Critique of Nondiffracting Beams

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Bessel beam
Electromagnetic directed energy pulse train

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CRITIQUE OF NONDIFFRACTING BEAMS

A number of researchers have discussed the possibility of generating electromagnetic beams or pulses which can propagate without the usual degree of transverse spreading. Nondiffracting directed radiation beams have been the subject of a number of special sessions at various conferences. Our intention in this note is to discuss i) the Bessel beam which has been called remarkably resistant to the diffractive spreading commonly associated with all wave propagation; and ii) the electromagnetic directed energy pulse train which is claimed to be significantly improved over conventional, diffraction-limited beams, and to defeat diffraction. In this note we show that diffraction is not eliminated or reduced in any of the proposed schemes and that conventional Gaussian beams will propagate at least as far for a given transmitting antenna dimension.

Durnin et al. have studied, analytically and experimentally, a solution of the scalar wave equation whose transverse profile is that of a Bessel function of order zero, \( J_0 \). The field \( \psi(r,z,t) = J_0(k_r r) \exp \{i(k_z z - \omega t) \} \) is a solution of the scalar wave equation where \( k_r \), the
transverse wave number, is given by $k_2^2 = \omega^2/c^2 - k_z^2$, $\omega$ is the frequency and $k_z$ is the axial wave number. Some of the properties of the solution are: i) the transverse profile of $\psi$ is independent of $z$, ii) the intensity function, $J_0^2(k_\perp r)$, falls off like $1/r$ for $r \gg 1/k_\perp$, iii) the power contained in each transverse lobe, between the adjacent zeros of $J_0^2(k_\perp r)$, is of the same order, and iv) $J_0(k_\perp r)$ is an axisymmetric superposition of plane waves propagating at an angle $-k_\perp/k_z$ to the $z$ axis. In most of what follows, we will be interested in the qualitative comparison and scaling of various quantities and, hence, will not be concerned with factors of order unity.

Using arguments based on geometric optics, Durnin et al.\textsuperscript{4} found the propagation distance of the central lobe of the Bessel beam to be given by $\sim 2Rr_0/\lambda$, where $R$ is the radius of the aperture (radius of clipped Bessel beam), $r_0$ is the typical spacing between adjacent zeros of $J_0$, i.e., $r_0 = \pi/k_\perp$, and $\lambda$ is the wavelength. They compare the propagation distance (diffraction length) of the apertured Bessel beam with a Gaussian beam which has a spot size of approximately $r_0$. That is, the center lobe of the Bessel beam nearly matches the Gaussian beam as shown in Fig. 1. The diffraction distance (Rayleigh length) of the Gaussian beam is given by the well-known formula, $Z_G = \pi r_0^2/\lambda$. Since $R \gg r_0$, they observed that the Bessel beam propagated further than the Gaussian beam, by a factor $- (2/\pi)R/r_0$.

Our interpretation of the results of Durnin and co-workers differs in a number of fundamental ways. We first note that the cartesian counterpart of $J_0(k_\perp r)$ is $\cos(k_\perp x)$, which is simply a plane wave propagating at an angle to the $z$ axis. Plane waves do not diffract since there is no spatial variation transverse to the propagation direction; however, when clipped or
apertured, plane waves diffract. Bessel beams, when apertured, will diffract in a similar way.

To be specific, let us consider a Bessel beam limited by a finite aperture of radius $R$. If $N >> 1$ is the number of lobes within the aperture radius, then $R = N r_0$. For the Bessel beam, the diffraction length is given by $Z_B = R/\theta_B$, where $\theta_B = k_\perp / k_z = \lambda / 2 r_0$ is the diffraction angle and $\lambda = 2\pi(k_z^2 + k_{\perp}^2)^{-1/2}$ is the wavelength. The diffraction length associated with the central lobe of the apertured Bessel beam is, therefore, $Z_B = 2 N r_0^2 / \lambda = 2 R r_0 / \lambda = (2/\pi)NZ_G$. Since the lobes carry about equal power, there is sufficient power in the off-axis lobes to replenish the central lobe. Each of the $N$ lobes diffract away sequentially starting with the outermost one. Roughly speaking, the outermost lobe diffracts after a distance $\sim \pi r_0^2 / \lambda$, the next one diffracts after a distance $2 \pi r_0^2 / \lambda$ and so on until the central lobe diffracts away after a distance $\sim N \pi r_0^2 / \lambda$, which is approximately equal to $Z_B$. The central lobe persists as long as there are off-axis lobes to replenish its diffraction losses.

A far more meaningful measure for comparing the diffractive properties of beams would be to ask the following question. For a given aperture (source) size and a target size, which is some distance away, what beam configuration or shape will maximize the power transmitted to the target? If this procedure is used to compare a Bessel beam with a Gaussian beam, we would use a Gaussian beam having a spot size equal to the aperture, $R = N r_0$. The power through the aperture is the same if the peak amplitude of the Bessel beam is $\sim N^{1/2}$ larger than that of the Gaussian beam. In this case the Gaussian beam will propagate $N$ times further than the Bessel beam. In addition, by appropriately focusing the Gaussian beam, nearly all the power can be focused on a target of dimension $r_0$ in a distance $Z_B$. For
the same total power through the aperture, a focused Gaussian beam delivers 
-N times more power on the target than the Bessel beam.

Another solution to the wave equation which has been studied for its
diffractive properties is the electromagnetic directed energy pulse
train.5-7 This pulse form is a superposition of fundamental Gaussian
pulses, ψk, which are exact solutions to the homogeneous wave equation
(\nabla^2 - c^{-2}a^2/\partial t^2)ψk = 0, where ψk(r,z,t) = (4πiV)^{-1}\exp(ikη - kr^2/V),
1/V = 1/A-i/R, A = z_0+ξ^2/z_0, R = ξ+z_0^2/ξ, η = z+ct, ξ = z-ct, k = ω/c and z_0
is a constant. The solution, ψk, which has been studied by Ziolkowski and
co-workers,6 represents a pulse train traveling to the left which is
modulated by an envelope traveling to the right (z direction). The
functions ψk, for all k, form a complete set and each basis function has
infinite energy, i.e., \int_{-\infty}^{\infty} d^3ξ |ψ_k|^2 → \infty. A finite energy pulse can be formed
by a superposition of the basis functions with the weight function F(k),
i.e., f = Re\int_{-\infty}^{\infty} dk ψ_k(r,z,t)F(k). Ziolkowski and co-workers6 have examined a
particular pulse form both numerically and experimentally. This pulse form
is called a modified power spectrum (MPS) pulse which is given by
f(r,ξ,η) = Re[(z_0+iξ)^{-1}(a+r^2/β(z_0+iξ)-in/β)^{-α} \exp(\text{ib}η/β-br^2/β(z_0+iξ))],
where the function f is an exact solution to the scalar wave equation for
the pulse amplitude and a, b, α, β, and z_0 are arbitrary constants. The
initial shape of the MPS pulse is shown in Fig. 2 for particular values of
a, b, α, β and z_0. The MPS pulse has finite energy and its shape evolves
as it propagates. To determine the propagation distance of this pulse,
Ziolkowski and co-workers note that the radial profile is dominated by the
factor \exp(-bz_0r^2/(β(z_0^2+ξ^2))). They use the minimum spot size, w_o =
(βz_0/b)^{1/2}, (which occurs at the pulse’s center, ξ=0) to calculate the
diffraction length, \( Z = \pi w_0^2/\lambda = \pi b z_0/(b\lambda) \). The numerical and experimental results indicate that the MPS pulse propagates significantly further than this diffraction length \( (Z = \pi w_0^2/\lambda) \) before the amplitude begins to fall off like \( 1/z \).

In our interpretation of their results, the diffraction length for the MPS pulse is not \( \pi w_0^2/\lambda \), but is properly given by \( Z_{MPS} = w_0 R/\lambda \), where \( R \) is the transmitting antenna dimension (radius of aperture) and \( R > w_0 \). This can be understood by first noting that the diffraction angle associated with a pulse having a typical transverse spatial variation of \( \sim w_0 \) is \( \Theta_{MPS} \approx \lambda/(\pi w_0) \). As in the case of the Bessel beam, we note that the energy in the MPS pulse is radially spread out, typically over the full width, \( R \), of the aperture. Consequently, the diffraction distance is given by \( Z_{MPS} = R/\Theta_{MPS} \). The numerical and experimental studies of the MPS pulse span a wide range of values for \( \lambda, w_0 \) and \( R > w_0 \). In all cases we find that the observed propagation distance is fully consistent with the length \( Z_{MPS} = w_0 R/\lambda \) for wavelengths within the main part of the spectrum.

Utilizing the same transmitting antenna radius \( R \), an unfocused Gaussian beam with spot size \( R \), would propagate a distance \( \sim \pi R^2/\lambda \); this is greater than the MPS pulse propagation distance since \( R > w_0 \). An appropriately focused Gaussian beam can be focused to a dimension \( \sim w_0 \) in the distance \( \sim Z_{MPS} \). Such a Gaussian beam focuses more power on the target than a corresponding MPS pulse.

Other researchers have considered alternative approaches for propagating electromagnetic beams or pulses. One such study indicates the possibility of generating wave packets with a broad frequency spectrum. The high-frequency end of the spectrum determines the furthest distance the pulse can propagate, in complete accord with our understanding of diffraction.
In conclusion, we find that when a proper comparison is made, Bessel Beams and Electromagnetic Directed Energy Pulse Trains have no particular range advantage over conventional Gaussian beams.

Acknowledgment

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References


5. R. W. Ziolkowski, in Ref. 1, p. 312.


Figure Captions

Fig. 1 Transverse intensity profiles of a Bessel beam and a Gaussian beam. The full width at half maximum (FWHM) of the Gaussian beam is the same as that of the central lobe of the Bessel beam. The parameters are the same as those in the experiment of Ref. 4: $\lambda = 6328\text{\AA}$ and FWHM = 70 $\mu$m.

Fig. 2 Plot of modified-power spectrum pulse $f(r,\xi,\eta)$ at $t = 0$, i.e., $\xi = \eta = z$. The parameters for this pulse are the same as those in Ref. 7: $a = 1 \text{ cm}$, $\alpha = 1$, $b = 1 \times 10^{10} \text{ cm}^{-1}$, $\beta = 6 \times 10^{15}$, and $z_0 = 1.667 \times 10^{-3} \text{ cm}$. 
Figure 1
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