of incidence. Quite possibly this something is associated with the curvature term in (3), since sharp surface features can be expected to have a much broader scattering pattern than the facet slopes described by \(<s^2>\). For a gaussian correlation function, however, the curvature term is unexciting, so it would seem that theories based on expressions like Eqn. (1) should consider statistical behavior more accurately descriptive of an active sea surface with its abundance of sharp features.

Although many papers have been written on this topic over the last 40 years, it appears that the status of GBVP approaches to electromagnetic scattering by the sea remains about as follows:

1. GBVP’s reduce to physical optics solutions for any reasonably active sea at microwave wavelengths, and under these conditions:
   2. Scattering correlation lengths on the surface are only of the order of the radar wavelength, so the concept of resonance in Bragg resonant scattering is open to question;
   3. The scattering predicted at medium to high angles of incidence (\(\theta > 30^\circ\)) is deficient, and if sea scatter is to be successfully described via a GBVP it would seem that surface sharpness must be introduced in some realistic manner.
The Bragg hypothesis is intuitively appealing, and provides a direct linear relationship between the radar cross section and the ocean wave spectrum (Eqn.(2)). Unfortunately, it is a small-perturbation result that applies globally only to surfaces satisfying \( kh << 1 \) (thus, for example, to HF scattering of wavelengths of tens of meters, where it works very well). The use of the concept of Bragg scatter at microwave frequencies traces (in the USA, at least) to wave tanl. measurements by Wright in the middle 1960's. Recognizing that the small perturbation solution that works in the tank cannot apply to real sea surfaces, Wright put forth a heuristic composite-surface model in which patches of Bragg-resonant wavelets were tilted (and later, strained) by the larger motions of the underlying surface, with these effects augmented by specular scatter at the lower incident angles. Interestingly, tank measurements, perturbation theories, and a composite surface model for the same wavelength regime were pursued in the acoustics community five years earlier, associated with the names Liebermann, Marsh, and Kuryanov - which says something about how disciplinary parochialism can lead to needless duplication of effort.

As generally conceived, the Composite Surface Model is not really a scattering theory, but rather a scattering picture, developed from a group of more-or-less plausible assumptions and supported by circumstantial evidence. The assumptions required to make this picture work are the following: (1) the surface wave spectrum is somehow separable into two parts, one containing the Bragg-scattering wavelets and having an integrated rms partial waveheight that satisfies the small perturbation assumption \( kh<<1 \), the other containing only the long waves that both tilt and otherwise modulate the short Bragg wavelets, and also contribute a specular return at low incidence angles as in the physical optics approximation described above (Eqn.(6)); (2) the correlation lengths of the Bragg wavelets are long enough to permit a resonant interaction, yet short enough that adjacent areas on the surface contribute to the scattered signal in random phase; (3) the long waves that tilt and otherwise modulate the Bragg wavelets have radii of curvature sufficiently large that the curvature over the correlation length of the Bragg patches is small in some sense. In its most elementary form, the Bragg part of the model takes a small perturbation cross section (e.g., from Eqn.(2) for \( kh<<1 \)), for the local incident angle \( (\theta+\alpha) \), where \( \theta \) is the mean incident angle and \( \alpha \) is the local slope angle, and simply averages over the slope distribution \( p(\alpha) \):

\[
\sigma^0(\theta) = \int \sigma^0_B(\theta+\alpha)p(\alpha)d\alpha
\]

It is of interest to note that in practice, this averaging affects mainly the horizontally polarized returns and tends to improve the agreement of predictions with experiment for H-polarization. For vertical polarization the effect is inconsequential, yet the predictions for V-polarization, even for very rough sea surfaces at high wind speeds, tend to be quite good (see, e.g., Valenzuela). This means, essentially, that the small perturbation approximation for V-polarization provides good agreement with measurement under conditions for which it could not possibly apply, thus leading us to the central paradox of the Bragg scatter hypothesis. On the other hand, for horizontal polarization the improvement afforded by using the CSM requires unrealistically high rms slopes to force reasonable agreement, and has increasingly been called into question. Nevertheless, the CSM in general tends often to provide quite reasonable scattering predictions at the medium to high incident angles where the GBVP expressions tend to fail in their present form. Moreover, it is also found that under certain conditions the Doppler shifts of the scattered signals are appropriate to the velocity of the Bragg resonant surface waves. It is these tendencies of the CSM to agree with experiment that keep it alive in spite of its conceptual difficulties.
Although the CSM cannot be derived from first principles, it is often discussed in terms of a GBVP expression like Eqn.(1) (e.g., Thompson\(^8\)). The arbitrary partition of the surface wavenumber spectrum referred to above leads to a division of \(C(x)\) into short wave and long wave components \(C_S(x)\) and \(C_L(x)\), where \(k^2h^2 C_S(x) \ll 1\), thereby separating the exponent in Eqn.(1) and allowing the part with \(C_S(x)\) to be brought down in a linear approximation. A few simple integrations involving the first two terms of Eqn.(3) for the long wave slopes yields the complete composite-surface model. Much has been written about how to choose the proper partition wavenumber separating the short and long wave components of the spectrum. Thompson's comparison of the GBVP result with the CSM with variable partition wavenumber suggests that the choice is not critical, although a value of about \(1/3\) the Bragg wavenumber brings the CSM close to the physical optics version of the GBVP. It is difficult to understand what this could possibly mean. The simple fact is that there is no logical way to separate a continuous spectrum into two disjoint sections that retain the essential conceptual picture of flat plates of little Bragg wavelets tossed about by the underlying sea surface. A detailed discussion of the CSM may be found in reference 21.

The present status of the CSM might be stated as follows:

1. Conceptually, the CSM remains today pretty much as it was originally proposed by Wright (and others) in the 1960's: a heuristic device for linking linear, small-perturbation Bragg scatter solutions to the motions of the larger waves on the sea. However, it cannot be derived from first principles, and there seems to be no rational way to extend, modify, or improve it.

2. It has provided reasonable predictions of sea scatter under certain conditions (vertical polarization, moderate wind speeds and incident angles), but has failed under others (low grazing angles, horizontal polarization, internal wave effects, high wind speeds).

3. It has provided an intuitive view of sea scatter as a Bragg resonant phenomenon which, quite possibly, has inhibited the exploration of other scattering mechanisms and theoretical approaches.

C. Scattering by Surface Features

In approaching the scattering problem in a purely statistical sense, with temporally and spatially averaged spectra and correlation functions, and mean-square slopes and curvatures, it is sometimes overlooked that \textit{spectra do not scatter}, and the actual scattering takes place from real scattering obstacles. Anyone looking at the sea will see the surface populated by wedges, cusps, microbreakers, hydraulic shocks, patches of turbulence and gravity/ capillary waves, punctuated on occasion by the sharp crest of a breaking wave, with plumes of water cascading down its face and a halo of spray above it.

Interest in scattering from surface features arose recently as a result of the failure of any of the conventional formalisms to account for phenomena observed at shallow grazing angles: large transient returns (sea spikes), unexpected polarization dependences, strange signal dynamics. It was found that many of these anomalies could be explained on the basis of scattering from structures associated with breaking waves and hydrodynamic non-linearities, both large-scale (breakers) and small scale (microbreakers and sloshes). (See Wetzel\(^2\^22\), for a discussion of this general topic.) Even the lowly wedge, simulating a Stokes wave, was found to provide polarization and grazing-angle dependences about as good as, and in some ways better than, the predictions of the Bragg and CSM models. The scattering formalisms can be quite primitive: standard diffraction theory for a wedge, physical optics for the hydrodynamic objects. Polarization dependence is reintroduced into physical optics calculations by
considering multiple scattering from the underlying wave surface to produce a polarization-dependent 
reflected field over the scattering obstacle. (Actually, this is the same way that polarization dependence is 
produced in the small-perturbation approximation, only there it is hidden in the formalism.) Wind speed 
effects are handled just as in the CSM (Eqn.(7)), by averaging the feature cross section over the wind-
dependent wave slope distribution. In a sense, then, the formalism of feature scattering simply replaces the 
fictitious Bragg patch of the CSM with an assortment of real surface scattering objects.

While consideration of feature scattering introduces the aspects of locality and sharpness that seemed 
missing from the current GBVP formulations, and permitted treatment of chaotic and specialized 
scatterers that get ignored in wide-scale averaging, the central problem with this approach is the difficulty 
in establishing the real-world morphology of the scatterers. In addition it must be determined how they are 
distributed in size, orientation and lifetime over the sea surface and how these distributions depend on 
environmental variables such as wind speed, presence of currents, air and sea temperatures, etc.

Feature scattering was originally introduced to account for the behavior of low grazing angle scatter, 
but there is no reason to believe that the scattering world becomes completely different at 10°, or 20° or 
30°; i.e., it might prove possible to describe all sea scatter in terms of discrete scattering elements on the 
surface.

Status of feature scattering:

1. Features describe a variety of scattering behaviors that are not explained by other formalisms, 
particularly the sea spikes, polarization inversions, and signal dynamics characteristic of radar returns at 
low grazing angles. They are also capable of expressing the behavior of sea scatter under more general 
conditions (see Fig. 2c.)

2. The distributions of feature parameters necessary for use in statistical predictions of sea scatter, not 
to mention the dependence of such distributions on environmental variables, are not yet available.

III. CONCLUSIONS AND FUTURE DIRECTIONS

It is not an exaggeration to say that electromagnetic scattering from the sea is not well understood. In 
fact, it is really quite surprising to see how little real progress has been made in this field after almost 40 
years of research. Part of the problem is the lack of carefully collected experimental data of sufficient 
variety to guide inductive conclusions, and to permit crucial comparisons of the various modeling 
approaches. The other part appears in the missing pieces noted above in the GBVP and surface feature 
approaches to sea scatter, and in the puzzling combination of success, failure and ambiguity in the 
composite surface model.

Figure 2 shows various models compared with an example of experimental data. The experimental 
curves are representative of sea clutter obtained at the higher wind speeds by the NRL/4FR (see 
Valenzuela19). In Fig.2a the model is the SPM of Eqn. (2) for V-polarization and the CSM as expressed 
by Eqn. (7) for H-polarization. Figure 2b contains several models: the qualitative behavior of a wedge 
model22; the standard Lambert’s law scattering by a perfectly rough surface; and the simple specular facet 
model embodied in Eqn. (6). Figure 2c shows cross sections inferred from a three-feature model 
containing microbreakers (contributions below 50°), small steep gravity waves (50° to 80°), and specular
facets (>80°) (Wetzel, unpublished.) All of them show reasonable agreement with the experimental curves, and since the assumptions and concepts underlying each of them are quite different, one might be tempted to regard sea scatter as a model-independent phenomenon.

Aside from a general plea for more and better data of the right type, work on the following topics might clarify some of the issues and improve our understanding of this difficult phenomenology:

1. Introduce surface sharpness into the GBVP approach by, for example, considering the higher-order terms in the expansion of the surface correlation function.

2. Make a determined and open-minded effort to establish the true role of Bragg resonance in sea scatter at microwave frequencies. To what extent is it real, and to what extent an artifact of the structure of the scattering integral?

3. Establish the hydrodynamic verisimilitude of feature models; e.g., how are the ubiquitous small steep gravity waves and microbreakers most faithfully represented mathematically? Determine how features are created and destroyed by environmental forces, how they are distributed, and how they can best be incorporated into a useful predictive model.

4. Find out whether there is anything useful to be gained in applying fashionable conceptologies like fractals and chaos to the study of sea scatter. The operative word here is useful. In a recent paper, for example, a sample of radar clutter data was examined for the presence of a chaotic attractor\(^2\). An attractor of low dimensionality (~7) was identified, and the authors concluded that radar clutter was not completely random, but rather the result of a limited number of independent processes. But of course, it is well known that radar clutter is not a totally random process, and all of the models of sea scatter discussed in this report are directed toward the identification of the actual (limited) physical processes responsible for it. Can application of these new methodologies lead to more than reaffirmation of what is already known?

5. Explore the significance of the choice of measurement instrument in the development of sea scatter concepts. That is, those who go to sea with averaging radar wave spectrometers come back convinced of the Bragg hypothesis and the central role of sea wave spectra, while those who use high resolution time-domain probes are led to conclude that scattering from the sea is dominated by spiky returns from highly localized features. How can these conflicting points of view be reconciled?

Quite obviously there is still much to be done before sea scatter can be said to be understood.

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**END NOTE:** This report is an expanded version of a Working Paper prepared for the NSF Workshop on Future Directions in Electromagnetics Research held in Boston, Mass. on July 27, 1989, which, in turn, was based on Section 13.4 of the author’s chapter Sea Clutter in the Radar Handbook, M. Skolnik Ed., published by McGraw-Hill, 1990. Each version of this work has attempted to bring the topic up to date by introducing new material and new interpretations of the old material.
REFERENCES


Fig. 1  Second-Order GBVP Solutions from Fung and Pan

Showing Transition from SPM to Physical Optics (Kirchhoff)
a. Comparison with the SPM Bragg (V-pol) and the CSM Bragg (H-pol)

b. Comparison with simple wedge and other generalized models

c. Comparison with behavior of 3-feature scattering model, assuming "reasonable" parameters for best fit for microbreakers, small steep gravity waves, specular facets.

Fig. 2 Comparison of Various Scattering Models with Experimental Data. (Solid curves based on NRL/4FR data for higher wind speeds, as reported in Valenzuela.)
Table 1
Characteristics of Dredging Operation

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* Overflow began at 0900 and ended at 1000.
** Overflow began at 1010 and ended at 1034.
† Overflow began at 1815 and ended at 1920.
Table 2
Physical Properties of Bucket and Overflow Samples

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Number of Samples</th>
<th>Atterberg Limits</th>
<th>USCS Classification</th>
<th>Average Solids Conc.*</th>
<th>Average D50**</th>
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<td>Plasticity Index, %</td>
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</table>

* D50 is the grain size for which 50 percent of the particles by weight are finer.
** Values in parentheses indicate the range of values.