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R.J. Carroll

Department of Statistics
University of North Carolina
Chapel Hill, North Carolina 27514
The material on pages 1 to 20 describes the research effort during the period 1 February 1979 through 31 January 1980, and the cumulative list of publications on pages 21 to 26 serves as a final report.

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Cumulative List of Publications Credited to this Grant

(A) Journal Publications

(B) Institute of Statistics Mimeo Series Technical Reports

(C) Manuscripts Submitted for Publication

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S. Cambanis: PROBLEMS IN STATISTICAL COMMUNICATION THEORY AND THE ANALYSIS OF STOCHASTIC SIGNALS AND SYSTEMS.

Research completed includes consistent estimation of (nonrandom) signals from nonlinear transformations of noisy samples; approximations of non-bandlimited signals using a finite number of samples and their rate of convergence; (infinite) sampling approximations for non-bandlimited signals, and sampling representations for bounded linear operations on bandlimited signals and for generalized bandlimited signals; the evaluation of linear estimates and regression estimates in stable processes, including regression and linear filtering of signals in noise; and certain probability and expectation inequalities.

R.J. Carroll: ROBUST ESTIMATION AND SEQUENTIAL ANALYSIS

Research completed includes work in the following areas: trimming least squares estimators in the linear model by using a preliminary estimator; tests for heteroscedasticity in the linear model; estimation of regression coefficients in a heteroscedastic linear model; almost sure properties of robust regression estimates with applications to sequential clinical trials; robust methods in factorial experiments; studying sequential procedures for estimating the largest of three normal means; non-parametric estimation of regression functions.
The work briefly described here was developed in connection with problems arising from and related to the statistical theory of communications and signal processing. Item 1 describes continuing joint work with Dr. F. Masry of the University of California at San Diego. Item 2 is a discussion of recent work on nonlinear systems with Gaussian inputs, leading to several open problems. Item 4 is the Ph.D. dissertation of Dr. M.K. Habib written under my direction, and Item 3 is the first of a series of papers which will be written from this work. Item 5 represents further joint work with Dr. G. Miller of the U.S. Army Material Systems Analysis Activity at Aberdeen Proving Grounds, and is included in an extensive expansion of earlier work. Item 6 describes joint work with my colleague, Dr. G. Simons, (about half of this work was completed during the previous funding year and reported in the last report). Finally, my student, Mr. N. Gerr, has been working in the area of Delta modulation of random signals and the complete work will be reported in the next annual report.

1. Consistent Estimation of Continuous-Time Signals from Nonlinear Transformations of Noisy Samples [1].

In general, a signal cannot be reconstructed from its sign, i.e., from its hardlimited version. However, by deliberately adding noise to samples of the signal prior to hardlimiting, it is shown that the signal can be estimated consistently from its hardlimited noisy samples as the sampling rate tends to infinity. In fact, such estimates are shown to converge with probability one to the signal and also, to be asymptotically normal. The estimates, which are generally nonlinear, can be made linear by a proper choice of the noise distribution. These rather unexpected results hold
for all bounded and uniformly continuous signals. In addition to the hardlimiter, such results are also established for certain monotonic and non-monotonic nonlinearities.

2. **Nonlinear Systems with Gaussian Inputs: Representation, Identification and Inverse Problems** [2].

 Certain recent results leading to several open problems are discussed for the following questions for nonlinear systems with input Gaussian processes. How can the system be represented? How can the system be identified from the input and output processes? Does knowledge of the way the system responds to a certain Gaussian input determine the way it will respond to another Gaussian input or to a deterministic input? Does knowledge of the system and the statistics of the output determine the statistics of the input?

3. **Finite Sampling Approximations for Non-Bandlimited Signals** [3].

 Finite sampling approximations, and their rate of convergence, are derived for certain deterministic and random signals which are not bandlimited. The merit of these results lies in the fact that in many practical situations, the signals under consideration are not bandlimited and only a finite number of samples are available.

4. **Sampling Representations and Approximations for Certain Functions and Stochastic Processes** [4]

 Sampling representations and approximations of deterministic and random signals are important in communication and information theory. Finite and infinite sampling approximations are derived for certain functions and processes which are not bandlimited, as well as bounds on the approximation errors. Also derived are sampling representations for bounded linear
operators acting on certain classes of bandlimited functions and processes; these representations enable one to reconstruct the image of a function (or a process) under a bounded linear operator using the samples of the function (or the process) rather than the samples of the image. Finally, sampling representations for distributions and random distributions are obtained.

5. **Regression and Linear Estimation in Stable Processes** [5].

Regression estimates and linear estimates, along with the estimation error, are evaluated for certain classes of stable processes. Included are evaluations of regression and linear filtering of signal in noise, and also noncausal linear prediction and filtering of harmonizable stable processes.

6. **Probability and Expectation Inequalities** [6].

A mathematical framework is introduced within which a wide variety of known and new inequalities can be viewed from a common perspective. Probability and expectation inequalities of the following types are considered: (a) $P(Z \in A) \geq P(Z' \in A)$ for some class of sets $A$, (b) $E_k(Z) \geq E_k(Z')$ for some class of pairs of functions $l$ and $k$. It is shown, sometimes using explicit constructions of $Z$ and $Z'$, that, in several cases, (a) $\iff$ (b) $\iff$ (c); included here are cases of normal and elliptically contoured distribution. A case where (a) $\implies$ (b) $\iff$ (c) is studied and is expressed in terms of "$n$-monotone" functions for (some of) which integral representations are obtained. Also, necessary and sufficient conditions for (c) are given.
References


3. S. Cambanis and M. Habib, Finite sampling approximations for non-bandlimited signals, manuscript submitted for publication.


The work described here was developed in connection with problems arising in robust estimation and sequential analysis. The goal of robust estimation is to develop estimates which are insensitive to realistic departures from model assumptions (such as outliers); many of the commonly used procedures do not possess this property. Our special focus this year has been on the linear model. We have considered a theoretical description of the applied statistician's common practice of fitting an equation and then eliminating points which could be outliers, and we have shown that such a practice can only rarely be recommended. We have also provided strong theoretical results which enable robust M-estimators to be applied in many sequential situations, including clinical trials and fixed width confidence intervals. We have also obtained results in testing for and estimations in the heteroscedastic linear model; for the latter we have obtained a practical methodology which is essentially as good as optimal (but unknowable) weighted least squares. Finally, we have obtained results for nonparametric regression curve fitting and the sequential selection of the largest mean.

1. Robust Estimation in the Linear Model.

a) Trimming the least squares estimator in the linear model by using a preliminary estimator [1].

Let \( \hat{\beta}_0 \) be an estimate of \( \beta \) in the linear model, \( Y_i = x_i^T \beta + \epsilon_i \). Define the residuals \( Y_i - x_i^T \hat{\beta}_0 \), let \( 0 < \alpha < 1 \), and let \( \hat{\beta}_L \) be the least squares estimate of \( \beta \) calculated after removing the observations with the \( [\alpha n] \) smallest and \( [\alpha n] \) largest residuals. This is a common practice of applied statisticians worried about possible outliers. By use of an asymptotic expansion, the limit distribution of \( \hat{\beta}_L \) is found under certain
regularity conditions. This distribution depends heavily upon the choice of \( \hat{\theta}_0 \). We discuss several choices of \( \hat{\theta}_0 \), with special attention to the contaminated normal model. If \( \hat{\theta}_0 \) is the median regression or least squares estimator then \( \hat{\theta}_0 \) is rather inefficient at the normal model. If \( F \) is symmetric, then a particularly convenient, robust choice is to let \( \hat{\theta}_0 \) equal the average of the \( \alpha \)-th and \((1-\alpha)\)-th regression quantiles (Koenker and Bassett, Econometrica (1978)). Then \( \hat{\theta}_0 \) has a limit distribution analogous to the trimmed mean in the location model, and the covariance matrix of \( \hat{\theta}_0 \) is easily estimated.

b) **Almost sure properties of robust regression estimates** [3]

We consider Huber's Proposal 2 for robust regression estimates in the general linear model. The estimates are first shown to be strongly consistent. We then develop an almost sure expansion of these estimates, approximating them (to order \( o(n^{-1}) \)) by a weighted sum of bounded random variables. The approximation is sufficiently strong to permit construction of sequential fixed-width confidence regions for the regression parameter, as well as for use in sequential clinical trials with repeated significance tests.

c) **On the asymptotic normality of robust regression estimates** [5]

Huber's 1973, *Annals of Statistics* proof of asymptotic normality of robust regression estimates is modified to include the estimates used in practice, which have unknown scale and only piecewise smooth defining functions \( \psi \).

d) **Robust methods for factorial experiments with outliers** [7]

Two factorial experiments with possible outliers (Joh (1978), *Applied Statistics*) are reanalyzed by means of robust regression techniques using M-estimates (Huber 1973, *Annals of Statistics*). We find that
M-estimates provide an easy and efficient methodology for experiments with possible outliers.

2. Robust Estimation in the Heteroscedastic Linear Model.

a) On Bickel's tests for heteroscedasticity [2]

The asymptotic distribution theory of Bickel's (1978) tests for heteroscedasticity is extended to a wider class of test statistics and distributions, when the number of regression parameters is fixed.

b) On robust tests for heteroscedasticity [4]

We extend Bickel's (1978) tests for heteroscedasticity to include wider classes of tests statistics and fitting methods. The test statistics include those based on Huber's function, while the fitting techniques include Huber's Proposal 2 (1977) for robust regression.

c) M-estimates for the heteroscedastic linear model [6]

We treat the linear model \( Y_i = C_i^T \beta + Z_i \) where \( C_i \) is a known vector, \( \beta \) is an unknown parameter, and \( \text{Var} Z_i \) is a function of \( |C_i^T \beta| \) which is known, except for a parameter \( C \). For simultaneous M-estimates, \( \hat{\beta} \) and \( \hat{\sigma} \), we show that \( (\hat{\beta} - \beta) \sim \mathcal{N}(0, \mathbf{V}) \), and find the limit distribution of \( \mathcal{N}(\hat{\beta} - \beta) \). For the special case of least squares estimation, this limit distribution is the same as the limit distribution of the weighted least squares using the weights, \( w_i = (\text{Var} Z_i)^{-1} \), and in general the distribution is that of a "weighted M-estimate" using these weights. Moreover, the covariance matrix of the limit distribution can be consistently estimated, so large sample confidence ellipsoids and tests of hypotheses concerning \( \beta \) are feasible.
3. Nonparametric Regression (Written by G. Johnston under the direction of R.J. Carroll.)

   a) Nonparametric estimation of a regression function by means of concomitants of order statistics [8]

   We study the properties of nonparametric estimates of a regression function based on concomitants of order statistics (Yang (1977), unpublished) and use results of Johnston's thesis to obtain uniform confidence bounds for her estimates.

   b) Smooth nonparametric regression analysis [9]

   We consider nonparametric estimation of the regression function $E[Y|X=x]$. For the Watson estimators (1964, Sankhya, Series A) we obtain nonparametric confidence bounds, and apply them to simulated data.


   We study the problem of estimating the largest mean from three populations when data are normally distributed. A Monte-Carlo study is performed to compare two sequential procedures, one of which eliminates populations during the experiment, while the other does not. The elimination procedure is shown to be preferable.

References


Other Activities

The Department was fortunate to have Drs. Keener and Lee visiting during the academic year 1979-80 and making substantial research contributions in the areas of U-statistics and sequential design of experiments and renewal theory.

Dr. Robert W. Keener is a Visiting Assistant Professor and received his Ph.D. degree in 1979 from the Massachusetts Institute of Technology under the direction of H. Chernoff. Dr. Alan J. Lee of the University of Auckland, New Zealand is a Visiting Associate Professor, and received his Ph.D. degree in 1973 from the University of North Carolina at Chapel Hill under the direction of S. Cambanis. Following are brief summaries of manuscripts completed by Drs. Keener and Lee while in Chapel Hill.

Dr. Robert W. Keener

1. **Renewal Theory and the Sequential Design of Experiments with Two States of Nature** [1].

   In the sequential design of experiments an experimenter is sequentially performing experiments to help him make an inference about the true state of nature. Using results from renewal theory, we derive approximations for the operating characteristics and average sample numbers for this problem when there are two states of nature. A critical problem in the sequential design of experiments is finding a good procedure. We investigate a Bayesian formulation of this problem and use our approximations to approximate the Bayes risk. Minimization of this approximate Bayes risk over procedures is discussed as a method of finding a good procedure, but difficulties are encountered due to the discrete time character of the sequential process. To avoid these difficulties, we consider minimization of an approximation related to a diffusion process. This leads to a simple rule for the sequential selection of experiments.

Standard theory is concerned with expectations related to sums of positive i.i.d. variables, \( S_n = \sum_{i=1}^{n} Z_i \). We generalize this theory to the case where \( \{S_i\} \) is a Markov chain on the real line with stationary transition probabilities satisfying a drift condition. The expectations we are concerned with satisfy generalized renewal equations, and in our main theorems, we show that these expectations are the unique solutions of the equations they satisfy.

3. Maximum Likelihood Regression of Rank-Censored Data [3].

Linear regression is a common method for analyzing cardinal data, but is inappropriate when the dependent variable is an ordinal ranking. The model we propose for analyzing these data sets is the general linear model \( u^* = \mathbf{X} \beta + \varepsilon \) where the rank of the dependent variable \( u^* \) is observed instead of its value. We describe a numerical algorithm for evaluating the likelihood function which is efficient enough to permit maximum likelihood estimation of normalized regression coefficients. This algorithm can be modified to evaluate the cumulative distribution function of any multivariate normal random vector with nonsingular tri-diagonal covariance matrix.

References


1. **A Note on the Campbell Sampling Theorem [1]**.

   Campbell's 1968 sampling theorem is examined and a more explicit formula for the truncation error is given. The result is shown to apply to random processes bandlimited in a general sense.

2. **On the Asymptotic Distribution of U-Statistics [2]**.

   The asymptotic distribution of a U-statistic is found in the case when the corresponding Von Mises functional is stationary of order 1. Practical methods for the tabulation of the limit distributions are discussed, and the results extended to certain incomplete U-statistics.

3. **On the Asymptotic Distribution of Certain Incomplete U-Statistics [3]**.

   Incomplete U-statistics based on the arithmetic mean of $m$ quantities $g_i$ where $g$ is the kernel of the complete U-statistic evaluated at a randomly chosen subsample of a sample of size $n$, are considered. The asymptotic distribution of the incomplete U-statistic is obtained in terms of that of the complete statistic, and the asymptotic relative efficiency of the incomplete statistic discussed. Comparisons are made with other incomplete U-statistics.

4. **On Incomplete U-statistics having Minimum Variance [4]**.

   Let $U$ be an incomplete U-statistic of order $k$, that is to say, an arithmetic mean of $m$ quantities $g(x_{i_1}, \ldots, x_{i_k})$ where $x_1, \ldots, x_n$ is a random sample from some distribution, $g$ is a function symmetric in its $k$ argu-
ments, and the sum is taken over m k-subsets \((i_1, \ldots, i_k)\) of \((1, \ldots, n)\).

The problem of how to choose the m subsets to make the variance of \(U\) a minimum is discussed. Some results on the asymptotic properties of \(U\) are given.

References


4. On incomplete U-statistics having minimum variance, manuscript under preparation.


INSTITUTE OF STATISTICS MIMB0 SHRI'S TECHNICAL REPORTS

1 February 1979 to Date

1220 D. Ruppert and R.J. Carroll, Trimming the least squares estimator in the linear model by using a preliminary estimator, April 1979.


1254 R.J. Carroll and R.A. Smith, A study of sequential procedures for estimating the largest of three normal means.


1271 G. Johnston, Probabilities of maximal deviations for nonparametric regression function estimates, August 1979.

LIST OF PROFESSIONAL PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT:

1. Faculty Investigators:  S. Cambanis
   R.J. Carroll

2. Dr. A.J. Lee of the University of Auckland (partially supported while spending the academic year 1979-80 as Visiting Associate Professor).

   Dr. Robert Keener (partially supported while spending the academic year as a Visiting Assistant Professor).

3. Graduate Students:  R. Frimmel
   P. Gallo
   N. Gerr
   M. Habib

DEGREES AWARDED:


S. Cambanis organized a session on Stochastic Processes, chaired another session, and gave an invited talk on "Inverse problems for nonlinear transformations of Gaussian processes," at the meeting of the Institute of Mathematical Statistics in East Lansing, Michigan, June 18-20, 1979.

S. Cambanis attended the International Symposium on Information Theory, Grigniano, Italy, June 25-29, 1979; chaired a session; and presented the following two talks: "p-th order and stable processes: Linear structure and path properties" (joint work with G. Miller), and "Signal identification after noisy nonlinear transformations," (joint work with E. Masry).

S. Cambanis gave an invited talk on "Consistent estimation of continuous-time signals from quantized noisy samples" (joint work with E. Masry) at the International Conference on Information Sciences and Systems, in Patras, Greece, July 9-13, 1979.

S. Cambanis gave an invited talk on "Nonlinear systems with Gaussian inputs: Representation, identification and inverse problems," at the 18th IEEE Conference on Decision and Control, in Fort Lauderdale, Florida, December 12-14, 1979.

R.J. Carroll presented the paper "M-estimates for the heteroscedastic linear model" to

(i) Institute of Statistics Special Topics Meeting, October 1979;
(ii) National Institutes of Health, November 1979;
(iii) General Motors Research Labs, December 1979;
(iv) University of Maryland, January 1980.
CUMULATIVE LIST OF PUBLICATIONS CREDITED TO THIS GRANT

(A) JOURNAL PUBLICATIONS

1976


1977


H.M. Leung and S. Cambanis, On the rate distortion function of a memoryless Gaussian vector source whose components have fixed variances, Information and Control 34 (1977), 198-209.

1978


R.J. Carroll, Sequential confidence intervals for the mean of a subpopulation of a finite population, J. Amer. Statist. Assoc. 73 (1978), 408-413.


1979

S. Cambanis, Nonlinear systems with Gaussian inputs: Representation, identification and inverse problems, Proceedings 18th IEEE Conference on Decision and Control (1979), 1035-1037.

(1979)


1980


Accepted for Publication

S. Cambanis and H.M. Leung, On the rate distortion functions of memoryless sources under a magnitude error criterion, Information and Control.

S. Cambanis and E. Masry, On the reconstruction of stationary Gaussian processes observed through zero-memory nonlinearity - Part II, IEEE Trans. Information Theory.


R.J. Carroll, On sequential elimination procedures, Sankhya, Series A.

R.J. Carroll, Robust transformation to achieve approximate normality, J. Roya Statist. Society, Series B.

R.J. Carroll, Robust methods for factorial designs with outliers, Applied Sta.

R.J. Carroll and D. Ruppert, Trimming the least squares estimator in the linear model by using a preliminary estimator. J. Amer. Statist. Assoc.


R.W. Keener, Renewal theory and the sequential design of experiments with two states of nature, Communications in Statistics.


(B) INSTITUTE OF STATISTICS Mimeo Series Technical Reports

(Excluding those reports which have been published in journals or accepted for publication, and including Ph.D. dissertations, M.S. essays, and recent reports which have been submitted for publication.)

1975

1028 S.T. Huang, Nonlinear analysis of spherically invariant processes and its ramifications (Ph.D. dissertation under the direction of S. Cambanis), September 1975.

1976

1057 H.M. Leung, Bounds and the evaluation of rate distortion functions (Ph.D. dissertation under the direction of S. Cambanis), February 1976.

1060 C. Segami, Power series distributions, a dual class and some extensions (Ph.D. dissertation under the direction of N.L. Johnson and G. Simons), April 1976.

1079 W.L. Lane, A survey of sequential approaches to the problem of selecting the best of k populations (M.S. essay under the direction of R.J. Carroll), July 1976.


1977

1121 G. Miller, Some results on symmetric stable distributions and processes (Ph.D. dissertation under the direction of S. Cambanis), May 1977.

1123 R. Fondren, A study of power transforms (M.S. essay under the direction of R. J. Carroll), May 1977.

1978


1189 R.J. Carroll, A study of sequential procedures for estimating the largest mean, August 1978.

1191 R.J. Carroll, On sequential confidence intervals for the largest normal mean, August 1978.
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<tr>
<td></td>
<td>C. Schoenfelder, Random designs for estimating integrals of stochastic processes (Ph.D. dissertation under the direction of S. Cambanis), November 1978.</td>
<td>C. Schoenfelder</td>
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<tr>
<td>1979</td>
<td>S.T. Huang, Characterizations of the exponential distribution by conditional moments, March 1979.</td>
<td>S.T. Huang</td>
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<tr>
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<td>G. Johnston, Smooth nonparametric regression analysis (Ph.D. dissertation under the direction of R.J. Carroll and M.R. Leadbetter), September 1979.</td>
<td>G. Johnston</td>
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<tr>
<td></td>
<td>R.J. Carroll and R.A. Smith, A study of sequential procedures for estimating the largest of three normal means, September 1979</td>
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1271 G. Johnston, Probabilities of maximal deviations for nonparametric regression function estimates, August 1979.


(C) MANUSCRIPTS SUBMITTED FOR PUBLICATION

S. Cambanis and M.K. Habib, Finite sampling approximations for non-bandlimited signals.

R.W. Keener, Renewal theory for Markov chains on the real line.

R.W. Keener, Maximum likelihood regression of rank-censored data.