Analog Automatic Control Loops in Radar and EW

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Automatic gain control, signal thresholding, and range tracking are basic building blocks for most radar and electronic warfare (EW) systems, active and semiactive missiles, and antiradiation missiles (ARMs). This report is intended to present the theory and design procedures for these “blocks” from a practical viewpoint, and to outline the evolution of the author’s efforts in this area during the past 20 years.

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Richard Smith Hughes

Abstract

This report discusses the design and analysis of the basic "building blocks" of many radar and electronic warfare (EW) systems, active and semiactive missiles, and anti-radiation missiles (ARMs): namely automatic gain control, signal thresholding, and range tracking. Analog rather than digital automatic tracking loops are discussed because knowledge of analog circuit design and operation is fundamental to digitizing of analog functions.

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Radar
Range track loops
Receivers

Tangential sensitivity
Thresholding
Variable gain control

19. (Contd.)

(U) Chapter 1 presents automatic gain control from theoretical and practical standpoints. Chapter 2 concentrates on the necessity of providing a noise riding threshold to compensate for the excess noise caused by IF/RF amplification preceding a basic crystal video receiver. Chapter 3 presents the analysis and design of both Type I and Type II range-tracking loops from a phase-lock loop perspective.

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Richard Smith Hughes
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Automatic gain control, signal thresholding, and range tracking are basic building blocks of many radar and electronic warfare (EW) systems, active and semi-active missiles, and antiradiation missiles (ARMs). This book is meant as a practical starting point for those who must analyze and design these blocks. The reader may well ask why analog, and not digital, automatic tracking loops? Certainly most modern radar, EW, and related systems employ digital (software) technology to perform many of the functions once performed by analog circuitry; however, the digitizing of analog functions requires a fundamental knowledge of analog circuit design and operation. The intent of this book is to help in providing that knowledge. The deviation from pure analog circuitry to a hybrid of analog, digital, and software is up to the readers' requirements; however, it is hoped that this book will provide a solid foundation upon which one can begin to build.

Chapter 1 presents automatic gain control from a theoretical and practical standpoint. Static regulation, dynamic regulation, loop bandwidth, and rise time are covered and the theoretical results verified with several design examples. Methods of varying IF and RF gains are discussed, as are the effects of square law and linear detection on the various AGC parameters and the effect of self-AGC (the IF or RF signal having a controlling influence on the circuits' gain).

Virtually all radar, EW, and related systems employ signal thresholding to identify the presence of a signal. Signal thresholds can be as simple as a comparator and a fixed signal threshold voltage or as complicated as circuitry that nulls any returns due to clutter or noise. Automatic thresholding loops to remove clutter are usually quite complicated and indeed a digital approach at the offset of design is often necessitated. Chapter 2 concentrates on the
necessity of providing a noise riding threshold to compensate for the excess noise due to IF/RF amplification preceding a basic crystal video receiver. A fairly simple and straightforward noise tracking threshold is presented to illustrate the techniques involved and the results that may be expected. An automatic noise tracking loop to optimize the detection process is often a necessity, even for simple receivers. This chapter is not meant as an all-inclusive presentation on signal thresholding, but rather a starting point for those who must define and design this important building block.

No book with the title *Analog Automatic Tracking Loops* would be complete without a presentation of range-tracking loops. Chapter 3 presents the analysis and design of both Type I and Type II range-tracking loops from a phase-locked loop perspective. Examples are presented to verify the design theory given.

Topics that deal with signal acquisition and reacquisition, signal duty cycle dependence, etc., have been purposely left untouched. As an engineer employed by the U.S. Government, I am acutely aware of the possibilities of inadvertently and unwittingly compromising Navy programs. To remove this risk, any topic which I felt could, in any way, compromise the work in which we are engaged at the Naval Weapons Center (NWC) was left out completely.

The author would like to extend formally his appreciation to John Daugherty and Paul Hilliard, who during the past 18 years, had the responsibilities of constructing and testing the circuits presented. Ms. Janet Pande, Bob Sutton, and Brad Wiitala reviewed this work, and I am most grateful for the time spent and suggestions offered. I wish to give special thanks to Mrs. Freddie Perry, my NWC editor, who had to take my unintelligible notes and transform them into a readable document.

This work is an evolution of design notes, NWC publications, and other documents, spread over the past twenty years. I wish to express my deepest appreciation to the management of NWC (past and present) who continue, with ever increasing money constraints, to create the practical and academic atmosphere that make works such as this possible.
A final thank you goes to the reader. Without your support, books such as this would not be possible. I hope this work provides some of the material you have been seeking. If it does, the past six months of nights and weekends will have been worthwhile.

Richard Smith Hughes
June 1988
Chapter 1

AUTOMATIC GAIN CONTROL.

Introduction

Many radars and electronic warfare (EW) systems employ automatic gain control (AGC) to normalize the received signal prior to signal processing (i.e., range tracking, signal acquisition, direction finding, etc.). This chapter presents the basic AGC theory and design philosophy from a practical standpoint. The theory covers static and dynamic regulation and AGC rise time, with respect to square law detection and linear detection. Various methods of variable gain control are presented and self-AGC (the controlled signal level effecting the gain) is discussed. The chapter concludes with several design examples.

Figure 1-1 illustrates a basic radar receiver employing AGC. As the received input signal varies, the input to the intermediate frequency (IF) amplifier changes. The AGC loop notes the change and varies the gain of the IF amplifier in such a way that the output of the detector remains constant (the AGC voltage could also vary the gain of the radio frequency (RF) amplifier).

Figure 1-2 illustrates the basic components of AGC tracking loops (continuous wave (CW) inputs will be assumed for now; however, operation with pulse inputs is basically the same and will be covered later). The only difference between Figures 1-2a and 1-2b is that one uses a low-pass filter (LPF) and the other an integrator. The differences between these two techniques are also discussed.
FIGURE 1-1. Basic Radar Receiver.

FIGURE 1-2. Basic Components of Automatic Gain Control (AGC) Tracking Loops.
The configurations of Figure 1-2 are quite general in that most AGC loops can be reduced to the components illustrated (necessary pulse stretching and timing circuitry for pulse AGC operation are not shown in the interest of simplicity). The input signal, $P_{in}(\text{dBm})$, is amplified by the variable gain IF amplifier. The variable gain IF output may be amplified or attenuated, depending on the desired normalized detector input power, $P_{dN}(\text{dBm})$. The detected signal is amplified by the video amplifier, $A_v$, compared with a reference voltage, $E_{Ref}$, and again amplified by the error amplifier, $A_e$. The resultant voltage drives the variable gain IF AGC input.

If $P_{in}(\text{dBm})$ should increase, $P_{dN}(\text{dBm})$, and thus the normalized video voltage, $e_N$, would increase, increasing the AGC voltage, and thus decreasing the gain until $e_N = E_{Ref}$.

Three basic parameters define the operation of an AGC loop:

1. Static regulation is the capability to compress large input variations into small output variations. This is the same concept as line regulation in a regulated power supply. The compressed output variation, $\Delta P_{dN}(\text{dB})$, divided by the input variation, $\Delta P_{in}(\text{dB})$, is called the compression ratio (CR). The change in the normalized video voltage, $\Delta e_N$, depends on detector type and will be discussed as we proceed.

   \[
   CR_{\text{tr}} = \frac{\Delta P_{dN}(\text{dB})}{\Delta P_{in}(\text{dB})} 
   \]

   \[
   CR_{\text{vid}} = \frac{\Delta e_N(\text{dB})}{\Delta P_{in}(\text{dB})} 
   \]

2. Dynamic regulation is the capability of an AGC loop to reduce the dynamic input modulation, $MP_{in}$, appearing at the output, $MP_{dN}$, and is called the input modulation reduction (IMR). This quality is especially important in conical scanning radars. IMR is dependent on the loop gain (LG) of the AGC loop and may be given as
\[
\text{IMR} = \frac{M_{P_{IF}}}{M_{P_{in}}} = \frac{1}{1 + LG}
\]  

(1-3)

3. Loop rise time \( (t_r) \) is the 10 to 90% loop-response time resulting from a step change in input power.

These three parameters are discussed in the next sections.

**Static Regulation**

Assume that the variable gain IF amplifier has a variable gain characteristic as illustrated in Figure 1-3. (Most variable gain IF/RF amplifiers have a rather linear relationship between gain (dB) and AGC voltage.) The equation relating IF gain to AGC voltage is

\[
A_{IF}(dB) = A_o(dB) - X(AGC \text{ Voltage})
\]

(1-4)

where

\[
X = \text{variable gain slope in dB/V}
\]
\[
A_o(dB) = \text{maximum gain}
\]

AGC loops involve power levels and ratios in the IF and detector portion and voltage levels and ratios in the video (or post detection) portion. Appendix 1A summarizes power-voltage relationships for easy reference.

One primary function of an AGC loop is to keep the output power, \( P_{IF}(dBm) \) (or video voltage, \( e_N \)), normalized to within a specified amount, \( \Delta P_{IF}(dB) \) or \( \Delta e_N(dB) \), despite large variations in the input power, \( \Delta P_{in}(dB) \). A predetector amplifier may be necessary to ensure that the detector is operated at the desired level, linear or square law (see Appendix 1B for detector characteristics pertinent to AGC design).
FIGURE 1-3. IF Amplifier Gain Versus AGC Voltage.

The static regulation characteristics of the low-pass filter AGC (Figure 1-2a) will now be discussed. Compression ratio has been defined in Equation (1-1). Appendix 1C presents the solution for $\Delta P_{IF}(dB)$ as a function of detector type, linear (Lin) or square law (SL), and the results are

$$\Delta P_{IF,SL}(dB) = 10 \log \frac{\Delta P_{in}(dB)}{X \Delta A_{e} + 1}$$

(1.5)
\[
\Delta P_{IF,lin}(dB) = 20 \log \left( \frac{\Delta P_{in}(dB)}{X A_\Delta A_e e_N} + 1 \right)
\]

Thus, to minimize the change in output power, \(e_N\), \(A_\Delta\), and \(A_e\) must be made as large as practicable (X is assumed constant for a given IF amplifier).

Solving Equations (1-5) and (1-6) for \(A_\Delta A_e e_N\),

\[
(A_\Delta A_e e_N)_{Sl} = \frac{\Delta P_{in}(dB)}{\Delta P_{IF}(dB)} \left( \frac{10}{10} - 1 \right)
\]

\[
(A_\Delta A_e e_N)_{lin} = \frac{\Delta P_{in}(dB)}{\Delta P_{IF}(dB)} \left( \frac{20}{10} - 1 \right)
\]

Thus, since X and \(\Delta P_{IF}(dB)\) are known, the necessary \(A_\Delta A_e e_N\) may be found.

To illustrate the discussion thus far, consider the following:

- Minimum input for AGC action (or AGC delay) \(P_{in, min}(dBm) = -70 \text{ dBm}\)
- Minimum output under AGC action \(P_{IF, min}(dBm) = -0.5 \text{ dBm}\)
- Maximum output under AGC action \(P_{IF, max}(dBm) = +0.5 \text{ dBm}\)
- Maximum input for AGC action (or AGC dropout level) \(P_{in, max}(dBm) = -20 \text{ dBm}\)

The input dynamic range is

\[
\Delta P_{in}(dB) = P_{in, max}(dBm) - P_{in, min}(dBm) = 50 \text{ dB}
\]

and the output dynamic range is

\[
\Delta P_{e}(dB) = P_{IF, max}(dBm) - P_{IF, min}(dBm) = 1 \text{ dB}
\]
Thus the compression ratio is

\[
CR = \frac{\Delta P_{\text{IF}}(\text{dB})}{\Delta P_{\text{in}}(\text{dB})} = 0.02
\]

or the output increases 0.02 dBm for each 1 dBm increase to the input.

Figure 1-4 illustrates the characteristics of the AGC system just presented.

This discussion has concerned the AGC of Figure 1-2a. Figure 1-2b illustrates an AGC loop that incorporates a true integrator. A true integrator has a very large gain at low frequencies; therefore, under normalized, unmodulated inputs, \(e_c = 0\). Thus,

\[
\Delta P_{\text{IF}}(\text{dB}) = 0
\]

and perfect regulation is obtained. (Further in the chapter the theoretical equations presented will be verified with a practical example.)

\[\text{FIGURE 1-4. Typical AGC Characteristics.}\]
The next section presents the behavior of the AGC loop, illustrated in Figure 1-2, under input modulation conditions.

Dynamic Regulation

Figure 1-5 illustrates the AGC loop of Figure 1-2a in classical feedback form. Using conventional feedback theory, Oliver [1]* has shown that

\[ \frac{\Delta e_{IF}(PP)}{e_{IF}(PP)} = \frac{1}{1 + AB} \left( \frac{\Delta e_{in}(PP)}{e_{in}(PP)} \right) \]  

(1-13)

where

\[ e_{IF}(PP) = \text{peak - to - peak IF output voltage} \]

\[ \frac{\Delta e_{IF}(PP)}{e_{IF}(PP)} \times (100) = \text{percent output modulation} \ (M_o \text{ or } M_{IF}) \]

\[ \frac{\Delta e_{in}(PP)}{e_{in}(PP)} \times (100) = \text{percent input modulation} \ (M_i) \]

\[ AB = \text{loop gain (LG)} \]

Equation (1-3) may be written as (assuming LG \( \geq 1 \))

\[ M_{IF} = \frac{M_i}{LG} \]  

(1-14)

or, the IF output modulation is reduced by the inverse of the loop gain. Thus the input modulation reduction is (for large loop gains)

---

*Numbers in brackets \[ \] refer to references. These references, along with a bibliography, are contained at the end of each chapter.
IMR = \frac{1}{LG} \quad (1.15)

and

M_{IF} = IMR(M_1) \quad (1.16)

\[\begin{array}{c}
\text{IMR} = \frac{1}{LG} \\
\text{and} \\
M_{IF} = IMR(M_1)
\end{array}\]

\[\begin{array}{c}
\text{FIGURE 1-5. Block Diagram of Feedback Amplifier.}
\end{array}\]

The loop gain (for constant input power) is found by breaking the AGC input to the variable gain IF amplifier and modulating the AGC voltage. The loop gain is thus the modulated output voltage, $\Delta AGC'$, divided by the modulated AGC voltage, $\Delta AGC$, as illustrated in Figure 1-6:

\[LG = \frac{\Delta AGC'}{\Delta AGC} \quad (1.17)\]

or

\[LG = A_{AGC}A_{PD}A_{v}A_{\Delta}A_{\epsilon} \quad (1.18)\]

*The effect of the low-pass filter will be neglected. Ideally this sets the frequency response of the loop.

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where

\[ A_{AGC} = \text{dynamic AGC gain, } \Delta e_f(PP)/\Delta AGC \ (V/V) \]
\[ A_{PD} = \text{predetector gain, } \Delta e_{PD}(PD)/\Delta e_f(PP) \ (V/V) \]
\[ A_D = \text{dynamic detector gain, } \Delta e_D/\Delta e_{PD}(PP) \ (V/V) \]
\[ A_v = \text{video gain, } \Delta e_N/\Delta e_D \ (V/V) \]
\[ A_e = \text{error gain } \Delta AGC'/\Delta e_e \ (V/V) \]

**FIGURE 1-6. Method of Finding Loop Gain.**

The dynamic AGC gain is a nonlinear quantity, but for small values of \( \Delta AGC \), may be given as (Appendix 1D)

\[ A_{AGC} = 0.115 X e_{IF}(PP) \ (V/V) \]  \hspace{1cm} (1-19)

The dynamic detector gain is also nonlinear; however, it may be assumed linear over small values of \( \Delta e_{PD}(PP) \). The dynamic detector gain for a square law detector (Appendix 1B) may be given as

\[ A_{D,SL} = 5 \times 10^{-3} K_{SL} e_{PD}(PP) \ (V/V) \]  \hspace{1cm} (1-20)

where \( K_{SL} \) is a square law detector constant.

Substituting Equations (1-19) and (1-20) into (1-18),

\[ L_{SL} = \left[ 0.115 X e_{IF}(PP) \right] A_{PD} \left[ 5 \times 10^{-3} \right] K_{SL} e_{PD}(PP) A_D A_e \]  \hspace{1cm} (1-21)
However, since

\[ e_{TP}(PP) = e_{PD}(PP)/A_{PD} \]  

Equation (1-21) may be written as

\[ LG_{SL} = 0.115 X [e_{PD}(PP)]^2 5 \times 10^{-3}K_{SL}A_{\Delta}A_{c} \]  

The detector output, \( e_{D} \), is (Appendix 1B)

\[ e_{D} = 2.5 \times 10^{-3} K_{SL} e_{PD}(PP)^2 \text{ (V)} \]  

thus, under normalized conditions \( e_{D} = e_{D,N} \)

\[ LG_{SL} = 0.23 X A_{\Delta}A_{c} e_{D,N} \]  

or, since

\[ e_{N} = e_{D,N} \cdot \cdot \cdot \]

\[ LG_{SL} = 0.23 X A_{\Delta}A_{c} e_{N} \]  

Equation (1-26) is simple but very accurate. Using the same methods, but for a linear detector (Appendix 1B),

\[ A_{D,lin} = 1.59 \times 10^{-3} K_{Lin} \text{ (V/V)} \]  

and

\[ e_{D,lin} = 1.59 \times 10^{-3} K_{Lin} e_{PD}(PP) \text{ (V)} \]  

thus the loop gain becomes

\[ LG_{Lin} = 0.115 X A_{\Delta}A_{c} e_{N} \]  

It can be seen from Equations (1-5) and (1-6) that the loop gain and static regulation (\( \Delta PI(dB) \)) are dependent on \( XA_{\Delta}A_{c}e_{N} \); thus, for a given IF amplifier, \( A_{\Delta}A_{c} \) can be maximized to give the necessary loop gain.
\[ \Delta P_{IF,SL}(dB) = 10 \log \left| \frac{\Delta P_{in}(dB)}{X A_\Delta A_\varepsilon e_N} + 1 \right| \]  \hspace{1cm} (1.30)  

and

\[ LG_{SL} = 0.23 X A_\Delta A_\varepsilon e_N \]  \hspace{1cm} (1.31)  

Solving Equation (1.31) for \( A_\Delta A_\varepsilon e_N \),

\[ A_\Delta A_\varepsilon e_N = \frac{LG_{SL}}{0.23X} \]  \hspace{1cm} (1.32)  

and substituting into Equation (1.30),

\[ \Delta P_{IF,SL}(dB) = 10 \log \left| \frac{0.23 \Delta P_{in}(dB)}{LG} + 1 \right| \]  \hspace{1cm} (1.33)  

For the linear detector this equation becomes

\[ \Delta P_{IF,ln}(dB) = 10 \log \left| \frac{0.12 \Delta P_{in}(dB)}{LG} + 1 \right| \]  \hspace{1cm} (1.34)  

Thus the loop gain uniquely determines the static regulation. Conversely, the static regulation uniquely determines the loop gain, as shown below.

Solving Equation (1.30) for \( A_\Delta A_\varepsilon e_N \),

\[ A_\Delta A_\varepsilon e_{N,SL} = \frac{\Delta P_{in}(dB)}{\Delta P_{IF}(dB)} \times \left( \frac{1}{10} \right)^{10} + 1 \]  \hspace{1cm} (1.35)  

Substituting Equation (1.35) into (1.31),

\[ LG_{SL} = 0.23 \left| \frac{\Delta P_{in}(dB)}{\Delta P_{IF}(dB)} \right| \left( \frac{1}{10} \right)^{10} + 1 \]  \hspace{1cm} (1.36)  

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and for the linear detector,

$$LG_{Lin} = 0.115 \left( \frac{\Delta P_{in}(dB)}{\Delta P_{in}(dB)} \right) ^{0.115} \left( \frac{10}{20} \frac{u}{v} - 1 \right)$$

(1.37)

The dynamic regulation for the true integrator AGC loop of Figure 1-2b is similar to the low-pass-filter loop, except that the loop gain must be multiplied by the frequency-dependent gain of the integrator, $A_{Int}$ (assuming a large low-frequency gain for the operational amplifier),

$$A_{Int} = \frac{-Z_f}{R}$$

(1.38)

where

$$Z_f = \frac{-j}{2\pi fC}$$

(1.39)

or

$$A_{Int} = \frac{0.159}{\tau RC} \angle -90^0$$

(1.40)

Thus,

$$LG_{sl} = \frac{0.036 \times A \times e_n}{fCR}$$

(1.41)

and

$$LG_{Lin} = \frac{0.018 \times A \times e_n}{fCR}$$

(1.42)

The frequency response for the low-pass-filter loop will be determined by the low-pass filter (assuming that the open loop frequency, neglecting the filter, is much larger than the filter frequency response, which is usually the case).
Loop Rise Time and Bandwidth ($f_{3dB}$)

The loop rise time, $t_r$, is defined as the 10 to 90% AGC response time to step changes in input power. The rise time is dependent on the nonlinear characteristics of the detector; thus, the equations presented (see Appendix 1E for the derivation) are valid only for small input steps (less than ±2 dBm).

The rise times and bandwidth for the integrator and low-pass-filter loops are the same, and may be given as

$$t_{r,Sl} = \frac{9.56 RC}{X A_\Delta A e_N}$$

$$f_{3dB,Sl} = \frac{0.35}{t_{r,Sl}}$$

$$t_{r,lin} = \frac{19.13 RC}{X A_\Delta A e_N}$$

$$f_{3dB,lin} = \frac{0.35}{t_{r,lin}}$$

As can be seen, the loop rise time is also dependent on $X A_\Delta A e_N$; however, the rise time can be calculated independently in terms of $R$ and $C$.

The rise times given in Equations (1-43) and (1-44) are for CW input loops. What is the rise time for a pulse loop rather than a CW loop? The effect a pulse AGC loop has on $t_r$ is quite easy to determine. Figure 1-7 illustrates a basic pulse AGC loop, and the pertinent timing is shown in Figure 1-8.

![FIGURE 1-7. Basic Pulse AGC Block Diagram.](image)
The basic operation of the pulse AGC loop is straightforward; the input pulse is amplified, detected, sampled, and compared to the reference voltage, $E_{Ref}$. The amplified video also triggers a comparator (threshold) that initiates the timing. The integrator update switch is closed for a given period, $T_u$, every pulse; thus the integrator is only allowed to update the AGC loop during $T_u$. The effective time constant ($T_{pulse}$) for the updated integrator is

$$T_{pulse} = RC \left( \frac{PRI}{T_u} \right)$$ \hspace{1cm} (1.45)

The update duty cycle, $D$, may be given as

$$D = \frac{T_u}{PRI}$$ \hspace{1cm} (1.46)

Thus, Equation (1.45) may be written as

$$T_{pulse} = \frac{RC}{D}$$ \hspace{1cm} (1.47)
All equations thus presented for loop rise time, $t_r$, may now be given in more general terms as

$$t_{r,SL} = \frac{9.56 \text{RC}}{XA_sA_t e_N D} \quad f_{MRL,SL} = \frac{0.35}{t_{r,SL}} \quad (1-48)$$

$$t_{r,lin} = \frac{19.13 \text{RC}}{XA_sA_t e_N D} \quad f_{MRL,lin} = \frac{0.35}{t_{r,lin}} \quad (1-49)$$

It should be noted that sampling the AGC loop makes it a sampled data system. This publication will assume that any input modulation frequency is much smaller (by at least ten) than the sampling rate (PRF in pulsed AGC). If this condition is not met, instability may well result.

**Automatic Gain Control Design Verification**

The preceding section presented the equations that characterize an AGC tracking loop. In this section, we will verify these equations using a simple AGC tracking loop. Nonideal parameters will be discussed (i.e., the IF amplifier's variable gain slope, $X$, is not linear over the full AGC range), and practical design equations will be presented with these nonideal parameters in mind.

**Square Law Detector Test Circuit**

Figure 1-9 illustrates the circuit used to validate the equations already presented. The philosophy here is to analyze an existing AGC loop to verify the equations rather than to verify by design (Chapter 2 presents the design for several practical AGC loops). The square law detector is discussed first, then the linear.
D1  D2  HP 5082 2800
RF = 2.4 MΩ (LOW PASS FILTER AGC)

FIGURE 1-9. Square Law Detector AGC Test Circuit.
Figure 1-10 illustrates the square law characteristics for the detector used (see Appendix 1B for a discussion of detector characteristics). A normalized video reference, $e_N$, of $–1$ volt will be used; thus the detector output, $e_D$, is

$$e_D = e_N A$$

or, since $A = 240$,

$$e_D = 1.240 = 4.2 \text{ mV}$$

The detector input power required to give an $e_D$ of $4.2 \text{ mV}$ is (Appendix 1B)

$$P_{p1}(\text{dBm}) = -10 \log \left( \frac{K_{sl}}{e_D(\text{mV})} \right)$$

or

$$P_{p1}(\text{dBm}) = -10 \log \left( \frac{350}{4.2} \right)$$

or

$$P_{p1}(\text{dBm}) = -19.2 \text{ dBm}$$

Detector $D_2$ is used as a temperature stabilizing element to minimize the dc offset effects of $D_1$. With no IF present, $R_a$ is adjusted until $e_N = 0$ volt. Resistor $R_h$ is adjusted to give the desired normalized video voltage, $e_N (-1$ volt). The $–6$ dB power splitter enables the output power variation to be monitored. The IF output power is

$$P_{p1}(\text{dBm}) = P_{p1}(\text{dBm}) + 6 \text{ dB}$$

where both outputs of the power splitter are the same ($P_o(\text{dBm}) = P_{p1}(\text{dBm})$). Resistor $R_F$ is used for the low-pass-filter AGC.
FIGURE 1-10. Detector Output Versus Input Power (HP 5082-2800).
Figure 1-11 illustrates the variable gain characteristics of the IF amplifier. It will be noticed that the slope varies with AGC voltage (which is typical of many commercial variable gain IF amplifiers). The variation in $X$ with AGC voltage and gain is given in Table 1-1. As can be seen, there is more than a three-to-one variation in $X$. Thus the loop gain, static regulation (for the low-pass-filter AGC), and rise time of the loop will be a function of AGC voltage and, thus, the input power, $P_{in}(\text{dBm})$. 

FIGURE 1-11. IF Amplifier Variable Gain Characteristics.
TABLE 1-1. IF Amplifier Variable Gain Slope.

<table>
<thead>
<tr>
<th>AGC voltage, V</th>
<th>Gain, dB</th>
<th>X, dB/V</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.75</td>
<td>40</td>
<td>18.2</td>
</tr>
<tr>
<td>-2.85</td>
<td>30</td>
<td>13.9</td>
</tr>
<tr>
<td>-3.25</td>
<td>20</td>
<td>9.1</td>
</tr>
<tr>
<td>-4.6</td>
<td>10</td>
<td>5.5</td>
</tr>
</tbody>
</table>

The static regulation for the low-pass-filter AGC was presented as (Equation (1-56))

$$\Delta P_{IF,SL}(dB) = 10 \log \left[ \frac{\Delta P_{in}(dB)}{X A_{AGC} e^{-N}} + 1 \right]$$  \hspace{1cm} (1-56)

which assumes a linear gain slope, $X$. Obviously the gain slope varies (Figure 1-11 and Table 1-1), and Equation (1-56) is not valid. This problem is easily corrected, however, by noting that

$$\Delta AGC = \frac{\Delta P_{in}(dB)}{X}$$  \hspace{1cm} (1-57)

where $\Delta AGC$ is the total change in AGC voltage for the total desired input dynamic range, $\Delta P_{in}(dB)$. Equation (1-57) may now be written as

$$\Delta P_{IF,SL}(dB) = 10 \log \left[ \frac{\Delta AGC}{A_{AGC} e^{-N}} + 1 \right]$$  \hspace{1cm} (1-58)

which defines the static regulation for a practical low-pass-filter AGC loop using a square law detector (the linear detector loop will be presented shortly). The static regulation for the integrator loop will still be near zero because of the large static (dc) gain of the integrator.

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The parameters for the low-pass-filter AGC loop may now be given as (see Figure 1-9)

\[ A_{PD}(dB) = -6; P_{PD}(dBm) = -19.2; P_{IF}(dBm) = -13.2 \]

\[ e_N = -1 \text{ (adjusted at -3 volts AGC); } A_v = 240 \]

\[ A_D = -1; A_c = -8.57; R_p C = 9.6 \]

The IF input was varied from -70 to -10 dBm (\( \Delta P_{in}(dB) = 60 \)), and the AGC voltage varied from -1.01 to -6.87 volts (\( \Delta AGC = 5.86 \) volts), as shown in Figure 1-12. The output power varied from -22.2 dBm at an AGC voltage of -1.01 volts to -19.3 dBm at an AGC voltage of 6.87 volts (\( \Delta P_o(dB) = 2.9 \)), as shown in Figure 1-13. Thus the change in output power, \( \Delta P_o(dB) \), is

\[ \Delta P_{o,SL}(dB) = 2.9 \tag{1-59} \]

The predicted change in output power (\( \Delta P_o = \Delta P_{IF} \)), using Equation (1-58), is

\[ \Delta P_{o,SL}(dB) = 10 \log \left| \frac{5.86}{(-1)(-8.57)(1)} + 1 \right| = 2.26 \text{ dB} \tag{1-60} \]
and is in excellent agreement with the measured value of 2.9 dB. The normalized video voltage, $e_N$, varied from $-0.78$ volt at an AGC voltage of $-1.01$, to $-1.42$ volts at an AGC voltage of $-6.87$ volts. Thus the measured change in $e_N$, $\Delta e_N(dB)$, is

$$
\Delta e_{N,SL}(dB) = 20 \log \left( \frac{1.42}{0.78} \right) = 5.2
$$

The predicted value is twice the change in output power, since the change in $e_P$ in decibels is twice the change in input power in decibels for a square law detector (see Appendix 1B). Thus,

$$
\Delta e_{N,SL}(dB) = 20 \log \left( \frac{\Delta AGC}{\Delta A_{e_N}} + 1 \right)
$$

or

$$
\Delta e_{N,SL}(dB) = 20 \log \left| \frac{5.86}{(-1)(-8.57)(1)} + 1 \right| = 4.5
$$

which also is in excellent agreement with the measured value of 5.2 dB.
The value for $e_N$ was changed to $-2$ volts by adjusting $R_b$, and the measured difference in $\Delta P_o(dB)$ for input variations from $-70$ to $-10$ dBm was

$$\Delta P_{o,SL}(dB) = 1.23$$

(1.64)

The predicted value, using Equation (1.58), is

$$\Delta P_{o,SL}(dB) = 10 \log \left( \frac{5.86}{(1)(8.57)^2} + 1 \right) = 1.28$$

(1.65)

which again is in excellent agreement with the measured result. The measured change in $e_N$, $\Delta e_{N,SL}(dB)$, was $2.15$ dB. The predicted change is in excellent agreement with the measured value, as shown in Equation (1.66):

$$\Delta e_{N,SL}(dB) = 20 \log \left( \frac{5.86}{(1)(8.57)^2} + 1 \right) = 2.55$$

(1.66)

The loop rise time was measured for input variations of $\pm 1$, $\pm 5$, and $\pm 10$ dB at three input levels. Table 1-2 summarizes the results.

<table>
<thead>
<tr>
<th>Table 1-2. Loop Rise Time Results.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_m$, dBm</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>-53</td>
</tr>
<tr>
<td>-37</td>
</tr>
<tr>
<td>-26</td>
</tr>
</tbody>
</table>

24
The equations predicting the loop rise time are valid only for small input deviations, as discussed in the preceding section and Appendix 1E. The rise time for the low-pass-filter loop was given as (Equation (1-43))

$$t_{r,\text{st}} = \frac{9.56 \cdot RC}{X A_{\Delta} A_{r} e_{N}} \quad (1-67)$$

The predicted values are compared to the measured values in Table 1-3 (± 1 dB), and as can be seen, there is good agreement.

**Table 1-3. Measured and Predicted Loop Rise Times.**

<table>
<thead>
<tr>
<th>$P_{\text{in}}, \text{dBm}$</th>
<th>$X, \text{dB/V}$</th>
<th>$t_{r} (\text{measured}), \text{s}$</th>
<th>$t_{r} (\text{predicted - Eq. 1-67}), \text{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-53</td>
<td>15.0</td>
<td>0.8</td>
<td>0.71</td>
</tr>
<tr>
<td>-37</td>
<td>9.1</td>
<td>1.0</td>
<td>1.18</td>
</tr>
<tr>
<td>-26</td>
<td>5.5</td>
<td>1.25</td>
<td>1.94</td>
</tr>
</tbody>
</table>

The deviation in loop rise time at large input variations (Table 1-2) is due to the nonlinear behavior of the detector. Two methods to minimize this effect are given later in the chapter.

The loop gain was measured as illustrated in Figure 1-6. The modulating frequency was well below the low-pass-filter bandwidth.

$$f_{\text{mod}} (\text{LPF}) = \frac{1}{2 \pi RC} \quad (1-68)$$

(The open loop frequency response for this circuit (with C removed) is in excess of 3 kilohertz; thus the $R_{P}C$ filter will determine the bandwidth for frequencies up to about 1 kilohertz.)
The measured loop gains are listed in Table 1-4. The predicted loop gain is (Equation (1-26))

\[ L_{G_{SL}} = 0.23 X A_{DA}^\Delta e_N \]  

(1-69)

Table 1-4 also compares the measured and predicted loop gains, which can be seen to be quite close.

<table>
<thead>
<tr>
<th>( P_{in}, \text{dBm} )</th>
<th>AGC</th>
<th>( X, \text{dB/V} )</th>
<th>Measured loop gain</th>
<th>Predicted loop gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50</td>
<td>-1.95</td>
<td>15.0</td>
<td>25.7</td>
<td>29.6</td>
</tr>
<tr>
<td>-35</td>
<td>-3.19</td>
<td>9.1</td>
<td>15.0</td>
<td>17.9</td>
</tr>
<tr>
<td>-25</td>
<td>-4.53</td>
<td>5.5</td>
<td>12.8</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Figure 1-9 (with \( R_F \) removed) illustrates the basic integrator AGC loop using a square law detector. Figure 1-14 illustrates the input power versus AGC voltage for \( e_N = -1 \) volt. The measured change in the output power, \( \Delta P_o(\text{dB}) \), was 0.12 dB for inputs from -65 to -10 dBm. The predicted change, from Equation (1-12), is 0 dBm, which is in excellent agreement with the measured result.

Table 1-5 summarizes the loop rise time results.

The predicted rise time, again valid only for small input variations (Equation (1-67)), is given in Table 1-6, and agrees favorably with the measured results. The value for \( e_N \) was doubled, and the loop rise time was halved, as predicted by Equation (1-67).

TABLE 1-5. Integrator AGC Loop Rise Time Results.

<table>
<thead>
<tr>
<th>$P_{in}, \text{dBm}$</th>
<th>$V_{AGC}$, V</th>
<th>$X$, dB/V</th>
<th>Loop rise time, $\tau_r$, s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$-1 \text{ dB}$</td>
</tr>
<tr>
<td>$-50$</td>
<td>1.95</td>
<td>15.0</td>
<td>0.58</td>
</tr>
<tr>
<td>$-35$</td>
<td>3.19</td>
<td>9.1</td>
<td>0.95</td>
</tr>
<tr>
<td>$-25$</td>
<td>4.53</td>
<td>5.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>$P_{in}, \text{dBm}$</th>
<th>$X$, dB/V</th>
<th>$\tau_r$ (measured), s</th>
<th>$\tau_r$ (predicted), s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-50$</td>
<td>15.0</td>
<td>0.58</td>
<td>0.71</td>
</tr>
<tr>
<td>$-35$</td>
<td>9.1</td>
<td>0.95</td>
<td>1.17</td>
</tr>
<tr>
<td>$-25$</td>
<td>5.5</td>
<td>1.5</td>
<td>1.94</td>
</tr>
</tbody>
</table>
The loop gain is frequency-sensitive, as discussed earlier.

\[ \text{LG}_{\text{SL}} = \frac{0.036 X A_X A_c e_N}{f RC} \] (1.70)

The measured loop gain for an input of \(-35\) dBm \((X = 9.1 \text{ dB/V})\) was 0.03 with an input frequency of 10 Hertz. The predicted value is

\[ \text{LG}_{\text{SL}} = \frac{(0.036 \times 9.1 \times 10^4)}{(10 \times 280 \times 10^3 \times 4 \times 10^{-6})} = 0.0293 \] (1.71)

which is in excellent agreement with measured results.

Linear Detector Test Circuit

Figure 1-15 illustrates the circuit used to verify the linear detector AGC equations. The same variable gain amplifier used for the square law detector test circuit is used, as is the detector. Figure 1-16 illustrates the linear characteristics for the HP 5082-2800 detector. As can be seen, this detector has linear qualities above 0 dBm.

A normalized video reference, \(e_N\), of \(-1\) volt is used; the detector output, \(e_D\), is

\[ e_D = e_N/A_v \] (1.72)

\(A_v = 3.3\); thus,

\[ e_D = -1/3.3 = -0.3 \text{ V} \] (1.73)
FIGURE 1-15. Linear Detector AGC Test Circuit.
The necessary input power to satisfy Equation (1-73) is (Appendix 1B)

\[ P_{PD}(\text{dBm}) = -20 \log \left( \frac{K_{Lin}}{e_D(\text{mV})} \right) \]  

\[ (1.74) \]
\[
P_{\text{in}}(\text{dBm}) = -20 \log \left( \frac{164}{300} \right)
\]

or

\[
P_{\text{in}}(\text{dBm}) = 5.3 \text{ dBm}
\]

and, referring to Figure 1-15,

\[
P_{\text{out}}(\text{dBm}) = 5.3 - 25 + 6 = -13.75 \text{ dBm}
\]

The low-pass-filter AGC will be discussed first \((A_v = 3.3, A_\Delta = -1, A_t = -20, e_N = -1 \text{ volt})\). The static regulation may be given as

\[
\Delta P_{\text{in},\text{lin}}(\text{dB}) = 20 \log \left( \frac{\Delta AGC}{A_\Delta A_t e_N} + 1 \right)
\]

The input power was varied from -69 to 0 dBm \((\Delta P_{\text{in}}(\text{dB}) = 69)\), and the AGC voltage varied from -0.82 to -7.03 volts \((\Delta AGC = 6.21 \text{ volts})\), as shown in Figure 1-17. The output power from the 6-dBm power divider varied from -19.6 to -17.49 dBm \((\Delta P_{\text{out}}(\text{dB}) = 2.11)\) over the AGC range, as shown in Figure 1-18. The nonlinearities for inputs larger than -10 dBm are due to the self-AGC effects of the input signal on the gain (the signal is large enough to have a controlling effect on gain). This condition must be avoided for linear operation (self-AGC will be discussed shortly). To avoid any effects of self-AGC, only inputs from -69 to -10 dBm will be used. Using Figures 1-17 and 1-18,

\[
\Delta P_{\text{in}}(\text{dB}) = -59 \text{ dB}
\]

\[
\Delta AGC = 5.64 \text{ V}
\]

\[
\Delta P_{\text{out}}(\text{dB}) = 2.11 \text{ dB}
\]
FIGURE 1-17. Input Power Versus AGC Voltage (LPF, Linear Detector).

The predicted change in output power is (Equation (1-78))

\[ \Delta P_{o, lin} (\text{dB}) = 20 \log \left( \frac{5.64}{(1X-1X-20)} + 1 \right) = 2.16 \text{ dB} \]  

(1.82)

which is in excellent agreement with the measured value. The normalized video output, \( e_N \), varied from \(-0.895 \text{ volt} \) (AGC = \(-0.82 \text{ volt}\)) to \(-1.143 \text{ volts} \) (AGC = \(-6.46 \text{ volts}\)). Thus the change in \( e_N \) is (in dB)

\[ \Delta e_N (\text{dB}) = 20 \log \left( \frac{1.143}{0.895} \right) = 2.12 \text{ dB} \]  

(1.83)

This is the same as the change in output power, which is to be expected for a linear detector. Thus

\[ \Delta e_{N,lin} (\text{dB}) = 20 \log \left( \frac{\Delta \text{AGC}}{A_{\Delta A_c} e_N} + 1 \right) = \Delta P_{o} (\text{dB}) \]  

(1.84)

The loop rise time was measured for input variations of \( \pm 1 \text{ dB} \) (AGC = 3.1; \( X = 9.1 \text{ dB/V} \)) and was 1.7 seconds. The predicted loop rise time is (Equation (1-44))

\[ t_{r, lin} = \frac{19.12 \frac{R_C}{C}}{X A_{\Delta A_c} e_N} \]  

(1.85)

or

\[ t_{r, lin} = \frac{19.12 \left(1 \times 10^6 \times 22 \times 10^{-6}\right)}{(9.1X-1X-20X1)} = 2.31 \text{ seconds} \]  

(1.86)

which is in good agreement with the measured value.

The loop gain was measured to be 22 (AGC = \(-3.1, X = 9.1 \text{ dB/V}\)). The predicted loop gain is (Equation (1-29))

\[ L_{G, lin} = 0.115 X A_{\Delta A_c} e_N \]  

(1.87)
or

\[ LG_{Lin} = 0.115(9.1 - 1 - 20)(1) = 20.93 \] (1.88)

which is in excellent agreement with the measured value.

The linear detector integrator AGC loop is illustrated in Figure 1-15, with \( R_F \) removed, \( C = 1 \ \mu F \), and \( R_I = 1 \ \Omega \). Figure 1-19 illustrates the input power versus AGC voltage. This curve also deviates from the expected at inputs larger than \(-10\) dBm due to the self-AGC effects.

The power output from the 6-dB power divider varied from \(-18.3\) dBm (\( P_{PP}(\text{dBm}) = 6.7 \text{ dBm} \)) at \(-65\) dBm input power to \(-18.53\) dBm (\( P_{PP}(\text{dBm}) = 6.47 \text{ dBm} \)) at \(-10\) dBm input power. Thus the output power changed:

\[ \Delta P_o(\text{dB}) = 6.7 - 6.47 = 0.23 \text{ dB} \] (1.89)

which is very close to the 0-dBm change predicted. The normalized video voltage, \( e_N \), varied from \(-1.042\) to \(-1.018\) volts over the same input power range. Thus,

\[ \Delta e_N(\text{dB}) = 20 \log \frac{1.042}{1.018} = 0.2 \text{ dB} \] (1.90)

which is similar to the change in output power, as is to be expected with the linear detector (\( \Delta e_N(\text{dB}) = \Delta P_{PP}(\text{dB}) \)).

The loop rise time measured for an AGC voltage of \(-3.1 \ (X = 9.1 \text{ dB/V}) \) at \( \pm 1 \) dB input power deviation was 1.8 seconds. The predicted value (Equation (1.85)) is

\[ \tau_r = \frac{19.12(1 \times 10^6)(1 \times 10^{-6})}{(9.1)(1)(-1)(-1)} = 2.1 \text{ seconds} \] (1.91)

and is in excellent agreement with the measured value.
The measured loop gain is 0.0147 for a modulation frequency of 10 Hertz. The predicted value (Equation (1-42)) is

$$LG_{Lin} = \frac{0.018 X A \Delta A_e e_N}{f RC}$$  \hspace{1cm} (1-92)

or

$$LG_{Lin} = \frac{0.018(9.1 \times 1 \times 1 \times 1)}{10(1 \times 10^{-6})} = 0.0164$$  \hspace{1cm} (1-93)

which is also in excellent agreement with the measured value.

A series switch was placed between the differencing amplifier, $A_\Delta$, and the integrator to verify Equations (1-48) and (1-49). A pulse repetition frequency of 1 kilohertz and update time, $T_u$, of 500 microseconds were used. The duty cycle is thus

$$D = \frac{T_u}{PRI} = T_u (PRF)$$  \hspace{1cm} (1-94)
or

\[ D = (500 \times 10^{-6})(1 \times 10^3) = 0.5 \]  

(1.95)

The expected loop rise times are, from Equations (1-48) and (1-49),

\[ t_{r,St} = \frac{9.56 \, RC}{X A \Delta A \epsilon_n D} \]  

(1.96)

\[ t_{r,lin} = \frac{19.12 \, RC}{X A \Delta A \epsilon_n D} \]  

(1.97)

or the loop rise times should double for a duty cycle of 0.5. Measured results show exact agreement.

It should be noted that a holding capacitor (sample-hold circuit) is needed to hold the error voltage, \( e_c \), for the low-pass-filter AGC in pulse-AGC applications (Figure 1-7).

Figures 1-20, 1-21, and 1-22 summarize the AGC loop design equations for easy reference.

A basic understanding of various methods of controlling amplifier gain will now be presented.

Variable Gain Elements

This section will present a basic discussion of several modern variable gain techniques. The approach to vary IF/RF amplifier gain in years past (early 1960s) was to vary the operating point of the amplifying transistor (forward
\[ e_N = A_p \delta_D \Delta P_{in} \text{dBm} = \frac{\Delta P_{in} \text{dBm}}{X} \]

**STATIC REGULATION**

\[
\Delta P_{PD} \text{dBm} = 10 \log \left[ \frac{\Delta AGC}{A_A A_D A_T} \right] + 1
\]

\[
\Delta P_{PD} \text{dBm} = 10 \log \left[ \frac{\Delta AGC}{A_A A_D A_T} \right] + 1
\]

\[
\frac{-P_{PD} \text{dBm}}{10} = \frac{\Delta P_{PD} \text{dBm}}{10}
\]

**DYNAMIC REGULATION**

\[
LG = 0.23 A_A A_D A_T D + 0.23 \times 10^{3} A_T K_{sl} A_A A_D A_T
\]

\[
A_A A_D A_T = 0.35 LG = 0.35 \times 10^{3} \text{dBm}
\]

**LOOP RISE TIME AND BANDWIDTH**

\[
\tau = \frac{2.2 \times 10^{3} RC}{K_{sl} A_A A_D A_T}
\]

\[
f_{3dB} = \frac{0.35 \times 10^{3} \text{dBm}}{R_C}
\]

*It is assumed that the loop frequency response is determined by the low-pass filter.*

**FIGURE 1-20. Low-Pass Filter, Square Law Detector Design Summary.**
\[ e_N = A_N e_i \], \[ \Delta e_N (dR) = \Delta P_{11}(dR), \] \[ \Delta AGC = \frac{\Delta P_{in}(dR)}{X}, \] \( D = \) low-pass filter update duty cycle

**STATIC REGULATION**

\[ \Delta P_{11}(dR) = 20 \log \left[ \frac{\Delta AGC \times \Delta e_i}{A_N A_s A_c e_D} \right] \]

\[ \Delta P_{11}(dR) = 20 \log \left[ \frac{\Delta AGC \times 10^{\Delta e_i}}{K_{Lin} A_N A_s A_c} \right] \]

**DYNAMIC REGULATION**

\[ P_{pin}(dBm) \]

\[ LG = 0.115 X A_N A_s A_c e_D = 0.115 \times 10^{-1} X K_{Lin} A_N A_s A_c 10^{20} \]

\[ \Delta P_{pin}(dRm) \]

\[ LG = \frac{8.7 \times K_{Lin} A_N A_s A_c 10^{20}}{X e_D} \]

\[ LG = \frac{0.115 \Delta P_{pin}(dR)}{\Delta P_{pin}(dR)} \]

**LOOP RISE TIME AND BANDWIDTH** *(f3dB)*

\[ \tau = 2.2 RC \]

\[ \tau = \frac{19.1 \times 10^3 RC}{X K_{Lin} A_N A_s A_c D} \]

\[ f_{3dB} = \frac{0.35}{\tau} = \frac{0.159(XGID)}{RC} \]

*It is assumed that the loop frequency response is determined by the low-pass filter.*

**FIGURE 1-21. Low-Pass Filter, Linear Detector Design Summary.**
\[ e_N = A_v e_D \]
\[ \Delta AGC = \frac{\Delta P_{in}(dB)}{X} \]
\[ D = \text{integrator update duty cycle} \]

**STATIC REGULATION**
\[ \Delta P_{PL}(dB) = 0 \]
\[ \Delta e_N(dB) = 0 \]

**DYNAMIC REGULATION**
\[ L_{GSL} = \frac{0.036 X A_d A_v e_N}{fCR} \]
\[ L_{GLin} = \frac{0.018 X A_d A_v e_N}{fCR} \]

**LOOP RISE TIME AND BANDWIDTH* (f_{3dB})**
\[ t_{r,SL} = \frac{9.56 R C}{X A_d A_v e_N D} \]
\[ t_{r,Lin} = \frac{19.17 R C}{X A_d A_v e_N D} \]
\[ f_{3dB} = \frac{0.35}{t_r} \]

*It is assumed that the loop frequency response is determined by the integrator (the frequency response of the AGC elements and associated circuitry much greater than the unity loop gain frequency).*

**FIGURE 1-22. Integrator Design Summary.**
and reverse AGC). This method has two drawbacks that become serious in modern high-performance systems:

1. The dynamic range of the transistor is altered as the operating point changes.

2. Changes in the transistor's frequency characteristics occur, which make phase and amplitude tracking most difficult.

In general, there exists an operating point at which a transistor (Bipolar or FET) exhibits its best performance (low noise, bandwidth, gain, etc.), and any deviation from this operating point will result in degraded performance.

What, then, can the designer do if a variable gain is desired without changing the bias of the amplifying transistor? The answer, obviously, is to change something else.

**P-N Junction Diodes as Variable Gain Elements**

If a diode is included in either the emitter or collector of a transistor, changing the diode voltage or current will vary its impedance, and thus change the circuit's gain without affecting the transistor itself. Furthermore, depending upon whether the diode is placed in the emitter or collector leg, and whether it is controlled by a current or a voltage, four different gain versus control-signal transfer characteristics can be obtained: linear, logarithmic, hyperbolic, and inverse logarithmic.

The first step in implementing this scheme is to determine the dynamic resistance of the diode as a function of its applied voltage or current. The voltage across a forward-based diode is given by

\[ V_F = \eta V_T \ln \left( \frac{I_F}{I_s} \right) + I_F R_s \]  

(1.98)
where

\[ V_F = \text{diode forward voltage drop} \]
\[ \eta = \text{diode constant (temperature insensitive)} \]
\[ \eta \approx 2 \text{ for silicon} \]
\[ \eta = 1 \text{ for Schottky} \]
\[ V_T = \frac{KT}{q} \]
\[ K = \text{Boltzmann’s constant} \]
\[ T = \text{absolute temperature} \]
\[ q = \text{electron charge} \]
\[ I_F = \text{diode forward current} \]
\[ I_s = \text{reverse saturation current (temperature sensitive)} \]
\[ R_b = \text{ohmic (bulk) resistance} \]

If Equation (1-98) is differentiated with respect to \( I_F \), the dynamic resistance (\( r_d \)) is obtained

\[ r_d = \frac{\eta V_T}{I_F} + R_s \] (1-99)

or, as a function of forward voltage,

\[ r_d = \frac{\eta V_T}{I_s} \exp \left( \frac{-V_F}{\eta V_T} \right) + R_s \] (1-100)

For modern diodes, the bulk resistance, \( R_s \), is on the order of 1 ohm and can be neglected in both equations.

To make use of Equations (1-99) and (1-100), the two constants \( I_s \) and \( \eta \) must be determined. \( I_s \) is found by examining a plot of \( \log I_F \) versus \( V_F \) (Figure 1-23) and extrapolating the straight portion (logarithmic region) until it intersects the ordinate. Alternatively, the reciprocal slope, \( m \), of the straight portion can be measured and then, using any voltage current pair, \( V_x \) and \( I_x \), \( I_s \) is given by

\[ \log I_s = (m \log I_x - V_x) \] (1-101)
HOW TO CALCULATE $I_s$: THE RECIPROCAL SLOPE, $m$, OF THE STRAIGHT PORTION OF THIS LOG $I$ VS. $V_F$ CURVE IS SEEN TO BE 0.2 VOLT PER DECADE. FOR A $V_x$ OF, SAY, 0.4 VOLT, $I_x$ IS $10^{-3}$ A, HENCE LOG $I_x = -3$. SUBSTITUTING THESE NUMBERS INTO EQUATION (1-101), A VALUE OF $-5$ IS FOUND FOR LOG $I_s$, HENCE, $I_s = 10^{-5}$ µA AS VERIFIED BY THE EXTRAPOLATED (DASHED) LINE.

FIGURE 1-23. Calculation of $I_s$.

The slope, $m$, is measured in volts per decade of current. It should be noted that the curve will change with temperature.

$I_s$ is a strong function of temperature while $\eta$ is relatively unaffected by it. Furthermore, it is found that $\eta$ is quite constant for a whole diode family, although $I_s$ may vary greatly from unit to unit.

Most amplifier gains can be expressed in the form $A_v = R_C/R_E$ where $R_C$ and $R_E$ are collector and emitter resistances, respectively. Therefore, a
variable-resistance diode placed across one of these gain-determining resistors may be used to control the amplifier's gain.

The four basic configurations shown in Figure 1-24 illustrate the flexibility of the technique [2].

One problem that this diode technique shares with more conventional methods of gain control is the undesired changing of the circuit gain by the gain-controlled signal itself (self-AGC). If the ac signal appearing across the gain-controlling diode is too large, it will strongly affect the circuit gain.

While this effect cannot be eliminated, it can be kept within set limits by making sure that the ac signal across the diodes does not exceed

\[ e_d = m \log \left( \frac{A_v}{A_x} \right) \]  

where

- \( e_d \) = the (instantaneous) value of the ac signal voltage across the diode
- \( m \) = the reciprocal slope discussed in Figure 1-23
- \( A_v \) = the desired gain of the amplifier
- \( A_x \) = the minimum acceptable gain including the self-AGC effect.

Consider, as an example, a gain-controlled amplifier biased to provide a voltage gain of 10 (20 dB). Suppose that loss of 1 dB due to self-AGC is the maximum that can be tolerated; this would reduce the voltage gain to 8.9 (19 dB). If the diode slope, \( m \), equals 120 mV/decade, then

\[ e_d = 0.12 \log \left( \frac{10}{8.9} \right) = 6 \text{ mV} \]

Thus the maximum ac signal across the diode should not exceed 6 mV.
FIGURE 1-24. Four Basic P-N Diode AGC Configurations.
It is often necessary to have several amplifiers track in gain and phase. If the variable gain control is a current, little problem in diode matching for amplitude tracking arises.

The Four Basic P-N Diode AGC Configurations

The four circuits shown in Figure 1-24 demonstrate the flexibility of the diode gain-control approach.

The gain equations all assume a transistor alpha of unity. Deviations from the ideal response at low values of $r_d$ are caused by diode saturation. At the other end of the scale, deviations arise as $r_d$ approaches the value of the bypassed resistance.

For the cases of the emitter-coupled diode (linear and logarithmic functions) $R_E'$ is the parallel combination of $R_E$ and $r_d$. When the diode is effectively across the collector resistor (hyperbolic and inverse log functions) $R_C'$ represents the parallel combinations of $R_C$ and $r_d$.

A good working estimate of the gain temperature coefficient for the voltage-controlled cases is $+2$ mV/°C. This means that, for decreasing temperatures, increasing the applied voltage by 2 mV/°C will keep the gain constant. For the current-controlled cases, the temperature coefficient is approximately $-0.17\%$/°C. Thus, for decreasing temperatures, decreasing I by 0.17%/°C will provide constant gain. (These numbers assume silicon diodes.)

Equation (1-99) indicates that the diode dynamic resistance, $r_d$, when driven from a current source, is not a function of $I_s$. Since $\eta$, for a given diode family, is quite constant and the diodes can be colocated to ensure they are at the same temperature, little problem in diode matching is encountered; simply ensure the diode slopes, $m$, meet the following criterion:
Thus, to ensure a 1-dB gain match, the diode slopes \( \frac{\Delta V_{F}(mV)}{\text{Decade } I_F} \) must be

\[
\frac{m_A}{m_B} = 10^{\frac{\text{dB match}}{20}}
\]

or the slopes must be matched to within 11.2\%, which is quite easily accomplished.

If the control signal is a voltage, however, the diode resistance will depend on \( I_s \), which is most temperature sensitive and varies from diode to diode. To ensure a given match, the diodes' forward voltage must be matched to within a specified limit, \( \Delta V_F \).

A simple equation relating \( \Delta V_F \) for a given gain match may be given as

\[
\Delta V_F = (\eta T)(\text{dB Match}) \times 10^{-5}
\]

Consider that two amplifiers must gain track to within 1 dB. Assuming Schottky diode (\( \eta = 1 \)) the necessary forward voltage match, \( \Delta V_F \), as a function of temperature, is given in Table 1-7.

To ensure proper phase tracking, the variable gain amplifiers must have a much wider bandwidth than the IF center frequency, and the variable diode

<table>
<thead>
<tr>
<th>Temperature, °C (°K)</th>
<th>( \Delta V_F ), mV</th>
</tr>
</thead>
<tbody>
<tr>
<td>+100 (373)</td>
<td>3.7</td>
</tr>
<tr>
<td>+27 (300)</td>
<td>3.0</td>
</tr>
<tr>
<td>−50 (223)</td>
<td>2.23</td>
</tr>
</tbody>
</table>
resistance should not have any affect on any frequency- or bandwidth-determining elements. A simple method to minimize any frequency or bandwidth degradation is to use variable attenuators following broadband fixed-gain stages as illustrated in Figure 1-25.

To ensure that, in an AGC loop, the low-pass filter or integrator (Figure 1-2) determines the frequency response, all capacitors must be small to ensure that the variable attenuation response is much faster than the desired AGC response time.

Figure 1-26 illustrates a block diagram of a commercial 60-MHz variable gain IF amplifier using P-N junction-diode controlled attenuators. Figure 1-27 illustrates the gain versus AGC voltage, and, as can be seen, the slope is fairly constant for gains from 10 to 55 dB. The self-AGC (for a 1-dB gain compression), as a function of gain and input power, is illustrated in Figure 1-28. The importance of this curve is that it clearly illustrates that a maximum output of $-15$ dB is permissible for a gain of 0 dB. Thus, over the full variable gain range of 60 dB, this amplifier does not have the output capability to drive a detector into its linear region. Thus the amplifier is suited only for square law detectors.
FIGURE 1-26. IF Amplifier Configuration.

FIGURE 1-27. IF Amplifier Variable Gain Characteristics (Commercial 60-MHz Variable Gain Amplifier).
P-N junction diodes have two serious limitations in modern EW systems: (1) self-AGC starts at low input voltage levels, and (2) they are generally frequency-limited to IFs of less than 100 MHz. The PIN diode, however, overcomes these limitations.

PIN Diodes as Variable Gain Elements

The PIN diode appears as an almost pure resistance at frequencies up to several thousand MHz, and is similar to the P-N junction diode in that this
resistance is a function of bias current or voltage. The general circuit concepts discussed earlier are valid for PIN diodes; however, they are most used as variable resistance elements in variable attenuators, usually in the \( n \), \( T \), or bridged \( T \) configuration, as illustrated in Figure 1-29. The designers of voltage variable attenuators using PIN diodes ensure that the 50 \( \Omega \) input and output impedances are fairly insensitive to attenuation level.

The variable resistance of PIN diodes behaves in a manner similar to that of P-N junction diodes, in that it is an exponential function of voltage and a linear function of current. Also, the variable resistance is a strong function of temperature when driven from a voltage source, and relatively constant when driven from a current source.
Figure 1-30 illustrates the variable resistance versus forward current for a commercial PIN diode suited for use in variable attenuators [3, 4, and 5]. The variable resistance, $r_{PIN}$, may be given as

$$r_{PIN} = \frac{K_{P1}}{K_{P2}}$$

where $K_{P1}$ and $K_{P2}$ are PIN diode constants and can be determined as illustrated in Figure 1-31.

The ease of using Figure 1-31 will become apparent by finding $r_{PIN}$ for the "typical" resistance characteristic illustrated in Figure 1-30:

$$I_{FA} = 10\mu A \quad r_{PINA} = 900\Omega \quad I_{FH} = 1\ mA, r_{PINA} = 15\Omega$$

![Figure 1-30. Variable Resistance Characteristics for an HP 5082-3004 PIN Diode.](image-url)

\[
K_{p2} = \frac{\log(\frac{900}{15})}{\log(\frac{1 \times 10^{-3}}{10 \times 10^{-6}})} = 0.889
\]

(1.108)

\[
K_{p1} = (900)(10 \times 10^{-6})^{0.889} = 0.032
\]

(1.109)

Thus the variable resistance for the "typical diode" is

\[
r_{PIN} = \frac{0.032}{I_F^{0.889}}
\]

(1.110)

Figure 1-32 illustrates the basic configuration for a commercial n attenuator using PIN diodes. The variable attenuation characteristics are illustrated in Figures 1-33 and 1-34. Figure 1-35 illustrates the self-AGC effects, and as will be noted, there is little self-AGC for input powers less than 0 dBm. The PIN diode is far superior to P-N junction diodes with respect to self-AGC.
FIGURE 1-32. WJ G-1 PIN Attenuator.

FIGURE 1-33. Voltage Drive Characteristics for PIN Diode Attenuator.
FIGURE 1-34. Current Drive Characteristics for PIN Diode Attenuator.

FIGURE 1-35. Attenuation Versus Input Power (Self-AGC Effect).
Figure 1-36 illustrates the effect of frequency on the variable attenuation characteristics of the PIN diode attenuator. The importance of this figure is that it is an indicator of reactive (rather than pure resistance) effects. For frequencies below 1,000 MHz, Figure 1-36 indicates that attenuations of less than 20 dB are due to the PIN diode resistance. This insensitivity to frequency greatly eases the amplitude and phase tracking from unit to unit. Amplitude tracking to within ±1 dB, and phase tracking to within less than ±5 degrees, is not difficult to obtain if the attenuation is limited to less than 20 dB and is via current control.

GaAs FETs as Variable Gain Elements

The advent of GaAs monolithic microwave integrated circuits (MMIC) extends the variable pure resistance needed for frequency insensitive variable attenuators to more than 10 GHz. Attenuator configurations using GaAs
FETs are similar to those illustrated in Figure 1-29; however, unlike PIN diodes, the FETs are monolithically integrated and very close matching would be expected.

Figure 1-37 [6] illustrates a modern voltage variable attenuator utilizing GaAs MMIC technology. The attenuator and operational amplifier within the box compare a reference attenuator that ensures a 50 Ω input and output impedance independent of attenuation. Assuming close matching between the reference and signal attenuators, the RF input sees a 50 Ω impedance (with RF output terminated in 50 Ω) independent of attenuation. The linearizing circuit compensates for the nonlinear attenuation versus control voltage (FETs are voltage-controlled, not current-controlled, devices). Figure 1-38 illustrates the variable attenuation characteristics.

Self-AGC effects depend upon FET design and geometry. Maximum input power of 0 dBm is typical (for 0.5-dB change in attenuation due to self-AGC).
FIGURE 1-38. WJ-RG45 Variable Attenuation Characteristics.
Matching amplitude and phase is not as simple as one would expect for a monolithic design; in fact, it is easier to match PIN diode attenuators (amplitude less than ±1 dB, phase less than ±5 degrees) than GaAs FET attenuators. This may change as GaAs MMIC technology matures.

Many modern EW receivers are designed using commercially available wide-bandwidth fixed-gain amplifiers (which can be chosen for optimum noise figure, output power, etc.) and variable attenuators that can be positioned to optimize the linear dynamic range. The desired bandwidth and center frequency can be obtained with properly placed filters. A typical monopulse receiver employing variable gain may well appear as shown in Figure 1-39. Amplitude and phase matching can be accomplished by testing at the component level.

Applications

Several applications will now be presented to illustrate the design concepts previously described. Only the basic circuits will be presented since a

![Diagram](image-url)

**FIGURE 1-39. Basic Monopulse Receiver.**
complete design, for even a simple AGC loop, would require more in-depth linear circuit design background than can be presented here.

To illustrate the design procedure, we will first consider a simple 30-MHz CW IF amplitude nulling loop.

A 30-MHz CW AGC loop is desired that will give a $-15$ dBm ($\pm 0.25$ dB) output. The loop is to operate in the square law region of the detector diode, and inputs to the IF amplifier are $-60$ dBm minimum and $-10$ dBm maximum. The loop should be of the low-pass filter design, with a closed loop bandwidth of at least 5 kHz (or loop rise time of 70 $\mu$s).

The variable gain characteristics for the 30-MHz amplifier is illustrated in Figure 1-40, and, as can be seen, the variable gain slope is nearly constant at $X = 70$ dB/V.

**FIGURE 1-40. IF Gain Versus AGC Voltage (30-MHz IF Amplifier).**
A back diode [7] will be used because of its near zero dc offset, thus minimizing the need for temperature stabilization. The diode constant for the detector (Ge BD-2) is

\[ K_{SL} = 500 \text{ mV/mW} \tag{1.111} \]

Figure 1-41 illustrates the AGC loop and \( A_c \), \( A_\Delta \), and \( A_V \) have been combined to minimize parts. Since an output power of \(-15 \text{ dBm}\) is desired, the power divider ensures the detector is also at \(-15 \text{ dBm}\) (this power is within the detector's square law region). The IF amplifier's output will be 6-dB above the output power, \( P_{IF} = -9 \text{ dBm} \), which is well within the power output capabilities of the amplifier.

![Diagram of Single Operational Amplifier 30-MHz AGC Loop](image)

FIGURE 1-41. Single Operational Amplifier 30-MHz AGC Loop.

Referring to Figure 1-20, the value for \( A_V A_\Delta A_c \) may be found

\[
A_x A_\Delta A_c = \frac{-P_o(dBm)}{\frac{10^{\Delta P_{in}(dB)} \times 10^3}{10}} \times K_{SL} \tag{1.112}
\]
or

\[
A_e A_s A_r = \frac{50 \times 10^3 \times 10^{-10}}{0.5} \frac{0.5}{(70)(500)} \left| \frac{10^{10} - 1}{10^{10}} \right|
\]

Thus,

\[
A_e A_s A_r = 370
\]

We will use a gain of 392 (392 kΩ feedback resistor, 1 kΩ series resistor). The feedback capacitor needed for a loop rise time of 70 μs (5 kHz closed loop bandwidth) may be given as (Figure 1-20)

\[
C = \frac{1}{r_e N K_s A_s A_r A_r} \frac{10^{-6}}{P_0 (\text{dBm})} 9.56 \times 10^3 R \left| \frac{10^{10}}{10^{10}} \right|
\]

or

\[
C = \frac{(70 \times 10^{-6})(70)(500 \times 392 \times 1)}{0.5} \frac{10^{-15}}{9.56 \times 10^3 (392 \times 10^3) 10^{10}}
\]

and

\[
C = 0.0081 \mu F \text{ (let } C = 0.01 \mu F)\]

61
$E_{Ref}$ in Figure 1-41 is adjusted for $P_0 = -15$ dBm at an input power of $-35$ dBm. Figure 1-42 illustrates the AGC voltage versus input power; inputs larger than $-10$ dBm have a controlling effect on the IF amplifiers' gain (self-AGC).

![AGC Voltage Versus Input Power](image)

**FIGURE 1-42. AGC Voltage Versus Input Power (30-MHz Continuous Wave AGC).**
Figure 1-43 illustrates the change in output power versus input power. The output power changes from $-14.6$ to $-15.2$ dBm ($P_{in} = -10$ dBm to $P_{in} = -60$ dBm) or $\Delta P_o$ (dB) = 0.6 dB, which is very close to our design value of 0.5 dB. The loop rise time was measured (with ±1 dB input steps) and was found to be 65 µs, which is very close to our design value. The measured loop gain is 90, with a predicted value (Figure 1-20) of

$$LG = 0.23 \times 10^{-3} K_{St} X A_s A_r A_r 10^{\frac{P_{PIL\text{dBm}}}{10}}$$

or

$$LG = (0.23 \times 10^{-3}) (500) (70) (392) 10^{\frac{-15}{10}}$$

or

$$LG = 99.8$$

which is very close to the measured value.

FIGURE 1-43. Output Power Versus Input Power.
This example was chosen not for its state-of-the-art importance, but to illustrate the accuracy afforded by the equations presented. The circuitry used to achieve a given AGC loop is entirely up to the designer.

An AGC loop for a pulsed conical scanning radar receiver will now be presented.

**Conical Scanning AGC Loop**

The AGC loop illustrated in Figure 1-44 is typical of many conical scanning radar receivers. The AGC loop keeps the video \( e_N \) constant for slowly varying intensity changes. However, the AGC loop must have minimum effect at the conical scanning frequency to avoid any plane rotation. Figure 1-45a illustrates a typical conical scanning radar. The angle return is dependent on the modulation envelope of the return pulses (Figure 1-45b) and phase, with respect to a reference voltage (Figure 1-45c). The resultant azimuth and elevation signals are defined in Figure 1-45d.

\[
A_E = -\frac{R_f}{R_i}
\]

**FIGURE 1-44. Conical Scanning Automatic Gain Control Loop.**
FIGURE 1-45. Typical Conical Scanning Radar.
The information necessary for angle tracking is usually obtained by comparing the sampled received signal with a reference signal (Figures 1-45b and 1-45c); thus, the signal modulation envelope should have minimum AGC at the scanning frequency. (If AGC were applied at the scanning frequency, no modulation envelope would result, due to the inherent input modulation reduction characteristics of the AGC loop.) Furthermore, any phase shift of the received modulation envelope must be kept small to avoid cross talk between the azimuth and elevation angle loops (plane rotation). The loop gain at the scanning frequency must be much less than unity.

Assuming a linear detector, the modulation ratio of the normalized video ($M_{eN}$) to the intermediate frequency (IF) input ($M_{ein}$) may be given as [8]:

$$\frac{M_{eN}}{M_{ein}} = \frac{1}{1 + LG \cdot Y_{AGC}(f)}$$  \hspace{1cm} (1.121)

where

$LG =$ low frequency loop gain (Figure 1-21)

$Y_{AGC}(f) =$ filter and sample hold transfer function at frequency $f$

Since $Y_{AGC}(f)$ is a complex quantity, $M_{eN}$ will also be complex. In certain conical scanning radar receivers, the scan modulation is extracted from the sample/hold output, and it is important that the phase shift between the input modulation and output modulation be insensitive to any dynamic receiver parameters, or the coordinates into which the target alignment is resolved will not coincide with the vertical and horizontal coordinates. (Any fixed phase shift can easily be compensated by a phase shift circuit, as illustrated in Figure 1-44.)

The low frequency loop gain for the AGC configuration of Figure 1-44 may be given as (Figure 1-21)

$$LG_{Lin} = 0.115 \times A \Delta \alpha e N$$  \hspace{1cm} (1.122)
The static regulation for a linear detector may be given as (Figure 1-21)

\[
\Delta e_N(dB) = 20 \log \left( \frac{\Delta P_{in}(dB)}{X A_A A_c e_N} + 1 \right) \tag{1-123}
\]

where

\[
\Delta e_N(dB) = \text{decibel change in normalized video for a given change in } \Delta P_{in}(dB).
\]

Equation (1-123) may be given as

\[
\Delta e_N(dB) = 20 \log \left( \frac{\Delta AGC}{A_A A_c e_N} + 1 \right) \tag{1-124}
\]

where

\[
\Delta AGC = \frac{\Delta P_{in}(dB)}{X} \tag{1-125}
\]

Equation (1-124) may now be solved in terms of \(A_A A_c e_N\) required for a desired \(\Delta e_N(dB)\) and a specified \(\Delta AGC\):

\[
A_A A_c e_N = \frac{\Delta AGC}{\left( \frac{\Delta e_N(dB)}{20} - 1 \right)} \tag{1-126}
\]

Now, since \(A_A A_c e_N\) is known for given static regulation requirements, the loop gain is also uniquely determined from Equation (1-122).

The plane rotation as a function of AGC loop dynamics is fairly complicated. However, provided that the signal PRF is much larger than the conical scanning frequency, the plane rotation may be reasonably approximated as [8]

\[
\theta_{PR} = \tan^{-1} \left( \frac{L G f_{\Delta dB}^{(LPF)}}{f_{es}} \right) \tag{1-127}
\]
where

\[ \theta_{PR} = \text{plane rotation due to AGC loop} \]

\[ f_{cs} = \text{conical scanning frequency} \]

\[ f_{3dB}^{\text{(LPF)}} = \text{low pass 3-dB frequency response necessary for given } \theta_{PR}. \]

Solving Equation (1-127), \( f_{3dB} \) may be given as

\[ f_{3dB}^{\text{(LPF)}} = \frac{f}{LG} \tan \theta_{PR} \quad (1.128) \]

A practical design example will now be given.

Design parameters: minimum input power for AGC action = -60 dBm, maximum input for linear operation = -10 dBm, conical scanning frequency = 175 Hz, plane rotation = 5 degrees, \( \Delta e_N = \pm 1 \text{ dB}, \) instantaneous input dynamic range = \( \pm 10 \text{ dB} \).

To obtain a \( \pm 10 \text{ dB} \) instantaneous input dynamic range, a linear detector will be utilized (\( \Delta e_N(dB) = \Delta P_{in}(dB) \)). Figure 1-46 illustrates the linear characteristics for a Schottky diode, and, as can be seen, the detector output is quite linear (dB out versus dB in) for inputs greater than -3 dBm. A normalized value of \( P_{PP}(dBm) = +7 \) will be used (\( P_{PP}(\text{max}) = +17 \text{ dBm}, P_{PP}(\text{min}) = -3 \text{ dB} \)). PIN diode attenuators (with a linearizing circuit) will be utilized to obtain the necessary gain variation as illustrated in Figure 1-47.

Figure 1-48 illustrates the basic AGC loop. A normalized video voltage (\( e_N \)) of -3 volts will be used. Thus, for a normalized \( P_{PI}(dBm) = +7 \text{ dBm} \) (Figure 1-46)

\[ A = \frac{2}{0.6} = 5 \quad (1.129) \]
2.0

\[ K_{\text{LIN}} = e_D(\text{mV}) \times 10 \]

\[ K_{\text{LIN}} = (460 \text{ mV}) \times 10^{-5} = 258 \text{ mV/mW} \]

\[ e_D = 460 \text{ mV} \]

\[ P_{\text{PD}} = 5 \]

FIGURE 1-46. Linear Detector Characteristics.
FIGURE 1-47. Basic Conical Scanning Receiver.

FIGURE 1-48. Basic Conical Scanning AGC Loop.
Figure 1-49 illustrates the AGC characteristics for the receiver \((A_v = 5\) and \(e_N = 3\) volts). The variable gain slope, \(X\), is \(5\) dB/volt, and \(A\Delta A e_N\) (Equation (1-126)) may now be found

\[
A\Delta A e_N = \frac{10}{10^{20} - 1} = 38.6
\]

and \(A_e\) is (for \(A\Delta = 1\))

\[
A_e = \frac{38.6}{10^3} = 12.9
\]
The loop gain necessary for our static regulation is (Equation (1-122))

\[ LG = (0.115)(5)(38.6) = 22.2 \]  

(1-132)

The low-pass filter 3-dB frequency response \( f_{3dB}(LPF) \) necessary to ensure the loop gain is low enough for the desired 5-degree plane rotation is (Equation (1-128))

\[ f_{3dB}(LPF) = \frac{175}{22.2} \tan 5 = 0.69 \text{ Hz} \]  

(1-133)

The values for \( R_f, C_f \) and \( R_I \) (Figure 1-48) will now be found

\[ f_{3dB}(LPF) = \frac{1}{2\pi R_f C_f} \]  

(1-134)

or

\[ R_f C_f = \frac{0.159}{f_{3dB}(LPF)} = 0.23 \]  

(1-135)

Letting \( C = 6.6 \mu F \),

\[ R_f = \frac{0.23}{6.6 \times 10^{-6}} = 34.8 \text{ k}\Omega \]  

(1-136)

and \( R_I \) (for \( A_c = 12.9 \)) is

\[ R_I = \frac{R_f}{A_c} = \frac{34.8}{12.9} = 2.71 \text{ k}\Omega \]  

(1-137)
Figure 1-50 illustrates the receiver and AGC loop characteristics. As can be seen, \( \Delta e_N(dB) \) is quite close to our design value (± 1 dB). The sample/hold signal is held for the pulse repetition interval, thus the duty cycle \( (D) \) is unity, and the predicted rise time (Figure 1-21) is

\[
\tau_r = \frac{2.2RC}{LG} = \frac{2.2(34.8 \times 10^3 \times 6.6 \times 10^{-6})}{22.1}
\]

or

\[
\tau_r = 23 \text{ msec}
\]

which compares favorably with the measured value of 27 msec.

**FIGURE 1-50. Receiver/AGC Variable Gain Characteristics.**
The sample/hold process complicates the loop stability, and the loop will oscillate half the sampling frequency (PRF) if the loop gain at this frequency exceeds unity [8].

Optimizing Loop Rise Times

The loop rise time calculations presented are valid only for small steps in input power, typically less than ±2 dB. Tables 1-2 and 1-5 illustrate the dependence of the input step and polarity on the loop rise time. As can be seen, the rise time decreases for positive input steps (increased power), and increases for negative steps (decreased power). This nonlinearity is due to the inherent nonlinear properties of the detector. Figure 1-51 illustrates the normalized instantaneous detector output for a square law detector, using

\[
ed_{i,Sy} = \frac{10^{\frac{\Delta P_{Pl}(dBm)}{10}}}{10}
\]  

(1.140)

FIGURE 1-51. Instantaneous Detector Voltage Versus \(\Delta P_{in}(dBm)\).
As can be seen, $e_d$ is very nonlinear for $\Delta P_{in}(dB)$ greater than $\pm 2$ dB. Thus the AGC integrator, or low-pass-filter input, will be large for $+\Delta P_{in}(dB)$ and small for $-\Delta P_{in}(dB)$ (e.g., for $\Delta P_{in}(dB) = +5$, $\Delta e_d = 3.2 - 1 = 2.2$, and for $\Delta P_{in}(dB) = -5$, $\Delta e_d = 1 - 0.3 = 0.7$). Thus the rise times for large values of $+\Delta P_{in}(dB)$ should be expected to be shorter than for large values of $-\Delta P_{in}(dB)$.

Certain AGC loops may require a more constant loop rise time with large values of $\pm \Delta P_{in}(dB)$. Two solutions to this problem are apparent from the previous discussion: (1) have a shorter RC time constant for $-\Delta P_{in}(dB)$ steps, or (2) linearize $e_d$ versus $\Delta P_{in}(dB)$. These two techniques will now be presented.

**Variable Time Constant**

Figure 1-52 illustrates a variable time constant circuit. This circuit replaces the differencing circuit and integrator of Figure 1-9. The diode, $D_1$, blocks positive voltages (or $+\Delta P_{in}(dB)$), and the loop behaves as it normally does; however, $D_1$ turns on for $-\Delta P_{in}(dB)$, thus decreasing the integrator's RC time constant ($R_1$ and $R_2$ are now in parallel). The transistor compensates for the 0.6-volt DC diode drop. $R_1$ may now be adjusted to give equal rise times for $+\Delta P_{in}(dB)$. This circuit works for $\Delta P_{in}(dB) = \pm 5$ dBm. A much better solution would be to linearize $e_N$ with respect to $\Delta P_{in}(dB)$.

**Linearized Time Constant**

A logarithmic amplifier preceding the differencing amplifier (Figure 1-53) will linearize $e_N$ with respect to $\Delta P_{in}(dB)$. A logarithmic amplifier has the following characteristics [9]

$$e_N = K_1 \log K_2 (A_v e_d)$$

(1.141)
FIGURE 1-52. Variable Time Constant Circuit.

FIGURE 1-53. AGC Loop Employing Logarithmic Amplifier.

where

\[ K_1, K_2 = \text{logarithmic amplifier constants} \]

Now, for a square law detector,

\[
\begin{align*}
  e_D &= K_{SL} \frac{P_{PD}(\text{dBm})}{10} \\
  e_D &= K_{SL} 10^{\frac{P_{PD}(\text{dBm})}{20}} \quad (1.142)
\end{align*}
\]
and substituting Equation (1-142) into (1-141),

\[ e_N = K_1 \log K_2 A_v \left| \frac{P_{PD}(dBm)}{K_{sl} 10} \right| \]  

(1-143)

or

\[ e_N = \frac{K_1 P_{PD}(dBm)}{10} = K_1 \log (K_2 A_v K_{sl}) \]  

(1-144)

Taking the derivative of Equation (1-144) with respect to \( P_{PD}(dBm) \),

\[ \frac{\Delta e_N}{\Delta P_{PD}(dB)} = \frac{K_1}{10} (V/dB) \]  

(1-145)

or, the logarithmic video output has a constant slope of \( K_1/10(V/dB) \), which is the desired result.

To calculate the static regulation, loop gain, and loop rise time, simply include the logarithmic amplifier in the calculations given in Appendices C, D, and E. Figures 1-54 and 1-55 illustrate the pertinent parameters for the logarithmic amplifier AGC loop.

Figure 1-56 illustrates a basic pulse AGC loop incorporating a logarithmic amplifier for optimizing loop rise times.
\[ X = \text{IF GAIN SLOPE (dB/V)} \]
\[ \text{LS} = \text{LOGARITHMIC SLOPE (V/dBm)} \]
\[ D = \text{UPDATE DUTY CYCLE} \]

**STATIC REGULATION**

\[
\Delta P_{o,\text{St.}}(\text{dBm}) = \frac{\Delta \text{AGC}}{2A_A A_c(\text{LS})} \quad \Delta e_{n,\text{St.}}(\text{dBm}) = \frac{\Delta \text{AGC}}{A_A A_c(\text{LS})}
\]

**LOOP RISE TIME (10 TO 90%)**

\[
\tau_{r,\text{St.}} = \frac{1.05 \cdot RC}{X A_A A_c(\text{LS})D}
\]

**LOOP GAIN**

\[
L_{G,\text{St.}} = 2X A_A A_c(\text{LS})
\]

\[
f_{\text{3dBV}(LG)} = \frac{0.159}{(LG)RC} \quad \tau_r = \frac{2.2 \cdot RC}{(LG)D}
\]

**FIGURE 1-54. Low-Pass Filter and Log Video AGC Design Summary.**
\[ P_o(dBm) \]

\[ P_{IN}(dBm) \]

\[ A_{IF}(dB) \]

\[ A_{PD}(dBm) \]

\[ AGC \]

\[ X = 1F \text{ GAIN SLOPE (dB/V)} \]

\[ LS = \text{LOGARITHMIC SLOPE (dB/V)} \]

\[ D = \text{INTEGRATOR UPDATE DUTY CYCLE} \]

**STATIC REGULATION**

\[ \Delta P_o(dBm) = 0 \]

\[ \Delta e_N(dBm) = 0 \]

**LOOP RISE TIME (10 TO 90%)**

\[ t_{r,St.} = \frac{1.05 RC}{X A_{A_c}(LS)D} \]

\[ t_{r,in} = \frac{2.1 RC}{X A_{A_c}(LS)D} \]

**LOOP GAIN**

\[ L G_{St.} = \frac{0.315 X A_{A_c}(LS)D}{f RC} \]

\[ L G_{in} = \frac{0.158 X A_{A_c}(LS)D}{f RC} \]

\[ (L G)(f) = \frac{0.331}{f} \]

\[ f = \text{INPUT MODULATION FREQUENCY} \]

**FIGURE 1-55. Integrator and Log Video AGC Design Summary.**
Basic Loop Stability

AGC loop dynamics have been given as (Figures 1-5 and 1-6 and Equation (1-13))

\[ M_o = \frac{M_I}{1 + LG(f)} \]  

where

- \( M_o \) = output modulation
- \( M_I \) = input modulation
- \( LG(f) \) = frequency dependent loop gain
Classical feedback theory tells us that the loop gain must be less than unity when its phase shift is 180 degrees. The AGC loop designer must ensure that this condition is satisfied. The loop frequency response is usually dominated by the low-pass filter or integrator; thus, to avoid instability, the variable gain element frequency response must be much larger than: (1) $f_{3dB}(\text{LPF})$ for the low-pass filter and (2) the frequency for unity loop gain for the integrator.
References


Appendix 1A

POWER-VOLTAGE RELATIONSHIPS
FOR A 50-OHM SYSTEM

\[ P = \frac{\left( \frac{e_{\text{rms}}}{R} \right)^2}{R} = 0.02 \left( \frac{e_{\text{rms}}}{R} \right)^2 \quad e_{\text{rms}} = 7.07 \sqrt{P} \]

\[ e(PP) = 2.83 e_{\text{rms}} \quad e(PP) = 20 \sqrt{P} \quad P = \left( \frac{e_{\text{PP}}}{20} \right)^2 \]

\[ e(dBV) = 20 \log e(PP) = 10 \log P + 26 \]

\[ P(dBM) = 10 \log P + 30 \quad P = 1 \times 10^{-3} \quad \frac{P(dBM)}{10} \]

\[ e(dBV) = P(dBM) - 4 \quad P(dBM) = 20 \log e(PP) + 4 \]

\[ e(PP) = 10 \quad \frac{P(dBM)}{10} = (0.63) \frac{P(dBM)}{10} \]

FIGURE 1A-1. Power-Voltage Relationships for a 50-Ohm System.
Appendix 1B

BASIC DETECTOR CHARACTERISTICS

The crystal diode detector converts the IF waveform into a dc waveform for continuous wave AGC, or into pulses for pulse AGC. The basic characteristics for detectors are square law for low input powers (the output voltage increases 2 dBV for each dBm increase in input power) and linear for high input powers (the output voltage increases 1 dBV for each dBm increase in input power). Watson [10] covers detector characteristics in detail; however, several important characteristics pertinent to AGC design will be presented here.

Figure 1B-1 illustrates the input-output characteristics for a typical Schottky barrier detector (HP 5082-2800). The square law characteristics will be covered first, then the linear.

The square law detector characteristics are valid for inputs of -15 dBm and lower. The detector output voltage, $e_D$, may be given as

$$e_D = K_{SL} P_{PD}$$

where

$e_D$ is in millivolts

$P_{PD}$ is in milliwatts

$K_{SL}$ is a square law detector constant

The diode constant, $K_{SL}$, is the detector static gain,

$$K_{SL} = \frac{e_D}{P_{PD}} \left( \frac{mV}{mW} \right)$$
FIGURE 1B-1. Detector Characteristics (see Figure 1-16).

and is fairly constant for detector inputs less than $-15$ dBm. The value of $K_{SL}$ may be found, provided a curve similar to Figure 1B-1 is given.
The detector input power, $P_{PI}$, may be given in terms of dBm as (Appendix 1A)

$$
P_{PI}\text{(dBm)} = P_{PI}(mW) \times 10^\frac{P_{PI}(dBm)}{10} \tag{1B.3}
$$

substituting Equation (1B.3) into (1B.2),

$$
K_{SL} = e_0 \times \frac{-P_{PI}(dBm)}{10} \left( \frac{mV}{mW} \right) \tag{1B.4}
$$

Referring to Figure 1B-1, $K_{SL}$ is (for $P_{PI} = -20$ dBm, $e_0 = 3.5$ mV)

$$
K_{SL} = (3.5) \times 10^\frac{-20}{10} \left( \frac{mV}{mW} \right) \tag{1B.5}
$$

or

$$
K_{SL} = 350 \left( \frac{mV}{mW} \right) \tag{1B.6}
$$

Knowing $K_{SL}$, $e_0$ for a $P_{PI}(dBm)$ may be found by solving Equation (1B.4) for $e_0$:

$$
e_{D,SL} = K_{SL} \times \frac{P_{PI}(dBm)}{10} \left( \frac{mV}{mW} \right) \tag{1B.7}
$$

The dynamic detector gain, $A_{D,SL}(V/V)$, is necessary in calculating the loop gain (Figure 1-6). The detector's dynamic gain will be found for a fixed input, $e_{PP}(PP)$:

$$
A_{D,SL}(V/V) = \frac{d e_{D}}{d e_{PP}(PP)} \left( e_{PP}(PP) \right) = \text{Constant} \tag{1B.8}
$$

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where (Appendix 1A)

\[
\frac{P_{pu(dBm)}}{10} = (0.63) \times 10^{-10}
\]

or

\[
\frac{P_{pu(dBm)}}{10} = (0.63) \times 10^{-10}
\]

and

\[
\frac{P_{pu(dBm)}}{10} = (0.63) \times 10^{-10}
\]

Thus,

\[
\frac{P_{pu(dBm)}}{10} = (0.63) \times 10^{-10}
\]

Substituting Equation (1B-12) into (1B-7),

\[
e_D = \left(2.51\right) K_{SL} \left| e_{pu(PP)}\right|^2 \text{ (mV)}
\]

The dynamic detector gain, \(A_{D,SL}(V/V)\), is

\[
A_{D,SL}(V/V) = \frac{d e_D}{d e_{pu(PP)}}
\]

Thus from Equation (1B-13),

\[
A_{D,SL}(V/V) = 5 \times 10^{-3} K_{SL} e_{pu(PP)}
\]

---

\*\(K_{SL}\) has been defined in terms of \(e_D\) in millivolts \((K_{SL} = e_D(mV)/P_{pu(mV)})\). Thus \(K_{SL}\) must be divided by 1,000 for \(e_D\) in volts.

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Since (Appendix 1A)

\[ e_{PD}(PP) = (0.63) \times 10^{-2} \]  

(1B-16)

\[ P_{PD}^{\text{dBm}} \]

\[ A_{D,SL}^{(V/V)} = 3.76 \times 10^{-3} K_{SL}^{10} \]  

(1B-17)

The dynamic detector gain in V/dBm, \( A_{D,SL}^{(V/dBm)} \), is needed when solving for the loop rise time, \( \tau_r \). This gain may be found by differentiating Equation (1B-7). The solution to this differentiation is

\[ A_{D,SL}^{(V/dBm)} = 0.23 \times 10^{-3} K_{SL}^{10} \]  

(1B-18)

The voltage gains for the diode illustrated in Figure 1B-1 are, assuming \( P_{PD}^{\text{dBm}} = -30 \text{ dBm} \) and \( K_{SL} = 350 \),

\[ A_{D,SL}^{(V/V)} = (3.16 \times 10^{-3})(350) \times 10^{30} = 0.035 \text{ V/V} \]  

(1B-19)

\[ A_{D,SL}^{(V/dBm)} = (0.23 \times 10^{-3})(350) 10^{10} = 80.5 \times 10^{-6} \text{ V/dBm} \]  

(1B-20)

Figure 1B-2 summarizes square law detector characteristics.
Linear detector characteristics extend for input powers larger than −2 dBm for the HP 5082-2800 detector (Figure 1B-1). The output voltage may be given as

$$e_{D,\text{lin}} = K_{\text{lin}} \sqrt{P_{PD}}$$

(1B-21)

where

$e_{D,\text{lin}}$ is in millivolts

$P_{PD}$ is in milliwatts

$K_{\text{lin}}$ is a linear detector constant

FIGURE 1B-2. Square Law Detector Summary.
Solving Equation (1B-21) for $K_{Lin}$,

$$K_{Lin} = \frac{e_{D, Lin}}{\sqrt{P_{PD}}} \left( \frac{mV}{mW} \right)$$  \hspace{1cm} (1B-22)

$P_{PD}$ (mW) may be given as (Appendix 1A)

$$P_{PD}^{dBm} = 10 \log_{10} P_{PD}^{mW}$$  \hspace{1cm} (1B-23)

Substituting Equation (1B-23) into (1B-21)

$$K_{Lin} = \frac{-P_{PD}^{dBm}}{e_{D, Lin} (mV) 10^{-20}}$$  \hspace{1cm} (1B-24)

Referring to Figure 1B-1, for $P_{PD} = +10$ dBm and $e_D = 520$ mV, $K_{Lin}$ is

$$K_{Lin} = (520) 10^{-20}$$  \hspace{1cm} (1B-25)

or

$$K_{Lin} = 164$$  \hspace{1cm} (1B-26)

The dynamic detector gain, $A_{D, Lin} (V/V)$, will be found in terms of $e_{PP} (PP)$

$$A_{D, Lin} (V/V) = \frac{d e_{D, Lin}}{d e_{PP} (PP)}$$  \hspace{1cm} (1B-27)

Using Appendix 1A

$$e_{PP} (PP) = (0.63) 10^{-20}$$  \hspace{1cm} (1B-28)

and substituting Equation (1B-28) into (1B-24), and solving for $e_D$,

$$e_D = 1.59 K_{Lin} e_{PP} (PP) (mV)$$  \hspace{1cm} (1B-29)
The dynamic linear detector gain may now be found (remember \( K_{Lin} \) is defined in terms of \( e_D \) in mV)

\[
A_{D,Lin}(V/V) = 1.59 \times 10^{-3} K_{Lin}
\]

Thus, for the detector characteristics illustrated in Figure 1B-1 \((P_{PD} = +10 \, \text{dBm}, e_D = 520 \, \text{mV}, K_{Lin} = 164)\)

\[
A_{D,Lin}(V/V) = 1.59 \times 10^{-3} (164)
\]

or

\[
A_{D,Lin}(V/V) = 0.26
\]

The dynamic linear detector gain in V/dBm, \( A_{D,Lin} \, (V/dBm) \), can be found by solving Equation (1B-24) for \( e_D \) and then differentiating.

\[
e_D = K_{Lin} 10^{\frac{P_{PD}(dBm)}{20}} \, (mV)
\]

\[
A_{D,Lin}(V/dBm) = \frac{d e_p}{d P_{PD}(dBm)}
\]

Thus,

\[
A_{D,Lin}(V/dBm) = 116 \times 10^{-6} K_{Lin} 10^{\frac{P_{PD}(dBm)}{20}}
\]

and for our linear detector \((P_{PD} = +10 \, \text{dBm})\),

\[
A_{D,Lin}(V/dBm) = (116 \times 10^{-6})(164) 10^{\frac{10}{20}}
\]

or

\[
A_{D,Lin}(V/dBm) = 60.16 \times 10^{-3}
\]
Figure 1B-3 summarizes linear detector characteristics.

\[ P_{PD} \text{(dBm)} \rightarrow e_D \text{(mV)} \]

\[ K_{Lin} = \frac{e_p}{\sqrt{P}} \left( \frac{\text{mV}}{\text{mW}} \right) \]

\[ e_D = K_{Lin} \left( \frac{P_{PD} \text{(dBm)}}{10^{20}} \right) \times 10^{-3} \text{(V)} \]

\[ K_{Lin} = e_p \text{(mV)} \times 10^{20} \left( \frac{\text{mV}}{\text{mW}} \right) \]

\[ A_{D,Lin} \text{(V/V)} = 1.59 \times 10^{-3} K_{Lin} \]

\[ A_{D,Lin} \text{(V/dBm)} = 116 \times 10^{-6} K_{Lin} \times 10^{20} \]

FIGURE 1B-3. Linear Detector Summary.
Appendix 1C
STATIC REGULATION CALCULATIONS

The static regulation of AGC loops will now be given. Figure 1.2a illustrates the basic loop configuration.

The AGC voltage is

\[ AGC = A_\Delta A_e (E_{\text{Ref}} - A_e e_D) \]  

(1C.1)

The voltage will vary as the input power is changed. The change in AGC voltage may be given as

\[ \Delta AGC = A_\Delta A_e A_v \Delta e_D \]  

(1C.2)

where \( \Delta e_D \) is the maximum permissible change in detector output for the desired change in input power. \( \Delta e_D \) may be given as

\[ \Delta e_D = e_{D,\text{max}} - e_{D,\text{min}} \]  

(1C.3)

\( e_D \) is related to detector input power, for a square law detector, by (Appendix 1B)

\[ e_D = K_{\text{SL}} \frac{P_{PD,\text{dBm}}}{10^{10}} \text{(mV)} \]  

(1C.4)

Substituting Equation (1C.4) into (1C.3),

\[ \Delta e_D = K_{\text{SL}} \left| \frac{P_{PD,\text{max,dBm}}}{10^{10}} - \frac{P_{PD,\text{min,dBm}}}{10^{10}} \right| \]  

(1C.5)
This equation may be rewritten as

$$\Delta e_D = K_{SL} 10^{10} \left| \frac{P_{PD_{\text{max}}}^{(\text{dBm})}}{10} - 1 \right|$$

or

$$\Delta e_D = e_{D,\text{min}} 10^{10} - 1$$

where

$$\Delta P_{PD_{\text{SL}}}^{(\text{dBm})} = P_{PD_{\text{max}}}^{(\text{dBm})} - P_{PD_{\text{min}}}^{(\text{dBm})}$$

Substituting Equation (1C-7) into (1C-2) yields

$$\Delta AGC = A_v A_e A_{\Delta t} e_{D,\text{min}} 10^{10} - 1$$

and solving Equation (1C-8) for $\Delta P_{PD}^{(\text{dBm})}$ gives

$$\Delta P_{PD_{\text{SL}}}^{(\text{dBm})} = 10 \log \left( \frac{\Delta AGC}{A_v A_e A_{\Delta t} e_{D,\text{min}}} + 1 \right)$$

Since the variable gain IF and predetector IF amplifiers are linear,

$$\Delta P_{PD_{\text{SL}}}^{(\text{dBm})} = \Delta P_{1F}^{(\text{dBm})}$$

and

$$\Delta P_{1F_{\text{SL}}}^{(\text{dBm})} = 10 \log \left( \frac{\Delta AGC}{A_v A_e A_{\Delta t} e_{D,\text{min}}} + 1 \right)$$
The average value for $e_{D,N}$ is

$$e_{D,N} = \frac{e_{D,max} + e_{D,min}}{2} \quad (1C-12)$$

If $e_{D,N}$ is adjusted in the mid-AGC range, $\Delta AGC/2$,*

$$\Delta P_{IF,SL} (dBm) = 10 \log \left( \frac{\Delta AGC}{A_C A_e e_{D,N}} + 1 \right) \quad (1C-13)$$

The same procedure can be used to find the static regulation for linear detectors:

$$\Delta P_{IF,lin} (dBm) = 20 \log \left( \frac{\Delta AGC}{A_C A_e e_{D,N}} + 1 \right) \quad (1C-14)$$

Noting that

$$A_v e_{D,N} = e_N \quad (1C-15)$$

Equations (1C-13) and (1C-14) become

$$\Delta P_{IF,SL} (dBm) = 10 \log \left( \frac{\Delta AGC}{A_C e_N} + 1 \right) \quad (1C-16)$$

and

$$\Delta P_{IF,lin} (dBm) = 20 \log \left( \frac{\Delta AGC}{A_C e_N} + 1 \right) \quad (1C-17)$$

The IF output may be divided (power split) to drive two or more circuits. Provided all circuitry is linear, the change in power-split output, $\Delta P_{d}(dBm)$, will be the same as $\Delta P_{IF}(dBm)$.

*If the AGC voltage does not start at zero volt, $\Delta AGC/2$ must be added to the voltage at which the AGC action does start.
Appendix 1D
AGC GAIN CALCULATION

The loop gain of the AGC loop is dependent on the AGC gain of the variable gain IF amplifier. The AGC gain is defined as the change in IF output voltage for a given change in AGC voltage, or (Figure 1-2).

\[
A_{AGC} = \frac{\Delta e_{IF}(PP)}{\Delta AGC} \left( P_{in} \text{ (dBm)} \right) = \text{constant}
\]

where

\[
\Delta e_{IF}(PP) = e_{IF, max}(PP) - e_{IF, min}(PP)
\]
\[
\Delta AGC = AGC_{max} - AGC_{min}
\]

AGC\_min = AGC voltage for minimum gain

AGC\_max = AGC voltage for maximum gain

The IF output power is

\[
P_{IF}(dBm) = X(AGC)
\]

The IF output voltage in terms of output power is (Appendix 1A)

\[
e_{IF}(PP) = (0.63) 10^{\frac{P_{IF}(dBm)}{20}}
\]

or, substituting Equation (1D 2),

\[
e_{IF}(PP) = (0.63) 10^{\frac{X(AGC)}{20}}
\]
The change in amplifier output voltage now may be written as

\[
\Delta e_{1p}(PP) = 0.63 \left( \frac{x_{\Delta AGC}^{\max}}{10} - \frac{x_{\Delta AGC}^{\min}}{10} \right)
\]

(1D.5)

or

\[
\Delta e_{1p}(PP) = 0.63 \left( \frac{x_{\Delta AGC}^{\max}}{20} \right)^{\frac{1}{10}} - 1
\]

(1D.6)

or, using Equation (1D-4) and simplifying,

\[
\Delta e_{1p}(PP) = e_{t_F,\min}(PP) \left( \frac{x_{\Delta AGC}}{20} - 1 \right)
\]

(1D.7)

The parenthetical term in Equation (1D-7) is of the general form \(a^Y\). Expanding this exponential function,

\[
a^Y = 1 + Y \ln(a) + \left[ \frac{Y \ln(a)}{2} \right]^2 + \left[ \frac{Y \ln(a)}{3} \right]^3 + \ldots
\]

(1D.8)

Provided \(Y\) is small \((Y < 0.5)\), Equation (1D-8) simplifies to

\[
a^Y = 1 + Y \ln(a)
\]

(1D.9)

Equation (1D-7) now becomes

\[
\Delta e_{1p}(PP) = e_{t_F,\min}(PP) \left[ \frac{x_{\Delta AGC}}{20} \ln(10) \right]
\]

(1D.10)

or

\[
\Delta e_{1p}(PP) = 0.115 x_{\Delta AGC} e_{t_F,\min}(PP)
\]

(1D.11)
Now Equation (1D-11) may be solved to find the AGC gain,

$$A_{AGC} = 0.115e_{1F-min}(PP)$$  \hspace{1cm} (1D-12)

and this equation is valid, to within 10%, for

$$\frac{X\Delta AGC}{20} < 0.5$$  \hspace{1cm} (1D-13)

The $e_{IF,min}(PP)$ term may be replaced by $e_{IF}(PP)$; thus,

$$A_{AGC} = 0.115e_{IF}(PP)$$  \hspace{1cm} (1D-14)

or, for $e_{IF}(PP)$ in dBm (Appendix 1A),

$$A_{AGC} = 72.4 \times 10^{-3} \times 10^{\frac{P_{IF\,dBm}}{20}}$$  \hspace{1cm} (1D-15)
Appendix IE

LOOP RISE TIME CALCULATIONS

The loop rise time for the integrator AGC loop (Figure 1-2b) will now be found. The general method will be to solve for the AGC voltage in terms of input power. This equation can then be differentiated to find the change in AGC voltage with change in input power.

The AGC error voltage, $e_c$ (Figure 1-2a), is zero under static conditions; however, if the input power is increased, the error voltage will increase and fall back toward zero as the loop nulls. This error voltage may be given as

$$e_c = A_v A_D e_D - A_D K_{Ref}$$  \hspace{1cm} (IE-1)

The value for $e_D$ may be given as $e_{D,N}$, the normalized or static value, plus $\Delta e_D$, the change in $e_D$ due to a change in input power. The static value for $e_D$ may be given, for a square law detector, as (Appendix IB)

$$e_{D,N} = K_{SL} \frac{10^{P_{PD,N}(dBm)}}{10} \text{ (mV)}$$  \hspace{1cm} (IE-2)

The value for $\Delta e_D$ is

$$\Delta e_D = \left| A_{D,SL} \frac{(V/dBm)}{dBm} \right| \Delta P_{PD}(dBm) \text{ (mV)}$$  \hspace{1cm} (IE-3)

or, from Appendix IB (Figure 1B-2),

$$\Delta e_D = 0.23 e_{D,N} \Delta P_{PD}(dB) \text{ (V)}$$  \hspace{1cm} (IE-4)

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where

\[ e_{D,N} = \text{normalized detector output voltage, volts} \]

The instantaneous detector output may now be given as

\[ e_{D} = e_{D,N} + 0.23 e_{D,N} |\Delta P_{PD}(dB)| \]  
(1E.5)

The value for \( \Delta P_{PD}(dB) \) may be given as

\[ \Delta P_{PD}(dB) = P_{PD}(dBm) - P_{PD,N}(dBm) \]  
(1E.6)

since

\[ P_{PD}(dBm) = P_{in}(dBm) + A_{IF}(dB) + A_{PD}(dB) \]  
(1E.7)

and (Figure 1-3)

\[ A_{IF}(dB) = A_0(dB) - X(AGC) \]  
(1E.8)

\[ P_{PD}(dBm) = P_{in}(dBm) + A_0(dB) - X(AGC) + A_{PD}(dB) \]  
(1E.9)

Equation (1E-5) now becomes

\[ e_{D} = e_{D,N} \left( 1 + 0.23 |P_{in}(dBm) + A_0(dB) - X(AGC) \right) \]

\[ + A_{PD}(dB) - P_{PD,N}(dBm) \]  
(V)  
(1E-10)

The error voltage may now be written as

\[ e_c = A_v A_\Delta \left( e_{D,N} \left( 1 + 0.23 |P_{in}(dBm) + A_0(dB) - X(AGC) \right) \right. \]

\[ + A_{PD}(dB) - P_{PD,N}(dBm) \right| - \frac{E_{Ref}}{A_v} \]  
(V)  
(1E-11)
The integrator output is

$$AGC(S) = \frac{e_c(S)A_c}{RCS}$$  \hspace{1cm} (1E-12)

Substituting Equation (1E-12) into (1E-11) and solving for $AGC(S)$,

$$AGC(S) = A_v A_d A_c \left[ e_{D,N} \left| 1 + 0.23 \left| P_{in}(dBm) + A_0(dB) + A_{P_0}(dB) \right| \right. \right.$$ 

$$- AGC(S) - P_{P_d,N}(dB) \left| - \frac{E_{Ref}}{A_v} \right| \left. \right] RCS + 0.23 X A_v A_d A_c e_{D,N} \right)$$  \hspace{1cm} (1E-13)

To find the change in AGC voltage for a given change in input power, Equation (1E-13) is differentiated with respect to $P_{in}(dBm)$, giving

$$\frac{\Delta AGC(S)}{\Delta P_{in}(dBm)} = \frac{0.23 A_v A_d A_c e_{D,N}}{RCS + 0.23 X A_v A_d A_c e_{D,N}}$$  \hspace{1cm} (1E-14)

or

$$\Delta AGC(S) = \frac{0.23 A_v A_d A_c e_{D,N}}{RCS + 0.23 X A_v A_d A_c e_{D,N}} = \Delta P_{in}(dBm)(S)$$  \hspace{1cm} (1E-15)

Assuming a step change in input power,

$$\Delta AGC(S) = \frac{0.23 A_v A_d A_c e_{D,N}}{RCS + 0.23 X A_v A_d A_c e_{D,N}} \left| \frac{\Delta P_{in}(dBm)(S)}{S} \right|$$  \hspace{1cm} (1E-16)

The inverse Laplace transform of Equation (1E-16) is

$$AGC(t) = \frac{\Delta P_{in}(dBm)}{X} \left| 1 - \exp \left( \frac{XRC}{0.23 X A_v A_d A_c e_{D,N}} \right) \right|$$  \hspace{1cm} (1E-17)

The time constant for Equation (1E-17) is

$$T = \frac{RC}{0.23 X A_v A_d A_c e_{D,N}}$$  \hspace{1cm} (1E-18)
The 10 to 90% rise time, \( t_r \), may be approximated as

\[ t_r = 2.2T \]  \hspace{1cm} (1E-19)

Thus

\[ t_{r,SL} = \frac{9.56 RC}{A_v \Delta A_c e_{D,N}} \]  \hspace{1cm} (1E-20)

or, since

\[ A_v e_{D,N} = e_N \]  \hspace{1cm} (1E-21)

\[ t_{r,SL} = \frac{9.56 RC}{XA_v A_c e_N} \]  \hspace{1cm} (1E-22)

Using the same procedure, the loop rise time for the linear detector is

\[ t_{r,lin} = \frac{19.13 RC}{X A_v A_c e_N} \]  \hspace{1cm} (1E-23)

The loop rise time for the low-pass-filter AGC loop is similar to that for the integrator AGC loop. One main difference, however, is that the error voltage is zero for the integrator AGC loop under static conditions, and is finite for the low-pass-filter loop. The error voltage is small for a well-designed low-pass-filter AGC loop, and may usually be neglected. The AGC voltage, for a square law detector, may be given as

\[ \Delta AGC(S) = \frac{0.23 A_v A_c e_{D,N}}{1 + RCS + 0.23 X A_v A_c e_{D,N}} \Delta P_{in}(dB)(S) \]  \hspace{1cm} (1E-24)

The AGC voltage for a step change in input power is

\[ \Delta AGC(S) = \frac{0.23 A_v A_c e_{D,N}}{RCS + 0.23 X A_v A_c e_{D,N} + 1} \frac{\Delta P_{in}(dB)}{S} \]  \hspace{1cm} (1E-25)
If

\[ 0.23 X A_v A_\Delta A_e e_{D,N} \gg 1 \]  

(1E-26)

\[ \Delta A_{GC}(S) = \frac{0.23 X A_v A_\Delta A_e e_{D,N}}{R C S + 0.23 X A_v A_\Delta A_e e_{D,N}} \left| \frac{\Delta P_{in}(dB)}{S} \right| \]  

(1E-27)

which is exactly the same equation as the integrator AGC loop, Equation (1E-16). Thus, provided the conditions of Equation (1E-20) are met, the low-pass-filter AGC loop has the same loop rise time as the integrator AGC loop, Equations (1E-22) and (1E-23).
Nomenclature

\( A \) \hspace{1em} \text{conventional gain term}
\( A_{AGC} \) \hspace{1em} \text{dynamic AGC gain}
\( A_D \) \hspace{1em} \text{dynamic detector gain}
\( A_{D,\text{lin}} \) \hspace{1em} \text{dynamic detector gain for linear detector}
\( A_{D,\text{SL}} \) \hspace{1em} \text{dynamic detector gain for square law detector}
\( AGC \) \hspace{1em} \text{automatic gain control}
\( A_{IP}(\text{dB}) \) \hspace{1em} \text{IF amplifier gain, in dB}
\( A_{\text{Int}} \) \hspace{1em} \text{integrator frequency dependent gain}
\( A_o(\text{dB}) \) \hspace{1em} \text{maximum IF amplifier gain, in dB}
\( A_{PD} \) \hspace{1em} \text{predetector amplifier gain}
\( A_v \) \hspace{1em} \text{video amplifier voltage gain}
\( A_{\Delta} \) \hspace{1em} \text{differencing amplifier gain}
\( A_c \) \hspace{1em} \text{error amplifier voltage gain}

\( C \) \hspace{1em} \text{capacitance}
\( CR \) \hspace{1em} \text{compression ratio}
\( CR_{\text{IF}} \) \hspace{1em} \text{compression ratio of the IF}
\( CR_{\text{vid}} \) \hspace{1em} \text{compression ratio of the video}
\( CW \) \hspace{1em} \text{continuous wave}

\( D \) \hspace{1em} \text{integrator or low-pass-filter loop update duty cycle}
\( dBm \) \hspace{1em} \text{decibel referenced to 1 milliwatt}
\( dBV \) \hspace{1em} \text{decibel referenced to 1 volt}

\( e_d \) \hspace{1em} \text{maximum ac signal across a diode for a given gain error}
\( E_{\text{Ref}} \) \hspace{1em} \text{AGC reference voltage}
\( e_D \) \hspace{1em} \text{detector output voltage}
\( e_{D,\text{lin}} \) \hspace{1em} \text{output of linear detector}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_{D,SL} )</td>
<td>output of square law detector</td>
</tr>
<tr>
<td>( e_{D,N} )</td>
<td>normalized detector output voltage</td>
</tr>
<tr>
<td>( e_{IF}(PP) )</td>
<td>peak-to-peak IF amplifier output voltage</td>
</tr>
<tr>
<td>( e_{N} )</td>
<td>normalized video output voltage</td>
</tr>
<tr>
<td>( e_{PP}(PP) )</td>
<td>peak-to-peak detector input voltage</td>
</tr>
<tr>
<td>( e_{rms} )</td>
<td>root mean squared voltage</td>
</tr>
<tr>
<td>( e_{V} )</td>
<td>video amplifier output voltage</td>
</tr>
<tr>
<td>( EW )</td>
<td>electronic warfare</td>
</tr>
<tr>
<td>( f )</td>
<td>frequency in Hertz</td>
</tr>
<tr>
<td>( f_{cs} )</td>
<td>conical scan frequency</td>
</tr>
<tr>
<td>( f_{3,db,LPF} )</td>
<td>low-pass-filter 3-dBV frequency response</td>
</tr>
<tr>
<td>( f_{3,db,ln} )</td>
<td>3-dB frequency response for linear detector</td>
</tr>
<tr>
<td>( f_{3,db,Sl} )</td>
<td>3-dB frequency response for square law detector</td>
</tr>
<tr>
<td>( IMR )</td>
<td>input modulation reduction</td>
</tr>
<tr>
<td>( IF )</td>
<td>intermediate frequency</td>
</tr>
<tr>
<td>( I_{F} )</td>
<td>diode forward current</td>
</tr>
<tr>
<td>( I_{s} )</td>
<td>diode reverse saturation current</td>
</tr>
<tr>
<td>( I_{T} )</td>
<td>constant current source in milliamperes</td>
</tr>
<tr>
<td>( j )</td>
<td>( \sqrt{-1} )</td>
</tr>
<tr>
<td>( K )</td>
<td>Boltzmann's constant</td>
</tr>
<tr>
<td>( K_{1}, K_{2} )</td>
<td>logarithmic video amplifier constants</td>
</tr>
<tr>
<td>( K_{Lin} )</td>
<td>linear detector diode constant</td>
</tr>
<tr>
<td>( K_{Sl} )</td>
<td>square law detector diode constant</td>
</tr>
<tr>
<td>( K_{P1}, K_{P2} )</td>
<td>PIN diode constants</td>
</tr>
<tr>
<td>( LG )</td>
<td>loop gain</td>
</tr>
<tr>
<td>( LG(f) )</td>
<td>frequency dependent loop gain</td>
</tr>
<tr>
<td>( LG_{lin} )</td>
<td>loop gain for linear detector</td>
</tr>
<tr>
<td>( LG_{Sl} )</td>
<td>loop gain for square law detector</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>LPF</td>
<td>low-pass filter</td>
</tr>
<tr>
<td>LS</td>
<td>logarithmic amplifier slope, in V/dB</td>
</tr>
<tr>
<td>m</td>
<td>diode slope (ΔV R/decade of IF)</td>
</tr>
<tr>
<td>MeN</td>
<td>normalized video modulation</td>
</tr>
<tr>
<td>M₀</td>
<td>output modulation</td>
</tr>
<tr>
<td>M₁</td>
<td>input modulation</td>
</tr>
<tr>
<td>MP₁ IF</td>
<td>IF output modulation</td>
</tr>
<tr>
<td>MP₁ in</td>
<td>IF input modulation</td>
</tr>
<tr>
<td>P₁(IF, max)(dBm)</td>
<td>maximum IF amplifier signal output power, in dBm, for AGC action</td>
</tr>
<tr>
<td>P₁(IF, min)(dBm)</td>
<td>minimum IF amplifier signal output power, in dBm, for AGC action</td>
</tr>
<tr>
<td>P₁n(dBm)</td>
<td>signal input power, in dBm</td>
</tr>
<tr>
<td>P₁n,max(dBm)</td>
<td>maximum signal input power, in dBm, for AGC action</td>
</tr>
<tr>
<td>P₁n,min(dBm)</td>
<td>minimum signal input power, in dBm, for AGC action (sometimes referred to as AGC delay)</td>
</tr>
<tr>
<td>P₀(dBm)</td>
<td>signal output power, in dBm</td>
</tr>
<tr>
<td>P₀N(dBm)</td>
<td>normalized signal output power, in dBm</td>
</tr>
<tr>
<td>P₁P(dBm)</td>
<td>detector input power, in dBm</td>
</tr>
<tr>
<td>PRF</td>
<td>pulse repetition frequency</td>
</tr>
<tr>
<td>PRI</td>
<td>pulse repetition interval</td>
</tr>
<tr>
<td>q</td>
<td>electron charge</td>
</tr>
<tr>
<td>R_C</td>
<td>collector resistor</td>
</tr>
<tr>
<td>R_E</td>
<td>dc emitter resistance</td>
</tr>
<tr>
<td>RF</td>
<td>radio frequency</td>
</tr>
<tr>
<td>R_s</td>
<td>diode bulk resistance</td>
</tr>
<tr>
<td>r_d</td>
<td>diode dynamic resistance</td>
</tr>
<tr>
<td>rPIN</td>
<td>PIN diode dynamic resistance</td>
</tr>
</tbody>
</table>
$S$ Laplacian $S$

$SL$ square law

$T$ absolute temperature (Kelvins)

$T_{pulse}$ integrator or low-pass-filter time constant

$T_u$ integrator or low-pass-filter AGC loop update time

$V_F$ diode forward voltage

$V_T$ $KT/q$

$X$ slope of variable gain amplifier, dB/V

$Y_{AGC}(f)$ conical scan filter and sample/hold transfer function

$Z_F$ operational amplifier feedback impedance

$\Delta AGC$ change in AGC voltage into IF amplifier

$\Delta AGC'$ change in AGC voltage at error amplifier output

$\Delta e_{IF}(PP)$ peak-to-peak change in IF amplifier output voltage

$\Delta e_N(dB)$ change in normalized video output in dB

$\Delta e_{N,lin}(dB)$ change in normalized video voltage, in dB, for linear detector

$\Delta e_{N,SL}(dB)$ change in normalized video voltage, in dB, for square law detector

$\Delta e_{pD}(PP)$ peak-to-peak change in detector input voltage

$\Delta e_c(PP)$ peak-to-peak change in error voltage

$\Delta P_{IF}(dB)$ change in IF amplifier output power, in dB

$\Delta P_{IF,lin}(dB)$ change in IF amplifier output power, in dB, for linear detector

$\Delta P_{IF,SL}(dB)$ change in IF amplifier output power, in dB, for square law detector

$\Delta P_{in}(dB)$ change in input power, in dB

$\Delta P_o(dB)$ change in output power, in dBm

$\Delta P_{o,lin}(dB)$ change in output power, in dBm, for linear detector
\[ \Delta P_{o,SL}(\text{dB}) \text{ change in output power, in dBm, for square law detector} \]

\[ \Delta PP_D(\text{dB}) \text{ compressed output variation} \]

\[ \eta \text{ diode constant} \]

\[ \theta_{PR} \text{ conical scan plane rotation due to AGC loop} \]

\[ t_r \text{ loop rise time} \]

\[ t_{r,\text{lin}} \text{ loop 10 to 90\% rise time for linear detector} \]

\[ t_{r,\text{SL}} \text{ loop 10 to 90\% rise time for square law detector} \]

\[ \phi \text{ conical scan modulation phase shift, in degrees} \]

\[ \Delta V_F \text{ diode forward voltage match necessary for a given gain match} \]
Bibliography

This bibliography is arranged in chronological order. My reason for this is to pay tribute to those whose works formed the foundation on which we build.


Rheinfelder, W. A. Designing Automatic Gain Control Systems;
Part 1 Design Parameters, EEE, December 1964, pp. 43-47.


Hughes, R. S. Semiconductor Variable Gain and Logarithmic Video Amplifiers. Continuing Education Institute, Inc., 1967.


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Chapter 2

AUTOMATIC NOISE TRACKING LOOPS

Virtually all radar and electronic warfare systems employ a signal threshold to start the necessary system timing process (signal sample/hold, analog-digital conversion, etc.). The signal threshold voltage must be large enough to prevent thresholding on noise, but not so large as to prevent excessive loss in signal sensitivity. This chapter will discuss signal thresholds from a basic, practical standpoint. The bibliography at the end of this chapter lists many excellent references on signal thresholding. It is not the intent of this chapter to be an all-encompassing thresholding tutorial, but rather a discussion of the need for, and basic design of, an automatic signal threshold that adjusts its voltage to keep the false alarm rate (signal crossings due to noise) constant, despite excess noise generated in the receiver (due to changes in gain due to automatic gain control (AGC)).

Figure 2-1 illustrates the basic configuration that will be presented. The received signal is detected and amplified by the video amplifier. The resultant signal and noise drive the signal threshold comparator.

The signal threshold (Vrs) is generally set for a given false alarm rate (FAR) in the absence of signal. The false alarm rate is dependent on individual system requirements; however, values generally range from one noise crossing every ten seconds (FAR = 0.1) to ten noise crossings every second (FAR = 10).
FIGURE 2-1. Basic Receiver Configuration.
The signal’s threshold voltage determines the receiver’s sensitivity.* Thus, a large threshold voltage (for a low false alarm rate) requires a large receiver input for signal detection (thus a loss in sensitivity). Signal sensitivity is defined here as that input signal required to give an 80% count of the input PRF (i.e., for a PRF of 2 kHz, the 80% probability of detection \( P_{d|80\%} \) occurs when the pulse count is 1.6 kHz).

The video amplifier output noise (for unity receiver gain) is a function of the detector, video amplifier noise figure, video bandwidth (\( B_v \)), and the video amplifier gain (\( A_v \)). The sensitivity obtained from such a configuration (commonly called a crystal video receiver) can be greatly increased by increasing the receiver gain (Appendix 2A). The video noise is independent of receiver gain until the receiver noise into the detector becomes dominant. Further increases in gain continue to provide an increase in sensitivity; however, the noise at the output of the video amplifier also increases due to increases in IF gain. Thus, the signal threshold must be increased to maintain the required false alarm rate. Setting the signal threshold (for the required false alarm rate) at large receiver gains results in a loss in sensitivity and dynamic range as the receiver gain, thus, video output noise, decreases.

Consider a receiver that has a bandwidth (\( B_R \)) of 120 MHz, a noise figure of 5 dB, and a video bandwidth (\( B_v \)) of 7.7 MHz (Appendix 2A). Figure 2-2 shows the sensitivity \( P_{d|dBm} \) and signal threshold voltage \( V_{Th}\) for the signal threshold set for a false alarm rate equal to one for several values of \( G_R \). The sensitivity with no receiver gain is \(-38\) dBm (with \( V_T \) set for a false alarm rate of one). The sensitivity increases 1 dB for each dB increase in receiver gain, as expected. The video amplifier output noise is constant for receiver gains less than about 30 dB; however, as the gain is increased beyond this.

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*The bibliography at the end of this chapter contains an extensive listing of sensitivity related material.
(a) $G_R = 0$ (Crystal Video Receiver),
$V_T = 0.162 \, \text{V}$, $P_d^{\text{dBm}} = -38$,
$rms \text{ noise} = 33 \, \text{mV}$

(b) $G_R = 25 \, \text{dB}$, $V_T = 0.163 \, \text{V}$,
$P_d^{\text{dBm}} = -62$,
$rms \text{ noise} = 33 \, \text{mV}$

(c) $G_R = 50 \, \text{dB}$, $V_T = 0.551 \, \text{V}$, $P_d^{\text{dBm}} = -82$,
$rms \text{ noise} = 73 \, \text{mV}$

(d) $G_R = 56 \, \text{dB}$, $V_T = 2.15 \, \text{V}$, $P_d^{\text{dBm}} = -83$,
$rms \text{ noise} = 277 \, \text{mV}$

FIGURE 2.2. Effect of Receiver Gain on Sensitivity and Signal Threshold ($V_T$)
for a FAR = 1. $B_R = 120 \, \text{MHz}$, $B_V = 7.7 \, \text{MHz}$, $N_{\text{FNA}} = 5 \, \text{dB}$, $PW = 0.5 \, \mu\text{sec}$. 

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value, the video noise increases, necessitating an increase in $V_T$ to keep a
unity false alarm rate (Figures 2-2c - 2-2d). The loss in dynamic range may
be given, for a square law detector (Appendix 1B), as (Appendix 2A, Equation
(2A-19))

$$\text{DR}_{\text{Loss}} = 10 \log \left( \frac{V_T^{A}}{V_T^{S}} \right) \quad (2-1)$$

where

- $\text{DR}_{\text{Loss}}$ = loss in input dynamic range
- $V_T^{A}$ = signal threshold needed for required false alarm rate at a
  particular receiver gain
- $V_T^{S}$ = signal threshold needed for required false alarm rate for
  no receiver gain

Thus, setting $V_T^{S}$ at the maximum expected receiver gain results in a
significant loss in input dynamic range as the receiver gain is decreased.

Figure 2-3 shows the signal sensitivity for two fixed thresholds: (1) $V_T^{S} = 2.15$, the value necessary for a false alarm rate of one for a receiver gain of 56
dB, and (2) $V_T^{S} = 0.551$, the value necessary for a false alarm rate of one for a
receiver gain of 50 dB. The loss in input dynamic range due to $V_T^{S}$ will now
be found.

A. Signal threshold voltage ($V_T^{S}$) set for a false alarm rate of one at a
receiver gain of 56 dB is 2.15 volts (Figure 2-2d). The measured sensitivity
(for $G_R = 0$) for this threshold setting is $-26$ dBm (Figure 2-3a). The
optimum sensitivity for $V_T^{S} = 0.162$ is $-38$ dBm (Figure 2-2a). Thus, there
is a loss in input dynamic range (and sensitivity) of

$$\text{DR}_{\text{Loss}} = -26 - (-38) = 12 \text{ dB} \quad (2-2)$$

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(a) \( G_R = 0, V_{T|S} = 2.15 \) V (see Figure 2-2d),
\[ P_d \mid_{dBm} = -26 \].

(b) \( G_R = 0, V_{T|S} = 0.551 \) V (see Figure 2-2c),
\[ P_d \mid_{dBm} = -32 \].

**FIGURE 2-3.** Effect of \( V_{T|S} \) on Sensitivity.
\( B_R = 120 \) MHz, \( B_V = 7.7 \) MHz, \( NF_R = 5 \) dB,
\( PW = 0.5 \mu sec \).
The predicted loss in input dynamic range is (Equation (2-1))

\[
\text{DR}_{\text{Loss}} = 10 \log \frac{2.51}{0.162} = 11.3
\] (2.3)

B. Signal threshold voltage set for a false alarm rate of one at a receiver gain of 50 dB is 0.551 volt (Figure 2-2c). The measured sensitivity (for \( G_R = 0 \) dB) for this threshold setting is \(-32 \) dBm (Figure 2-3b). The optimum sensitivity for \( V_{TS} = 0.162 \) is \(-38 \) dBm (Figure 2-2a). Thus the loss in input dynamic range is

\[
\text{DR}_{\text{Loss}} = -32 - (-38) = 6 \text{ dB}
\] (2.4)

The predicted loss is (Equation (2-1))

\[
\text{DR}_{\text{Loss}} = 10 \log \frac{0.551}{0.162} = 5.3 \text{ dB}
\] (2.5)

which also is in excellent agreement with the 6-dB measured value.

Figure 2-4 shows the increase in sensitivity and loss in input dynamic range as a function of receiver gain.

Figure 2-5 shows the increase in false alarm rate with increased receiver gain for a signal threshold of 0.162 volt (Figure 2-2a). It is obvious that for receiver gains in excess of about 40 dB, the false alarm rate increase is intolerable. The system engineer must make a decision about decreasing receiver gain and losing sensitivity, or increasing the signal threshold and losing dynamic range. What to do?

If a radar or EW system requires the maximum sensitivity available, and the loss in input dynamic range accompanying a fixed signal threshold cannot be tolerated, a noise riding threshold may be the only solution.
FIGURE 2-4. Effect of Receiver Gain on Sensitivity and Dynamic Range. \( B_R = 120 \text{ MHz}, B_V = 7.7 \text{ MHz}, \)
\( \text{NF}_R = 5 \text{ dB}, \text{PW} = 0.5 \mu\text{sec}, \text{rms noise (gain < 30 dB)} \)
\( = 35 \text{ mV}, \text{FAR} = 1. \)

One method to obtain a noise riding threshold is illustrated in Figure 2-6. Circuit operation is straightforward: the noise comparator is triggered for noise voltages greater than the noise threshold voltage \( (V_{TH}) \). The noise comparator output triggers the noise one-shot (to ensure reliable triggering of the noise counter). The resultant noise is counted (provided no signal is present) during the counter enable time \( (T_C) \) and digitally converted to an analog voltage \( \text{DAC} \). The \( \text{DAC} \) analog voltage (which is controlled by the noise count during the noise counter enable time) is compared with a reference voltage \( (-V_{\text{Ref}}) \) and the resulting error voltage \( (V_e) \) is

\[
V_e = \frac{V_{DAC} - V_{Ref}}{2}
\]  

(2.6)
which is integrated during the noise integrator enable time ($T_1$). The noise threshold loop, via feedback, ensures that the noise integrator input voltage ($V_n$) is driven to zero. Since there is only one DAC input (noise count) that will ensure $V_{DAC} = V_{Ref}$, the noise comparator output count is constantly independent of input noise amplitude. The noise threshold voltage ($V_{TIN}$) is increased by the signal threshold amplifier ($AV|S_T$). The larger the signal threshold amplifier gain, the larger the signal threshold voltage; thus, the lower the signal comparator false alarm rate. To ensure that the noise count is insensitive to the input signal, the noise counter is disabled via the signal threshold comparator. The signal comparator one-shot ensures that the first signal threshold crossing determines the sensitivity. Without this one-shot (or similar digital element), a threshold count well in excess of the input PRF will be obtained for large receiver gains (thus large noise) as illustrated in Figure 2-7.
FIGURE 2-6. Noise Riding Threshold.
Appendix 2A shows that for a noise count (false alarm rate) of 250 kHz, the threshold voltage is reasonably insensitive to receiver gain, and this value will be used for the normalized noise threshold count in Figure 2-6. The noise count is only enabled for time, $T_C$ (Figure 2-6), since a count of 250 kHz would require an excessively large DAC. Equation (2-7) presents the relationship between the desired normalized noise threshold count ($FAR_N$), counter enable time ($T_C$), and normalized noise count ($N$)

$$N = |FAR_N| \times 10^{-3} \left( \frac{T_C}{\mu s} \right)$$  \hspace{1cm} (2.7)

or

$$T_C (\mu s) = \frac{N \times 10^3}{FAR_N (k Hz)}$$  \hspace{1cm} (2.8)
where

\[ N = \text{normalized noise counter count for a} \]
\[ T_C = \text{DAC enable time (in } \mu\text{sec) for a} \]
\[ \text{FAR}_N = \text{normalized noise threshold count (in kHz).} \]

Assume the following:

\[ \text{FAR}_N = 250 \text{ kHz, } N = 8, \quad T_C (\mu\text{sec}) = \frac{8 \times 10^3}{250 (\text{kHz})} = 32 \mu\text{sec} \quad (2.9) \]

The noise integrator update time \((T_I)\) should be as small as practical to ensure that the noise riding threshold can track noise changes due to changes in receiver gain. A value of \(T_I = T_C\) will be used for the example to follow.

The noise riding threshold illustrated in Figure 2-6 will be designed for receiver/detector video amplifier characteristics illustrated in Figures 2-2, 2-3, 2-4, and 2-5.

Design characteristics:

\[ \text{FAR}_N = 250 \text{ kHz} \quad N = 8 \quad T_C = 32 \mu\text{sec} \quad T_I = T_C = 32 \mu\text{sec} \]

Figure 2-8 illustrates the functional noise riding threshold schematic. The three least significant bits for the DAC are grounded and only the four most significant bits are used. The DAC08 has a current output and the voltage integrator error (Figure 2-6) may be represented as shown in Figure 2-9. The noise counter and DAC (Figure 2-7) are configured such that a noise count of eight in 32 \(\mu\text{sec}\) (see Equation (2-8)) is obtained. Table 2-1 gives \(V_r\) as a function of noise counts, \(N\). Thus, as the noise count is increased \((N > 8)\), the effective noise integrator error voltage \((V_e)\) becomes negative, increasing the noise threshold voltage, and thus decreasing \(N\) and driving \(V_r\) to zero. The signal threshold amplifier \((A_{VT})\) increases the noise threshold voltage and adjusts for the desired maximum false alarm rate.

\[ V_T \bigg|_S = \left( A_V \bigg|_{ST} \right) \left( V_T \bigg|_N \right) \quad (2.10) \]
Diodes $D_1$ and $D_2$ comprise a lower noise threshold clamp to prevent the noise riding threshold from attempting to lock on negative noise crossings.

Figures 2-10, 2-11, and 2-12 illustrate the measured results. The signal threshold amplifier determines the maximum signal false alarm rate and the results of three values of $AV_{ST}$ gain are shown.

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A. Figure 2-10 gives the sensitivity \( (P_d|_{80\%}) \) as a function of receiver gain. As can be seen, the data compare quite well with the manually adjusted signal threshold data of Figure 2-4.
B. Figure 2-11a shows the noise count as a function of receiver gain, and it is quite insensitive to changes in receiver gain (input noise). Figure 2-11b shows the signal false alarm rate for three values of $A_{V|ST}$. Increasing $A_{V|ST}$ decreases the FAR at the expense of sensitivity (Figure 2-10).

FIGURE 2-11. Signal False Alarm Rate and Noise Threshold Count as Functions of Receiver Gain.
C. Figure 2-12 shows the signal threshold and noise threshold voltages as a function of receiver gain.

\[ V_{TS} = 3.75 \times V_{TN} \]

\[ V_{TN} \]

FIGURE 2-12. Threshold Voltages as a Function of Receiver Gain.

A basic model to determine loop closure time for a change in video noise (due to changes in receiver gain) is presented in Figure 2-13. The false alarm rate is modeled as a voltage controlled oscillator having an output frequency that is the normalized false alarm rate (250 kHz for our design example). The oscillator has a slope of

\[ S_F = \frac{FAR}{Volt} \]
and is a nonlinear function of voltage (see Figures 2A-9, 2A-11, 2A-13, and 2A-16). Figure 2-14 illustrates false alarm rate as a function of threshold voltage (see Figure 2A-9). At large false alarm rate values (greater than 1,000) the curves can be approximated as

\[ \text{FAR} = a \exp(-nV_T) \]  

where

\[ a = \text{FAR for } V_T = 0 \]  

\[ n = \left( \frac{1}{V_T} \right) \ln \left( \frac{a}{\text{FAR}} \right) \]
and

\[ S_F = \frac{d \text{FAR}}{d V_T} = -an \exp(-n V_T) \left( \frac{\text{FAR}}{\text{Volt}} \right) \]  \hspace{1cm} (2.14) 

\[ V_T = \left( \frac{1}{n} \right) \ln \left( \frac{a}{\text{FAR}} \right) \]  \hspace{1cm} (2.15)

Table 2.2 summarizes the results of Equations (2-11) to (2-15) for the curves of Figure 2.14.

**TABLE 2.2. FAR Results from Figure 2.14.**

<table>
<thead>
<tr>
<th>Curve</th>
<th>(n) (Equation (2-13))</th>
<th>(\text{FAR}) (Equation (2-11))</th>
<th>(V_T) (FAR = 250 kHz) (Equation (2-15))</th>
<th>(S_F) (FAR = 250 kHz) Equation (2-14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100 (\times) 10^6</td>
<td>46</td>
<td>100 (\times) 10^6 exp (-46 V_T)</td>
<td>0.13</td>
</tr>
<tr>
<td>B</td>
<td>200 (\times) 10^6</td>
<td>17</td>
<td>200 (\times) 10^6 exp (-17 V_T)</td>
<td>0.39</td>
</tr>
<tr>
<td>C</td>
<td>100 (\times) 10^6</td>
<td>8.2</td>
<td>100 (\times) 10^6 exp (-8.2 V_T)</td>
<td>0.73</td>
</tr>
</tbody>
</table>

The loop closure time will now be found for Figure 2.13.

The loop gain (LG) is

\[ LG(f) = \frac{(S_F)(S_{DAC})D}{2nfRC} \]  \hspace{1cm} (2.16)

where

\[ D = \text{integrator update duty cycle (see Equation (1-94))} \]
The loop transfer function is

\[
\frac{V_T(f)}{V(f)} = \frac{LG(S)}{1 + LG(S)} = H(S)
\]  \hspace{1cm} (2.17)

substituting Equation (2-16) into (2-17) and simplifying

\[
H(S) = \frac{1}{\frac{fS_{DAC}D}{2nRC} + j}\n\]

The 3-dB bandwidth may be given as

\[
f_{3dB} = \frac{S_F S_{DAC}D}{2nRC}
\]  \hspace{1cm} (2.19)
and the loop rise time may be given as

\[ t_r = \frac{0.35}{f_{3dB}} \]  

(2.20)

or, upon substituting Equation (2-19) into (2-20),

\[ t_r = \frac{2.2RC}{S_FSDACD} \]  

(2.21)

and then using the parameters of the design example (Figure 2-7),

\[ R = 12.1k\Omega \quad C = 1\mu F \quad D = 0.5 \]

The DAC scale factor, \( S_{DAC} \), may be approximated as (Table 2-1)

\[ S_{DAC} = \frac{2V_{CC}}{N_{(max)}} \]  

(2.22)

or

\[ S_{DAC} = \frac{24}{15} = 1.6 \text{ Volts/bit} \]  

(2.23)

and \( S_F \) is obtained from Table 2-2.

The loop routine (for a receiver gain of 45 dB) is

\[ t_r = \frac{2.2(12.1 \times 10^3)(1 \times 10^{-6})}{(4 \times 10^6)(1.6)(0.5)} = 8.3 \times 10^{-9} \]  

(2.24)

To say the least, this predicted loop rise time is ridiculously fast. A much more realistic approach is to assume that the DAC is hard limited for any increase or decrease in false alarm rate (\( V_e = V_{CC} \) for FAR < 250 kHz and \( V_e = -V_{CC} \) for FAR > 250 kHz). This is a most reasonable assumption as the noise riding threshold of Figure 2-7 is only counting 15 noise crossings out of 250,000 (in fact, the statistical nature of the noise ensures that \( V_e \) (Figure 2-9) will toggle between \( \pm V_{CC} \), with an average value of zero, for a constant input noise and the loop nulled). Since the DAC is limited, the integrator has a
constant slew rate for increasing or decreasing input noise. Thus, the change in integrator output voltage may be given as

\[ \frac{\Delta V}{\Delta t} = \frac{V_{CC}D}{RC} \]  

(2.25)

or for the circuit of Figure 2-7

\[ \frac{\Delta V}{\Delta t} = \frac{12(0.5)}{(12.1 \times 10^3)(1 \times 10^{-6})} = 495 \text{ Volt/sec} \]  

(2.26)

or

\[ \frac{\Delta V}{\Delta t} = 0.495 \text{ Volt/sec} \]  

(2.27)

Referring to Figure 2-14, the threshold voltage difference (for a FAR = 250 kHz) is 0.6 volt as the gain is changed from 50 dB (Figure 2-2(c)) to 0 dB (Figure 2-2(a)). Thus the expected loop closure time is (Equation (2.27))

\[ \Delta t = \frac{\Delta V}{0.495} \]  

(2.28)

or

\[ \Delta t = \frac{0.6}{0.495} = 1.2 \text{ msec} \]  

(2.29)

which is quite close to the measured value shown in Figure 2-15.

The noise riding threshold technique presented is intended only as a starting point. There are many concepts that will achieve automatic signal thresholding and they are limited only by the originality of the designer.

Achieving the optimum sensitivity that can be obtained for a given receiver configuration may necessitate a noise riding threshold if excessive loss in dynamic range is to be avoided. The system engineer must take a close look at system requirements to justify the need for the system level complications of a noise riding threshold.
FIGURE 2-15. Noise Riding Threshold Closure Time. Top trace: video noise (0.5 V/div); bottom trace: noise threshold (0.5 V/div); horizontal time, 2 msec/div.
References


Appendix 2A

THE EFFECT OF RECEIVER GAIN
ON SIGNAL SENSITIVITY AND NOISE

Introducing receiver gain prior to signal detection increases signal sensitivity; however, the video noise increases as the receiver gain is increased beyond a certain point. This increase in noise may well necessitate the need for a noise riding threshold. This appendix presents a basic discussion of signal sensitivity and increased video noise as a function of receiver gain, receiver bandwidth, and video bandwidth, and is based on [1] and [2].

Background

Figure 2A-1 illustrates the basic configuration that will be discussed. The main restrictions that apply for the theory to be presented are (1) the detector is operated in the square law region (Appendix 1B), and (2) the receiver bandwidth (BR) is at least twice the video bandwidth (BV) (see Appendix 2B for BR = BV).

The RF/IF variable gain amplifier amplifies and limits the bandwidth (BR) of the incoming signal. The resulting signal (and noise) drives the square law detector and video amplifier (see Figure 2A-2 for a brief summary of square law detectors). The resulting signal and noise drives a comparator. If the detected video signal is greater than the signal threshold voltage (VTₜₛ), the comparator is triggered. A practical method of determining the relationships of the factors that determine signal threshold triggering will be presented.
FIGURE 2A-1. Basic Configuration, Detector-Video Receiver.
**FIGURE 2A-2. Square Law Detector Summary.**

\[ \Delta P_D \text{ (dBm)} = 2 \Delta P_D \text{ (dBm)} \]

\[ K_{SL} = \frac{P_D \text{ (mW)}}{P_D \text{ (mW)}} = \frac{P_D \text{ (mV)}}{10} \]

\[ R_V = \text{DETECTOR VIDEO RESISTANCE} \]

\[ M = \frac{K_{SL}}{\sqrt{R_V}} \]
Maximum Tangential Sensitivity and Optimum Maximum RF Gain

If the RF/IF receiver is removed, the input signal \( P_{in|dBm} \) drives the detector directly. An excellent "goodness" factor for the sensitivity of this configuration is the tangential sensitivity, \( T_{SS} \). Tangential sensitivity represents a signal-to-noise power ratio of 6.31 (8 dB) at the video amplifier output, and refers to the condition in which the signal plus noise waveform (on an oscilloscope) protrudes above the noise baseline such that the bottom edge of the combined terms just touches the top edge of the baseline noise as shown in Figure 2A-3. Figure 2A-4 summarizes the measurement technique for determining \( T_{SS} \). [3]

![FIGURE 2A-3. Tangential Signal.](image-url)
DECREASE THE INPUT POWER AND BRING THE TWO TRACES TOGETHER UNTIL THE DARK BAND BETWEEN THEM JUST DISAPPEARS.

INCREASE THE INPUT POWER UNTIL THE DARK BAND BETWEEN THE PULSES JUST APPEARS.

INCREASE THE INPUT POWER 1 dB THE RESULT IS THE TANGENTIAL SENSITIVITY (CHANNEL B).

Tangential sensitivity depends on the noise figure of the amplifier following the detector, the video 3-dB bandwidth, and the detector characteristics. The TSS in dBm, \( T_{SS_{dBm}} \), may be given as [3,4]

\[
T_{SS_{dBm}} = -35 + 5 \log B_V + \frac{NF_V}{2} - 10 \log M
\]  

(2A.1)

where

- \( B_V \) = video 3-dB bandwidth in MHz
- \( M \) = detector constant (see Figure 2A-2)
- \( NF_V_{dB} \) = video amplifier noise figure (10 \( \log F_V \))
- \( G_0 \) = receiver gain = 0 dB.

Equation (2A.1) accurately predicts the TSS for a given configuration and is an excellent comparison factor for crystal video receivers (provided \( B_V \) is specified along with TSS).

The detector "effective" input signal-to-noise ratio for a TSS condition is 4 dB (remember the detector is square law, Figure 1A-2 and Appendix 1B). Thus the "effective" noise appearing at the detector’s input is**

\[
N_D_{dBm} = T_{SS_{dBm}} - 4
\]  

(2A.2)

---

*Noise figure will be defined in dB (NFIdB) while noise factor will be a power ratio. Thus, NFIdB = 10 \( \log F \).

**For small gains the actual detector input noise is much less (\( N_D_{dBm} = -114 + 10 \log B_R + G_{0dBm} \)). As the receiver gain is increased, \( N_D_{dBm} \) is increased. When this noise power exceeds the "effective" noise power of Equation (2A.2), the receiver noise dominates.
and this is termed the gain-limited tangential sensitivity. If receiver gain is provided (and this gain is not sufficient to supply noise in excess of that defined by Equation (2A-2)), the detector input signal-to-noise ratio will increase 1 dB for each dB increase in gain (increasing the detector-video amplifier output signal-to-noise ratio at twice the rate of increased gain). If the receiver gain is excessively large, the noise contributed by the receiver becomes the dominant issue, and any further increase in gain results in no increase in sensitivity, but rather an equal increase of signal and noise at the input to the detector. This is termed "noise-limiting" (TSS$_{\text{Max}}$ dB) and is the best that can be obtained for a given configuration. Figure 2A-5 illustrates the effect of receiver gain on TSS (the configuration of Figure 2A-1 is used). The TSS for no gain (gain-limited TSS) is −41 dBm. As the gain is increased, the video noise remains constant (and TSS increases 1 dB for each dB increase in gain) until the gain approaches 45 dB (Figure 2A-5d, and note the change in voltage scale factor). As the gain is increased above 45 dB (Figure 2A-5e and f), the video noise increases and little improvement in TSS is gained (this is the noise-limited TSS$_{\text{Max}}$ dB). The obvious questions now are (1) what is the noise-limited TSS and (2) how much receiver gain is needed to achieve this noise-limited TSS? These two questions will now be answered.

The only assumption that will be made for the following analysis is that $B_R \approx 2 B_V$. Lucas [4] presented an excellent analytic discussion on this subject; however, his equations are most difficult to use. Tsui [3] has extended Lucas' results to a more practical conclusion. The equation for TSS$_{\text{GR}}$ (this is the TSS for a given receiver gain, GR) may be given as (see Appendix 2B for derivation of the equations to follow)
Note: $T_{SS} = -41$ dBm not $-43$ dBm

**FIGURE 2A-5.** Effect of Receiver Gain on $T_{SS}$ and Output Noise, $(BR = 40$ MHz, $NF_{dB} = 5, BV = 7.7$ MHz).
\[ T_{SS\mid dBm} = \begin{align*} &\left[ -114 + NFR_{dBm} \right] \left[ 6.31 B_V + 2.5 \sqrt{2 B_R B_V - B_V^2} + \frac{T_{SS\mid dBm} + 110}{10} \right. \\ &+ \left. 10 \log \frac{G_R}{B_R F_R R^2} \right] \end{align*} \]  

(2A-3)

where

\[ NFR_{dB} = \text{receiver total noise figure in dB} \]

\[ B_V = \text{video 3-dB bandwidth in MHz} \]

\[ B_R = \text{receiver 3-dB bandwidth in MHz} \]

\[ G_R = \text{total receiver gain (power ratio)} \]

\[ F_R = \text{total receiver noise factor (power ratio, remember} \]

\[ NFR_{dB} = 10 \log F_R \]

\[ T_{SS\mid dBm} = \text{TSS for detector-video amplifier with no receiver gain} \]

Two limiting cases of Equation (A-3) will now be discussed.

**Case 1**: \( G_R F_R \) small (gain-limited sensitivity), or the right-hand term of the square root term much greater than the left. This is not obvious at first glance; however, consider the receiver parameters shown in Figure 2A-5:

\[ B_R = 40 \text{ (MHz)} \]

\[ B_V = 7.7 \text{ (MHz)} \]

\[ NFR_{dB} = 5 \text{ dB} \]

\[ T_{SS\mid dBm} = -41 \text{ dBm (see Figure 2A-5a)} \]
The left term of the square root term is

$$2B_R B_V - B_V^2 = 2(40)(7.7) - (7.7)^2 = 556.7$$  \hspace{1cm} (2A.4)

The right term is (if $GRF = 1$)

$$\frac{T_{SS}^{G0}_{dBm} + 110}{10^{10}} = \left(\frac{-41 + 110}{10^{10}}\right)^2 = 63.1 \times 10^{12}$$  \hspace{1cm} (2A.5)

Thus, neglecting the left term (Equation (2A.5)) is quite valid.

Equation (2A-3) may now be written as

$$T_{SS}^{G_R}_{dBm} = T_{SS}^{G0}_{dBm} - G_R^{dB}$$  \hspace{1cm} (2A.6)

Equation (2A.6) is most important, as it shows that increasing the gain increases the $T_{SS}$ by the same amount. Figure 2A-5 shows that the $T_{SS}$ with no gain is $T_{SS}^{G0}_{dBm} = -41$. If 20 dB of receiver gain is added, the expected sensitivity is

$$T_{SS}^{G_R}_{dBm} = -41 - 20 = -61 \text{ dBm}$$  \hspace{1cm} (2A.7)

and this agrees well with the measured value of -62 dBm (Figure 2A-5b).

The maximum sensitivity is obtained when $GRF$ in Equation (2A-3) is increased such that the left term in the square root is dominant.
Case 2: GRFR large, noise-limited sensitivity \((2BRB_V - By^2\) much greater than the term to the right). Equation (2A-3) may be given as (Appendix 2B)

\[
T_{\text{SS}}^{\text{Max}} dBm = -114 + NF_R + 10 \log B_V
\]

\[
+ 10 \log \left[ 6.31 + 2.5 \sqrt{\frac{B_R}{B_V}} - 1 \right]
\]

and the maximum \(T_{\text{SS}}\) is not only dependent on the receiver noise figure, but upon the video bandwidth and the ratio of receiver to video bandwidth.

Again referring to the parameters of Figure 2A-5, we find that the maximum sensitivity expected is

\[
T_{\text{SS}}^{\text{Max}} dBm = -114 + 5 + 10 \log 7.7
\]

\[
+ 10 \log \left[ 6.31 + 2.5 \sqrt{\frac{40}{7.7}} - 1 \right]
\]

or

\[
T_{\text{SS}}^{\text{Max}} dBm = -88.7 dBm
\]

and this agrees quite well with the measured value of 88 dBm (Figure 2A-5f).

Later in this appendix we will discuss thresholding, and will find that increasing the receiver gain when the system is noise-limited reduces the instantaneous signal dynamic range at the detector output; thus, it is most worthwhile to find the maximum practical receiver gain. There is no pat answer as to the maximum practical receiver gain; however (as is shown in Appendix 2B), if \(G_R\) is increased (Equation 2A-3) such that
the sensitivity obtained is within 1.5 dBm of that predicted by Equation (2A-8). Solving Equation (2A-11) for $G_{R}^{\text{Max}}$ dBm,

$$2B_{R}B_{V} - B_{V}^{2} = \left( \frac{T_{SS}^{\text{dBm}} = 110}{10} \right)^{2}$$

$$G_{R}^{\text{Max}} \text{ dBm} = 110 + T_{SS}^{\text{dBm}} - NF_{R}^{\text{dBm}} - 10 \log \left( \sqrt{2B_{R}B_{V} - B_{V}^{2}} \right) \quad (2A-12a)$$

or

$$G_{R}^{\text{Max}} \text{ dB} = 110 + T_{SS} - NF_{R}^{\text{dB}} - 10 \log B_{V}$$

$$- 10 \log \left( \sqrt{2B_{R}B_{V} - 1} \right) \quad (2A-12b)$$

Appendix 2B illustrates this is quite close to the gain that increases the video rms noise by a factor of two over that with no receiver gain. When, in the discussions to follow, the measured value of $G_{R}^{\text{Max}}$ is mentioned, it will be that receiver gain that increases the video noise by a factor of two.

The maximum predicted gain for the parameters previously discussed (Figure 2A-5) is

$$G_{R}^{\text{Max}} \text{ dB} = 110 - 41 - 5 - 10 \log \left( \sqrt{2(40 \times 7.7)^{2} - (7.7)^{2}} \right) \quad (2A-13)$$

or

$$G_{R}^{\text{Max}} \text{ dB} = 50.3 \text{ dB} \quad (2A-14)$$

The measured value for $G_{R}^{\text{Max}}$ was 49.5 dB, which gives a $T_{SS}^{\text{dBm}}$ within 1.5 dBm of the maximum as predicted from the theory.
Several receiver configuration results are given in Table 2A-1; a plot of $TSS_{dBm}$ versus $GR_{dB}$ is given in Figure 2A-6. As will be seen, the predicted and measured results are quite close. Figure 2A-7 is a summary of the theory presented thus far. Applying this theory to the more useful area of thresholding will be presented shortly; however, it is worthwhile to discuss how the material presented relates to the work of others.

It will be seen from Figure 2A-5e that the noise riding on the pulse is greater than the baseline noise. This condition is predicted by Lucas [4], and the situation becomes more pronounced as $R/B_V$ decreases. Figure 2A-8 shows $TSS_{dBm}^{Max}$ for a receiver with $R/B_V = 80$, and as can be seen, the noise riding on the pulse is indeed nearly equal to the standing noise. This situation is inconsequential, however, as Lucas shows that $TSS_{dBm}^{Max}$ is fairly independent (0.5 dBm or so) of this "excess" noise.

Ayer [5] was one of the first to present a coherent discussion of detector-video amplifiers preceded by RF (or IF) amplification. His work is a pioneering effort, and he defines an "effective bandwidth," $B_e$, as

$$B_e = \sqrt{2 B_R B_V - B_V^2} \quad (2A-15)$$

This is based on the assumption that $R/B_V$ is quite large, which is often not the case. Many subsequent authors neglected this assumption, and one can have serious errors in the predicted $TSS_{dBm}^{Max}$. Referring to Equation (2A-12), we see that our predicted maximum gain is a function of Ayer's $B_e$; however, $TSS_{dBm}^{Max}$ (Equation 2A-8) is not a direct function of $B_e$. Appendix 2B shows that, if $R/B_V > 50$ (for $R/B_V = 50$ the error using $B_e$ is 0.98 dBm), $TSS_{dBm}^{Max}$ may be approximated as

$$TSS_{dBm}^{Max} = -110 + NF_{dB} + 10 \log B_e \quad (for \ R/B_V > 50) \quad (2A-16)$$
TABLE 2A-1. Comparison of Predicted and Measured Results ($PW = 1\mu$sec).

<table>
<thead>
<tr>
<th>$BR_{MHz}$</th>
<th>$BV_{MHz}$</th>
<th>$NF_{RdB}$</th>
<th>$TSS_{dBm}$</th>
<th>$\max G^0_{dBm}$</th>
<th>Max $TSS_{dBm}$</th>
<th>Max $G_R_{dB}$</th>
<th>$BR/BV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>7.7</td>
<td>5</td>
<td>-41</td>
<td>-88</td>
<td>-88.7</td>
<td>51</td>
<td>50.27</td>
</tr>
<tr>
<td>120</td>
<td>7.7</td>
<td>6</td>
<td>-41</td>
<td>-86</td>
<td>-87.1</td>
<td>48</td>
<td>47.8</td>
</tr>
<tr>
<td>40</td>
<td>0.5</td>
<td>5</td>
<td>-44c</td>
<td>-95</td>
<td>-95.0</td>
<td>53</td>
<td>53.0</td>
</tr>
<tr>
<td>120</td>
<td>0.5</td>
<td>6</td>
<td>-44c</td>
<td>-92</td>
<td>-93.0</td>
<td>50</td>
<td>50.6</td>
</tr>
</tbody>
</table>

a Measured at $G_R_{dB}$ (measured value).

b This is the receiver gain needed to double the output video rms noise (see Appendix 2B).

c Equation (2A-1) is a function of $5 \log_{10} BV$; thus, if $BV$ is decreased from 7.7 (MHz) to 0.5 (MHz) the $TSS_{dB}$ should increase by $5 \log_{10} 7.7/0.5 = 5.9$ dB. This assumes the $M$ and $NF_{V_{dB}}$ remain constant. It was found that as $BV$ was decreased, $NF_{V_{dB}}$ increased by 2.5 dB, thus accounting for the 3 dBm, rather than 5.9 dBm, increase in $TSS$. 

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which is the same as predicted by Ayer (provided one adds +4 dB to his equation to account for the 4-dB signal-to-noise ratio at the detector input for a TSS condition).
Basic Thresholding

$T_{ss|dBm}$ is a reasonable parameter to compare receiver-detector-video amplifiers with similar $B_R$ and $B_V$; however, the real world of electronic warfare and radar requires a threshold to be exceeded to indicate the presence of a signal. The effect of $B_R$, $B_V$, and $G_R$ on this threshold will now be presented.
FIGURE 2A-8. $T_{SS}^{\text{Mat \ dBm}}$ for $B_R/B_V = 80$.

Figure 2A-1 illustrates the general configuration to be presented. The detector-video amplifier drives a threshold comparator as shown. With no signal present, if signal threshold voltage, $V_{T|S}$, is too low, noise due to the receiver-detector-video amplifier will trigger the threshold, giving a large noise-count termed false-alarm rate (FAR). As $V_{T|S}$ is increased, this false signal count decreases. We will be concerned with the threshold voltage that gives a FAR count of one per second (FAR = 1). Figure 2A-9 illustrates the FAR count versus $V_{T|S}$ as the receiver gain of Figure 2A-1 is varied (this is the same configuration used for Table 2A-1). Several observations concerning Figure 2A-9 should be noted:

1. For a given gain, FAR is a strong function of $V_{T|S}$.

2. The FAR curves are a function of $G_{R|\text{dB}}$. This is to be expected at large gains ($G_{R|\text{dB}}^{\text{Mat}}$) since the receiver is contributing noise.
FIGURE 2A-9. False Alarm Rate Versus $V_T|_S$
($B_R = 120$ MHz, $B_V = 7.7$ MHz, $NF_{RdB} = 6$).
3. For receiver gains less than 45 dB ($G_R^{Max} = 47.8$, from Table 2A-1), the FAR (log scale) curves versus $V_T|S$ are fairly linear.

4. For receiver gain in excess of $G_R^{Max}$, the threshold voltage necessary to give FAR = 1 becomes excessive.

Item 4 is most important in receiver systems employing variable gain. If $V_T|S$ is set for 0.4 volt (FAR = 1 for $G_R$ (dB) = 0 dB), the FAR will be 100,000 if the gain is increased to 45 dB. This is obviously an unreasonable situation. One obvious solution would be to set $V_T|S = 1$ volt, which gives FAR = 1 for a gain of 45 dB. However, as the gain is decreased (decreasing the noise), only 0.4 volt of $V_T|S$ is needed. Obviously, the FAR for 0-dB gain and $V_T|S = 1$ volt is quite low (one count every 15 minutes or so); however, the input signal to the comparator must increase to 1 volt (rather than 0.4 volt) to trigger the threshold. Thus a loss in detector output dynamic range occurs. This loss in output dynamic range, referred to the input of the square law detector, may be given as

$$D_R^{Loss}_{dBm} = 10 \log \frac{V_T^A}{V_T^B}$$

(2A-17)

where

- $V_T^A|S$ = signal threshold for gain A (FAR = 1)
- $V_T^B|S$ = signal threshold for gain B (FAR = 1)

As an example (refer to Figure 2A-9), the threshold for a gain of 50 dB is 2 volts, and for a gain of 0 dB is 0.4 volt. The loss in dynamic range is

$$D_R^{Loss}_{dBm} = 10 \log \frac{2}{0.4} = 7 \text{ dB}$$

(2A-18)

or if the threshold is fixed at 2 volts and the gain is decreased to 0 dB, the receiver input must be increased 7 dBm to exceed the 2-volt threshold.
Reasonable Criterion for Signal Sensitivity

You are probably wondering at this point what criterion should be used to determine if a signal is present: A threshold crossing for all pulses? 50%? Or what? We will use the 80% probability of detection (PdI₈₀) criterion--80% of the signal pulse repetition frequency (PRF) will trigger the signal threshold (for a PRF of 2 kHz, a count of 1.6 kHz is the 80% PdI₈₀). Figure 2A-10 shows the 80% probability of detection sensitivity (PdI₈₀), as a function of receiver gain, for a FAR = 1 (with no signal, VTJS is increased until FAR = 1; then the signal is increased until an 80% signal count is obtained). A plot of TSS is also given, and it should be noted that PdI₈₀ is between 1 and 2 dBm lower than TSSdBm. The loss in instantaneous dynamic range, DRLoss (Equation (2A-17)) and the rms video noise are also shown. The important point here is that for receiver gains larger than GrMax, the video noise increases, necessitating an increase in VTJS to keep FAR = 1, thus increasing DRLoss. A reasonable estimate in the loss in DRLoss for gains in excess of GrMax is:

\[
DRLoss = G_{R_{dBm}} - G_{R_{dBm}}^{Max} + 6 \tag{2A 19}
\]

Figures A-11 through A-16 illustrate FAR versus VTJS and PdI₈₀ for the receiver parameters given in Table 2A-1. It is interesting to note that PdI₈₀ for large receiver gains is within 1 dBm of the measured TSSMaxdBm. At lower gains (gain-limited sensitivity), the difference between TSSMaxdBm is between 2 and 3 dB.

Table 2A-2 is a summary of the data taken for the receivers presented. Several important observations should be noted:

1. PdI₈₀ measured at GrMax is within 1 dBm of the maximum possible.

2. PdI₈₀ maximum is within 2 dB of the maximum tangential sensitivity.

3. The loss in dynamic range is approximately 6 dB for GrMax.
Thus, from a practical standpoint, the 80% probability of detection sensitivity (FAR = 1) may be approximated as:

\[
\left. P_d \right|_{\text{dBm}}^{80\%} = T_{\text{SS}} \left. \right|_{\text{dBm}}^{\text{Max}} - 2
\]

(2A 20)

and the maximum gain for this sensitivity is equal to the maximum gain obtained from Equation (2A-12).
FIGURE 2A-11. False Alarm Rate Versus $V_{\text{TH}}$ 
($BR = 40 \text{ MHz}, BV = 7.7 \text{ MHz}, NF_{\text{RdB}} = 5$).
FIGURE 2A-13. False Alarm Rate Versus $V_{THS}$ ($B_R = 120$ MHz, $B_V = 0.5$ MHz, $NF_{RdB} = 6$).
FIGURE 2A-14. Sensitivity Versus Receiver Gain
($B_R = 120$ MHz, $B_V = 0.5$ MHz, $NF_R/d_R = 6$).
FIGURE 2A-15. FAR Versus VTIS ($B_R = 40$ MHz, $B_V = 0.5$ MHz, $N_{FRdB} = 5$).
FIGURE 2A-16. Sensitivity Versus Receiver Gain
\( (B_R = 40 \text{ MHz}, B_V = 0.5 \text{ MHz}, N_{FRdB} = 5) \).
Appendix 2B

DERIVATIONS OF $T_{SS}$ | $GR_{\text{dBm}}$, $T_{SS}$ | $GR_{\text{Max dBm}}$

Tsui solves Lucas' $T_{SS}$ equation as (for $BR > 2 BV$)

$$T_{SS\,\text{dBm}} = -114 + NF_R\,\text{dB} + 10 \log \left| 6.31 R_V + 2.5 \sqrt{(2BR_V - B_V^2) + \frac{AB_V}{(G_K F_V)^2}} \right| \quad (2B.1)$$

where

$$A = \frac{4 F_V R_V}{KTK_{SL}} \times 10^{-6} \quad (2B.2)$$

and

$R_V = \text{detector video resistance}$

$T = \text{temperature in degrees Kelvin}$

$K_{SL} = \text{detectors square law K in mV/mW}$

$F_V = \text{video amplifier noise factor}$

$K = \text{Boltzmann's constant} \ (1.38 \times 10^{-23} \text{J/K})$

If $GR_R = 1$, $AB_V$ is found to be $> 2BR_V - B_V^2$, and Equation 2B.1 may be solved for $A$

$$A = \frac{\left( \frac{T_{SS\,\text{dBm}} + 110}{10} \right)^2}{10 B_V} \quad (2B.3)$$

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Limiting Case 1, $ABV/(GRF)^2 \gg (2BRBV - BV^2)^*$

$$T_{SS} \bigg|_{dBm} = -114 + NF_R \bigg|_{dB} + 10 \log \left( 2.5 \sqrt{\frac{ABV}{(GRF)^2}} \right)$$  \hspace{1cm} (2B-4)

Substituting Equation (2B-3) for $A$

$$T_{SS} \bigg|_{dBm} = -110 + \left[ 10 \log \sqrt{\left( \frac{T_{SS} \bigg|_{dBm} + 110}{10} \right)} - GR \bigg|_{dB} \right]$$  \hspace{1cm} (2B-5)

or

$$T_{SS} = T_{SS} \bigg|_{dBm} - GR \bigg|_{dB}$$  \hspace{1cm} (2B-6)

Limiting Case 2, $(2BRBV - BV^2) \gg ABV/(GRF)^2$

$$T_{SS} \bigg|_{dBm}^{\text{Max}} = -114 + NF_R \bigg|_{dB} + 10 \log \left( 6.31 BV + 2.5 \sqrt{2RV - BV^2} \right)$$  \hspace{1cm} (2B-7)

or

$$T_{SS} \bigg|_{dBm}^{\text{Max}} = -114 + NF_R \bigg|_{dB} + 10 \log \left( 6.31 + 2.5 \sqrt{2RV - BV^2} \right)$$  \hspace{1cm} (2B-8)

Many authors neglect the $6.31 BV$ term in the equation. This assumption, valid only if $BR \gg BV$, yields a $T_{SS}$ (Equation (2B-7)) of

$$T_{SS} \bigg|_{dBm} = -114 + NF_R \bigg|_{dB} + 10 \log (2.5) \left( \sqrt{2RV - BV^2} \right)$$  \hspace{1cm} (2B-9a)

or

$$T_{SS} \bigg|_{dBm}^{\text{Max}} = -110 + NF_R \bigg|_{dB} + 10 \log \left( \sqrt{2RV - BV^2} \right)$$  \hspace{1cm} (2B-9b)

*It should be noted that $\sqrt{ABV/(GRF)^2}$ is much greater than 6.31 BV.
The term $\sqrt{2R_B V - B_V^2}$ has been termed the "effective bandwidth" and, if it can be used, simplifies $T_{SS}$ calculations since a single effective bandwidth can be assigned to a given receiver. Comparing Equations (2B-8) and (2B-9a), we see that the only error is to the right of the 10 Log term. Equation (2B-9a) may now be given as

$$T_{SS}^{\text{Max}}_{\text{dbm}} = -114 + NF_R + 10 \log (2.5 B_V) \left( \sqrt{2 \frac{R}{B_V}} - 1 \right)$$  \hspace{1cm} (2B-10)$$
or

$$T_{SS}^{\text{Max}}_{\text{dbm}} = -114 + NF_R + 10 \log B_V + 10 \log \left( 2.5 \sqrt{2 \frac{R}{B_V}} - 1 \right)$$  \hspace{1cm} (2B-11)$$

Thus the error from the actual predicted $T_{SS_{\text{dbm}}}$ (Equation (2A-8)) and approximate $T_{SS}^{\text{Max}}_{\text{dbm}}$ (Equation (2B-10)) is

$$\text{Error}_{\text{db}} = \text{Equation B-10} - \text{Equation B-11}$$  \hspace{1cm} (2B-12)$$
or

$$\text{Error}_{\text{db}} = 10 \log \left( 6.31 + 2.5 \sqrt{2 \frac{R}{B_V}} - 1 \right) - 10 \log \left( 2.5 \sqrt{2 \frac{R}{B_V}} - 1 \right)$$  \hspace{1cm} (2B-13)$$

Table 2B-1 gives this error for various $R/B_V$ ratios and, as can be seen, $R/B_V > 50$ has an error within 1 dB. Thus, using Equation (2A-9b) to predict $T_{SS_{\text{Max}}_{\text{dbm}}}$ can give larger than expected predicted $T_{SS}^{\text{Max}}_{\text{dbm}}$. 

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TABLE 2B-1. Error in Assuming $BR/B_V \gg 1$.

| $BR/B_V$ | $10 \log \left( 6.31 + 2.5 \sqrt{2} \frac{BR}{B_V} - 1 \right)$ | $10 \log \left( 2.5 \sqrt{2} \frac{BR}{B_V} - 1 \right)$ | Error $|dB$ |
|----------|--------------------------------------------------|--------------------------------------------------|----------|
| 2        | 10.27                                            | 6.37                                             | 3.9      |
| 5        | 11.4                                             | 8.75                                             | 2.65     |
| 10       | 12.36                                            | 10.37                                            | 1.98     |
| 50       | 14.94                                            | 13.96                                            | 0.98     |
| 100      | 16.19                                            | 15.47                                            | 0.71     |

Determining the maximum gain necessary to provide $TSS_{dBm}^{dBI}$ is not, as stated earlier, a straightforward matter; however, solving for the gain necessary to give Equation (2B-1):

$$\frac{AB_B}{dG_R^2 + 1} = 2B_BR_V - R_V^2$$

(2B-14)

does give quite reasonable results. Substituting Equation (2B-3) into Equation (2B-14) and solving for $G_R$:

$$G_R_{\text{Max}} = \left. TSS\right|_{dBm}^{G_0} + 110 - \left. NF_R\right|_{dBm}$$

(2B-15)

$$- 10 \log \sqrt{2B_BR_V - R_V^2}$$
The TSS\textsubscript{dB} for this condition may be easily solved as

\[ T_{\text{SS}} \left| dB_{\text{m}} \right|^\text{Max} = -114 + N \text{FR}_\text{dB} \]

\[ + 10 \log \left( B_V \right) \left( 6.32 + 3.54 \sqrt{2 \frac{R}{B_V}} - 1 \right) \]

The error in TSS\textsubscript{dB\textsubscript{m}} predicted (Equation (2B-16)), and the maximum TSS\textsubscript{dB\textsubscript{m}} expected (Equation (2B-8)), is summarized in Table 2B-2. As can be seen, the TSS\textsubscript{dB} predicted using \( GR_{\text{dB}} \) is slightly lower than expected; however, this error is at worst \(-1.5\) dB (for large \( B_R/B_V \) ratios). It should be noted that increasing \( GR_{\text{dB}} \) 1 dB above our calculated value does not increase TSS\textsubscript{dB} by 1 dB, as the two terms in Equation (2B-14) are still nearly equal. To actually obtain the TSS\textsubscript{Max\textsubscript{dB\textsubscript{m}}} predicted by Equation (2B-8), \( GR_{\text{dB}} \) must be increased 5 dB or so above \( GR_{\text{Max\textsubscript{dB}} \text{dB}} \) given in Equation (2B-15). This increase in gain increases the video noise and causes serious thresholding problems. It has been found experimentally that the value for \( GR_{\text{Max\textsubscript{dB}}} \) of Equation (2B-15) approximately doubles the rms noise out of the video amplifier (with respect to the noise with no receiver gain). Thus \( GR_{\text{Max\textsubscript{dB}}} \) may be regarded, in principle at least, as that gain needed to produce a receiver noise power equal to the detector-video amplifier noise power at the detector's input.
<table>
<thead>
<tr>
<th>BR/BV</th>
<th>10 Log (6.31 + 2.5 \sqrt{2BR/BV - 1})</th>
<th>10 Log (6.31 + 2.5 \sqrt{2BR/BV - 1})</th>
<th>Error dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10.27</td>
<td>6.37</td>
<td>3.9</td>
</tr>
<tr>
<td>5</td>
<td>11.40</td>
<td>8.75</td>
<td>2.65</td>
</tr>
<tr>
<td>10</td>
<td>12.36</td>
<td>10.37</td>
<td>1.98</td>
</tr>
<tr>
<td>50</td>
<td>14.94</td>
<td>13.96</td>
<td>0.98</td>
</tr>
<tr>
<td>100</td>
<td>16.19</td>
<td>15.47</td>
<td>0.71</td>
</tr>
<tr>
<td>&gt;100</td>
<td>...</td>
<td>...</td>
<td>-1.5</td>
</tr>
</tbody>
</table>
Appendix 2C

DERIVATION OF $T_{SS|dBm}$, $T_{SS|dBm}^{Max}$, AND $G_R|dB$ FOR $B_V \leq B_R \leq 2B_V$


$$T_{SS|dBm} = -114 + NF_R|dB$$

$$+ 10 \log \left| 3.15B_R + 2.5 \sqrt{B_R B_V - B_V^2} + \frac{AB_V}{(G_R F_R)^2} \right|$$

(2C-1)

The only difference between Equation (2C-1) and Equation (2B-1), is the first term after the $\log$ (6.31$B_V$ for Equation (2B-1) and 3.15$B_V$ for Equation (2C-1)). The A term is defined in Equation (2B-3). The limiting cases may now be given as

Limiting Case 1: $\frac{AB_V}{(G_R F_R)^2} \gg 2B_R B_V - B_V^2$

$$T_{SS|dBm}^{Max} = T_{SS|dBm} + G_R|dB$$

(2C-2)

which is the same as Equation (2B-6) and is to be expected.

Limiting Case 2: $(2B_R B_V - B_V^2) \gg \frac{AB_V}{(G_R F_R)^2}$

$$T_{SS|dBm}^{Max} = -114 + NF_R|dB + 10 \log (B_V)$$

$$+ 10 \log \left| 3.15 + \sqrt{\frac{B_R}{B_V} - 1} \right|$$

(2C-3)

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This equation gives a $TSS$ some 1.5 to 2.3 dBm better than the $TSS$ if Equation (2B-8) is used.

The receiver gain necessary to give $TSS_{dBm}^{Max}$ is found in accordance with Equation (2B-14)

$$\frac{A_B V^+ V}{G_R F_R^2} = 2B_R B_V - B_V^2$$

and Equation (2B-15) is obtained

$$G_R_{dBm}^{Max} = T_{SS_{dBm}} + 110 - NF_R_{dB} - 10\log \sqrt{2B_R B_V - B_V^2}$$

or

$$G_R_{dBm}^{Max} = T_{SS_{dBm}} + 110 - NF_R_{dB} - 10\log B_V$$

$$- 10\log \sqrt{\frac{B_R}{B_V} - 1}$$

It has been found experimentally that for $B_V > B_R$, replacing $10\log B_V$ with $10\log B_R$ and neglecting the $\sqrt{2B_R/B_V} - 1$ terms in Equations (2C-3) and (2C-5b) gives quite reasonable results.
Nomenclature

\( A_{V/ST} \)  signal threshold noise amplifier gain

\( B_e \)  effective bandwidth

\( B_R \)  receiver Bandwidth

\( B_V \)  video bandwidth

\( D \)  noise integrator duty cycle

\( D_R \)  receiver instantaneous dynamic range loss

\( f_{3dB} \)  noise riding threshold loop 3-dB frequency response

\( FAR \)  false alarm rate

\( FAR_N \)  normalized FAR count

\( F_V \)  video amplifier noise factor

\( G_0 \)  receiver gain = 0 dB

\( G_R |_{dB} \)  receiver gain (dB)

\( H \)  noise loop transfer function

\( LG \)  loop gain

\( M \)  detector diode constant

\( N \)  normalized noise count

\( n \)  noise loop contrast

\( N_{D_{dB}} \)  effective detector noise input for \( G_R = 0 \)

\( NF_{R_{dB}} \)  receiver noise figure (dB)
\( o/s \) one shot

- \( P_{D\text{dBm}} \): detector input (dBm)
- \( P_{d\text{dBm}} \): signal input for 80% probability of detection
- \( P_{\text{in\text{dBm}}} \): receiver input power (dBm)
- \( PRF \): signal pulse repetition frequency
- \( PW \): signal pulse width

- \( S_{DAC} \): DAC scale factor (volt/bit)
- \( S_F \): noise loop scale factor (FAR/volt)

- \( T \): temperature in degrees Kelvin
- \( T_C \): counter enable time
- \( T_I \): integrator enable (update) time

- \( T_{SS\text{dBm}} \): tangential signal sensitivity
- \( T_{SS\text{dBm}}^{\text{Co}} \): \( T_{SS} \) for no receiver (receiver gain = 0)
- \( T_{SS\text{dBm}}^{\text{Max}} \): maximum possible \( T_{SS} \) obtainable

- \( V_{DAC} \): digital to analog converter effective output voltage
- \( V_e \): error voltage
- \( V_{\text{Ref}} \): reference voltage
- \( V_{T/N} \): noise threshold voltage
- \( V_{T/S} \): signal threshold voltage

- \( \Delta t \): noise loop closure time
Bibliography


Chapter 3

RANGE-TRACKING LOOPS

Many radar systems (both airborne and land or ship based) employ range trackers as an integral part of their pulse processing. This chapter will present the basics of range tracker operation, and a detailed analysis and design procedures (with examples) for the heart of range tracking: the range tracking integrators.

Target acquisition and reacquisition techniques will not be covered, and the discussions to follow assume that range tracker lock-on has occurred (as will be shown, the range track loop is, in essence, a phase-locked loop and the acquisition and lock-on concepts are similar).

Figure 3-1 illustrates a simplified active pulse radar. The transmit-receive (T/R) switch allows the transmitted signal to be radiated by the antenna. The T/R switch then connects the antenna to a mixer and the received signal is mixed to the IF frequency, amplified, detected, and fed to the pulse processing circuitry. The range-tracking loop enables the AGC loop, angle tracker, and target detection and acquisition circuitry only when an expected return pulse is present.

The range tracker must be capable of following any changes in closing velocity between the radar and target, and if the target return should fade, the range enable gates should still be generated until the target reappears or reacquisition is initiated. The loop bandwidth should also be as low as possible to limit the effectiveness of countermeasure techniques used against range-tracking loops.
Figure 3-2 illustrates a basic airborne radar-target configuration. The radar transmits at time $T_0$, and the transmitted pulse reaches the target at time $T_1$. The pulse is then reflected and reaches the radar at time $T_2$. It is obvious that $T_1 = T_2$. Thus, the range to the target, $R$ (assuming the pulse travels at the speed of light), is

$$R = \frac{CA_1}{2} \text{ (ft)}$$  \hspace{1cm} (3-1)
where
\[ c = \text{speed of light (984 ft}/\mu\text{sec}) \]
\[ \Delta t = T_1 + T_2 \] (total time of pulse travel in μsec)

**FIGURE 3-2. Typical Radar Target Configuration.**

Figure 3-3 illustrates the basic range-tracking loop configuration (it should again be emphasized that target acquisition has occurred). At time \( T_0 \), the T/R switch allows signal transmission and also starts the linear range ramp. When the range ramp voltage equals the range integrator voltage, \( ER \) (Figure 3-4a), the comparator output changes state, triggering the range enable and early-late gate. The early-late gates enable the range discriminator, the output of which is continually "zeroed" by the feedback action of the loop (i.e., the range gate is driven such that the target return is centered in the range gate). The loop filter determines the dynamic loop performance.

The configuration shown in Figure 3-3 does not lend itself to straightforward analysis. Appendix 3A discusses the range-tracking loop from a phase-locked loop perspective, and the range-tracking loop can be presented (for analysis purposes) as illustrated in Figure 3-5. We will first do a general analysis of the functional range-tracking loop (without specifying the loop filter) and then replace \( F(S) \) with two filters that are in general use.
FIGURE 3-3. Basic Range Track Loop (Type II, Zero Velocity Error).
(a) Range ramp range voltage timing.

(b) Discriminator volt-error curve.

FIGURE 3-4. Range Ramp and Discriminator Timing.
FIGURE 3-5. Functional Range-Tracking loop.

The general loop gain equation may be given as (Figure 3-5)

\[ L(G(S)) = (D) F(S) \left( \frac{1}{RCS} \right) S_R \]  

or

\[ L(G(S)) = \frac{DS_R F(S)}{R_1 S} \]  

The loop transfer function, \([R_o(S)]/[R_{in}(S)]\), may be given as

\[ H(S) = \frac{R_o(S)}{R_{in}(S)} = \frac{L(G(S))}{1 + L(G(S))} \]  

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Substituting Equation (3-3) into (3-4),

\[ H(S) = \frac{DS_R F(S)}{S + DS_R F(S)} \]  
(3.5)

The normalized range error is

\[ \frac{R_{in}(S) - R_{in}(S)}{R_{in}(S)} = \frac{R_{in}(S)}{R_{in}(S)} \]  
(3.6)

or

\[ 1 - \frac{R_{in}(S)}{R_{in}(S)} = \frac{R_{in}(S)}{R_{in}(S)} = 1 - H(S) \]  
(3.7)

Substituting Equation (3-5) into (3-7),

\[ \frac{R_{in}(S)}{R_{in}(S)} = \frac{S}{S + DS_R F(S)} \]  
(3.8)

Solving Equation (3-8) for the range error, we have

\[ R_{in}(S) = R_{in}(S) \frac{S}{S + DS_R F(S)} \]  
(3.9)

The importance of Equation (3-9) lies in the fact that the loop dynamics to step input changes in range (\(\Delta R_{in}S\)), velocity (\(\Delta Vel \cdot S^2\)), and acceleration (\(\Delta Accel \cdot S^3\)) may be obtained. The final range error can then be found by the final value theorem:

\[ \lim_{t \to \infty} R_{in}(t) = \lim_{S \to 0} S|R_{in}(S)| \]  
(3.10)
The first loop filter that will be covered (and the one generally used) is illustrated in Figure 3-6. This filter ensures a zero final range error to a velocity step as will be shown.

\[
\begin{align*}
R_2 C_v & = \frac{1}{\tau_1} = R_1 C_v \\
R_1 & = \frac{1}{\tau_2} = R_2 C_v
\end{align*}
\]

FIGURE 3-6. Zero Velocity Error (Type II) Loop Filter.

The transfer function, assuming a large operational amplifier gain, may be given as:

\[
F(s) = \frac{E_c(s)}{E_v(s)} = \frac{1 + S \tau_2}{S \tau_1}
\]  

(3-11)

Substituting Equation (3-11) into (3-3), the loop gain becomes

\[
LG(s) = \frac{D_S R}{\tau_R \tau_1} \left| \frac{1 + S \tau_2}{S^2} \right|
\]

(3-12)

*The inverting sign for the filter (and the range integrator) will be omitted as it is easily accounted for in the final design.*
Letting

\[ K = \frac{DS_R}{t_{RI}} \quad (3.13) \]

\[ LG(S) = \frac{K}{S^2} \left[ \frac{1 + S t_2}{S t_1} \right] \quad (3.14) \]

The loop transfer function, \( H(S) \) (Equation (3.4)), may now be given as

\[ H(S) = \frac{K}{S} \left[ \frac{1 + S t_2}{S t_1} \right] \quad (3.15) \]

or, upon simplifying,

\[ H(S) = \frac{K + \frac{K t_2}{t_1}}{S + \frac{K}{t_1}} \quad (3.16) \]

This is the classic equation of a second-order system*

\[ H(S) = \frac{S + \omega_n^2 + 2 \zeta \omega_n S}{S^2 + 2 \zeta \omega_n + \omega_n^2} \quad (3.17) \]

* Order is defined as the highest degree of the denominator of the characteristic equation. Since we have an \( S^2 \) in the denominator of Equation (3.17), this is a second order system.

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where $\omega_n$ is the loop natural frequency and $\zeta$ is the damping factor. From Equations (3-16) and (3-17),

$$\omega_n = \sqrt{\frac{K}{t_1}} = \sqrt{\frac{DS_R}{t_{RI} t_1}}$$  \hspace{1cm} (3-18)

$$\zeta = \frac{t_2}{2} \sqrt{\frac{K}{t_1}} = \frac{t_2}{2} \sqrt{\frac{DS_R}{t_{RI} t_1}}$$  \hspace{1cm} (3-19)

Equations (3-18) and (3-19) illustrate that the natural frequency, $\omega_n$, can be made independent of $\zeta$ by ensuring

$$\frac{t_2}{2} \omega_n = \text{constant}$$  \hspace{1cm} (3-20)

The importance of Equation (3-20) will be discussed momentarily.

In addition to being a second-order system, Equation (3-20) is also a Type II loop.* Thus from classic servo theory we expect the final value for the range error, $R_e$, found in Table 3-1.

<table>
<thead>
<tr>
<th>Input</th>
<th>Final Error ($R_e(\text{final})$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step range</td>
<td>0</td>
</tr>
<tr>
<td>Step velocity</td>
<td>0</td>
</tr>
<tr>
<td>Step acceleration</td>
<td>Constant</td>
</tr>
</tbody>
</table>

---

*Loop type is defined as the number of poles of the loop gain (Equation (3-15)) located at the origin.

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Substituting the filter Equation (3-11) into the range error Equation (3-9) and solving for the range error as a function of time yields complicated results. Fortunately, these equations have been solved and graphed for phase-locked loops [1 and 2], and, since the configuration of Figure 3-5 (and the previous discussion) is based on phase-locked loop theory, they are equally valid for range-tracking loops.

Figures 3-7 through 3-9 are plots of normalized range error for a step in range, velocity, and acceleration, respectively. As expected, in Figures 3-7 and 3-8, the final range error is zero. In Figure 3-9 the final range error is nonzero and approaches

$$R_e(\text{final}) = \frac{\Delta \text{Accel}}{\omega_n^2}$$  \hspace{1cm} (3-21)

Figure 3-10 summarizes the general equations for a second-order, Type II range-tracking loop for easy reference.

Second-Order, Type II, Range-Tracker Design Procedure

The following paragraphs describe the procedure for designing a second-order, Type II, range tracker. Figure 3-11 illustrates the dual bandwidth (fast for velocity acquisition, slow for acceleration tracking), second-order, Type II, range-tracking loop that will be discussed.

A. Knowing maximum range in feet ($R_{\text{max}}(\text{ft})$), determine range ramp sweep width ($T_{SW}$) and scale factor, $S_v$.

$$T_{SW} = 1.016 \times 10^9 \left( \frac{\text{sec}}{\text{ft}} \right) R_{\text{max}}(\text{ft}) \text{ sec}$$  \hspace{1cm} (3-22)
FIGURE 3-7. Range Error, $R_e(t)$, due to a Step in Range, $\Delta R$ (Second-Order, Type II Loop).

FIGURE 3-8. Range Error, $R_e(t)$, due to a Step in Velocity, $\Delta \text{Vel}$ (Second-Order, Type II Loop).
FIGURE 3-9. Range Error, $R_e(t)$, due to a Step in Acceleration, $\Delta$Accel (Second-Order, Type II Loop).

The maximum value for the sweep voltage ($E_{SM}$) is only limited to the supply voltages available.

$$S_v = \frac{E_{SM}}{T_{SW}} \text{ V/sec}$$  \hspace{1cm} (3-23)

B. Knowing the range ramp scale factor, calculate $S_R$ (ft/volt)

$$S_R = \frac{R_{max}(ft)}{E_{SM}} = \frac{R_{max}(ft)}{S_v (\text{volt/ sec}) T_{SW} (\text{sec})} \text{ ft/V}$$ \hspace{1cm} (3-24)
\[
\frac{R_o(S)}{R_{in}(S)} = \frac{\omega_n^2 + 2 \zeta \omega_n S}{S^2 + 2 \zeta \omega_n + \omega_n^2}
\]

\[
\omega_n = \sqrt{\frac{DS_R}{\tau_{RI} \tau_1}} = \frac{\tau_2}{2 \omega_n}
\]

NOISE BANDWIDTH, \( BW_N = \frac{\omega_n}{2} \left( \frac{1}{\tau_2} + \frac{1}{4\tau_1} \right) \) Hz

<table>
<thead>
<tr>
<th>INPUT</th>
<th>FINAL ERROR, ( R_\xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta R )</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta ) VELOCITY</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta ) ACCELERATION</td>
<td>( \Delta ) ACCEL/( \omega_n )^2</td>
</tr>
</tbody>
</table>

FIGURE 3-10. Second-Order, Type II, Range-Tracker General Equation Summary.
FIGURE 3-11. Second-Order, Type II, Range-Tracking Loop.
C. Calculate the range integrator time constant, \( \tau_{RI} \). The range integrator output voltage may be given as:

\[
\frac{d E_R}{dt} = \frac{E_v}{\tau_{RI}} \tag{3-25}
\]

Since \( E_v \) represents the target range-tracker closing velocity, the range voltage will be a linear ramp (remember a Type II loop has zero velocity error, and the range voltage, \( E_R \), must keep up with the target velocity). Referring to Figure 3-5, the velocity of \( R_o \) must be the same as the velocity of \( R_{in} \) \( (Vel_{in}) \); thus,

\[
\frac{d R_o}{dt} = \frac{d E_R}{dt} = \frac{E_v}{\tau_{RI}} \tag{3-26}
\]

or

\[
\frac{Velocity}{S_R} = \frac{E_v}{\tau_{RI}} \tag{3-27}
\]

Knowing the maximum velocity (this must be given as a system specification), we are free to choose the maximum value for \( E_v \). Thus \( \tau_{RI} \) may now be found.

\[
\tau_{RI} = R_{RI} C_{RI} = \frac{E_v^{(max)} S_R}{Vel^{(max)}} \tag{3-28}
\]

D. The wideband velocity integrator, \( \tau_{IW} \) and \( \tau_{2W} \), may be found knowing the maximum velocity step \( (\Delta Vel) \), the desired system damping factor \( (\xi) \), \( S_R \), the discriminator scale factor \( (D) \), and the maximum range error \( (R_e) \). Using Figure 3-8, find the maximum value (for the desired system \( \xi \)) for

*The minus sign for the integrator will be omitted as discussed earlier.*
[\text{R}_c(\tau)/\text{Vel}/\omega_n]. \text{ Let } X_{\text{Vel}} \text{ equal this value. The wideband system } \omega_{nw} \text{ may now be found as}

\[ \omega_{nw} = \frac{X_{\text{Vel}} \text{Vel}}{R_c(\text{max})} \]  

(3-29)

From Figure 3-10,

\[ \omega_{nw}^2 = \frac{D S_R}{t_{RI} t_{1w}} \]  

(3-30)

or

\[ t_{1w} = R_1 C_{vw} = \frac{D S_R}{t_{RI} \omega_{nw}^2} \]  

(3-31)

also,

\[ \zeta = \frac{t_{2w} \omega_{nw}}{2} \]  

(3-32)

or

\[ t_{2w} = R_2 C_{vw} = \frac{2 \zeta}{\omega_{nw}} \]  

(3-33)

The loop closure time, \( t_c \), may be found knowing \( \omega_n \), \( \zeta \), and using Figure 3-8 as follows: find \( \omega_n t \) where the relative range error goes to zero and divide by \( \omega_{nw} \).

E. Knowing \( \omega_{nw} \) and \( \zeta \), the loop noise bandwidth may be found:

\[ BW_{nw} = \frac{\omega_{nw}}{2} \left( \zeta + \frac{1}{4 \zeta} \right) \text{Hz} \]  

(3-34)

F. The loop bandwidth can be decreased after the target has been acquired in the wideband mode. Decreasing the loop bandwidth has the advantage of making range gate stealing (a common countermeasure
technique) more difficult. The loop should just be fast enough to follow any acceleration changes. Referring to Figure 3-9, find the maximum value for $|R(t)|/|\Delta\text{Accel}/\omega_n^2|$ for the system $\zeta$. Call this value:

$$X_{\text{Accel}} = \frac{R_{r}(\text{max})}{\Delta\text{Accel}/\omega_n^2} \quad (3.35)$$

The narrowband loop natural frequency may now be found:

$$\omega_{nN} = \sqrt{\frac{X_{\text{Accel}}\Delta\text{Accel}}{R_{r}(\text{max})}} \quad (3.36)$$

Now, from Figure 3-10

$$t_{1N} = R_1 C_{VN} = \frac{DS_R}{t_R \omega_{nN}^2} \quad (3.37)$$

The value for $R_1$ is determined from the wideband case and a unique value of $C_{VN}$ can be found.

$$C_{VN} = \frac{DS_R}{t_R \omega_{nN}^2 R_1} \quad (3.38)$$

The system damping ratio, $\zeta$, should be independent of $\omega_n$, thus a unique value of $R_{2N}$ may be determined.

$$\zeta = \frac{t_{2N}}{2 \omega_{nN}} \quad (3.39)$$

Solving for $t_{2N}$

$$t_{2N} = R_{2N} C_{VN} = \frac{2\zeta}{\omega_{nN}} \quad (3.40)$$

thus,

$$R_{2N} = \frac{2\zeta}{\omega_{nN} C_{VN}} \quad (3.41)$$

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The final range error is

\[ R_c(\text{final}) = \frac{\Delta \text{Accel}}{\omega_N^2} \]  \hspace{1cm} (3.42)

G. Knowing \( \omega_n N \) and \( \zeta \), the loop noise bandwidth may be found

\[ H_{WN} = \frac{\omega_n N}{2} \left( \zeta + \frac{1}{4 \zeta} \right) \text{ Hz} \] \hspace{1cm} (3.43)

H. The range discriminator is discussed in Appendix 3B, and the equations presented are excellent approximations; however, the discriminator differencing amplifier gain, \( A_\Delta \), may have to be adjusted to give the desired scale factor. Practice has shown that a value for \( D \) of 50 mV/ft gives reasonable values for the range and velocity integrator component values. Thus, practical designs assume a value for \( D \), and calculating the range discriminator differencing amplifier gain, \( A_\Delta \), is necessary to give \( D \). The value for \( A_\Delta \) may be given as (Appendix 3B)

\[ A_\Delta = \frac{D_{EG} \text{ (nsec)}}{4.1 e_{in}} \] \hspace{1cm} (3.44)

Figures 3.12 through 3.16 summarize the design procedure as an easy reference for the design to follow.
To
TARGET
RETURN
TARGET
RANGE
RADIUS
11
RANGE
MYAXIMUM
TARGET
RANGE
R(MAX), TR(MAX)--
N/RESET
TIME (TRS)
Sv (VOLT/SEC)
RANGE RAMP
TR(MAX), SEC
R(MAX), FT

$T_{R}(\text{MAX}) = 2.03 \times 10^{-9} \ R_{\text{MAX}}(\text{FT}) \ \text{SEC}$

$S_{V} = \frac{E_{S}(\text{MAX})}{T_{R}(\text{MAX})} \ \text{VOLT/SEC}$

$S_{R} = \frac{R_{\text{MAX}}(\text{FT})}{E_{S}(\text{MAX})} \ \text{FT/VOLT}$

$S_{R} = \frac{R_{\text{MAX}}(\text{FT})}{S_{V} T_{R}(\text{MAX})} \ \text{FT/VOLT}$

THE RANGE TRACK LOOP RESET TIME ALLOWS RANGE RAMP RESET AND INITIALIZATION, AND, DEPENDING ON DESIGN, IS IN THE ORDER OF 10 $\mu\text{SEC}$.

$PRI = \frac{1}{PRF} = T_{R}(\text{MAX}) + \text{RESET TIME (TRS)}$

FIGURE 3.12. Range Tracking Loop Scale Factor Design Equations.
Design Example:

A range-tracking loop will be designed to meet the following specifications:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum range, $R_{\text{max}}$</td>
<td>$180 \times 10^3$ ft</td>
</tr>
<tr>
<td>Transmitted pulse width</td>
<td>200 nsec</td>
</tr>
<tr>
<td>Maximum closing velocity, $V_{\text{el(max)}}$</td>
<td>1250 ft/sec</td>
</tr>
<tr>
<td>Maximum velocity step, $\Delta V_{\text{el}}$</td>
<td>400 ft/sec</td>
</tr>
<tr>
<td>Maximum acceleration, $\Delta A_{\text{ccel(max)}}$</td>
<td>5.6 g ($180 \text{ ft/sec}^2$)</td>
</tr>
<tr>
<td>Early/Late gate width, $t_{\text{FG}}$</td>
<td>250 nsec</td>
</tr>
<tr>
<td>Maximum range error due to $\Delta V_{\text{el}}, R_e$</td>
<td>10 feet</td>
</tr>
<tr>
<td>Maximum range error due to $\Delta A_{\text{ccel}}, R_e$</td>
<td>5 feet</td>
</tr>
<tr>
<td>System damping factor, $\zeta$</td>
<td>0.7</td>
</tr>
<tr>
<td>Normalized video input, $e_{\text{in}}$</td>
<td>2 volts</td>
</tr>
</tbody>
</table>

FIND THE MAXIMUM VALUE OF X, FOR THE SYSTEM $\zeta$, FROM THE ABOVE GRAPH.

$$\omega_{nW} = \frac{X \Delta VEL}{R_e (\text{MAX})}$$

$$R_1 C_{VW} = \frac{D S_R}{t_{rel} \omega_{nW}^2}$$

$$R_{2w} C_{VW} = \frac{2 \zeta}{\omega_{nW}}$$

$$BW_{nW} = \frac{\omega_{nW}}{2} \left( \zeta + \frac{1}{4 \zeta} \right) \text{Hz}$$

THE LOOP CLOSURE TIME MAY BE FOUND, KNOWING $\zeta$ AND $\omega_{nW}$, FROM THE ABOVE GRAPH.

FIND THE MAXIMUM VALUE OF $X$, FOR THE SYSTEM $\xi$, FROM THE ABOVE GRAPH.

$$\omega_{nN} = \sqrt{\frac{X \Delta \text{ACCEL}}{R_c(\text{MAX})}}$$

$$R_1 C_{VN} = \frac{D S_R}{t_{RI} \omega_{nN}^2} \quad R_{2N} = \frac{2 \xi}{\omega_{nN} C_{VN}}$$

$$BW_{nW} = \frac{\omega_{nW}}{2} \left( \xi + \frac{1}{4 \xi} \right) \text{Hz}$$

THE LOOP CLOSURE TIME MAY BE FOUND, KNOWING $\xi$ AND $\omega_{nW}$, FROM THE ABOVE GRAPH.

RC > 10 \tau_{EG} \quad \text{RC} < 0.02 \tau_{EG} \cdot \text{PRF}

\tau_c = \text{LOOP CLOSURE TIME}

\text{PRF} = \text{RADAR PULSE REPETITION FREQUENCY}

\text{DISCRIMINATOR SCALE FACTOR, } SE_e

\[ SE_e = \frac{2 A \Delta e_{in}}{t_{EG} \text{nsec}} \text{VOLT} \]

\text{SYSTEM RANGE DISCRIMINATOR SCALE FACTOR, } D

\[ D = SE_e \left( \frac{\text{volt}}{\text{nsec}} \right) \frac{2.03 \text{nsec}}{\text{ft}} = \frac{4.06 A \Delta e_{in}}{t_{EG} \text{nsec}} \text{VOLT} \]

\text{FIGURE 3-16. Range Discriminator Design Equations.}
Determine $TR(\text{max})$, $S_v$, $S_R$, and PRF (Figure 3-12)

$$TR(\text{max}) = 2.03 \times 10^{-9} |R(\text{max})| = 2.03 \times 10^{-9} (180 \times 10^3)$$

$$TR(\text{max}) = 365.8 \mu \text{sec}$$

Let the maximum range sweep voltage, $SE(\text{max}) = 15$ volts

$$S_v = \frac{E_v(\text{max})}{TR(\text{max})} = \frac{15}{365.8 \times 10^{-6}}$$

$$S_v = 41 \times 10^3 \text{ V/sec}$$

$$S_R = \frac{R(\text{max})}{E_v(\text{max})} = \frac{180 \times 10^3}{15}$$

$$S_R = 12,000 \text{ ft/V}$$

$$PRI = TR(\text{max}) + TR_S$$

Let the system reset time $= 34.2 \mu \text{sec}$ (a more than reasonable value).*

$$PRI = 365.8 \mu \text{sec} + 34.2 \mu \text{sec} = 400 \mu \text{sec}$$

$$PRF = \frac{1}{400 \mu \text{sec}} = 2.5 \text{ kHz}$$

B. Calculate $R_{RI}$ and $C_{RI}$ (Figure 3-13). Let $E_v(\text{max}) = 10$ volts for a $Vel(\text{max}) = 1250 \text{ ft/sec}$

$$R_{RI} C_{RI} = \frac{E_v(\text{max}) S_R}{Vel(\text{max})} = \frac{10 (12,000)}{1250}$$

$$R_{RI} C_{RI} = 96$$

* seldom is the PRF determined by the system reset time; however, sufficient system reset time must be accounted for in the design.
Equation (3-55) yields unrealistic values, since, for a $C_{RI}$ of 1 µF

$$R_{RI} = \frac{96}{1 \times 10^{-6}} = 96 \text{ M} \Omega$$  \hspace{1cm} (3-56)

The problem may be corrected by attenuating $E_V$ as shown in Figure 3-17.

\[\text{FIGURE 3-17. Range Integrator Variation.}\]

If $R_3 \ll R_I$

$$E_V' = \frac{R_2 E_v}{R_2 + R_3}$$  \hspace{1cm} (3-57)

or

$$E_V' = \frac{R_3}{R_2 + R_3} = \text{Atten.}$$  \hspace{1cm} (3-58)

Let $C_{RI} = 2.2$ µF and $R_I = 1$ MΩ

$$R_1 C_{RI} = \frac{E_{v, (\text{max})} S_R}{\text{Vel}(\text{max})} = 2.2$$  \hspace{1cm} (3-59)

$$E_{v, (\text{max})} = \frac{R_1 C_{RI} \text{ Vel}(\text{max})}{S_R}$$  \hspace{1cm} (3-60)
\[
E'_v = \frac{2.2(250)}{12,000} = 0.229
\]  
(3.61)

\[
\frac{E'_v}{E_v} = \frac{R_3}{R_2 + R_3}
\]  
(3.62)

Let \(R_3 = 1 \text{k}\Omega\) (this is much less than \(R_1\))

\[
R_2 = \frac{R_3(1 - E'_v/E_v)}{E'_v/E_v}
\]  
(3.63)

\[
R_2 = \frac{1 \times 10^3(1 - 0.229/10)}{0.229/10} = 42.6 \text{k}\Omega
\]  
(3.64)

To verify this result,

\[
\tau_{RI} = \frac{R_1 C_{RI}}{R_3 + R_2}
\]  
(3.65)

\[
\tau_{RI} = \frac{(1 \times 10^3)(2.2 \times 10^{-6})}{1 \times 10^3} = 96
\]  
(3.66)

\[
\frac{(42.6 \times 10^3) + (1 \times 10^3)}{42.6 \times 10^3}
\]

C. Calculate wideband \(R_I\), \(R_{2w}\), \(C_{wv}\) and \(BW_{nw}\) (Figure 3-14). The maximum value of \(X\), for \(\zeta = 0.7\) is (Figure 3-14)

\[
X = 0.45
\]  
(3.67)

\[
\omega_{nw} = \frac{X \Delta v e l}{R_{c(\text{max})}} = \frac{(0.46)(400)}{10}
\]  
(3.68)
\[ \omega_{nw} = 18.4 \text{ rad/sec} \]  

\[ R_1 C_{vw} = \frac{DS_R}{\epsilon R_1 \omega_n^2} \]  

Let \( D = 50 \text{ mV/ft} \)

\[ R_1 C_{vw} = \frac{(0.05)(12,000)}{97(18.4)^2} = 18.27 \times 10^{-3} \]  

Let \( C_{vw} = 0.25 \mu F \)

\[ R_1 = \frac{18.27 \times 10^{-3}}{0.25 \times 10^{-6}} = 73 \text{k}\Omega \]  

\[ R_{zw} = \frac{2\zeta}{\omega_{nw} C_{vw}} \]  

\[ R_{zw} = \frac{2(0.707)}{(18.4)(0.25 \times 10^{-6})} = 304 \text{k}\Omega \]  

\[ BW_{nw} = \frac{\omega_{nw}}{2} \left( \zeta + \frac{1}{4\zeta} \right) \text{ Hz} \]  

\[ BW_{nw} = \frac{18.4}{2} \left( 0.7 + \frac{1}{4(0.7)} \right) = 9.7 \text{ Hz} \]  

The loop closure time for a step velocity input can be found for the loop \( \zeta \) by noting the value of \( \omega_n t \) for the normalized range error, \( X \), to cross zero. From Figure 3-14 it can be seen that for \( \zeta = 0.7 \),

\[ \omega_n t = 3.8 \]
the wideband loop closure time is

\[ t_{cw} = \frac{3.8}{\omega_{nw}} = \frac{3.8}{18.4} \]  

(3-78)

\[ t_{cw} = 0.21 \text{ sec} \]  

(3-79)

D. Calculate narrowband \( R_{2N} \), \( C_{uN} \) and \( BW_{wN} \) (Figure 3-15). The maximum value of \( X \), for \( \zeta = 0.7 \), is (Figure 3-15)

\[ X = 1.05 \]  

(3-80)

\[ \omega_{nN} = \sqrt{\frac{\Delta \text{Accel}}{R_e (\text{max})}} = \sqrt{\frac{(1.05)(180)}{5}} \]  

(3-81)

\[ \omega_{nN} = 6.15 \text{ rad/sec} \]  

(3-82)

\[ C_{vn} = \frac{DS_R}{R_1 t_{R1} (\omega_{nN})^2} = \frac{(50 \times 10^{-3})(12 \times 10^3)}{(73 \times 10^3)(97)(6.15)^2} \]  

(3-83)

\[ C_{vn} = 2.24 \mu\text{F} \]  

(3-84)

\[ R_{2N} = \frac{2 \zeta}{\omega_{nN} C_{vn}} = \frac{2(0.70)}{(6.15)(2.24 \times 10^{-6})} \]  

(3-85)

\[ R_{2N} = 102 \text{ k\Omega} \]  

(3-86)

\[ BW_{wN} = \frac{\omega_{nN}}{2} \left( 1 + \frac{1}{4 \zeta} \right) = \frac{6.15}{2} \left( 0.7 + \frac{1}{4(0.7)} \right) \]  

(3-87)

\[ BW_{wN} = 3.2 \text{ Hz} \]  

(3-88)
The final range error due to acceleration is

\[
R_c(\text{final}) = \frac{\Delta \text{Accel}}{(\omega_n)^2} = \frac{180}{(6.15)^2}
\]

\[
R_c(\text{final}) = 4.7 \text{ ft}
\]  

The loop closure time may be found, for the loop \( \zeta \), by noting the value of \( \omega_n t \) for the normalized range error, \( X \), to cross zero in Figure 3-15. For \( \zeta = 0.7 \)

\[
\omega_n t = 3.2
\]  

\[
t_CN = \frac{3.2}{\omega_n} = \frac{3.2}{6.15}
\]

\[
t_CN = 0.52 \text{ sec}
\]

E. Calculate range discriminator \( R, C, \) and \( A_\Delta \) (Figure 3-16)

\[
RC > 10 t_{BG}
\]

\[
RC > 10 (250 \times 10^{-9})
\]

\[
RC > 2.5 \times 10^{-6}
\]

\[
RC < 0.02 t_c t_{BG} PRF
\]

The minimum value for the loop closure time, \( t_c \), will be used (\( t_{CW} = 0.21 \) second)

\[
RC < 0.02 (0.21)(250 \times 10^{-9})(2.5 \times 10^3)
\]

\[
RC < 2.6 \times 10^{-6}
\]
Let $C = 0.001 \, \mu F \quad R = 2.5 \, k\Omega$

\[ HC = 2.5 \times 10^{-6} \]  

(3.99)

If $RC$ is less than $10 \, t_{EG}$, the discriminator scale factor will be low (which is easily corrected by adjusting $A_\Delta$ for the proper value). If $RC$ is greater than $0.02 \, t_c \, t_{EG} \, PRF$, range track loop dynamics may be affected.

The range discriminator gain may now be found (Figure 3-16).

\[
A_\Delta = \frac{D t_{EG} (\text{nsec})}{4.06 \, e_N} 
\]  

(3.100)

\[
A_\Delta = \frac{(0.05)(250)}{(4.06)(2)} = 1.54 
\]  

(3.101)

F. Design the range ramp generator.

There are many methods to generate a linear voltage ramp. One straightforward method is illustrated in Figure 3-18. The switch is closed during the reset time, $T_{RS}$ (Figure 3-12), and the range ramp voltage is zero. At time, $T_0$, the switch opens and the output linearly ramps up according to the following equation:

\[
\frac{d E_{out}}{dt} = -\left( \frac{-V}{RC} \right) = \frac{V}{RC} \, V/sec
\]  

(3.102)

The output slope is (Figure 3-12)

\[
S_V = \frac{V}{RC} \, V/sec
\]  

(3.103)
The system $S_v$ has been previously determined,

$$S_v = 41 \times 10^3 \text{ V/sec} \quad (3.104)$$

Let $V = -10$ volts and $C = 0.1 \mu\text{F}$

$$R = \frac{V}{S_v C} = \frac{10}{(41 \times 10^3)(0.1 \times 10^{-6})} \quad (3.105)$$

$$R = 2.44 \text{ k}\Omega \quad (3.106)$$

It is a simple task to adjust $R$ to give the exact $S_v$ needed.

Figures 3-19 and 3-20 illustrate the range tracking loop. Figure 3-19 is straightforward, but a few words are warranted about the comparator (Figure 3-20). The range integrator scale factor is
FIGURE 3.19. Second-Order, Type II, Range Track Loop.
FIGURE 3-20. Comparator.
\[ \text{SR} = 12,000 \text{ ft/V} \]  

or put in a different perspective

\[ \text{SR} = 12 \text{ ft/mV} \]  

To avoid any range timing jitter, the comparator must be quite accurate (1 mV of comparator uncertainty represents 12 feet of range uncertainty, or range jitter). The comparator shown in Figure 3-19 reduces range jitter to less than 5 feet.

The loop natural frequency, \( \omega_n \), and damping, \( \zeta \), are a function of the discriminator scale factor, \( D \), which in turn is a function of the target video return, \( e_{in} \). There is generally some variation in \( e_{in} \) due to target modulation (see Chapter 1 for AGC limitations and Reference 3 if the range track loop is driven by a logarithmic amplifier). Figure 3-21 shows the effect on \( \omega_n \) and \( \zeta \) due to variations in \( D \). Table 3-2 summarizes the design.

Verifying a range tracking loop design requires test equipment seldom available except in specialized laboratories. A simple method, however, exists to measure a range track loop's natural frequency and damping. It is a fairly simple matter to simulate a target return pulse at some arbitrary time (range) from a simulated transmit pulse (see Figure 3-12). Stepping the simulated return \( \pm 50 \text{ nsec} \) simulates a range change of \( \pm 24.6 \text{ feet} \) (range \( = 0.49 \text{ ft/nsec} \)). Measuring the range error voltage, \( E_r \) (Figure 3-19), a curve similar to Figure 3-7 is obtained. There is a unique relationship between the first zero crossing and the percent of overshoot (Figure 3-22), and the loop's natural frequency, \( \omega_n \), and damping, \( \zeta \). Figure 3-23 is a plot of \( \zeta \) and \( W_n L_n \) for a measured percent of overshoot. The usefulness of Figure 3-23 is shown by the following:
FIGURE 3-21 Effect of Varying D on Natural Frequency, $\omega_n$, and Damping, $\zeta$. 

DESIGN CENTER

WIDEBAND

NARROWBAND

$\omega_n$, RAD/SEC

D, mV/FT

1.4

1.2

1.0

0.8

0.6

0.4

0.2

0

20 40 60 80 100 120 140 160 180 200

D, mV/FT
Table 3-2. Design Summary.

<table>
<thead>
<tr>
<th></th>
<th>Wide bandwidth</th>
<th>Narrow bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$</td>
<td>0.707</td>
<td>0.707</td>
</tr>
<tr>
<td>$\omega_n$, rad/sec</td>
<td>18.4</td>
<td>6.15</td>
</tr>
<tr>
<td>$B_{W_n}$, Hz</td>
<td>9.7</td>
<td>3.2</td>
</tr>
<tr>
<td>Loop closure time, sec</td>
<td>0.21</td>
<td>0.52</td>
</tr>
</tbody>
</table>

FIGURE 3-22. Range Error Voltage, $R_e(t)$, Due to a Step Change in Range (See Figure 3-7).
FIGURE 3-23. Overshoot and $\omega_{nto}$ Versus Damping ($\zeta$).

Given $\%$ overshoot = 50%

- $t_o = 0.3$ second
- $\zeta = 0.25$
- $\omega_{nto} = 1.32$

$$\omega_n = \frac{1.32}{0.3} = 4.4 \text{ rad/sec}$$

The measured results of the loop are within 5% of the designed values.

Some designs use a Type I range tracking loop (finite velocity error), and the basic design procedure will now be covered.
Second Order, Type 1, Ranging Tracking Loop

The loop filter illustrated in Figure 3-24 will now be discussed. The loop filter transfer function may be given as (Av is included for amplitude scaling or buffering as necessary).

\[ F(S) = \frac{(1 + t_2 S)}{(1 + t_1 S)} \]  \hspace{1cm} (3-109)

Substituting Equation (3-109) into Equation (3-3), the loop gain is

\[ LG(S) = \frac{DS_R Av}{\tau_{RI} S} \frac{1 + t_2 S}{1 + t_1 S} \]  \hspace{1cm} (3-110)

The denominator of Equation (3-110) has a single pole at the origin, thus, this is a Type 1 loop (finite velocity error). The loop transfer function (Equation (3-5)) may be given as

\[ H(S) = \frac{DS_R Av t_2}{\tau_{RI} \tau_1} S + \frac{DS_R Av}{\tau_{RI} \tau_1} S + \frac{DS_R Av}{\tau_{RI} \tau_1} \]  \hspace{1cm} (3-111)

FIGURE 3-24. Type I Loop Filter \( F(S) = Av \frac{(1 + t_2 S)}{(1 + t_1 S)} \).
Defining

\[ \frac{DSR_AV}{t_{RI}t_1} = \omega_n^2 \]  \hspace{1cm} (3-112)

or

\[ \omega_n = \sqrt{\frac{DSR_AV}{t_{RI}t_1}} \]  \hspace{1cm} (3-113)

and

\[ \frac{1}{t_1} + \frac{DSR_AV}{t_1t_{RI}} = 2\zeta\omega_n \]  \hspace{1cm} (3-114)

Solving for \( \zeta \),

\[ \zeta = \frac{1}{2\omega_n t_1} \left( 1 + \frac{DSR_AV}{t_{RI}} \right) \]  \hspace{1cm} (3-115)

Substituting Equations (3-115) and (3-112) into Equation (3-111), \( H(S) \) may be given as

\[ H(S) = \frac{\omega_n^2 + \left( 2\zeta\omega_n - \frac{\omega_n^2 t_{RI}}{DSR_AV} \right)S}{S^2 + 2\zeta\omega_n + \omega_n^2} \]  \hspace{1cm} (3-116)

The normalized range error, \( R_e(S) \) (Equation (3-8)), may now be found:

\[ \frac{R_e(S)}{R_{in}(S)} = \frac{S^2 + \omega_n^2 \left( \frac{t_{RI}}{DSR_AV} \right)S}{S^2 + 2\zeta\omega_n S + \omega_n^2} \]  \hspace{1cm} (3-117)

The loop dynamics for step changes in range, velocity, and acceleration (Equation (3-10)) are quite complex [1], however, in summary:
A. Step change in range ($\Delta R$). Provided

\[
\frac{D S_{RA_v}}{t_{RI}} > \frac{\omega_n}{z}
\]  

the curves generated for the Type II loop (Figures 3-7 and 3-23) are valid for the Type I loop. The final range error is

\[
R_{z\text{ (final)}} = 0
\]  

B. Step change in velocity ($\Delta Vel$). The final range error is

\[
R_{z\text{ (final)}} = \frac{\Delta Vel}{D S_{RA_v}}\frac{\omega_n}{t_{RI}}
\]  

and provided $R_{z\text{ (final)}}$ is small (which can be ensured by making $A V$ large), the curves generated for the Type II loop (Figure 3-8) are valid.

C. Step change in acceleration ($\Delta Accel$). If $(D S_{RA_v})/t_{RI}$ is large, the curves generally follow those of the Type II loop (Figure 3-9); however, the range error increases with time which will eventually cause loss of lock.

\[
R_{z\text{ (t)}} = \frac{(\Delta Accel)(t)}{D S_{RA_v}}\frac{\omega_n}{t_{RI}}
\]

The design procedure is similar to that of the Type II loop.

**Type I Tracking Loop Design Equations**

A. Knowing $R_{max}$ (ft) determines the range ramp sweep width ($T_{SW}$) and scale factors, $S_v$ and $S_R$. This procedure is exactly the same as for the Type II loop (see Figure 3-12).
B. Calculate the range integrator time constant, $t_{RI}$. This differs considerably from Type II design as there is a finite range error as a function of velocity. The range integrator voltage must meet the same criteria as the Type II loop (Equation (3-26)),

\[
\frac{d E_R}{dt} = \frac{E_{V\text{max}}}{t_{RI}} = \frac{V_{\text{el\text{max}}}}{S_R}
\]

(3-122)

(it will be assumed that $A_V$ is unity, as this is most often the case for practical Type I systems).

\[
E_{V\text{max}} = R_c(V_e)D = E_{V\text{max}}
\]

(3-123)

where

\[R_c(V_e) = \text{Velocity range error}\]

Solving Equations (3-121) and (3-122) for $t_{RI}$

\[
E_{V\text{max}} = \frac{V_{\text{el\text{max}}}}{S_R} = R_c(V_e)D
\]

(3-124)

\[
t_{RI} = R_{RI} C_{RI} = \frac{R_c(V_e)DS_R}{V_{\text{el\text{max}}}}
\]

(3-125)

Choose a reasonable value for $C_{RI}$ and calculate $R_{RI}$ using Equation (3-125).

C. Calculate $R_1$, $R_2$, and $C_V$ (Figure 3-24) knowing the desired loop $\omega_n$ and $\zeta$. The solutions for $t_1$ and $t_2$ are complicated by the fact that $\omega_n$ and $\zeta$ are functions of $t_1$. Solving Equation (3-112) for $t_1$
\[ \iota_1 = \frac{DS_R}{\iota_{RI} \omega_n^2} \]  

(3-126)

Solving Equation (3-114) for \( \iota_2 \) and substituting Equation (3-126) for \( \iota_1 \),

\[ \iota_2 = R_2 C_V = \frac{2\zeta}{\omega_n} - \frac{\iota_{RI}}{DS_R} \]  

(3-127)

choose a reasonable value for \( C_V \) and solve Equation (3-127) for \( R_2 \).

The value for \( R_1 \) may be found via Equation (3-113).

\[ \iota_1 = (R_1 + R_2) C_V = \frac{DS_R}{\iota_{RI} \omega_n^2} \]  

(3-128)

\[ R_1 = \frac{1}{C_V} \left( \frac{DS_R}{\iota_{RI} \omega_n^2} - \iota_2 \right) \]  

(3-129)

A loop will now be designed using the Type II specification with the following exceptions:

\[ R_{(\text{max})} = 50 \text{ ft} \left( \Delta \text{Vel} = 400 \text{ ft/sec} \right), \quad R_{(\text{final})} = 10 \text{ ft} \left( \text{Vel} = 1250 \text{ ft/sec} \right). \]

Acceleration error will be calculated (Equation (3-119)) and is not a design parameter.

A. Determine \( T_R(max) \), \( S_V \), \( S_R \) and PRF (same as Type II design)

\[ S_V = 41 \times 10^3 \text{ V/sec} \]

\[ S_R = 12,000 \text{ ft/sec} \]

\[ \text{PRF} = 2.5 \text{ kHz} \]
B. Calculate $R_{RI}$ and $C_{RI}$ (Equation (3-125))

$$t_{RI} = R_{RI} C_{RI} = \frac{R_c (\text{Vel}) D S_R}{\text{Vel}(\text{max})}$$

(3.130)

$$R_{RI} C_{RI} = \frac{10 (0.05) (12 \times 10^3)}{1250}$$

(3.131)

$R_{RI} C_{RI} = 4.8$

(3.132)

Let $C_{RI} = 10 \mu F$

$$R_{RI} = \frac{4.8}{10 \times 10^{-6}} = 480 \text{ k}\Omega$$

(3.133)

C. Calculate $R_1, R_2,$ and $C_V$

The final desired range error is small (10 feet), thus Figure 3-8 can reasonably be used for our damping factor of 0.707 and a maximum $\Delta \text{Vel}$ error of 50 feet.

$$X = 0.45$$

(3.134)

$$\omega_n = \frac{X \Delta \text{Vel}}{R_c (\text{max})} = \frac{0.45 (400)}{50}$$

(3.135)

or

$$\omega_n = 3.6$$

(3.136)
From Equation (3-127) (using $D = 0.05$ volt/ft)

$$\tau_2 = R_2 C_V = \frac{2 \zeta}{\omega_n} - \frac{\xi R_1}{D S_R}$$

(3.137)

or

$$\tau_2 = R_2 C_V = \frac{2(0.707)}{3.6} - \frac{4.8}{(0.05)(12 \times 10^3)}$$

(3.138)

$$\tau_2 = R_2 C_V = 384.8 \times 10^{-3}$$

(3.139)

Let $C_V = 10 \mu F$

$$R_2 = \frac{384.8 \times 10^{-3}}{10 \times 10^{-6}} = 38.5 \text{ k\Omega}$$

(3.140)

From Equation (3-127)

$$R_1 = \frac{1}{C_V} \left| \frac{D S_R}{\xi R_1 \omega_n^2} - \tau_2 \right|$$

(3.141)

$$R_1 = \frac{1}{10 \times 10^{-6}} \left| \frac{(0.05)(12 \times 10^3)}{(4.8)(3.6)^2} - 384.8 \times 10^{-3} \right|$$

(3.142)

$$R_1 = 964 \text{ k\Omega}$$

(3.143)

The rest of the design (range ramp generator, range discriminator, etc.) is the same as the Type II loop (Figure 3-19).
The loop can be verified the same way as a Type II loop (Figures 3-22 and 3-23) provided (Equation (3-118) with $A_V = 1$)

$$\frac{DS_R}{t_{RI}} > \frac{\omega_n}{\zeta}$$ (3.144)

$$\frac{DS_R}{t_{RI}} = \frac{(0.05)(12 \times 10^3)}{4.8} = 125$$ (3.145)

and

$$\frac{\omega_n}{\zeta} = \frac{3.6}{0.707} = 5.1$$ (3.146)

Thus Equation (3.144) is well satisfied.

The range error as a function of time due to acceleration has been given as (Equation (3-120))

$$R_t(t) = \frac{\Delta \text{Accel}(t)}{DS_R\frac{t_{RI}}{t_{RI}}}$$ (3.147)

Solving Equation (3.147) for time, for a given maximum range error due to acceleration:

$$t = \frac{DS_R \cdot R_{t,\text{max}}}{t_{RI} \cdot \Delta \text{Accel}}$$ (3.148)

Assuming a 50-foot maximum range error for an acceleration of $180 \text{ ft/sec}^2$, the time is given by

$$t = \frac{(0.05)(12 \times 10^3)(50)}{(4.8)(180)} = 34.7 \text{ sec}$$ (3.149)

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The loop noise bandwidth for a Type I loop may be approximated by the
Type II loop Equation (3-43)

\[ BW_n = \frac{\omega_n}{2} \left( 4 + \frac{1}{4} \right) \text{Hz} \]  \tag{3-150}

or

\[ BW_n = \frac{3.6}{2} \left( 2 \cdot 0.707 + \frac{1}{4 \cdot 0.707} \right) \]  \tag{3-151}

\[ BW_n = 3.18 \text{ Hz} \]  \tag{3-152}

Figure 3-25 illustrates the Type I range-tracking loop. Note that an
instrumentation amplifier has replaced the Type II range discriminator
amplifier. Modern instrumentation amplifiers have input currents that are
quite low and prevent excessive capacitor discharge; also, the discriminator
gain, \( A_\Delta \), can be varied by a single resistor.

The two range tracking loops presented are inherently stable. Reference 1
presents an excellent discussion of loop stability from a phase-locked loop
perspective.
FIGURE 3-25. Second-Order, Range Tracking Loop.
References


Appendix 3A

RANGE-TRACKING LOOP
AND PHASE-LOCKED LOOP ANALOGY

Figure 3A-1 illustrates the block diagram for a phase-locked loop. The basic operation of this loop is straightforward: the phase detector output is a function of the phase difference between the input, $\theta_{in}$, and output, $\theta_{out}$, and is given as

$$E_1 = K_I (\theta_{in} - \theta_{out})$$

(3A-1)

where

$K_I = \text{phase detector scale factor (volts/radian)}$

FIGURE 3A-1. Phase-Locked loop.
The phase error is filtered (by $F(S)$) to suppress noise and determine dynamic loop performance.

The frequency of the voltage controlled oscillator (VCO) is controlled by the filter output voltage, $E_2$. The deviation of the VCO is

$$\Delta \omega = K_2 E_2 \quad (3A-2)$$

where

$$K_2 = \text{VCO sensitivity, Hz/volt} = 1/2n \text{ (rad/sec/volt)}$$

Since frequency is the derivative of phase, the VCO output may be given as

$$d \theta_{out}/dt = K_2 E_2 \text{ rad/sec} \quad (3A-3)$$

and by taking the Laplace transform we obtain

$$L \left[ \frac{d \theta_{out}(f)}{dt} \right] = S \theta_{out}(S) = K_2 E_2(S) \quad (3A-4)$$

Thus, the phase output of the VCO is

$$\theta_{out}(s) = \frac{K_2 E_2(s)}{S} \quad (3A-5)$$

or the phase of the VCO output is proportional to the integral of the control voltage, $E_2$. We will now relate the range tracking to that of a phase-locked loop.

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Referring to Figures 3-3 and 3-4, at time, \( T_o \), the T/R switch allows the transmitter signal to be radiated, and starts the range ramp. The range ramp slope, \( S_v \), is

\[
S_v = \frac{E_{R,max}}{T_{R,max}} \text{ V/sec} \tag{3A 6}
\]

where

\[
E_{R,max} = \text{maximum ramp voltage}
\]

\[
T_{R,max} = \text{maximum ramp width}
\]

The maximum ramp width must be at least as long as the maximum time for a signal return, or

\[
T_{R,max} \geq \left( \frac{\text{Maximum range in feet}}{492 \times 10^3 \text{ ft}} \right) \tag{3A 7}
\]

The range ramp slope may now be given as

\[
S_v = \frac{(E_{R,max})(492 \times 10^3)}{R_{max}} \text{ V/ft} \tag{3A 8}
\]

Referring to Figures 3-3 and 3-4a, the range gate trigger occurs when the range ramp voltage equals the target range voltage. The target range voltage, \( E_R \), is

\[
E_R = \frac{E_V}{S_{R1} C_{R1}} = \frac{E_V}{S_{I1}} \tag{3A 9}
\]
where

\[ E_V = \text{range integrator input voltage (or velocity voltage)} \]

\[ t_{RI} = \text{integrator time constant}, R_{RI} C_{RI} \]

The measured range voltage, \( R_0 \), may now be given as

\[
R_o = \frac{E_R \text{ (volt)}}{S_V \text{ (volt/ft)}}
\]

(3A-10)

or substituting Equation (3A-10) into (3A-9),

\[
R_o(S) = \frac{E_V \text{ (volt)}}{S t_{RI} \text{ (sec)} S_V \text{ (volt/ft)}} \text{ ft/sec}
\]

(3A-11)

letting

\[
S_R = \frac{1}{S_V} \text{ ft/V}
\]

(3A-12)

and

\[
R_o(S) = \left( \frac{S_R}{t_{RI}} \right) \frac{E_V}{S}
\]

(3A-13)

where \( S_R/t_{RI} \) for the range-tracking loop is the \( K_2 \) for the phase-locked loop (Equation (3A-2)).

Thus, the range integrator and range ramp may be replaced, for reasons of analysis, by Figure 3A-2.
FIGURE 3A-2. Range Integrator Analysis Simplification.
Appendix 3B

RANGE DISCRIMINATOR ANALYSIS

Figure 3B-1 illustrates the basic early-late gate configuration. With the video centered as shown, the charge on each capacitor is the same and $e_c$ is zero. If more of the received signal is in the early gate, the difference amplifier will be positive (and vice-versa).

![Range Discriminator Diagram]

**FIGURE 3B-1. Range Discriminator.**
The range discriminator scale factor will be found with the aid of Figure 3B-2. The output voltage is a function of $RC$, $PW$, $e_{in}$, $t$, and the number of switch closures (effective closure time).

\[ t_{\text{eff}} = \frac{PRI}{t_{\text{EG}}} (RC) \]  

(3B.1)

where

\[ PRI = \frac{1}{PRF} \]  

(3B.2)

FIGURE 3B-2. Early Gate Timing.
the output voltage (after 5 \( t_{eff} \) and assuming \( RC \gg t_E \))

\[
e_o = \frac{PW/2}{t_{EG}} e_{in}
\]  

(3B-3)

If the received pulse shifts into the early gate by \( t \) \( \mu \text{sec} \), Figure 3B-2, the output voltage is

\[
e_o(t) = \frac{PW/2 + t}{t_{EG}} e_{in}
\]  

(3B-4)

Thus the net output of the range discriminator, Figure 3B-1 will be

\[
E_r(t) = A_D \left| \frac{PW/2 + t}{t_{EG}} - \frac{PW/2 - t}{t_{LG}} \right| e_{in}
\]  

(3B-5)

and letting \( t_{EG} = t_{LG} \),

\[
E_r(t) = A_D \left| \frac{2t}{t_{EG}} \right| e_{in}
\]  

(3B-6)

The range discriminator scale factor may be found by differentiating Equation (3B-5) with respect to \( t \)

\[
\frac{dE_r(t)}{dt} = \frac{2A_D e_{in}}{t_{EG}} \text{ (V/sec)}
\]  

(3B-7)

Multiplying Equation (3B-6) by 1 sec/492 \( \times 10^6 \) ft, the range discriminator slope is obtained

\[
D = \frac{2A_D e_{in}}{t_{EG} (492 \times 10^6)} \text{ (V/ft)}
\]  

(3B-8)
or if \( t_{EG} \) is in nanoseconds,

\[
D = \frac{2 A_D e_{in}}{(0.492) t_{EG} \text{(nsec)}} \quad \text{(3B-9)}
\]

or

\[
D = \frac{4.1 A_D e_{in}}{t_{EG} \text{(nsec)}} \quad \text{(3B-10)}
\]

The value for \( RC \) must be much greater than \( t_{EG} \), and 5 \( t_{eff} \) must be much less than the loop closure times \( (t_c) \). Assume the following loop conditions:

- \( t_{EG} = 100 \) nsec
- \( e_{in} = 3 \) volts
- \( RF = 2 \) kHz (or PRF = 50 \( \mu \)sec)
- loop closure time = 0.3 second

Letting

\[
RC \geq 10 t_{EG} \quad \text{(3B-11)}
\]

or

\[
RC \geq 1 \mu \text{sec} \quad \text{(3B-12)}
\]

Thus,

\[
RC \geq 1 \times 10^{-6} \quad \text{(3B-13)}
\]

Let \( R = 1 \) k\( \Omega \)

\[
C \geq 0.001 \mu \text{F} \quad \text{(3B-14)}
\]
Letting

\[ 5 t_{\text{eff}} \leq \frac{t_c}{10} \]  \hspace{1cm} (3B-15)

\[ 5 \left( \frac{R_C}{t_{EG}^{PRF}} \right) \leq \frac{t_o}{10} \]  \hspace{1cm} (3B-16)

\[ R_C \leq \frac{t_o t_{EG}^{PRF}}{50} \]  \hspace{1cm} (3B-17)

or

\[ R_C \leq 0.02 t_o t_{EG}^{PRF} \]  \hspace{1cm} (3B-18)

and

\[ (1 \text{ K})(0.001 \times 10^{-6}) \leq (0.02)(0.3)(100 \times 10^{-9})(2 \times 10^3) \]  \hspace{1cm} (3B-19)

or

\[ 1 \times 10^{-6} \leq 4 \times 10^6 \]  \hspace{1cm} (3B-20)

and the necessary range discriminator conditions are met.

Figure 3B-3 summarizes the range discriminator equations.

Figures 3B-4, 3B-5, and 3B-6 illustrate the general shape of the range discriminator output for several \( PW/t_{EG} \) conditions \( (t_{EG} = t_{LG} = t) \).
A DISCRIMINATOR

\[ A_{\Delta} \]

\[ \text{D} = \frac{4.1 A_{\Delta} e_{\text{in}}}{t_{\text{EG}} \text{ (nsec)}} \]

\[ \text{VOLT} \]

\[ \frac{\text{FT}}{} \]

**FIGURE 3B-3.** Range Discriminator Design Summary.
FIGURE 3B-4. Range Discriminator Output for PW > t.
FIGURE 3B-5. Range Discriminator Output for $PW < t$. 

$E_e = \frac{e_{in}PW}{t}$

$E_e(\text{MAX}) = \frac{e_{in}PW}{t}$

SLOPE $= \frac{e_{in}}{t}$

SLOPE $= \frac{2e_{in}}{t}$

PULL IN RANGE
FIGURE 3B-6. Discriminator Output for PW > 2t.
Nomenclature

\( A_\Delta \) differencing amplifier gain

\( BW_n \) loop noise bandwidth

\( c \) speed of light (it is obvious when not meant as capacitance)

\( D \) range discriminator scale factor (volt/ft)

\( E_c \) range error voltage

\( E_R \) target range voltage

\( E_{d(max)} \) maximum range ramp voltage

\( E_V \) target velocity voltage

\( E_{SM} \) maximum range ramp sweep voltage

\( e_c \) range discriminator output voltage

\( e_{in} \) target video return

\( F(S) \) filter transfer function

\( H(S) \) loop transfer function

\( K \) dc loop gain \((D_{SR}/\tau_{R})\)

\( K_I \) phase-locked loop phase detector scale factor (volt/radian)

\( K_2 \) phase-locked loop VCO sensitivity (Hz/volt)

\( LG \) loop gain
**PRF**  pulse repetition frequency

**PRI**  pulse repetition interval

**PW**  pulse width

*R*  range (it is obvious when not meant as resistance)

*R_e*  range error

*R_in*  effective input range

*R_{max}(ft)*  maximum range in feet

*R_o*  effective output range

*S*  Laplacian S

**SE_e**  discriminator scale factor

**S_R**  range integrator scale factor (ft/volt)

**S_o**  range ramp scale factor (volts/sec)

*\(T_0\)*  master trigger time (range ramp start)

*\(T_R\)*  target range time

*\(T_{RS}\)*  track loop reset time

*\(T_{SW}\)*  range ramp sweep width

*\(t\)*  Early gate and late gate widths

*\(t_c\)*  loop closure time

*\(t_{EG}\)*  early gate width

*\(t_{LG}\)*  late gate width

*\(\Delta t\)*  time change

*\(\Delta R\)*  range step

*\(\Delta Vel\)*  velocity step

*\(\Delta Accel\)*  acceleration step

*\(\zeta\)*  loop damping factor

*\(\tau_{1,2}\)*  velocity integrator time constants

*\(\tau_{RI}\)*  range integrator time constant

*\(\theta\)*  phase-locked loop phase

*\(\omega_n\)*  loop natural frequency
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