REPAIR, EVALUATION, MAINTENANCE, AND REHABILITATION RESEARCH PROGRAM

TECHNICAL REPORT REMR-GT-6

GEOTECHNICAL APPLICATIONS OF THE SELF POTENTIAL (SP) METHOD

Report 4

NUMERICAL MODELING OF SP ANOMALIES: DOCUMENTATION OF PROGRAM SPPC AND APPLICATIONS

by

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<table>
<thead>
<tr>
<th>Problem Area</th>
<th>Problem Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>Concrete and Steel Structures</td>
</tr>
<tr>
<td>GT</td>
<td>Geotechnical</td>
</tr>
<tr>
<td>HY</td>
<td>Hydraulics</td>
</tr>
<tr>
<td>CO</td>
<td>Coastal</td>
</tr>
<tr>
<td>EM</td>
<td>Electrical and Mechanical</td>
</tr>
<tr>
<td>EI</td>
<td>Environmental Impacts</td>
</tr>
<tr>
<td>OM</td>
<td>Operations Management</td>
</tr>
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</table>

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**COVER PHOTOS**

TOP: View of a portion of Beaver Dam, Arkansas

MIDDLE: Concept of mapping anomalous subsurface flow paths.

BOTTOM: Concept of self potential generation by fluid flow.
This manual describes the theory and operations of program SPPC. This code is a microcomputer (PC) version of program SPXCPL which calculates the self potential anomaly due to fluid and thermal sources in geologic media. Fluid and heat flow generate significant electrical potentials due to cross-coupling between fluid and heat flow and the flow of electrical current. The SP anomalies are largest near the primary flow sources and in regions where the cross-coupling coefficients change (geological boundaries). Program SPPC allows for the forward model calculation of primary potential (pressure or temperature) and secondary potential (voltage) for discrete sources of heat or pressure in a two-dimensional model of the earth. The input for the program involves a small mesh where values are assigned for the permeability or thermal conductivity, cross-coupling coefficients, and electrical resistivity. The magnitude and location of the primary sources (fluid or heat flow) must also be specified. The output file provides values for the primary potential (Continued)
6A. NAME OF PERFORMING ORGANIZATION (Continued).

Engineering Geosciences
University of California
Geotechnical Laboratory
USA EWHES

6C. ADDRESS (Continued).

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11. TITLE (Continued).


16. SUPPLEMENTARY NOTATION (Continued).

A report of the Geotechnical problem area of the Repair, Evaluation, Maintenance, and Rehabilitation (REMR) Research Program. This report is available from National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia 22161.

19. ABSTRACT (Continued).

and voltage on the surface and throughout the two-dimensional model. The manual provides a discussion of the input parameters, a section on how to read the output, and some guidance on applying the program to physical problems. The program was validated with an analytical solution for a point pressure source in a homogeneous halfspace. A field example is also shown where SP data collected near a leaky dam site is used to locate the zone of leakage. The potential user of program SPPC is urged to read Report 3 ("Geotechnical Applications of the Self Potential Method; Development of Self Potential Interpretation Techniques for Seepage Detection," (Corwin 1989)), in this series for guidance on obtaining the required modeling parameters.
PREFACE

The work described in this report was authorized by Headquarters, US Army Corps of Engineers (HQUSACE), as part of the Geotechnical—Rock Problem Area of the Repair, Evaluation, Maintenance, and Rehabilitation (REMR) Research Program. The work was performed under Work Unit No. WUI32315, "Geophysical Techniques for Assessment of Existing Structures and Structural Foundations," for which Dr. Dwain K. Butler and Mr. Jose L. Llopis (CEWES-GG) are Principal Investigators. Mr. Lewis A. Gustafson (CECW-EG) was the REMR Technical Monitor for this work.

Mr. Jesse A. Pfeiffer, Jr. (CERD-C) is the REMR Coordinator at the Directorate of Research and Development, HQUSACE; Mr. James E. Crews (CECW-OM) and Dr. Tony C. Liu (CECW-ED) serve as the REMR Overview Committee; Mr. William F. McCleese (CEWES-SC-A), US Army Waterways Experiment Station (WES) is the REMR Program Manager. Mr. Jerry S. Huie (CEWES-GS) is the Problem Area Leader.

The work was performed at the University of California, Berkeley, and this report was prepared by Mr. Michael J. Wilt, Department of Engineering Geoscience, with consultation and input from Dr. Butler, while Dr. Wilt was employed as a Contract Student. The work was performed under the general supervision of Dr. William F. Marcuson, Chief, Geotechnical Laboratory (CEWES-GL) and the direct supervision of Dr. Arley G. Franklin, Chief, Earthquake Engineering and Geosciences Division (CEWES-GG).

Commander and Director of WES is COL Larry B. Fulton, EN. Technical Director is Dr. Robert W. Whalin.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREFACE</td>
<td>1</td>
</tr>
<tr>
<td>CONVERSION FACTORS, NON-SI (METRIC) UNITS OF MEASUREMENT</td>
<td>3</td>
</tr>
<tr>
<td>PART I: INTRODUCTION</td>
<td>4</td>
</tr>
<tr>
<td>Background</td>
<td>4</td>
</tr>
<tr>
<td>Scope</td>
<td>5</td>
</tr>
<tr>
<td>PART II: THEORETICAL BACKGROUND</td>
<td>7</td>
</tr>
<tr>
<td>General Principles</td>
<td>7</td>
</tr>
<tr>
<td>Numerical Modeling Scheme</td>
<td>8</td>
</tr>
<tr>
<td>PART III: PROGRAM OPERATIONS</td>
<td>9</td>
</tr>
<tr>
<td>File Preparation and a Sample Problem</td>
<td>9</td>
</tr>
<tr>
<td>Program Notes</td>
<td>20</td>
</tr>
<tr>
<td>PART IV: FIELD EXAMPLE</td>
<td>21</td>
</tr>
<tr>
<td>PART V: CONCLUSIONS AND RECOMMENDATIONS</td>
<td>26</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>27</td>
</tr>
<tr>
<td>APPENDIX A: PHYSICAL PARAMETERS</td>
<td>A1</td>
</tr>
</tbody>
</table>
CONVERSION FACTORS, NON-SI TO SI (METRIC)
UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

<table>
<thead>
<tr>
<th>Multiply</th>
<th>By</th>
<th>To Obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fahrenheit degrees</td>
<td>5/9</td>
<td>Celsius degrees or kelvins*</td>
</tr>
<tr>
<td>gallons</td>
<td>3.785412</td>
<td>cubic decimetres</td>
</tr>
<tr>
<td>inches</td>
<td>2.54</td>
<td>centimetres</td>
</tr>
</tbody>
</table>

* To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula:  
  \[ C = \frac{5}{9}(F - 32) \]. To obtain Kelvin (K) readings, use \[ K = \frac{5}{9}(F - 32) + 273.15 \].
This report, Report 4, documents a microcomputer program for the numerical modeling of SP anomalies caused by flow of fluid or heat (primary flows) through a porous medium. The program is applied to the modeling of SP data acquired at Beaver Dam, Arkansas. This model calculation is the first known numerical modeling simulation of SP phenomena caused by anomalous seepage through an earth dam and foundation.
2. Program SPPC is a FORTRAN code used to calculate self-potential (SP) anomalies due to thermal and pressure sources. This code is a version of program SPXCPL (Sill 1983; Sill and Killpack 1982) that has been modified for use on an IBM compatible personal computer (PC) running MSDOS. It is a 2-1/2 d solution—that is, the program restricts the user to a two-dimensional distribution of physical properties but allows for a three-dimensional source function.

3. The program calculates voltages and primary potentials (pressure or temperature) throughout the two-dimensional space. The final solution is generated using a three part process. First, the pressure or temperature distribution is determined from the source parameters and the permeability or thermal conductivity distribution. After this primary potential (pressure or temperature) has been calculated, the locations and strengths of the induced electrical sources are determined. These electrical sources are determined from the primary potentials (pressure or temperature) and the distribution of the cross-coupling coefficients which relate the electrical and the hydraulic or thermal potentials. At this point the problem is reduced to electrical conduction equation. That is, the final SP voltages are computed from the induced electrical sources and the two-dimensional resistivity distribution.

4. Although the code is fairly simple to operate, it is not quite so simple to use it to model physical problems. The difficulty comes from the necessity to know two-dimensional distributions of primary impedance (permeability or thermal conductivity), cross-coupling coefficients, and electrical resistivity. Some practice is also required to assign appropriate boundary conditions and source parameters and to determine if the computed solution is physically reasonable. This manual will guide the user through several examples so that the modeling process is clear.

5. The manual is divided into three sections in addition to Appendix A. The first section briefly develops the field equations and describes the method of solution. It should provide the user with a basic understanding of the workings of the program. More complete treatments of these topics are given in Sill (1983) and Marshall and Madden (1959). The second section describes the input-output operations for running the program and provides some details on the actual code. A sample problem is used to show how a
physical model may be translated into the appropriate input file and how the output can be examined to see if it is producing a physically reasonable result. The final section of the manual presents a field example case for the modeling of SP associated with a leaky dam. This section should provide the user with a glimpse at the power and limitations of this code. Appendix A contains some values for permeability, thermal conductivity, thermal and hydraulic cross-coupling coefficients and electrical resistivity of earth materials that might be reasonably encountered in field surveys. The tables are brief because, for many rocks, some of these physical property values are not well known.

6. Throughout this manual it is assumed that the user has a working knowledge of the operation of an IBM PC compatible computer. The program requires the user to modify and store input files and to retrieve and print or plot output files. Included with this manual is a 5-1/4-in.* disc containing five files. These include source and executable code, and sample input, output and plotting files. We strongly recommend that you make a copy of this disc when you first begin to use the program and use the original disc as a back-up. There are no restrictions on modifying the program.

* A table of factors for converting non-SI to SI (metric) units of measurement is presented on page 3.
PART II: THEORETICAL BACKGROUND

General Principles

7. The basic principle relating self potential and fluid or heat flow processes is that the flow of electrical current and heat or fluid are coupled. That is, there are electrical potentials $\phi$ related to fluid or heat flow processes and hydraulic or thermal $P$ associated with electrical current flow. The mathematical relationship between the potentials is given in Equations (1), (2) (Nourbehecht 1963; Marshall and Madden 1959).

\[
Q = -C_{11}VP - C_{12}V\phi
\]

\[
J = -C_{21}VP - C_{22}V\phi
\]

where

- $Q$ and $J$ are primary (heat or fluid) flow and current flow vectors, respectively.
- $C_{11} = K$, the hydraulic or thermal conductivity.
- $C_{22} = \sigma$, the electrical conductivity.
- $C_{12} = C_{21}$, the cross-coupling coefficients.

8. The cross-coupling terms in Equations (1), (2) are typically much smaller than the primary flow terms. Fluid or heat flow processes do not generate very large electrical currents and imposition of electrical potential does not generate large fluid flows (although this method which is called electroosmosis has been used for dewatering low permeability material). Note that we neglect the cross-coupling terms in Equations (1), (2) the equations decouple into the more familiar Darcy's law and Ohm's law.

9. In the absence of external electrical current sources we note that the current is divergenceless

\[
\nabla \cdot J = \frac{dq}{dt} = 0
\]

where $q$ is electrical charge. Applying this relation to Equation 2 in addition to some algebra, we obtain
\[- \nabla \times (\sigma \nabla \phi) = \nabla C_{21} \times \nabla \phi + C_{21} \nabla^2 \phi \quad (4)\]

10. This is the classical dc conduction equation with electrical potential and conductivity on the left-hand side and the induced current sources on the right-hand side. Equation (4) shows that induced current sources occur if (a) the cross-coupling coefficient \( C_{21} \) changes in the direction of primary flow and/or (b) there are primary flow sources \( (\nabla^2 \phi \neq 0) \).

11. Because the effects of secondary electrical potential on fluid or heat flow are small, the pressure (or temperature) distribution may be obtained by a straightforward application of Darcy's law or the heat conduction equation. This primary potential (pressure or temperature) is then used in combination with the coupling coefficient distribution to calculate the electrical source terms on the right-hand side of Equation (4). Once the source terms are known, the electrical potential may be calculated from Equation (4) using numerical methods.

**Numerical Modeling Scheme**

12. Program SPPC uses a transmission surface analogy to solve Equation (4). The transmission surface approach draws an analogy between the physical process to be modeled and a power transmission network (Swift 1967; Madden 1971). With this approach the model space is transformed into a lumped element network and the terms of Equation (4), for example, are replaced by impedances and admittances at each node. By imposing Kirchoff's circuit law and the continuity of current at each node of the network, we end up with an array of finite difference equations.

13. Because the program uses a three-dimensional source function and two-dimensional physical property distributions, the equations must first be Fourier transformed to eliminate the three-dimensional source dependence (Swift 1967; Sill 1983). The two-dimensional finite difference equations (in Fourier domain) are then solved implicitly using Greenfield's algorithm (Swift 1967). Solutions are obtained at several Fourier wavenumbers and the solution in three-dimensional space is obtained by inverse Fourier transform.
PART III: PROGRAM OPERATIONS

File Preparation and a Sample Problem

Problem description

14. In this section, program SPPC is used to calculate the self potential anomaly for a simple problem where an analytical solution is available. This provides the user with an example of how the model parameters are translated into a computer input deck. Comparing the analytical and numerical solutions for this problem also provides a validation of the computer code.

15. The problem considered is a point pressure source in a homogeneous halfspace. For the case the input deck is completely described and the model parameters are matched one by one to the program variables in the input deck. The output is then compared with the analytical results.

16. Figure 1 schematically illustrates the problem of a point pressure source in a homogeneous halfspace. This may correspond to calculating the SP anomaly around an injection well, for example. An analytical solution of this

![Diagram of a pressure source in a homogeneous halfspace](image.png)

**Figure 1.** Schematic model of a pressure source in a homogeneous halfspace
problem is easily obtained by placing an image source above the surface at a height equal to the depth of the true source. The superposition of the true source and the image source satisfies the governing equation (Darcy's law) and also matches the boundary condition at the surface (no vertical flow). The derivation below follows the treatment of Sill and Johng (1981).

**Analytical solution**

17. The pressure distribution may be obtained by applying Darcy's law to the true source and the image source

\[ Q = \frac{kA}{\mu} \nabla P \]  

(5)

where

- \( Q \) = flow rate
- \( k \) = intrinsic permeability,*
- \( A \) = cross-sectional area
- \( \mu \) = fluid viscosity
- \( P \) = pressure

For a single point source in a homogeneous earth, Equation (5) may be solved for the pressure to give

\[ P(r) = \frac{Q \mu}{4\pi r k} \]  

(6)

where \( r \) is the distance from the source to the observation point.

18. For a homogeneous halfspace, the pressure from the image well must also be calculated.

\[ P(r,r') = \frac{Q \mu}{4\pi r k} + \frac{Q \mu}{4\pi r' k} \]  

(7)

---

* The hydraulic conductivity \( K \) is related to the intrinsic permeability \( k \) by \( K = k g / \mu \), where \( g \) is the acceleration of gravity and \( \sigma \) and \( \mu \) are the density and viscosity, respectively, of the fluid.
At the surface \( r \) and \( r' \) are equal in magnitude and, therefore, the solution at the surface is

\[
P(x, z = 0) = \frac{Q\mu}{2\pi k(x^2 + z_0^2)^{1/2}}
\]  

(8)

where

\[ x = \text{horizontal distance from the source to the observation point} \]
\[ Z_0 = \text{source depth} \]
\[ r = r' = (x^2 + Z_0^2)^{1/2} \]

19. The electrical potential at the surface follows from Equation (2). This also has a simple form

\[
\phi(x, z = 0) = -\frac{\rho C_0 Q\mu}{2\pi k(x^2 + Z_0^2)^{1/2}}
\]  

(9)

where

\[ \rho = \text{resistivity of the halfspace} \]
\[ C_0 = \text{cross-coupling coefficient} \]

20. One difficulty in SP calculations is maintaining consistency in the units. Because fluid flow units are usually expressed in the cgs system (i.e., darcies and centipoise for \( k \) and \( \mu \)) and electrical units are most often given in the MKS system (i.e., ohm-metres and volts for \( \rho \) and \( V \)) unit conversions are needed to obtain results in one system or the other. Program SPPC utilizes the standard set of units for fluid or heat flow and electrical parameters and makes unit conversions within the program. The units for the program input are given in Table 1. Remember that all input values must be given in these units or the solution is invalid.

Program input and output

21. Figure 2 illustrates the input deck for program SPPC for the problem of a point fluid source of 1 l/s in a halfspace at a depth of 22.5 m. The halfspace parameters are given as follows: \( \rho = 10 \text{ ohm-m}, C_0 = \text{mv/atm}, k = 10.0 \text{ millidarcies}. \) These are typical values for unconsolidated sedimentary rocks (Appendix A). The section below describes the input parameters in detail.
Table 1
Units in Program SPPC

<table>
<thead>
<tr>
<th>Units for Sources</th>
<th>Fluid flow</th>
<th>liters/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heat flow</td>
<td>watt</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Units of Physical Properties</th>
<th>Permeability*</th>
<th>darcies (1 darcy = $9.87 \times 10^{-9}$ cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-coupling coefficient (fluid)</td>
<td>mmv/atmosphere</td>
<td></td>
</tr>
<tr>
<td>Cross-coupling coefficient (thermal)</td>
<td>mv/°C</td>
<td></td>
</tr>
<tr>
<td>Resistivity</td>
<td>ohm-m</td>
<td></td>
</tr>
</tbody>
</table>

Calculated Data

<table>
<thead>
<tr>
<th>Pressure</th>
<th>atmosphere (1 atm $= 1.013 \times 10^5$ Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>Electrical potential</td>
<td>volt</td>
</tr>
</tbody>
</table>

* For water at 20°C, 1 darcy (permeability unit) is equivalent to $9.613 \times 10^{-4}$ cm/s (hydraulic conductivity unit).

HALFSPACE
1 1 2
10.0
1
26
6
1.0
0

PHYSICAL PROPERTIES
5
1 10.0E-3 20.0 10.0
2 10.0E-3 20.0 10.0
3 10.0E-3 20.0 10.0
4 10.0E-3 20.0 10.0
5 10.0E-3 20.0 10.0

5 10 15 20 25 30 35 40 45 50

Figure 2. Sample SPPC input file for the case of a point pressure source in a homogeneous halfspace
Input deck

22. Program SPPC uses a mesh-type input which is common to many numerical modeling programs. It offers great flexibility but requires some adjustment if you are unfamiliar with this type of input. With this program the locations of all source and measurement points are referred to coordinates in the input mesh. You must therefore keep track of the "true" locations by using a distance scale factor. For example, if the scale factor is 100 m, then each "unit" within the mesh represents 100 m on or within the earth. A scale "unit" corresponds to 4 mesh points in the X direction and between 0.5 and 0.4 points in the Z direction depending on the depth.

23. The program works by assigning each individual mesh coordinate a set of physical properties: permeability of thermal conductivity, cross-coupling coefficient, and electrical resistivity. It assumes that these parameters are constant within the mesh block. The model therefore consists of a two-dimensional array of these mesh-blocks. If the blocks are small enough, then the discontinuous model will approximate the more continuous earth, but keep in mind that the voltages, temperature, and pressures are known only at these mesh points.

24. The input is format free, that is input values do not have to appear in specific columns in the deck. Comments appearing in the input deck are preceded by!. Note that the two lines giving the linear scale must be included in the input deck as the program expects to see them. The individual input variables are described below:
LINE 1 : TITLE
Profile title (15 characters max)
This input consists of alphanumeric characters corresponding to the title of the profile. The program
will ignore characters beyond the maximum limit of 15.

LINE 2 : ITYPE, Inline, LLENGTH
Type flag, line source flag, line length
ITYPE=1 for pressures sources, =2 for thermal sources. Values other than 1 or 2 are reset by the pro-
gram to 1 (pressure sources).
ILINE=0 Calculate for point sources only, =1 Calculate line sources also.
LLENGTH = Length of the line source in model units; the source is centered around Y=0. The source
is uniformly distributed throughout the line. NOTE: The output for a line source is given in cross-
section form at the end of the file. The cross-sections and areal plots at the front of the output refer to
point sources only.

LINE 3 : ASCALE
Length in meters of one model unit
Note that 1 model unit corresponds to 4 mesh points in the X direction. In the Z direction the mesh
spacing depends on the Z coordinate. For the default spacing, given in Figure 2, the first 3 Z coordi-
nates are spaced .25 units the next 4 are spaced .5 units, the next 3 are spaced 1 unit and the final spac-
ing is 2 units.

LINE 4 : NT
Number of sources (pressure or thermal)
The maximum number of sources allowed is 9.

LINE 5 : NTX(I), I=1, NT
X coordinate of each source
The X location of each source is given as a coordinate in the mesh (see sample deck in Figure 2).

LINE 6 : NTZ(I), I=1, NT
Z coordinate of each source
The Z location of each source is given as a coordinate in the mesh (see sample deck in Figure 2).

LINE 7 : TS(T), I=1, NT
Strengths of the various sources
Source strengths for pressure sources are given in liters per sec. (Note that one liter per sec is roughly equal to 16 gallons per minute). The source strength for thermal sources is given in watts.

LINE 8: IZ
Flag for changing Z mesh
If IZ=1 then a new mesh is input for Z, if IZ=0 then the default mesh is used. The default mesh is shown in Figures 2 and 3.

LINE 9: DELV(I)J=1,11
New mesh spacing for Z
If IZ=1 then input 11 new Z mesh spacings. Spacings do not need to increase with depth but the final spacings need to be large to insure an accurate solution.

LINE 10: NU
Number of individual model units
The maximum allowable number of units is 9. In the example shown in Figure 2 NU=5.

LINE 11,11+NU:1,PERM(I),CCP(I),RES(I)
Physical property values
PERM(I) permeability, CCP(I) cross-coupling coefficient and, RES(I) resistivity of the various model units.
PERM(I), is the permeability, in darcies, or thermal conductivity in mwatts/ m-deg C.
CCP(I), is the cross-coupling coefficient of the various units in mwatm or mvolt/ deg C.
RES(I), is the resistivity of the various units in ohm-meters.
In the example shown in Figure 2 all five units have the same permeability, cross-coupling coefficient and resistivity.

LINES 12+NU:33+NU
Gridded mesh
Each number corresponds to mesh unit which has the three physical property values as shown above.
This mesh is the representation of the input model, the center of the mesh (node 26) is given as the coordinate 0 in the output. In the X-direction all mesh units are spaced .25 units apart in the Z direction the mesh spacing varies from .25 units near the surface to 2 units at depth.
Running the program

25. After preparing an input deck as shown above and saving it on the disk of your PC you may run the program simply by typing "SPPC fn" where "fn" designates the name of the input file to be computed (the .dat extension is automatically provided). To run the sample problem provided simply type SPPC HALF. The program will automatically assign HALF.DAT as the input file name, HALF.RES as the output file name and HALF.PLT as the plotting file name. For this sample problem the program requires about 8 min to run on an IBM-PCAT; additional time is required if more than one source is used.

26. The plotting file contains only the model name, the distance scale factor, and the primary potential and voltage at individual mesh points on the surface. No plotting software is provided with this manual but a number of commercial plotting programs written for PC compatible microcomputers accept this type of file for input.

27. The output for the above example is shown in Figure 3. The first section of the output file contains a summary of the input parameters and a cross section of the physical parameter distribution. The results of the model calculation are then given as a profile of primary potentials and voltages at the surface and then as detailed profiles of primary potentials and voltages. The third section is an areal distribution (plan view) of the surface voltage. The final section of the output gives cross-sections of the voltages from a line source. For these sections the line sources are centered about Y=0 and the strength of the source in distributed evenly along the length of the line. The cross sections will appear only if ILINE = 1 in line 1 of the input file.

28. A comparison of the numerical model and the analytical results for points at the surface is shown in Figure 4. Examining both the pressure and voltage curves in Figure 4 we find a good agreement between numerical and theoretical results. Errors of 1 to 5 percent are observed at the farther mesh point. These are acceptable for this type of numerical modeling although they may be reduced by providing a finer mesh discretization.

29. Before accepting the numerical solution it is important to determine that the output is providing physically reasonable results. Note, for example, in the cross-sectional plots, the pressure is highest near the source location. Also notice, from the surface areal output, that the potential is symmetrical about the surface coordinate of the source point.
<table>
<thead>
<tr>
<th>LOCATION</th>
<th>PRESSURE (ATM)</th>
<th>VOLTAGE (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>215.01</td>
<td>450.02</td>
</tr>
<tr>
<td>2</td>
<td>210.01</td>
<td>452.02</td>
</tr>
<tr>
<td>3</td>
<td>205.01</td>
<td>454.02</td>
</tr>
<tr>
<td>4</td>
<td>200.01</td>
<td>456.02</td>
</tr>
<tr>
<td>5</td>
<td>195.01</td>
<td>458.02</td>
</tr>
<tr>
<td>6</td>
<td>190.01</td>
<td>460.02</td>
</tr>
<tr>
<td>7</td>
<td>185.01</td>
<td>462.02</td>
</tr>
<tr>
<td>8</td>
<td>180.01</td>
<td>464.02</td>
</tr>
</tbody>
</table>

Figure 3. SPPC output file for a point pressure source in a homogeneous halfspace (Continued)
Figure 4. Comparison of analytic and calculated results for model given in Figure 1
30. A few rules of thumb are useful in applying this program to model physical problems. First, when constructing an input mesh, it is a good practice to sketch out your model with paper and pencil before you type in the input. This allows a check on the input to see if it is consistent with the model that you want. Second, when constructing an input mesh, it is recommended that you begin at the center of the model and work outwards. The center point is located at node 26 and remember that the X nodes are spaced 0.25 units apart throughout the visible input mesh. Although the complete X mesh actually consists of 68 nodes, only the central 52 nodes are used to input models; the remaining nodes extend the mesh laterally to eliminate numerical errors. The output file provides an expanded plot of the voltage and primary potential for the central part of the mesh; therefore, it is useful to have your model centered around this region. The expanded region is located between -2 and 2 units and the voltages and primary potentials are plotted at each node (0.25 unit) in this region; whereas, outside of this area, they are plotted every 4 nodes (1 unit).

31. Modeling SP data for a thermal source is often more difficult than fluid flow modeling because the parameters are poorly constrained. Because the nature of heat sources are usually less well known than fluid flow sources, it is sometimes useful to initially assume a source strength (100,000 watts for example) and examine the temperatures in the output file to see if they are reasonable for your model. See the papers by Sill (1983) and Sill and Johng (1981) for examples of thermal source modeling.
32. The section below illustrates the use of this code for a more complex example. In this case the code is used to develop a model for a leaky dam site where a field SP survey has been conducted. This example will highlight some of the considerations in using the code to model field results.

33. Figure 5 shows a self potential anomaly map for the Beaver damsite in the state of Arkansas in the southern part of the USA (Llopis and Butler 1988). This is an earthen dam through which significant seepage occurs in the region labeled "new wet area". The seepage in the southern part of the survey area is thought to be controlled by an east-west trending fault zone that obliquely crosses the dam structure. The fault forms the southern boundary of a graben structure and is associated with a string of negative SP anomalies (Butler 1988).

34. The SP map shows some of the general characteristics of a leakage problem (Figure 5). That is, there are negative anomalies where fluid is leaking into the dam and reservoir boundary (near A) and positive anomalies over the surface discharge areas (new wet area). The association of the anomalies with the known fault zone strongly suggests that the fault provides a low impedance leakage pathway for fluids. The map also shows several other anomalies, some of which are related to other leakages and some are due to topography which can have a significant effect (Corwin 1988).

35. We selected a single profile from the map (A-A') that connects source and seep areas and fit the observed data to calculate data for a two-dimensional geometry. The permeability and resistivity distributions for the model shown in Figure 6 were obtained from field measurements (Butler 1988). Values given for the cross-coupling coefficients are "educated guesses" based on measurements reported in Ishido and Mizutani (1981) and Noubehecht (1963) (Appendix A). Figure 6 is a very simple model for the dam and only crudely matches the geometry and construction details of the dike. The model features single source and seep locations and a single coupling-coefficient contrast corresponding to where the graben fault crosses the profile line. The magnitude of the leak is about 0.5 l/sec (about 8 gal/min) and about one-half of this amount is discharging in the new wet area.

36. The input deck corresponding to the Beaver Dam model is given in Figure 7; it has several interesting features. Since water must flow out of
Figure 5. Observed SP data at the Beaver damsite in Arkansas
BEAVER DAM

Permeability: k = 100 m\text{darcies}

Coupling Coeff: C = 170 \text{mv/atm}

50 \text{mv/atm}

Seepage

Leak (z = 20 m)

p = 300

8 m

p = 40

p = 1000 \Omega \cdot \text{m}

C = 20 \text{ mv/atm}

k = 100 \text{ m\text{darcies}}

Figure 6. Model used to fit observed Beaver Dam data for profile A-A' in Figure 5

Figure 7. Input file for the Beaver Dam field example
the dam in the downstream direction, "permeability" of the water in the reservoir (property set 2) is assumed to be very low. This ensures that water from the source will not flow back into the reservoir. Also, since the dam is a topographic structure, measurements must be made within the mesh and not on the surface (see marked input in Figure 7).

37. For the Beaver Dam model 15 iterations of forward modeling were required to match field results. The fit between calculated and observed SP is shown in Figure 8. A very good match was achieved even though the assumed model is considerably simpler than the known geology. This suggests that the SP anomalies are dominantly controlled by the locations and magnitudes of the sources and seeps and not by complex flow geometry or rock-type variations. However, this is not a unique model and the parameters are not well constrained.
Figure 8. Comparison of calculated to observed SP results for the model in Figure 6
38. This paper describes the theory and operation of computer program SPPC. The program calculates SP anomalies due to fluid and heat flow processes in a two-dimensional earth. The model uses a mesh-type input and may require some practice before modeling can be done routinely; it runs on an IBM PC-AT computer in about 8 min.

39. This program is potentially a powerful tool for geoscientists and engineers using the SP method to locate areas of subsurface ground-water flow such as leaks in earthen dams, dikes, or evaporation ponds. It is also a useful tool in geothermal exploration. Future applications may include such diverse problems as nuclear repository monitoring or tracking of subsurface contaminant flow. Note that the program, in its present form, is designed for use on simple problems. Due to the memory restrictions and limited computing power on a PC, the mesh is quite small so only fairly simple models are possible. Also, the potential user is encouraged to consult the References and particularly Report 3 (Corwin 1989) for guidance on obtaining required model input parameters.
REFERENCES


APPENDIX A: PHYSICAL PARAMETERS

1. Some representative physical property values that may be used in modeling SP data are provided in Table A1. Included are values for resistivity, permeability, thermal conductivity, electrokinetic (fluid flow), and thermoelectric cross-coupling parameters. These data were compiled from a number of different sources; the values given are given as broad ranges. Values for permeability are compiled from Freeze and Cherry (1979) thermal conductivity information is obtained in Nourbehecht (1963) and Welch et al. (1981). Values for electrical resistivity are obtained from Keller and Frischknecht (1966), and Keller (1988). Cross-coupling coefficients are obtained from Nourbehecht (1963), Ishido and Mitzutani (1981) and Fitterman and Corwin (1982).

2. The effects of porosity, pore water salinity, and specific mineral composition are all important in determining these physical properties. These effects account for the wide ranges of values for these physical parameters. In general, the fluid flow coupling coefficients increase with porosity and decreases with pore water salinity. The resistivity decreases with increased porosity, pore water salinity, and temperature. The thermal coupling coefficient decreases with increased porosity and increased fracture density.

3. The parameters units are given as follows: resistivity values are given in ohm·m, thermal conductivities are in watts/m·deg C, thermal coupling coefficients (CCth) are in mv/deg C, permeabilities (k) are in millidarcies (md), and fluid coupling coefficient (CCfl) are in mvolts/atm. This table is far from complete and certainly for many applications it will prove to be inadequate. It does provide the user with a starting point for modeling a number of physical systems.

A1
Table A1
Representative Physical Property Ranges

<table>
<thead>
<tr>
<th>Rock</th>
<th>Resistivity (ohm-m)</th>
<th>Thermal Conductivity (watts/m°C)</th>
<th>Conductivity (mv/°C)</th>
<th>Permeability (md)</th>
<th>Capacitance (mv/atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sedimentary Rock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconsol</td>
<td>2-100</td>
<td>0.06-0.8</td>
<td>0.01-0.1</td>
<td>10-1000</td>
<td>30-150</td>
</tr>
<tr>
<td>Sandstone</td>
<td>5-100</td>
<td>0.10-1.5</td>
<td>0.04-0.3</td>
<td>5-100</td>
<td>15-100</td>
</tr>
<tr>
<td>Shale</td>
<td>2-50</td>
<td>0.20-2.0</td>
<td>0.05-0.5</td>
<td>0.02-5.</td>
<td>1-20</td>
</tr>
<tr>
<td>Limestone</td>
<td>20-500</td>
<td>0.15-1.5</td>
<td>0.05-0.5</td>
<td>0.01-5.</td>
<td>0.1-20</td>
</tr>
<tr>
<td>Igneous and Metamorphic Rock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Granitic</td>
<td>50-1000</td>
<td>0.50-10.0</td>
<td>0.10-2.0</td>
<td>0.001-1.</td>
<td>0.1-5.</td>
</tr>
<tr>
<td>Volcanic</td>
<td>20-500</td>
<td>1.0-15.0</td>
<td>0.10-2.0</td>
<td>0.01-10.</td>
<td>0.1-50.</td>
</tr>
<tr>
<td>Metamorphic</td>
<td>50-1000</td>
<td>0.50-10.0</td>
<td>0.10-2.0</td>
<td>0.01-10.</td>
<td>0.1-10.</td>
</tr>
<tr>
<td>Fractured</td>
<td>10-100</td>
<td>0.05-5</td>
<td>0.10-2.0</td>
<td>0.1-100.</td>
<td>0.5-100.</td>
</tr>
</tbody>
</table>