DEDICATED VERSUS HALF-DUPELEX RECEIVER TOPOLOGY IN JAMMED, FULLY-CONNECTED CDMA NETWORKS: THEORY AND NUMERICAL COMPARISONS

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We examine in this report the impact of halt-duplex operation on the throughput/delay performance of fully connected, spread-spectrum, slotted ALOHA-type networks under topology-selective stochastic jamming.
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Interim Technical Report

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1. INTRODUCTION

In our previous report [PoCh87] we introduced the notion of topology-selective jamming in conjunction with a fully-connected network topology, equipped with CDMA capability. Spread-spectrum modulation was employed, both as a multiple-access tool as well as an antijam aid. In particular, we assumed an arbitrary topology of dedicated transmitters (TR) and receivers (RVCR) in a statistically symmetrical model, where every TR can be heard by every RCVR; in this sense, the network is fully-connected. The number of TRs ($N_T$) was fixed* and, in general, different from the number of RCVRs, $N_R$. A special case is when $N_T = N_R$, which is essentially equivalent to a network of full duplex modems. Since full-duplex is but a special case, we prefer the nomenclature "dedicated", which is applicable to more general scenarios such as multi-receiver satellites, two-level hierarchical networks with dedicated receivers/repeaters, etc. In such environments, the present theory would provide performance estimates (throughput-delay) within a "local" subnet of connected radios.

The goal of the present report is to extend that theory to the case where the modems are nondedicated, i.e., half-duplex. This means that a unit will alternate between the receiving mode and the transmitting mode; it is not dedicated solely to either of the two functions and it cannot do both at the same time, as in the full-duplex case. We shall assume a TR-mode priority, meaning that whenever a unit has a packet scheduled for channel access the transmitter-function takes over and an active transmission ensues, blocking the receiving capability of the unit under consideration at that point in time. If there is no transmission scheduled for a particular slot, so that the unit is not an active TR in that slot, then it is set in the receiving mode.

We will keep all other assumptions about the networks the same as in [PoCh87]. In particular, the access protocol is of the slotted ALOHA type with $p_0$, $p_r$ denoting the

* Recall that the number of potential TRs, $N_T$, is fixed and different from the number of active TRs, $M_T$, which is a random variable. The same is, in general, true for the RCVRs also.
new-packet transmission and backlogged-packet transmission, respectively. No buffering will be assumed; thus, users are always busy (never idle because of an empty buffer), implying that they are either in the originative mode (with a new packet) or the backlogged mode (with a packet that failed sometime previously). In the status-classification presented in Figure 1, this implies that the "idle" branch is not considered here. The spreading-code distribution will be of either of the common-code or the TR-based-code type, specially excluding the RCVR-based-code case; the theory for the latter is different and will be entertained in a future report. In terms of topology, this code distribution results in a competitive scenario, whereby each active TR must try to secure some RCVR's attention, assuming that there exists at least one available RCVR at that particular slot. This is different from the paired-off situation, where TRs and RCVRs form distinct pairs and suffer only from secondary multi-user interference (plus jamming), but where RCVR attention is warranted for each individual packet.

The report is organized as follows: we will proceed with the analysis of the present model in section 2, while we will present the numerical results and comparisons in section 3.
Figure 1. General classification of packet-radio unit status.
2. Analytical Model

For ease of access, we include here Table 1 of the previous report which summarizes all the key parameters of the network under consideration. If $U$ denotes the total number of half-duplex radios which are within range, then the random numbers of active TRs ($M_T$) and active RCVRs ($M_R$) are related by

$$M_R = U - M_T$$

(1)

Note that if $U = M_T$ in a particular slot, implying that all units act as TRs (this is a likely event in a heavily backlogged period, where TRs give priority to "cleaning up" their backlog status by retransmitting frequently), then there will be no active RCVRs available to capture the transmitted packets. The implications of such events will be analyzed below.

We will proceed with our analysis in two steps: first, we discuss network performance when there is only multi-user interference plus thermal noise (part 2a). Then, we shall also address the jamming case in part 2b.

2a. Noise-Only Performance

When thermal noise is the only deterrent in addition to the omni-present multi-user interference, then the required analytical expressions are a straightforward extension of the results in [PoSi87], with $N_R$ substituted by $M_R$ as per equation (1). For instance, the basic equation (4) of [PoSi87, Proposition 1] for the throughput $\beta$ remains the same:

$$\beta = \sum_{m_T=1}^{N_T} m_T p^T_{s}(m_T) f_{M_T}(m_T)$$

(2a)

where now

$$p^T_{s}(m_T) = 1 - \left(1 - \frac{P^R_A(m_T)}{m_T}\right)^{U-m_T}$$

(2b)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>$U$</td>
<td>Total number of radio units in the local channel (fixed)</td>
</tr>
<tr>
<td>$N_T$</td>
<td>Maximum number of potential transmitters in a slot (fixed)</td>
</tr>
<tr>
<td>$N_R$</td>
<td>Maximum number of potential receivers in a slot (fixed)</td>
</tr>
<tr>
<td>$M_T$</td>
<td>Number of active transmitters in a particular slot (r.v.)</td>
</tr>
<tr>
<td>$M_R$</td>
<td>Number of active receivers in a particular slot (r.v.)</td>
</tr>
<tr>
<td>$B$</td>
<td>Number of backlogged users at the beginning of a slot (r.v.)</td>
</tr>
<tr>
<td>$M_B$</td>
<td>Number of backlogged users retransmitting in a slot (r.v.)</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Number of new users transmitting in a slot (r.v.)</td>
</tr>
<tr>
<td>$S$</td>
<td>Number of &quot;channel successes&quot; in a slot (r.v.)</td>
</tr>
</tbody>
</table>

$f_{M_T}(m)$: Prob $\{M_T = m\}$, unconditional pdf

$\Delta(\xi, \Xi, p)$: Pr$\{\xi \text{ successes in } \Xi \text{ trials}\}$, binomial with parameter $p$
assuming, as in [PoSi87, Proposition 2], independently operating receivers and success per packet at most one. In equation (2b), \( P^R_A(m_T) \) is the probability that an active RCVR will capture a packet, given \( m_T \) packet transmissions in that slot, while \( f_{m_T}(m_T) \) in equation (2a) is the composite traffic in a slot (i.e., the probability of \( M_T = m_T \) transmissions). The analytic procedure for computing \( f_{m_T}(m_T) \) via the appropriate Markovian model is identical to that of Appendix A in [PoSi87], except that the fundamental set of probabilities \( \{ P_{s_{mT}} \} \) should be evaluated based on \( m_R = U - m_T \) receivers. A recursive way to compute this set has been proposed in [PoCh87, section 3b]; thus, the quantity \( N_R \) in the latter algorithm should be simply substituted by \( m_R \) which, given the quantity \( m_T \) in a particular slot, can be considered fixed.

2b Noise Plus Jamming

We adopt here the same stochastic, slot-by-slot independent, topology-selective jamming model as in [PoCh87, section 2b]. Again, the jamming status of the network in a particular slot is summarized by the indicator vector \( A^j_U \), whose \((0,1)\)-valued components \( A^j_i \); \( i=1,...,U \) indicate whether a unit is jammed or not. The jamming strategy is again manifested in the joint probability distribution function (pdf) \( Pr[A^j_U = a_U] \) of \( A^j_U \).

However, we have a significant difference in this half-duplex case compared to the dedicated-RCVR case: we will assume that the jammer does not know which units will be the active TRs in a given slot; consequently, he cannot know which are the active RCVRs in order to target only those. This is very different from the previous case, where the set of dedicated RCVRs was assumed fixed, known, and exclusively targeted by the jammer. In the absence of similar information, the jammer will randomly select a subset of units to jam in each slot, despite the fact that some of that power might go wasted on active TRs within that slot.* The jammer has no alternative but to suffer this random loss per slot. Of

* We make the reasonable assumption that a TR's function is not affected in any way if jammed.
course, the jammer still has at his disposal for optimization the probabilistic law \( \Pr[A^I_U = a_U] \), and we examine below this optimization aspect.

The first consequence of the aforementioned modeling difference is the fact that the spatial duty factor \( \rho_{sp} \), defined in [PoCh87] as the average fraction of jammed RCVRs per slot, cannot be defined explicitly here, simply because the number of active RCVRs per slot is not fixed but random. Instead, we define the half-duplex spatial duty factor \( \rho_{sp}^{hd} \) as the average fraction of the units jammed per slot, i.e.,

\[
\rho_{sp}^{hd} = \frac{\mathbb{E}[M^I_U]}{U}
\]

where \( M^I_U \) is the r.v. signifying the number of such jammed units per slot. This quantity is a direct byproduct of the stochastic jamming law and can be chosen by the jammer in an optimal way. It can be argued, of course, that to every probabilistic jamming choice (and its concomitant \( \rho_{sp}^{hd} \)) there corresponds an effective jamming duty factor, which would be the average fraction of the jammed active RCVRs per slot. This can be computed after the system of equations for throughput, composite traffic, etc, has been solved and the average number of active transmissions per slot has been determined. However, this is an indirect result of the total model and not something that can be determined apriori, as \( \rho_{sp}^{hd} \) can; hence, we opt to express everything in terms of the latter.

We recall from [PoCh87] that a number of different probabilistic jamming scenarios can be devised which correspond to the same value of \( \rho_{sp} \), and this is also true here. Presumably, each such scenario would produce a different throughput for the system, and an exhaustive search would require optimizing over \( \rho_{sp}^{hd} \) as well as the underlying jamming pdf. However, a major conclusion of the previous report was that, for the two specific scenarios analyzed therein (Scenario 1 \( \equiv \) Bernoulli jamming and Scenario 2 \( \equiv \) fixed subset jamming), the difference in throughput was insignificant, once the jamming duty factor was kept the same. This offers a strong evidence (although not a proof) that the dominant
parameter is $\rho_{sp}$ and not the specific probabilistic jamming law inducing it. In view of this, we will only entertain the Bernoulli model here since it is somewhat more amenable to analysis. Thus, we assume that each unit is jammed with probability $p_j$ independently of all others. It follows that $\mathcal{E}\{M_U\} = p_j U$, so that $\rho_{sp}^d = p_j$.

A second major analytical consequence of the considered jamming model is that the jamming status of the set of active RCVRs in each slot is not statistically independent of the number of active TRs $M_T$, as in the previous dedicated case, simply because $M_R$ depends on $M_T$ via (1). We must, therefore, rework the derivation of throughput in [PoCh87] in order to account for this fact.

In particular, let $\Delta_{U-M_T}^I$ denote the jamming indicator vector within the random set $M_R = U-M_T$ of active RCVRs in a slot, with $\Pr[\Delta_{U-M_T}^I = a_{U-M_T} | M_T] = \Pr[A_1 = a_1, ..., A_{U-M_T} = a_{U-M_T} | M_T]$ denoting the conditional probability distribution function of that vector, given $M_T$ active TRs in the slot. We can then follow the steps of [PoCh87, section 3] and write

$$\beta = \mathcal{E}(S) = \mathcal{E}_{M_T, \Delta_{U-M_T}^I} \left\{ \mathcal{E}\{S | M_T, \Delta_{U-M_T}^I\} \right\}$$

$$= \mathcal{E}_{M_T} \left\{ \mathcal{E}_{\Delta_{U-M_T}^I | M_T} \mathcal{E}\{S | M_T, \Delta_{U-M_T}^I\} \right\}$$

$$= \sum_{m_T=0}^{N_T} f_{M_T}(m_T) \sum_{\tilde{a}_{m_R}} \Pr[\Delta_{U-M_T}^I = \tilde{a}_{U-M_T} | M_T = m_T] p_s^T(m_T, \tilde{a}_{m_R}) \ (m_R = U-m_T)$$

where $p_s^T(m_T, \tilde{a}_{m_R})$ is the probability of success from a typical TR's viewpoint, given another $(m_T-1)$ packets and a specific jamming pattern $\tilde{a}_{m_R} = \tilde{a}_{U-m_T}$ in a slot. Again, if we define

$$\frac{1}{p_s^T(m_T)} \triangleq \sum_{\tilde{a}_{U-m_T}} \Pr[\Delta_{U-m_T}^I = \tilde{a}_{U-m_T} | m_T] p_s^T(m_T, \tilde{a}_{U-m_T})$$

(5)
we arrive at the expression

$$\beta = \sum_{m_T}^{m_T} m_T \frac{p_s^T(m_T)}{p_s^T(m_T) f_M(m_T)}$$  \hspace{1cm} (6)$$

(see [PoCh87], equation (11)). Equation (12a) of [PoCh87] remains the same

$$\frac{p_s^T(m_T)}{p_s^T(m_T)} = \frac{1}{m_T} \sum_{s=1}^{m_T} \frac{1}{p_{s|m_T}}$$  \hspace{1cm} (7a)$$

where now

$$p_{s|m_T} = \sum_{s=1}^{m_T} \frac{1}{p_{s|m_T}}$$  \hspace{1cm} (7b)$$

Furthermore, utilizing the approximation of conditional independence between RCVRs, we arrive at

$$\frac{p_s^T(m_T)}{p_s^T(m_T)} = \sum_{m_R} \Pr [M_R = m_R | m_T] \cdot$$

$$\left[ 1 - \left( \frac{p_{A}(m_T;J_c)}{m_T} \right)^{U-m_T-m_R} \left( \frac{p_{A}(m_T;J) - p_{A}(m_T;J_c)}{m_T} \right)^{m_R} \right]$$  \hspace{1cm} (8)$$

with $M_R$ the Hamming-weight transformation of the random vector $A_{U-M_T}$.

To proceed, we must evaluate the conditional quantity $\Pr [M_R = m_R | m_T]$ in (8). For any specific stochastic jamming law, this is the solution to a combinatorial problem which can be addressed with the help of the Vennian diagram of Figure 2 and the total-probability law

$$\Pr [M_R = m_R | m_T] = \sum_{m_U} \Pr [M_U = m_U] \Pr [m_R | m_U, m_T]$$  \hspace{1cm} (9)$$
Figure 2. Vennian Diagram for Evaluating $\Pr \left[ m_R^t | m_T, m_U^t \right]$
where \( \Pr[M'_U = m'_U] \) depends solely on the jamming stochastic model. Although this can be a fairly complicated task, things simplify significantly in a Bernoulli jamming scenario, because then

\[
\Pr[M'_R = m'_R | m_T] = \begin{cases} 
\Delta \left( m'_R, U - m_T, \rho_{sp}^{hd} \right) ; m'_R \leq m_R = U - m_T \\
0 ; \text{otherwise}
\end{cases}
\]  

(10)

If we combine (8) with (10), we arrive at

\[
\overline{p_T}^R(m_T) = 1 - \left[ 1 - \rho_{sp}^{hd} p_A^R(m_T;J) + (1 - \rho_{sp}^{hd}) p_A^R(m_T;J^c) \right]^{U-m_T}
\]

(11)

which is completely analogous to equation (16) of [PoCh87] for the Bernoulli scenario. We note that Scenario 2 would be considerably more complicated to analyze, although the basic theoretical path we have provided herein should be adequate for the job.

As we also mentioned in [PoCh87], the above results suffice to calculate the throughput of (6) in the case where \( p_0 = p_r = p \), because then the composite traffic \( f_{MT}(m_T) \) is simply the binomial \( \Delta(m_T,N_T,p) \) and \( \overline{p_T}^R(m_T) \) is given by (11). The more general case \( p_0 \neq p_r \) requires the evaluation of \( \overline{P}_{slm}^T \) and \( f_{MT}(m_T) \), as per section 3b of [PoCh87]. A careful review of the analysis therein for Scenario 1 will convince us that all the algorithms remain intact, except that \( N_R \) should be substituted by \( U - m_T \). In other words, the recursion in equation (24) of [PoCh87] will indeed produce \( \overline{P}_{slm}^T \), if we replace \( p_{slm,m_R}^{(k)} \) by \( p_{slm,m_R}^{(k)} \) and \( p_{slm,m_R}^{(U-m_T)} \) by \( p_{slm,m_R}^{(U-m_T)} \), as long as we use the average acceptance probability

\[
\overline{P}_{A}^R(m_T) = \rho_{sp}^{hd} p_A^R(m_T;J) + (1 - \rho_{sp}^{hd}) p_A^R(m_T;J^c)
\]

instead of \( p_{A,R}^{pk} \). The rigorous proof would follow the steps in Appendix A of [PoCh87].
3. **Numerical Results**

The theory developed herein has been applied to a fully-connected network with $U=10$ half-duplex units. Figure 3 shows the normalized throughput $r_\beta(r)$ versus the jamming duty-factor $\rho_{sp}^{hd}$, parameterized by the new-packet transmission probability $p_0$. Both uncontrolled ($p_0=p_r$) and controlled ($p_0\neq p_r$) ALOHA protocols have been considered, where the retransmission probability $p_r$ has been optimized in the latter case. The system parameters are identical to those in Figure 3.

Certain interesting conclusions can be drawn from this figure: first, there is again an optimal $\rho_{sp}^{hd}$ which, for these parameter values, is around $(\rho_{sp}^{hd}) = 0.5$. Furthermore, comparing Figure 3 with Figure 3 of [PoCh87], we conclude that half-duplex has consistently lower throughput than the full-duplex version, as expected. The throughput in Figure 3 reaches its asymptotic maximum at $p_0 = 0.5$ (for the whole range of $\rho_{sp}^{hd}$); in fact, no noticeable gain exists in the range $p_0 = 0.3 \sim 0.5$, particularly around the minimum value. The throughput for the uncontrolled protocol eventually decreases as $p_0$ is larger than 0.6. This is because when $p_0$ and $p_r$ are large, only a few units can be RCVR's; and thus the throughput is reduced. On the other hand, the throughput for the controlled and optimized protocol keep increasing as $p_0$ increases. This is because when $p_0$ becomes larger, the corresponding optimum $p_r$ becomes smaller; and thus the throughput is kept essentially constant.
Figure 3. Normalized Throughput versus Spatial Duty-Factor For Half-Duplex Units, Scenario 1.
References
