Research accomplishments were registered in three areas. The first is lifetime estimation with truncated data. Maximum likelihood estimators and their asymptotic properties were studied. A Monte Carlo comparison of several estimators was performed when only interval data on lifetimes is available. The second area is under what conditions a "burn-in" was developed and tested using sequential estimation. Thirdly, distributions other than Poisson were tried, and it was found that in some cases a binomial approximation provides a more robust testing scheme for series systems with aging components.
Theory System Reliability
Demonstration and Burn-In Design

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I. ESTIMATION WITH TRUNCATED DATA


We are given \(N\) random variables \(X_1, \ldots, X_N\) with common density \(f(x; \theta)\) where both \(N\) and \(\theta\) are unknown. The values of the \(X\)'s can be observed only if they lie in the interval \((0, T)\). The problem addressed is estimation of \(N\) and \(\theta\). Maximum likelihood estimators and variants of these, namely Bayes modal estimators are derived and asymptotic properties are studied.

In [1], an additional restriction is imposed, namely that the interval \((0, T)\) is subdivided and only the number of \(X\)'s falling in each subinterval is reported, with all other details about actual values being lost. General techniques to analyze data in this form are developed, and restrictions on the estimation procedures are derived to assure the stochastic consistency of the estimator of \(N\). A widely used iterative computing scheme is shown to fail the consistency conditions and several alternative schemes which are consistent are examined. An analysis of a software reliability data set is carried out, and Monte Carlo comparison of several estimators is performed, leading to one of them being recommended for use.

In [2], instead of grouping at fixed time points and observability on an interval of length \(T\), it is assumed that only a fixed subset of the quantiles of the \(X\)'s can be
observed, the largest one being the \((n/N)^{th}\), with \(n < N\). Problems of the type considered in [1] are examined for this alternative model.

The paper [3] is a review of recent work on all aspects of estimation with truncated data, but concentrates on applications of this work to estimating the number of undetected errors remain in computer software after an initial testing period. Both the papers in the software reliability literature which lead to truncated data models, and the papers discussing the statistical analysis of these models are reviewed.

II. BURN-IN.


Closely related to the problem described in Part I is the burn-in problem. In its simplest form, a batch of \(M\) items contains \(N\) (unknown) defectives with known failure distribution. A sequential stopping rule is desired for removing enough of the defectives to assure that with a specified high probability the reliability of the remaining items in the lot after burn-in exceeds some given lower bound. Various stopping rules have been studied for both exact and approximate (large lot size) properties. The technical report [1] is based on the dissertation of Dr. Pan. In [2], a procedure for burn-in based on a sequential estimation scheme published previously by the proposer is extracted from [1], additional computations of its small lot size properties are made and some additional analytic properties are developed. This report is currently being readied for submission for possible publication. In [3], the characterization of the fixed burn-in time procedure which achieves the after burn-in reliability goal is extracted from [1]. This fixed time deper is on knowing more than is generally available, but serves as a benchmark to which to compare the sequential schemes to see how much extra time they take because the extra information is not available. This too, will soon be ready for submission to a journal.
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III. RELIABILITY DEMONSTRATION.


Series systems exhibit wear out if the components tend to wear out as they age. Acceptance sampling criteria are usually stated in terms of Mean Time Between Failures (MTBF) for an aged (or equilibrium) system. However, the acceptance test is often administered to a newly produced system. If the test does not take into account the fact that the new system has a much larger MTBF than it will have after aging, too high a proportion of poor quality systems will be accepted. In [1], we have developed tests which take this aging phenomenon into account. They are based on a Poisson approximation for the distribution of the number of failures a new system will have in a fixed time period, and on the assumption that the component failure distribution is known except for the value of a scale parameter. We have also looked into the robustness of the O.C. curves of these procedures in case the failure distribution is a gamma or Weibull with unknown shape parameter.

In [2], we have studied alternatives to the Poisson approximation and found that for moderate size systems, a binomial approximation fits the true distribution more closely than the Poisson or any of several other possible approximations. We are now looking at test design using the binomial.

In [3], we are studying a two stage testing procedure for gamma failure densities which develops an estimate of the shape parameter based on the first stage data, and then uses that estimate to develop an appropriate test time and acceptance criterion for the second stage so that a desired O.C. can be achieved.