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This final report summarizes research activities in applied mathematics, including control of distributed systems, optimization, and inverse problems. The report summarizes results published in 23 journal articles. Results include a new method for computing solutions to ill-posed problems via optimal filtering and new results for the analysis and control of switching systems. (KR)

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FINAL REPORT

During the grant period the PI has been involved in a considerable variety of research investigations within the grant areas [Control and system theory, Optimization, and Ill-posed problems]. The annotated bibliography attached to this report, representing those[1] 'projects' on which the PI expended some significant amount of effort during the grant period in research, writing or revision, consists of 23 items. (At this point we note that at least half of these are directly attributable to the stimulus and interaction provided by the Principal Investigator's opportunities for travel.)

We begin by summarizing the 'status' of these 23 items: many, [1], [2], [3], [4], [5], [6], [9], [10], [20], [21] involved results previously obtained so that it was *primarily* writing (but also, in some cases, additional research) which was done during the grant period; the others all involve significant new research done during the grant period. Of these others, we note that both research and writing are now essentially complete for the papers [8], [11], [14], [12], [15], [13], [17] while research but not yet writing is essentially complete for the papers [7], [18], [19], [20], [16]; the two remaining items, [22] and [23], represent 'work still in progress' but with substantial results already obtained. In addition, we note that several additional items *might* have been listed here (e.g., joint work with S. Jensen of UMBC on computational aspects of [10], initiation with M. Freidlin of UMCP of an investigation of differentiability with respect to a control parameter for stochastic differential equations with thermostat nonlinearities as in [15], joint work with A. Lehtonen of Jyväskylä on some differential-algebraic systems, etc.) but these have been omitted as there seemed to be insufficient progress in each as yet to warrant their inclusion. One hopes that these, as well as [22], [23], will continue to develop in time.

Each of the items is briefly 'annotated' in the bibliography itself but we now provide a somewhat more detailed exposition for some 'major' developments: free boundary problems, switching systems, optimal filtering.

Optimal control of free boundary problems

A considerable variety of important applied problems involve free boundaries — one need only glance at Crank's book or at the proceedings of the four International Conferences on Free Boundary Problems. Many of these are dynamic problems for which considerations of optimal control are appropriate; essentially similar ideas also arize in connection with 'shape optimization' (structural design). As an initial foray into this area, the Principal Investigator attended the ICFBP at Irsee and presented a paper [8] on a problem involving a robot arm with a prismatic joint (stimulated by earlier work by P.K.C. Wang). The paper formulated some problems (which remain open) for a flexible rod with boundary control through boundary variation and provided a rather pretty solution for one subproblem.

Collaboration with F. Conrad and D. Hilhorst was initiated at Irsee, leading to [9], [10]; Conrad had previously considered a steady-state version of a crystal growth problem and was interested in the corresponding time-dependent version. This, in turn, led to the consideration of related control-theoretic questions, especially optimal boundary control. In [11] the control-theoretic ideas were comparatively 'standard' but substantial work was needed on relevant pde analysis beyond that of [10]. The subsequent paper [12] concentrated on development of the 'optimality system' (first order necessary conditions) for this. One interesting aspect was a reformulation, evading the lack of known uniqueness for the direct problem, by decoupling the boundary variation to get differentiability and the applying the Implicit Function Theorem to justify the re-coupling; this idea may turn out to be significant for later work. Another major technical difficulty involved accounting for the domain variation when the control is varied which, finally, led to a pde problem of a fairly standard form for the adjoint state (Lagrange multiplier) except for the somewhat surprising inclusion of nonlocal terms involving integrals over the domain.[2] Collaboration with P. Neit-

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[2]We also note that conversation at Voran with J. Simon has somewhat clarified the relation of this variational computation to his 'calculus' for shape variation — which may

taanmäki while visiting at Jyväskylä involved the adaptation of a code he had written for shape optimization to the approximate computation of the optimal boundary control.

Switching systems

The notion of a 'switching system' is an outgrowth of the Principal Investigator's efforts at modeling the discontinuous and hysteretic nonlinearity occuring in the analysis of a thermostat. Essentially, one has a continuous-time dynamical system (alternate 'modes': ode's) with event-driven switching when the state encounters the appropriate switching surface. The interplay between the continuous and the discrete then requires new mathematical techniques for analysis.[3] Although a thermostat is a device for feedback control, the initial work by the Principal Investigator has focused on the behavior of the dynamical system obtained by implementation, particularly on the existence of periodic solutions [14]; it is expected that this line of investigation will continue with analysis of the occurrence of chaotic behavior.

The paper [15] considers optimal (open loop) control for a system already involving a thermostat, both in deterministic and in stochastic contexts. The interesting conjecture, for continued exploration, is that one may actually have differentiability of a standard cost functional in the stochastic case (thus permitting 'easy' formulation of necessary conditions for optimality) despite the lack of even continuity in the deterministic case. Finally, the work [16] looks at the 'degenerate' case in which the distance between the switching surfaces goes to 0 so the hysteresis disappears. One result already obtained is that when both mode are directed 'in' (toward the limit surface from each side) the behavior goes to a 'chattering' ('sliding mode') limit, independent of certain aspects of implementation, under quite general circumstances. Also considered is the relation of switching systems to certain singular perturbation problems.

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permit later extension to higher-dimensional problems for which one may no longer be able to map to a fixed domain.

[3] A 'fundamentals' paper by the PI is to appear in *Control and Cybernetics*. We note that continuation of this research effort will be partially funded by the NSF for 1989-'90.

The method of 'optimal filtering'

This is a new computational approach to certain ill-posed problems. The setting for the approach is a problem embedded (*à la* dynamic programming) in a one-parameter family of similar problems. The underlying idea is to view the procedure as continuation with respect to this 'time' variable, with the specific parameters of the infinitesimal generator ('differential' filter) viewed as controls. One simultaneously derives, in terms of these parameters, a differential inequality for the associated error bound and considers an 'optimal control problem' in which these 'controls' are to be chosen so as to minimize the growth of the error bound.

In the two problems, [18] and [19], for which the details have already been worked out, the availability of suitable 'spectral' representations (the Fourier transform and eigenfunction expansion, repectively) permit particularly elegant and explicit solutions. For these two problems, the method lives up to its name — giving approximation with estimates coinciding with known 'log-convexity' estimates for the theoretical uncertainty.

While one cannot always expect to obtain such explicit formulas as for the cases now worked out, the underlying idea seems clearly capable of wider applicability. The determination of the range of utility of this approach is, of course, a subject for continued exploration. Already from [18] and [19] the prospect looks hopeful that this will become an extremely valuable approach for some yet-to-be-specified class of ill-posed problems.

References

[1] *Periodic solutions of a parabolic quasi-variational inequality from stochastic optimal control* (with S. Belbas), *Applicable Anal.* **29**, pp. 301–327 (1988). [The possibility of voluntary stopping gives a one-sided obstacle problem with a periodicity condition. Existence of a periodic solution is obtained using the Schauder Theorem but it is shown that there can only be one bounded solution.]

[2] *Equilibrium states of a nonlinearly elastic conducting rod in a magnetic field* (with P. Wolfe), *Arch. Rat. Mech. Anal.* **102**, pp. 307–329 (1988). [The variational formulation gives a minimization problem for which the domain has empty interior, necessitating development of a new argument for justification of the optimality conditions (Euler equation).]

[3] *An inverse eigenvalue problem with rotational symmetry, Inverse Problems* **4**, pp. 1093–1115 (1989). [This extends a previous result by the PI to a higher-dimensional setting.]

[4] *The linear complementarity problem for general cones* (with M. S. Gowda), *Mathematical Programming*, to appear. [A result of Lemke's for the positive orthant in $\mathbb{R}^k$ is extended to more general cones.]

[5] *Well-posedness and convergence of some regularization methods for nonlinear ill-posed problems* (with C. Vogel), *Inverse Problems*, to appear. [Convergence results are well-known in the linear case for, e.g., Tikhonov regularization but a somewhat different argument is provided to show this in the nonlinear case.]

[6] *Some problems of distributed parameter control with unilateral constraints, J. Math. Anal. Appl.*, to appear. [This is a revision, containing significant new material, of an earlier conference paper *A one-sided miscellany*. The subject is a geometric characterization of certain constrained quadratic optimal control problems as boundary control of the heat equation using non-negative controls.]

[7] *A note on feedback stabilization* (with Houshi Li). [This is a revision of an earlier manuscript, including some new results and examples.]

[8] *Some free boundary problems arising in the control of a flexible robot arm*, in *Proc. 4th International Conf. on Free Boundary Problems*, (Irsee: 1987) (K.-H. Hoffmann and J. Sprekels, eds.), to appear. [The principal result is exact controlability of longitudinal vibration in a robot arm with a prismatic joint.]

[9] *On a reaction-diffusion equation with a moving boundary* (with F. Conrad and D. Hilhorst), in *Problèmes elliptiques et paraboliques non linéaires*, to appear. [This is a preliminary version of [10], presented at a conference at the Univ. de Nancy, I, in March, 1988.]

[10] *Well-posedness of a moving boundary problem arising in a dissolution/growth process*, (with F. Conrad and D. Hilhorst). [This does not specifically relate to control but formed the background for the investigation reported in [11] and [12].]

[11] *Some control-theoretic questions for a free boundary problem*, in Proc. IFIP Conf. (Santiago de Compostela: 1987) (A. Bermúdez, ed.), to appear. [In the setting of [10], a number of control-theoretic questions are posed and discussed — e.g., existence of an optimal boundary control is demonstrated.]

[12] *Optimal solutions for a free boundary problem for crystal growth* (with P. Neittaanmäki), for Proc. Vorau conf. on DPS (10-26 July, 1988), to appear. [Continuing the analysis of [11], this paper provides the (somewhat surprising) optimality conditions for the problem of optimal boundary control and gives some numerical computation.]

[13] *The coefficient map for certain exponential sums, II*, (with M. Gowda), submitted. [This does not specifically relate to control but extends an earlier paper which provided the technical infrastructure for results on the observability and exact controlability of a vibrating plate and seems potentially related to analysis, for some possible DPS problems, of the asymptotic sensitivity for short observation times.]

[14] *Switching systems and periodicity*, in *Nonlinear Semigroups, PDE, and Attractors* (Proc. 2nd Howard Univ. Sympos.: 1987) (T. Gill and W.W.

Zachary, eds.), Springer-Verlag, to appear. [This is a survey, with some new results, of the existence of periodic solutions for certain equations arising in the use of a thermostat for feedback control.]

[15] *Some problems with thermostat nonlinearities*, in *Proceedings IEEE 1988 CDC* (Austin, TX: Dec., 1988), pp. 1255–1259. [Optimal control is considered for deterministic and stochastic problems involving a thermostat with a pre-scheduled time-dependent setting.]

[16] *Local analysis of sliding modes (chattering) as degenerate switching.* [This looks at the 'degenerate' case in which the distance between the switching surfaces goes to 0 and also considers the interpretation of a switching system as a reduced problem from singular perturbation.]

[17] *Instabilities of CAF flow control policies for flexible manufacturing systems* (with P. R. Kumar). [A particular class of flow control policies was proposed by Kumar and shown to be 'stable' under certain conditions; an attempt to generalize this led to the construction of unstable examples when the original conditions did not hold.]

[18] *Optimal filtering for the sideways heat equation* (with L. Eldén). [A new computational approach gives an approximate solution with accuracy matching the theoretical 'log-convexity' bound.]

[19] *Optimal filtering for the backwards heat equation* (with R. Ewing), [Much as for [18]. Here the algorithm involves an eigenfunction expansion and the error estimate includes the effect of errors in computationally obtained eigenfunctions.]

[20] *A construction of stabilizing control laws* (with S.-P. Chen). [This approach and some of the results were obtained several years ago but the collaboration with Chen, supported through the grant, brought this to a reasonably complete form — although the manuscript is now still in the process of rather extensive revision. The thrust of the paper is the use of control-theoretic information (rather than, e.g., spectral information) in the construction of stabilizing feedback laws for distributed parameter systems.]

[21] *Numerical computation of minimum-energy nullcontrols for the heat equation* (with K.-H. Hoffmann). [This approach and some of the results were

obtained by the Principal Investigator several years ago, in connection with the 1979 Delaware conference on ill-posed problems. The proof that the approximating (computational) 'lumped parameter' problems support exact nullcontrolability is also about 9 years old. The new work, other than writing, consists primarily in the computational implementation.]

[22] *Some 'complexity' issues for ill-posed problems.* [This is concerned with the appropriate measure of computational complexity for ill-posed problems.]

[23] *A semigroup approach to Colton's kernel* (with W. Rundell). [Positive existence results for the rather unusual equation $w_t = w_{xx} - w_{yy} + qw$ would be useful for considering exact boundary nullcontrolability of $u_t = u_{xx} + q(t,x)u$ and for coefficient identification. At present we have some partial results but a 'new idea' is still needed.]