Research on systems reliability and inference were focused on three major areas: linear approximations for estimating first and second moments for a variety of different models; delayed reporting models, with emphasis on predicting unreported counts generated over a fixed exposure interval; and generalized models of life testing with incomplete data.
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**Approximations for Bayesian Estimation and Prediction**

A continuing research theme has been the development of approximation methods for Bayesian estimation and prediction. Our primary approach has been the use of "credibility theory" to provide linear approximations to the predictive means and variances of various applied models where prior information is available. In these applications, the availability of simple point prediction formulae and the resulting analytic insight is more important than obtaining full-distributional numerical results.

A case in point is the simultaneous prediction of co-product quality, for example, in the fabrication of semiconductor microchips, where output is sorted into "different" product types, based upon performance. A natural model for making predictions of this "binned" output is the multinomial distribution; however, the underlying sorting probabilities are only imprecisely known after each "tweaking" by the process engineers. Another difficulty, from an analytic point of view, is that the usual "natural-conjugate" Dirichlet prior is much too "thin" to describe possible process variation, since preliminary examination of industrial data indicates that, in fact, some prior output covariances may be positive. Since only the means and covariances of co-product are useful in production planning, the use of linear approximation techniques is justified. This work led to a Ph.D. dissertation (Chou, 1988) and has been reported in Jewell and Chou (1988).

A general approach to linear approximations for Bayesian predictive means and covariances in problems which are of high dimensionality, or in which exact analysis requires impractically detailed prior opinion and knowledge may be found in Jewell (1989a). In this paper, a number of special cases, including the multinomial and multinormal model densities, are outlined in which the required number of hyper-parameters is substantially reduced; these models are currently under investigation. Jewell (1989c) shows how to predict the first and second moments in a two-stage hierarchical model; comparison is made with exact results previously obtained for the multinormal hierarchical model (Jewell, 1988). A substantial reduction in dimensionality is obtained through analysis of the inversion of a large matrix with a new special covariance structure, reported in Jewell (1990b).
Delayed Reporting Models

An interesting partial information problem occurs when an unknown number of samples is generated from a point process during some fixed exposure interval, but the reporting of the events is delayed; both the process and delay parameters are assumed to be unknown, a priori. The problem is to predict the actual number of events that occurred, based only upon the partial number of events reported by some fixed date. This model describes, *inter alia*: the delayed reporting of effects from some catastrophe or environmental insult (Chernobyl, asbestosis, etc.); the financially important problem of estimating IBNYR (Incurred But Not Yet Reported) claims in insurance; problems in predicting with "incomplete returns" in sample surveys, voting; *et cetera*.

Jewell (1989b) assumes that events are generated according to a stationary Poisson process; delays are independent samples from an exponential distribution, and that reporting consists of exact measurements of various combinations of (i) occurrence dates, and/or reporting dates for events recorded by the end of the sampling interval. The exact predictive distribution is obtained in a recursive form that involves the ratio of certain integrals; these ratios are then accurately approximated using a Gammoid approximation, in a manner similar to the approach of Morris (1988). The paper examines the effect of four different assumptions on the availability of data. The two worst cases lead to identifiability problems with MLE estimators or to convergence problems with Bayesian estimators; this result has important practical significance for data gathering for IBNYR prediction.

Jewell (1990) considers the same problem with quantized delayed reporting, where the process is sampled only periodically. This variation is of great practical interest in applications where the volume of data precludes gathering exact dates of event occurrence and reporting. The main foci of the paper are comparing the effects of the two possible types of quantized data reporting, and investigation of the prediction accuracy of increasingly longer reporting cycles. Comparisons with our previous continuous data results are also made. The results of this investigation were reported at the ASTIN Colloquium in New York City, November, 1989.

Currently in progress is an investigation of the effect of adding cohort data from earlier exposure intervals (the so-called *IBNR Triangle* data); our hope here is to show that, with sufficient data, the Bayesian predictive mode for the total number of unreported events from all exposure years converges to one of the many heuristic IBNR Triangle estimators currently used in the risk industry. We will also develop Bayesian predictive densities using the Gammoid approximation used in the earlier papers.
Another research topic currently in progress is the investigation of partial information samples. For instance, the classical protocol for predicting future lifetimes of unreliable components is the censored test, terminated at some fixed time T with certain of the components having failed and the remainder still functioning. (A wide variety of model variations are possible, include renewal testing, withdrawn components, etc.) As is well known, it is important to use not only the completed failure data for estimation and prediction, but also the number of remaining components still "on test"; for exponential lifetimes, this leads to the classic "total-time-on-test" statistic and estimator. With the choice of a Gamma prior for the failure rate, Bayesian prediction of the lifetime distribution can then be made in closed form. This approach works even for a slightly generalized model, called the proportional hazard family [Jewell (1977)]. There remains the question as to the effect of censoring or truncating the samples in other models and densities; this problem is analytically difficult because of the presence of the tail distribution function in the likelihood.

A related problem occurs when it assumed that components to be tested include two or more types with different failure rates; usually, the proportion of defective parts and the failure rates are all unknown, a priori. Standard engineering practice is to "burn in" all the components for a fixed time, possibly under accelerated stress conditions, with the hope that the proportion of "good" parts will be higher in the remaining population. Here again, both the data furnished by the failed lifetimes and the number of survivors is important in making useful predictions.

*Truncated tests* are also of interest. For example, in the pricing of *stop-loss* coverage in reinsurance, only the the *excess losses* above a certain, usually high, limit are insured by the primary carrier. Because of sampling costs, values of losses below this limit are not reported, leaving reinsurers in the unenviable position of estimating the tail of a distribution only from the largest samples, with no information about the bulk of the distribution! In reliability terms, this would be the same as estimating mean residual life after an extended mission duration, using only a few samples of lifetimes that were longer than the mission duration. Clearly, a Bayesian approach that integrates prior experience is mandatory here. Jewell (1990c) presents a preliminary approach to approximate prediction in this difficult problem.

PH.D. THESIS SUPPORTED IN PART BY AFOSR-86-0208


ADDITIONAL REFERENCES


May 11, 1990

Dr. Jon Sjogren  
AFOSR/NM  
Bolling Air Force Base  
Washington, D. C. 20332-6448

Dear Dr. Sjogren:

Enclosed is a report of AFOSR supported research for the last two years. Current work in draft was not mentioned.

As you know, there is currently no money in the AFOSR budget for Ph.D. thesis student support. As a consequence none of our Ph.D. thesis students mentioned in our reports have been supported since Summer 1989. We hope that somehow it will be possible to restore AFOSR funding for graduate U. S. research assistants in our next year's budget.

Sincerely,

Richard E. Barlow

Richard E. Barlow
Our best research results in the 1989-90 time period concern algorithms for computing posterior probabilities in probabilistic influence diagrams. Influence diagrams are one of the chief tools used by consulting companies concerned with decision analysis; e.g. the Strategic Decisions Group, Menlo Park, CA. This is also used in decision analysis courses at Berkeley in the IEOR department and the Engineering Economic Systems department at Stanford among others. As a consequence, elaborate software packages to analyze influence diagrams have been created at Stanford and also Aalborg, Denmark. For example, DAVID is a program developed by Prof. R. Shachter and Leonard J. Bertrand at Stanford. A more elaborate program has been developed by the Strategic Decisions Group, Menlo, Park, California. Another competing program, HUGIN, has very recently been advertised by Hugin Expert in Aalborg, Denmark. This program is based on the research of Lauritzen and Spiegelhalter (1988). DAVID is based on the directed graph and arc reversals while HUGIN is based on the underlying undirected graph. Although the program codes are not available, the algorithms are in the published literature.

In the AFOSR sponsored technical report Decomposable Probabilistic Influence Diagrams (ESRC 90-2, Jan. 1990) the theoretical connection between these different approaches is laid bare. Although it has been claimed that the Lauritzen and Spiegelhalter approach is superior, it is shown that the two approaches are essentially the same. An improved algorithmic approach based on this research is given. This work was first reported at the DIMACS Workshop on Reliability of Computer and Communication Networks at Rutgers University Dec. 2-4. A copy of
the technical report will appear in the Proceedings of this Workshop. This and further work was reported at the recent, May 7-9 meeting of ORSA/TIMS in Las Vegas by Steven Chyu. It will form the core of his Ph.D. thesis. Professor Shachter also reviewed this work in an expository paper, "Directed Reduction Algorithms and Decomposable Graphs" at the March 1990 Conference on Uncertainty in Artificial Intelligence.

In the 1988-89 period perhaps the best work was the paper on environmental stress screening. Models and methods for determining optimal environmental stress screening procedures were analyzed and a new approach suggested.

There are many loopholes in the modeling and practical application of Environmental Stress Screening (ESS) for electronic parts. The most serious problem is that of optimizing the design and conduct of ESS programs. ESS is conducted to purge parts populations of parts that would experience early failure if used. ESS is also used later in the production phase to eliminate weak parts and production flaws in subassemblies and assemblies. One of the unanswered questions is how long and/or how strong ESS should be applied to products coming from a production line so as to obtain products which meet the specified reliability level. We summarize below some results from our paper "Classical and Bayesian Approaches to ESS: A Comparison" (1989).

Let $T$ be the optimal duration for the screen (under a given stress level $I$). Let $c_b$ be the cost of having a "bad" part escape the screen and $c_g$ be the cost of having a "good" part destroyed by the screen. The costs $c_b$ and $c_g$ are "decision" costs in the sense that they describe the cost of wrong "decisions" regarding the part. The cost $c_b$ is usually much larger than $c_g$. In any event, it will only be necessary to specify the ratio $c_b/c_g$. Set $K = \tau^{-1} (b/a) c_g/c_b$, then

$$T = K^{-1} \left[ 1 - K\theta + \sqrt{1 + K(\theta - \tau)} \right]$$

is the optimal duration for the screen where $1/\theta$ ($1/\theta + 1/\tau$) is the expected failure rate of good (bad) parts and $\frac{a}{a+b}$ is the expected fraction defective.
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ADDITIONAL REFERENCE


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