A STUDY OF SECOND AND THIRD ORDER MODELS FOR THE TRACKING SUBSYSTEM OF A RADAR GUIDED MISSILE

by

John W. Williams

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Thesis Advisor: H. A. Titus

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This thesis is a study of missile and target parameters used in second and third order modeling of the tracking subsystem used in radar guided missiles. Guidance methods are analyzed to determine which method is optimum in a search for an "ideal" missile. Target parameters which have an effect on the missile tracking system are analyzed and a target acceleration probability model is discussed. A two dimensional third order tracking model is simulated utilizing a Kalman filter for target parameter estimation and prediction. Linear second and third order tracking models are simulated and compared with the third order Kalman filter tracker. This thesis concludes that a proportional navigation guidance method, with a non linear third order tracking Kalman filter, is the better model. Benefits of using a non linear third order Kalman filter may not override the cost and complexity of implementation of the model.
A STUDY OF SECOND AND THIRD ORDER MODELS FOR THE TRACKING SUBSYSTEM OF A RADAR GUIDED MISSILE

by

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ABSTRACT

This thesis is a study of missile and target parameters used in second and third order modeling of the tracking subsystem used in radar guided missiles. Guidance methods are analyzed to determine which method is optimum in a search for an "ideal" missile. Target parameters which have an effect on the missile tracking system are analyzed and a target acceleration probability model is discussed. A two dimensional third order tracking model is simulated utilizing a Kalman filter for target parameter estimation and prediction. Linear second and third order tracking models are simulated and compared with the third order Kalman filter tracker. This thesis concludes that a proportional navigation guidance method, with a non linear third order tracking Kalman filter, is the better model. Benefits of using a non linear third order Kalman filter may not override the cost and complexity of implementation of the model.
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<td>proportional navigation</td>
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RNG matrix of range state space values
RPHI PHI matrix for range channel
S Sigma, estimate of line of sight
S matrix of state space sigma values
SDEL matrix of error values
SG gain matrix for bearing channel for Kalman filter
SMCOV measurement covariance matrix, bearing channel
SOHDG missile heading of second order simulation (rad)
SOLOS line of sight for second order simulation (rad)
SOR range for second order simulation (ft)
SOU commanded acceleration for second order simulation
SOXM missile position X coordinate for second order
SOYM missile position Y coordinate for second order
TEMP1 temporary matrix for storing values
TEMP2 temporary matrix for storing values
TEMP3 temporary matrix for storing values
TGTA acceleration of target (ft/sec/sec)
TGTAX target acceleration in X direction
TGTAY target acceleration in Y direction
TGTHDG target heading (rad)
TGT V velocity of target, 667 ft/sec
TGT VX0 initial velocity of target X parameter (ft/sec)
TGT VY0 initial velocity of target Y parameter (ft/sec)
THDG initial target heading (rad)
TK sample interval (sec)
TK square of the sample interval
TTG0 initial time to go (sec)
TTG time to go until impact (sec)
TVELX target velocity in X direction (ft/sec)
TVELY target velocity in Y direction (ft/sec)
U commanded missile acceleration
VM velocity of missile, 2500 ft/sec
VT velocity of target (kts)
XT target position X coordinate
YT target position Y coordinate
I. INTRODUCTION

Aviation plays an extensive role in current combat scenarios. An aircraft, because of its capability to carry missiles, is a very formidable weapon platform. Missiles provide offensive killing power which change tactics in battle scenarios. In order to have the edge in the air to air arena, an aircraft must possess the best defensive and offensive capabilities. One of the main weapons in the aircraft arsenal is the radar guided missile.

Radar guided missiles are all weather capable, can be launched outside of visual range and are less susceptible to countermeasures, compared with other missile types. Improvement of the missile is a constant necessity to maintain air superiority.

Improvements in aircraft maneuverability dictate the need for missiles to increase performance and capabilities. A rule of thumb for design is that the missile must have a 4:1 acceleration advantage over the target. With modern aircraft able to sustain lateral accelerations of ten times the force of gravity (G) the missile must be capable of 40G.

Guidance methods are chosen which optimize the missile capabilities to destroy the target. The guidance method selected has a large impact on the design of other missile subsystems. For any missile to guide to the target, the sensor subsystem must track the target. To optimize the guidance and tracking of radar guided missiles a predicting filter can be used.

One of the simplest missile guidance techniques is a second order system which compensates for bearing error and bearing rate error. This thesis will look at third order models to help optimize the missile sensor subsystem to provide better guidance command inputs. A major impetus for
finding an optimum predicting guidance method is to improve missile performance in the final guidance stages.

There is a region at the end of the missile flight path, where the time to intercept is so short that inputs to the missile control surfaces will not be effective. If the missile has an exact solution of target parameters, it can predict the future target position, through the time where no inputs will be effective. The missile is flying to the projected position of the target, compensating for the time delay of control effectiveness.

Section II will look at some guidance methods and target parameters to aid in finding the optimum missile guidance. Section III looks at target models and how to implement them in the computer simulation. Section IV derives the second and third order models. A Kalman Predicting Filter is discussed in Section V. Computer simulation and implementation of two dimensional third order models with the Kalman Filter is given in Section VI. Section VII contains conclusions and recommendations. Program listings of the computer simulation are in Appendix A and Appendix B.
II. THE IDEAL MISSILE

Definition of an ideal missile is very difficult. Cost or performance functions can be generated to account for miss distance, fuel expended, control inputs, flight path and numerous other parameters. To the operator the ideal missile is one that destroys the target. The designer must try to account for a variety of scenarios and targets to design the optimum missile. Tradeoffs of performance and costs will dictate what the final sub-optimum missile will be.

In an attempt to find an ideal missile insight may be gained by looking at the various flight paths and parameters for different guidance methods. Three methods to look at are pure pursuit, lead pursuit and proportional navigation. Pure pursuit would entail a missile always flying directly at the target. Lead pursuit would be the case of a missile always flying to a point slightly ahead of the target. The magnitude of the lead may vary from one time constant, to the total missile flight time, ahead of the target. The latter would produce a lead collision intercept. Proportional navigation utilizes line of sight rate to guide the turn of the missile, to zero any further line of sight rate. The commanded acceleration to the missile is given by the equation:

\[ AM = k \dot{\beta} V_c \]  

where \( AM \) = missile acceleration
\( k \) = constant of proportionality
\( \dot{\beta} \) = antenna angle rate
\( V_c \) = closing velocity

The constant of proportionality is determined by the designer. Observing the effects of the proportionality
constant on the line of sight rate shown in Figure II-1, any constant above 3 is an appropriate value. For \( k \) less than 3 the line of sight rate has its largest slope at the end of the intercept. For \( k \) greater than 3 the line of sight rate will be very small prior to the impact point.

![Figure II-1](image)

**Figure II-1. Prop Nav Proportionality Constant** [Ref. 1]

Three guidance methods are analyzed to compare flight paths, heading changes, line of sight angle and line of sight rate. A fourth missile is simulated and plotted, identified by "direct".

The direct path missile is programmed using "a posteriori" knowledge of the target flight path. The direct path missile goes to the point of impact, in a straight line, from the point of launch.

If the "ideal missile" is defined such that there is no missile maneuvering, burns minimal fuel, has the greatest launch distance, minimum intercept time and accounts for all target maneuver, it will be a direct path missile.
Two programs were written using Dynamic Simulation Language, DSL, to analyze the guidance methods and produce plots. The programs are included in Appendix A.

Three scenarios are used to compare the guidance methods:
- head-on aspect
- tail aspect
- beam aspect

Three target accelerations are used to determine the effect of the target maneuver on the parameters. The selected target accelerations are:
- 0 g's
- 3 g's
- 6 g's

A second order proportional navigation missile with a proportional navigation guidance constant of four, where the line of sight angle and angle rate is estimated by the antenna parameters of angular position and angular rate. The governing equations for tracking in bearing are:

\[
\beta = \int_{0}^{T} \beta \, dt + \beta_0 \quad (2)
\]

\[
\dot{\beta} = \int_{0}^{T} \dot{\beta} \, dt + \dot{\beta}_0 \quad (3)
\]

\[
\ddot{\beta} = -20\dot{\beta} + 100\beta \, (\text{LOS} - \beta) \quad (4)
\]

where
- \( \beta \) = antenna position angle
- \( \dot{\beta} \) = antenna angle rate
- \( \ddot{\beta} \) = antenna angular acceleration
- \( \text{LOS} \) = actual target bearing

Pure pursuit and lead pursuit guidance missiles are initialized heading directly at the target. Pure pursuit guidance maintains heading directly at the target by
calculating the heading at each step of the discrete simulation. The equation for pure pursuit heading is:

\[ \text{PPHDG} = \text{atan}(\frac{\text{yt} - \text{ym}}{\text{xt} - \text{xm}}) \]  

(5)

where \( \text{yt} \) = current target Y coordinate  
\( \text{xt} \) = current target X coordinate  
\( \text{ym} \) = current missile Y coordinate  
\( \text{xm} \) = current missile X coordinate

Lead pursuit maintains a heading in front of the target, with a variable lead, calculated using half the time to go, given by:

\[ \text{LPHDG} = \text{atan}(\frac{\text{yt} + \text{tvely} \times 0.5 \times \text{ttg}}{\text{xt} + \text{tvelx} \times 0.5 \times \text{ttg}}) \]  

(6)

where \( \text{yt} \) = current target Y coordinate  
\( \text{xt} \) = current target X coordinate  
\( \text{tvely} \) = target velocity in Y direction  
\( \text{tvelx} \) = target velocity in X direction  
\( \text{ttg} \) = time to go in the intercept

The results of the comparisons are given in graph form in Figure II-2 through Figure II-37.

A. HEAD-ON ASPECT

Figures II-2 through II-13 are the results of missile guidance comparisons for head-on aspect initial condition with 0G, 3G and 6G constant target acceleration. The missile begins at the origin of the graph. The target initial position is at \( x = 10000 \text{ ft} \), \( y = 500 \text{ ft} \). Applied lateral target acceleration is away from the missile.
Figure II-2: Position Plot for Head-on

Legend:
- ◯ Target
- ○ Prop. Nav
- △ Pure Pursuit
- + Lead Pursuit
- × Direct
Figure II-3  Missile Heading Plot for Head-on Aspect No Target Turn
Figure II-4 Line of Sight Angle Plot for Head-on Aspect No Target Turn
Figure II-5 Line of Sight Rate Plot for Head-on Aspect No Target Turn
HEAD-ON 3G TGT
POSITION PLOT

LEGEND
- TARGET
- PROP NOV
- PURE PURSUIT
- LEAD PURSUIT
- DIRECT

Figure II-6: Position Plot for Head-On Aspect Constant 3G Target Turn

0.0 250.0 500.0 750.0 1000.0 1250.0 1500.0 1750.0

0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0 x10^3

Y (FT)
X (FT)
HEAD-ON 3G TGT
MISSILE HEADINGS

Figure II-7 Missile Heading Plot for Head-on Aspect Constant 3G Target Turn
HEAD-ON 3G TGT
LINE OF SIGHT

Figure II-2  Line of Sight Angle Plot for Head-on Aspect Constant 3G Target Turn
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TAIL 6G TGT
MISSILE HEADINGS

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Aspect Constant 6G Target Turn
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Figure II-37 Line of Sight Rate Plot Beam Aspect Constant 6G Target Turn
By applying the acceleration away from the missile the pilot will lose sight of the missile. This is undesirable, but a turn into the missile will help the missile, in the early stages, more than a turn away. With a turn into the missile, the pilot will also lose sight of the missile, during a constant acceleration turn.

1. OG Target Acceleration

From the position plot, Figure II-2, it is seen that proportional navigation guidance is essentially the same as the direct path guidance. Any errors are due to initialization of the heading for the proportional navigation guidance. The pure pursuit guidance missile and the lead pursuit guidance missile fly curvilinear paths to target intercept. The curve for the lead pursuit guidance missile is less than the pure pursuit missile due to target lead.

The missile headings graph, Figure II-3, shows the relative heading changes involved for each missile guidance. After the initialization errors have been corrected, the proportional navigation guidance missile parallels the direct path missile. The heading changes for the lead pursuit are less than for the pure pursuit guidance method. Large increases in missile headings at the end of the intercept implies large lateral accelerations are required for the missile to complete the intercept.

The line of sight graph, Figure II-4, shows what would be expected for this case. The proportional navigation guidance and direct path missile maintain constant line of sight, approximately, while the line of sight increases for lead pursuit and pure pursuit guidance methods. The large change in line of sight at the end of the intercept also correlates to a high lateral acceleration required by the missile.

The line of sight rate graph, Figure II-5, gives some insight to the control inputs to the missile guidance.
subsystem. The values from, Figure II-5, are the slew rated for the sensor subsystem. A positive slew rate is seen for the prop nav guidance only at the final stages of the intercept. Lead pursuit and pure pursuit guidance methods have accelerating positive slew rates throughout the intercept. The direct path missile has a negative slew rate, caused by the missile speed advantage (2500 : 667 ft/sec).

2. 3G Target Acceleration

The position graph, Figure II-6, shows a curvilinear path for all three guidance methods. The curvature of the target flight path is misleading, because of the axis scaling. The target is maintaining a constant acceleration. All three guidance methods appear to end up in a tail chase. The scaling is misleading again. Target heading change is approximately 50 degrees. The proportional navigation missile impacts in the beam while lead pursuit will be rear quarter and pure pursuit will be a tail chase.

The missile headings graph, Figure II-7, shows the proportional navigation guidance has the lowest heading slope and is approximately linear at the end of the intercept. Pure pursuit and lead pursuit guidance methods have accelerating missile heading slopes requiring higher missile acceleration.

The line of sight graph, Figure II-8, is similar to the Figure II-4, proportional navigation guidance method, which has low line of sight angles, slightly increasing due to target acceleration. Lead pursuit and pure pursuit guidance methods have line of sight angles which increase at an accelerated rate throughout the intercept. The direct missile has decreasing line of sight. The large negative LOS for the direct missile at the end of the intercepts is caused by the miss distance and heading initialization.

Line of sight rates, Figure II-9, correlate with the line of sight plot, Figure II-8. Line of sight rates are
small but show an acceleration at the end of the intercept, due to decreasing range. Proportional navigation guidance methods are reducing the line of sight angle while lead pursuit and pure pursuit increase the line of sight angle.

3. 6G Target Acceleration

Comparing the position plot, Figure II-10, with that of the 3G case, Figure II-6, similar statements can be made about all of the missile paths. Scaling is slightly deceiving; the target has made approximately 100° heading change. All flight paths are curvilinear with the proportional navigation guidance method being the shorter of the three methods.

Figure II-11, shows the heading changes for the missiles and the smaller missile maneuvering required for the proportional navigation missile. Figure II-12 and Figure II-13 show larger magnitudes for line of sight angle and line of sight rate than the 3G case, but follow the same trends. The direct path missile shows a reversal in line of sight rate as the target heading change is greater than 90°.

B. TAIL ASPECT

Figure II-14 through Figure II-25 are the results of missile guidance comparisons for tail aspect initial condition with 0G, 3G and 6G constant target acceleration. The missile begins at the origin of the graph. The target initial position is X=10000 ft. and Y=1000 ft., with an initial heading of 090°, parallel to the X axis. Applied target acceleration is directed into the missile, perpendicular to the target heading.

1. 0G Target Acceleration

The position plot, Figure II-14, shows the proportional navigation guidance missile flies a similar path as the direct path missile. The difference in the flight paths is due to errors in initialization. The
missile heading plot, Figure II-15, shows that the heading for proportional navigation guidance and direct path missiles are parallel after the initialization errors are corrected. Pure pursuit and lead pursuit guidance methods have continually changing headings with accelerating slopes at the final stage of the intercept.

Line of sight angles for the proportional navigation guidance and direct path are approximately constant and equal to the initial line of sight angle, giving a constant bearing decreasing range trajectory as seen by the target. The line of sight angle for pure pursuit and lead pursuit guidance decrease, but non linearly, as seen in Figure II-16 and Figure II-17.

2. 3G Target Acceleration

With target acceleration, all three missiles fly a curvilinear path. The proportional navigation guidance method has the shortest flight path, as seen in Figure II-19. Proportional navigation guidance gives a linear heading change, as seen in Figure II-19. Pure pursuit and lead pursuit guidance methods give higher heading slopes when the target applies lateral acceleration as compared to the 0G heading plot, Figure II-15.

Line of sight angle changes are small for proportional navigation guidance, as shown in Figure II-20. Pure pursuit and lead pursuit guidance have decreasing line of sight angles, with corresponding decreasing line of sight rates, as seen in Figure II-20 and Figure II-21. Line of sight rates increase for proportional navigation guidance, as would be expected from the path the missile flies. The direct path gives both a nonlinear line of sight angle and line of sight rate throughout the intercept.

3. 6G Target Acceleration

When the target acceleration is increased, flight paths have a larger curvature, as seen in Figure II-22. The scaling gives some distortion, the target has gone through
approximately 100° of heading change. The missile heading changes, as per Figure II-23, are similar to those observed for the 6G head-on aspect, Figure II-11.

The line of sight angle and line of sight rate are larger for an increase in lateral target acceleration, as seen by comparing Figure II-24 and Figure II-25 with the 3G case, Figure II-20 and Figure II-21. The line of sight and line of sight rate for the direct path have a very large slope at the final intercept due to effects of decreased range. The reversal of line of sight rate for the direct path in Figure II-25 is where the target heading change is 90°.

C. BEAM ASPECT

Figures II-26 through II-37 are the result of missile guidance comparisons for beam aspect initial conditions with 0G, 3G and 6G constant target accelerations. The missile begins at the origin of the graph. The target initial position is X=15000, Y=0. Applied acceleration is directed into the missile.

1. 0G Target Acceleration

As in the two previous cases, with no lateral target acceleration, proportional navigation guidance and direct path missiles have similar flight paths. Pure pursuit and lead pursuit guidance have curvilinear flight paths, as seen in Figure II-26. Heading changes are small for proportional navigation guidance and direct flight path missiles but not zero as what might be inferred from Figure II-27, because of scaling. The heading change for pure pursuit and lead pursuit guidance is accelerating throughout the flight time with the intercept ending in a tail chase.

Line of sight and line of sight rate, Figure II-28 and Figure II-29, are similar to the two previous cases, for no target acceleration and the analysis is the same.

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2. **3G Target Acceleration**

For the beam aspect initial condition, when lateral acceleration is applied, the difference between flight paths for proportional navigation guidance and direct path is opposite from the two previous cases for lateral acceleration. The proportional navigation guidance missile flight path is on the opposite side of the direct path from pure pursuit and lead pursuit guidance flight paths, Figure II-30, with the opposite curvature. Headings for proportional navigation guidance continually decrease while pure pursuit and lead pursuit guidance increase.

Differences between the methods are enhanced by the line of sight angle and line of sight rate plots in Figure II-32 and Figure II-33. Proportional navigation guidance decreases line of sight while the others have an increasing line of sight and appropriate line of sight rate.

3. **6G Target Acceleration**

Increasing the target lateral acceleration magnifies the flight path differences between the guidance methods. As has been seen from the previous cases, the larger lateral acceleration increases the magnitudes of the values for Figure II-33 through Figure II-37, compared with similar graphs from the other cases, but the trends remain the same. Proportional navigation guidance parameters have smaller changes than pure pursuit and lead pursuit guidance, with parameters generally decreasing instead of increasing for pure pursuit and lead pursuit guidance.

D. CONCLUSIONS

For scenarios with no applied target lateral acceleration the proportional navigation missile is the same as the direct missile. The pure pursuit and lead pursuit missiles finish in a 'tail chase' where a missile speed advantage is required to complete the intercept. The line
of sight remains constant for the proportional navigation missile but increases with the pursuit missiles. The line of sight rate increases with decreased range for the pursuit missiles but is zero for the proportional navigation and direct missiles.

When target lateral acceleration is applied, there is a deviation between the direct and the proportional navigation missiles. Since the target is turning into the missile, the line of sight angle decreases at an accelerated rate as range decreases. The proportional navigation missile accounts for the change of line of sight by turning into the target. The pursuit missiles fly a tail chase profile with higher line of sight accelerations due to the target turn.

For the direct missile, when target acceleration is applied, the line of sight is not constant and the rate of change depends on the applied acceleration. Implementation of a direct missile is impossible because the parameters used to guide the missile are dependent on the target flight path.

If an optimum missile is to be designed, proportional navigation guidance is the closest to an "ideal" missile. The better the proportional navigation missile can compensate for the effects of the target acceleration, the closer to "ideal" the missile will become.
III. TARGET MODEL

A complete target model for use in computer simulation is very involved, time consuming and computer intensive. To simplify target simulation the target flight profile is based on the fact that the missile sees only the effects of the target command inputs and resultant flight path. The target model was simplified to include only the flight profile desired and not be concerned with the full target modeling. A constant speed, constant acceleration target is assumed for the simulation. A variable speed, variable acceleration target can be added at a later time. The missile simulation estimates and predicts target parameters of range, range rate, range acceleration, bearing, bearing rate, and bearing acceleration. Therefore, for proper evaluation of the missile guidance and missile flight profiles the target parameters in missile coordinates for all these parameters must be computed.

Complete analysis of target motion is obtained from a three dimensional derivation, but insight can be gained from two dimensional modeling. Three dimensional flight profiles are easily implemented on the computer but graphic display of the results are difficult. Two dimensional displays are easier to implement and comprehend. A two dimensional target model is assumed.

A target can accelerate at values ranging from negative maximum instantaneous acceleration to positive maximum instantaneous acceleration, A(max inst). A(max inst) is defined as the aerodynamic acceleration given by the maximum deflection of control surfaces. A(max inst) is dependent on airspeed and air density. High speeds and low altitudes produce the highest instantaneous accelerations. Maximum sustained acceleration, A(max sust), is defined as the aerodynamic acceleration to maintain constant airspeed and
constant altitude, at full thrust. $A(\text{max sust})$ can be exceeded but must be compensated for by a reduction in airspeed or altitude.

In three dimensional maneuvering cross coupling exists between horizontal and vertical angle and angle rate components of target velocity and target acceleration. Applied accelerations and velocity changes in one direction will affect the parameters, seen by a missile, in the two other directions.

Thrust capabilities have a direct correlation to $A(\text{max sust})$ and the airspeed of an aircraft. An aircraft with higher thrust can maintain a higher speed and compensate for drag induced by the applied acceleration. A modern aircraft with a relatively high thrust to weight ratio will have a very high $A(\text{max sust})$ which is close to $A(\text{max inst})$.

Airspeed is a key element for maneuverability and survivability. Tactics incorporate optimum techniques for maintaining airspeed or recovering lost airspeed. Pilots learn to compensate for limitations of $A(\text{max sust})$ by intentionally decreasing altitude and use the effects of gravity to maintain airspeed when lateral acceleration is applied. Another technique is to apply the lateral acceleration required to perform a maneuver then to reduce the acceleration, allowing excess thrust to restore the airspeed and altitude lost during the maneuver. There is a recovery time for the thrust to restore the lost energy, so to aid in restoring airspeed, a pilot will normally go to zero acceleration, reducing any induced drag, effectively increasing the aircraft thrust. This maneuver causes a loss in altitude due to gravity but improves airspeed restoration.

The probability of aircraft acceleration is used to determine parameters for the target model used in missile designs and simulations. Figure III-1 shows a typical acceleration probability graph used in missile design. The
figure does not account for pilot tendencies nor the difference between $A_{\text{max sust}}$ and $A_{\text{max inst}}$. The graph assumes that a pilot will maneuver primarily at zero acceleration, straight and level, or maximum acceleration, for a turn, with some probability for any other possible acceleration. The design engineer assigns probabilities for the impulse functions at zero acceleration and at maximum acceleration depending on the type of target aircraft. A large bomber may have an $A_{\text{max sust}}$ half that of a fighter aircraft, with less probability of turning than flying straight and level.

![Figure III-1 Probability of Aircraft Acceleration](Ref. 2,3)

A proper target maneuver model should include some pilot tendencies and known tactics. A pilot is not always able to move the control surface to a precise location to cause a precise acceleration at an optimal time. A pilot will move the control surface, judge the acceleration induced then move the control surface to achieve a desired acceleration. The feel a pilot receives from the 'stick' is a prime feedback source to allow the pilot to set the desired acceleration. The more force the pilot applies, the more the control surface moves, and the higher is the acceleration. Modern aircraft may use computers to achieve the commanded acceleration, reducing any pilot induced errors on input.
A. TACTICS FOR MISSILE DEFENSE

When a missile is fired at the target the battlefield scenario changes to an immediate survival situation. If the missile is undetected, the acceleration probabilities of Figure III-1 may be an adequate target model. If a pilot sees the missile, pilot reactions will change the probabilities. How the probabilities change may be of consequence to the missile guidance. An optimum missile design may be able to use pilot tendencies to increase missile performance. The overall acceleration probability from Figure III-1 is a zero mean function with a variance, $\sigma^2$, dependent on the probabilities assigned. Reference 2 discusses obtaining the parameters for the target acceleration probability model of Figure III-1.

When the pilot imposes a missile defense, the overall acceleration may not be zero mean, nor maintain the same variance. To account for the changes in acceleration probability a function similar to Figure III-2 might be used. This model accounts for some variance to the acceleration which the pilot is trying to achieve centered around $A_{\text{max sust}}$. If the pilot is trying to achieve $A_{\text{max inst}}$, it is assumed he will be decreasing airspeed and reducing actual acceleration until the applied acceleration is decreased to $A_{\text{max sust}}$ or below. A smart pilot will either fly at a maximum acceleration or at zero acceleration, increasing the aircraft maneuverability.

Last ditch maneuvers are performed at $A_{\text{max inst}}$ to avoid the missile, neglecting any adverse effects of applying acceleration, in order to increase survivability. If the last ditch maneuver is performed too soon, acceleration is decreased, due to loss of airspeed, negating the effectiveness of the maneuver. Further studies can be made correlating the use of $A_{\text{max inst}}$ versus $A_{\text{max sust}}$ for missile defense tactics.
Figure III-2 Probability of Acceleration

If pilot reaction is taken as Gaussian when trying to achieve a desired acceleration, the overall acceleration probability will be Gaussian with a non-zero mean and a variance dependent on the combination of the two Gaussian terms. The probability assigned to each term determines the mean and variance.

Missile defense includes placing the missile on the beam, to utilize the largest acceleration vector, with maneuvers made out of phase, out of plane with the missile. The largest acceleration component comes from the elevator, perpendicular to the wings. By placing the wings parallel with the plane of the missile, the largest acceleration component is used to create the largest missile corrections, perpendicular to the plane of the missile. The plane of the missile is defined by three points: target position, missile position and the projected impact point.

A graph of a target acceleration, while performing missile evasion, might look like Figure III-3. The pilot commands A(max sust) or A(max inst) for a short time, then
reduces the acceleration to zero to regain lost energy, before applying the acceleration again. This process may be repeated 2, 3 or more times during the missile flight time. The graph attempts to incorporate transient response induced by system time delays, transient responses of the control surfaces and pilot tendencies for maneuvering and control of the target.

![Figure III-3 Target Acceleration](image)

The resultant average acceleration is non-zero, with a non-zero variance. The mean and variance of Figure III-3 can be estimated by the parameters assumed in Figure III-2.

Figure III-1 and Figure III-2 show total aircraft acceleration. The parameters as seen by the missile, in antenna coordinates, will vary depending on the three dimensional maneuver employed. The missile tracking subsystem must be designed to handle the maximum acceleration possible for each of the orthogonal components of the reference frame.

Proportional navigation missiles compensate for the applied target acceleration by decreasing the line of sight rate induced by the change of target velocities. With a good target model and detection of maneuvering effects, the missile guidance can predict target motion and position.
IV. SYSTEM MODEL

From the section on the ideal missile, it was ascertained that target parameters are not constant if lateral acceleration is applied. This thesis will attempt to incorporate as much sensor information as is available in defining the system models for an optimum missile. Current radars allow the measurement of range, range rate and off boresight bearing error. Measurements taken by the radar are referenced to a radar axis system. In order for the missile to use the information supplied by the radar, a common reference frame must be established.

A. COORDINATE SYSTEM

Each entity in the missile-target intercept problem has its own coordinate system. The overall geometry as seen from an "eye in the sky" would view it in space coordinates. An observer on the ground would view it in earth coordinates. The launching aircraft, missile and target aircraft will view it in an individual coordinate system, referenced to that specific platform. Trying to equate each coordinate system is not an easy task but one which is done. By use of Euler angles any reference coordinate system can be related with Earth coordinates. By use of a directional cosine matrix transformation any reference coordinate system can be transformed to another reference coordinate system [Ref. 3].

The missile is concerned with flying to a point in space that will hopefully be occupied by the target at the completion of the intercept. The object is to guide the missile to the proper point where the target will be. The missile is concerned with its coordinate system and not that of the target. But on the missile itself there are
various coordinate system reference points. Each sensor has its own location on the missile and where it is mounted is its reference point. Any moving sensor, like the antenna, will have its special reference coordinate system. Missile parameters are normally referenced to the missile body frame of reference while target parameters are referenced to the antenna frame of reference. While very complicated, the frames of references can be transformed and equated. [Ref. 3]

To simplify simulation and evaluation of desired parameters, an inertial frame of reference will be used which is centered at the radar antenna location. This simplification will aid in better evaluation of the effects of the target parameters and the missile guidance without encumbrance of transformation errors. Although the simplification assumes ideal missile parameters, time delays can be incorporated later to account for first order modeling of the missile.

B. EQUATIONS OF MOTION

In cartesian coordinates missile and target motion is described by the standard motion equation:

$$X(T) = X_o + \int_0^T \dot{X}(t) \, dt + \int_0^T \ddot{X}(t) \, dt \, dt$$  \hspace{1cm} (7)

The equation is based on a fixed reference point. The orthogonal directions (Y and Z) will have the same equation.

The antenna frame of reference uses polar coordinates which have the equations:

$$r(t) = r_o + \int_0^T \dot{r}(t) \, dt + \int_0^T \ddot{r}(t) \, dt \, dt$$  \hspace{1cm} (8)

$$\theta(t) = \theta_o + \int_0^T \dot{\theta}(t) \, dt + \int_0^T \ddot{\theta}(t) \, dt \, dt$$  \hspace{1cm} (9)

$$\phi(t) = \phi_o + \int_0^T \dot{\phi}(t) \, dt + \int_0^T \ddot{\phi}(t) \, dt \, dt$$  \hspace{1cm} (10)
where \( r(t) \) = the radial component of motion
\( \Theta(t) \) = horizontal component of motion
\( \Phi(t) \) = vertical component of motion

When the reference point is not fixed, extra terms and cross coupling are introduced into equation dynamics. The coriolis equation accounts for the moving reference point. Depicted in Figure IV-1 a change in the vector \( R \) is accounted for by both changes in the magnitude and the rotation effects by the moving reference point. Using the terms as defined in Figure IV-1 we can obtain the necessary equations to find \( r(t) \), \( \Theta(t) \) and \( \Phi(t) \). For simplicity, only the derivation for \( r \) and \( \Theta \) are shown with \( r \) and \( \Phi \) relationships being a duality of derivation of \( r \) and \( \Theta \). The simulation of Section VI will be two dimensional.

\[ \dot{R} = \dot{r} + w \times R \]  
\[ \text{(11)} \]

where \( R \) = the directional vector
\( r \) = the magnitude of the directional vector
\( w \) = the angular rate of motion
The total change of the vector \( \mathbf{R} \) is the sum of the change in the magnitude of \( \mathbf{R} \) due to changes along the original vector \( \mathbf{R} \) given by \( \mathbf{r} \) and the angular rotation due to the moving coordinate frame, given by \( \mathbf{wxR} \).

Utilizing the general rule for differentiating a vector\(^1\), an expression is obtained for the acceleration of the \( \mathbf{R} \) vector.

\[
\dot{\mathbf{R}} = \dot{\mathbf{r}} + \dot{\mathbf{wxr}} + \dot{\mathbf{wxr}} + \dot{\mathbf{wxwxr}} \quad (12)
\]

\[
\ddot{\mathbf{R}} = \ddot{\mathbf{r}} + \ddot{\mathbf{wxr}} + \ddot{\mathbf{wxr}} + \ddot{\mathbf{wxwxr}} \quad (13)
\]

\[
\dddot{\mathbf{R}} = \dddot{\mathbf{r}} + \dddot{\mathbf{wxr}} + 2(\mathbf{wxr}') + \dddot{\mathbf{wxwxr}} \quad (14)
\]

where \( \mathbf{R} \) = directional vector
\( \mathbf{r} \) = the acceleration of the magnitude of \( \mathbf{R} \)
\( \mathbf{wxr} \) and \( \mathbf{wxr} \) are perpendicular to the \( \mathbf{R} \) vector
\( \mathbf{wxwxr} \) is centrifugal acceleration

This equation gives the cross correlation of range and angle to implement in a second order model. Applying the rule of differentiating a vector again will yield the equations for a third order model.

\[
\frac{d}{dt}(\dddot{\mathbf{R}}) = \dddot{\mathbf{r}} + \dddot{\mathbf{wxr}} + \dddot{\mathbf{wxr}} + 2(\mathbf{wxr}') + 2(\mathbf{wxr}''') + \dddot{\mathbf{wxwxr}} \\
+ \mathbf{wxwxr}' + \mathbf{wxwxr} + \mathbf{wxwxr} + \mathbf{wxwxr} + \mathbf{wxwxr} \quad (15)
\]

\[
\frac{d}{dt}(\dddot{\mathbf{R}}) = \dddot{\mathbf{r}} + \dddot{\mathbf{wxr}} + \dddot{\mathbf{wxr}} + 2(\mathbf{wxr}') + 2(\mathbf{wxr}'''') + \dddot{\mathbf{wxwxr}} \\
+ \mathbf{wxwxr}' + \mathbf{wxwxr}' + \mathbf{wxwxr}' + \mathbf{wxwxr}' + \mathbf{wxwxr}' + \mathbf{wxwxr}' \quad (16)
\]

\[
\dddot{\mathbf{R}} = \dddot{\mathbf{r}} + 3(\mathbf{wxr}') + 3(\mathbf{wxr}''') + \mathbf{wxr} + \mathbf{wxr} + \mathbf{wxr} \\
+ 2(\mathbf{wxr}') + 3(\mathbf{wxwxr}') + \mathbf{wxwxr} \quad (17)
\]

where \( \dddot{\mathbf{R}} \) is the acceleration jerk of vector \( \mathbf{R} \)

\(^1\) rule for differentiating a vector

\[
\frac{d}{dt} \mathbf{A} = \dot{\mathbf{A}} + \mathbf{wxA}
\]

the total derivative is the sum of the time rate of change of the vector and the rotation of the vector.
r is the magnitude of R
wxr is perpendicular to R
wxwxr is in the negative direction of R
wxwxwxr is perpendicular to R

C. SECOND ORDER MODEL

Beginning with equations 8-10, a second order state space system can be set up which would have the form:

\[
\begin{align*}
\mathbf{r} &= \mathbf{r}_0 + \int_0^T \mathbf{r} \, dt + \int_0^T \mathbf{r}' \, dt \\
\mathbf{\theta} &= \mathbf{\theta}_0 + \int_0^T \mathbf{\theta} \, dt + \int_0^T \mathbf{\theta}' \, dt
\end{align*}
\]  

(18)  

\[
\begin{bmatrix}
\dot{r} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
\dot{r} \\
\theta \\
\dot{\theta}
\end{bmatrix}
\]

(19)  

(20)  

The range portion of the second order system may be accomplished totally by the radar, since no other subsystems require the information. The radar receiver is designed to track the target in the radial direction without the need for an additional filter.

The more difficult state equations to implement in the missile are the angular directions. The second order, time invariant, constant velocity, zero acceleration, state feedback model makes the tracking much easier. The continuous system model can be given in a time derivative form as:

\[
\dot{\mathbf{X}}' + k_1 \cdot \dot{\mathbf{X}} + k_2 \cdot \mathbf{X} = 0
\]

(21)  

If the term given in the equation as X is actually the error of the angular position then

\[
\mathbf{X} = \text{(line of sight angle - antenna position angle)}
\]
where \( X \) can be directly measured by the antenna. The values of the \( k \)'s in the time derivative equation depend on the designer and the response desired. For the simulations of the proportional navigation missiles of Section II and Section VI, \( k_1 = 20 \) and \( k_2 = 100 \) were used. These constants give a response time constant of 0.1 sec.

The use of the coriolis equation to derive the second order model gives a time varying solution. Implementing time varying equations are difficult and often avoided by using a time invariant system and state feedback to cancel errors. The time variant space state model derived from equation 14 is shown below:

\[
\dot{R} = \dot{r} + \dot{w} + 2(\dot{w} - 1) + \dot{w} \dot{w} r 
\]

Separating into orthogonal components of radial and transverse with scalar multiplication:

\[
a_r = \dot{r} + \dot{w} r 
\]

\[
a_T = \dot{w} r + 2(\dot{w} r) 
\]

Rearranging into equations to implement into a system:

\[
\dot{r} = -\dot{w} r + a_r 
\]

\[
\dot{\theta} = -2\dot{\theta} r + a_T 
\]

The state space model is difficult to represent unless divided into two channels with cross coupling given in the \( A \) matrix.

\[
\begin{bmatrix}
\dot{r} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\dot{w} & 0
\end{bmatrix}
\begin{bmatrix}
\dot{r} \\
\dot{\theta}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\begin{bmatrix}
a_r \\
a_T
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -2\dot{\theta} r
\end{bmatrix}
\begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\begin{bmatrix}
a_r \\
a_T
\end{bmatrix}
\]
D. THIRD ORDER MODEL

The time invariant model, derived from equations similar to those deriving the time invariant second order model (equations 18 and 19), in state space form is given by:

\[
\begin{bmatrix}
\dot{\mathbf{r}} \\
\mathbf{r} \\
\mathbf{e} \\
\mathbf{e}\\
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{r} \\
\dot{\mathbf{r}} \\
\ddot{\mathbf{r}} \\
\dot{\mathbf{e}} \\
\ddot{\mathbf{e}}
\end{bmatrix}
\]  
(29)

Ranging may be accomplished by the radar receiver, as in the second order model, for similar reasons. Tracking angular positioning requires knowledge of the target angular acceleration as well as angular velocity. A filter is normally used to maintain a track of the target angular parameters. Common filters are α-β, Weiner, and Kalman. As compared in Reference 4, the Kalman is the best filter suited for air to air missiles, but also the most costly to implement. For the time invariant third order model a simple constant gain Kalman Filter can be used. The Kalman Filter will be discussed in the next section.

The time variant third order model is obtained from the second derivative of the coriolis equation derived in the previous section.

\[
\dot{\mathbf{R}} = \dot{\mathbf{r}} + 3(wx\dot{r}) + 3(wx\dot{r}) + w\dot{x}r + w\ddot{x}r
+ 2(wx\dot{x}r) + 3(wx\dot{x}r) + wx\dot{x}xw\dot{x}r
\]  
(30)

The \(\dot{\mathbf{R}}\) term is the change in the acceleration of the vector or a "jerk" term, a simple comprehension is the rate at which the pilot applies the commanded acceleration. Separating the equation into radial and tangential terms, the two orthogonal scalar equations are:

\[
\dot{a}_r = \dot{\mathbf{r}} - 3w^2\dot{r} - 3ww\dot{r}
\]  
(31)
\[ \ddot{r} = 3w\dot{r} + 3w^2 \dot{r} + w^3r \]  

(32)

Converting the equation into primary coordinate axis, the equations obtained are:

\[ \ddot{r} = \ddot{a}_r + 3\dot{\theta}^2 \ddot{r} + 3\dot{\theta}(\ddot{\theta}) \dot{r} \]  

(33)

\[ \ddot{\theta} = \ddot{a}_r - \frac{3(\dot{\theta})\ddot{r}}{r} - \frac{3\dot{\theta}(\ddot{r} - \dot{\theta})}{r} - \dot{\theta}^2 \]  

(34)

The disassociated space state model looks like:

\[
\begin{bmatrix}
\dot{\mathbf{r}} \\
\ddot{\mathbf{r}} \\
\dddot{\mathbf{r}}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
3\dot{\theta}(\dot{\theta}) & 3\dot{\theta}^2 & 0
\end{bmatrix} \begin{bmatrix}
\mathbf{r} \\
\dot{\mathbf{r}} \\
\ddot{\mathbf{r}}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} \ddot{\mathbf{a}}_r
\]  

(35)

\[
\begin{bmatrix}
\dot{\mathbf{\theta}} \\
\ddot{\mathbf{\theta}} \\
\dddot{\mathbf{\theta}}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -\frac{3\dot{\theta}}{r} & -\frac{3\ddot{r}}{r}
\end{bmatrix} \begin{bmatrix}
\mathbf{\theta} \\
\dot{\mathbf{\theta}} \\
\ddot{\mathbf{\theta}}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} \ddot{\mathbf{a}}_r
\]  

(36)

It is readily observed that a very high cross coupling of the radial and transverse components exists. The range, range rate and range acceleration are required to adequately compute the angular acceleration. The angular velocity and acceleration is required in computing the range acceleration. All of these quantities are time varying requiring a time varying filter to implement this model.

The cubic term of angle rate in the angle channel is insignificant compared to the other terms and is neglected. The second order model uses the simplifying assumption of constant velocity and constant acceleration. For the full third order model, no simplifying assumptions will be made. This third order model should account for all of the cross coupling between the bearing and angle channels.

A Kalman Filter can be employed to track the target in both range and bearing to implement the time variant third order model. The Kalman Filter will be discussed in the next section.
V. KALMAN FILTER

Given a system model, where the plant can be modeled by a set of first order differential equations and the output can be measured, a set of state equations can be defined similar to:

\[
\dot{X} = AX + BU + W \tag{37}
\]
\[
Y = CX + V \tag{38}
\]

where \( X \) is the state vector
\( Y \) is the system output vector
\( A, B, C \) and \( D \) are matrices
\( U \) is the system input
\( W \) is plant disturbances
\( V \) is measurement noise

The system can be modeled in discrete time as:

\[
X(k+1) = \Phi X(k) + \Gamma U(k) + W(k) \tag{39}
\]
\[
Y(k) = H X(k) + V(k) \tag{40}
\]

A Kalman Filter is the best filter to track the output of a discrete system [Ref. 3]. The Kalman Filter equations are given as:

\[
\hat{X}(k | k) = \hat{X}(k | k-1) + G(k) \cdot \left[ Y(k) - \hat{Y}(k | k-1) \right] \tag{41}
\]
\[
\hat{X}(k+1 | k) = \Phi \cdot \hat{X}(k | k) + \Gamma \cdot U(k) \tag{42}
\]
\[
\hat{Y}(k+1 | k) = H \cdot \hat{X}(k+1 | k) \tag{43}
\]

where \( \hat{X}(k | k) \) = the state estimate at time \( k \) given information through time \( k \).
\( \hat{X}(k+1 | k) \) = the state estimate at time \( k+1 \) given information through time \( k \).
\( \hat{Y}(k+1 | k) \) = the output estimate at time \( k+1 \) given information through time \( k \).
\( G(k) \) = the filter gain at time \( k \).
For linear, time invariant systems, the $\Phi$ and $\Gamma$ matrices are easy to calculate and follow directly from the state space model, where $\Phi = e^{At}$ and $\Gamma = \int e^{At} \, dt$. For non linear systems, an extended Kalman filter can be used. For the extended Kalman filter, the $\Phi$ and $\Gamma$ matrices are linearized about the projected operating point. One method of estimating the linearization is to take the partial derivatives of the non linear state space matrices:

$$\Phi = \begin{bmatrix} A & X \\ X &= X_0 \\ U &= U_0 \\ W &= 0 \end{bmatrix}$$  \hspace{1cm} (44)

$$\Gamma = \begin{bmatrix} B \\ U \\ X &= X_0 \\ U &= U_0 \\ W &= 0 \end{bmatrix}$$  \hspace{1cm} (45)

The gain matrix $G(k)$ will vary with the parameters of the filter. $G(k)$ is the weighting factor for the system error. The solution to the filter gain $G(k)$ requires the solution of Riccati equations:

$$G(k) = P(k|k-1) \, H^T \left[HP(k|k-1) \, H^T + R(k)\right]^{-1}$$  \hspace{1cm} (46)

$$P(k|k-1) = \Phi \, P(k|k-1) \, \Phi^T + \delta \, Q \, \delta^T$$  \hspace{1cm} (47)

$$P(k|k) = P(k|k-1) - G(k) \, P(k|k-1)$$  \hspace{1cm} (48)

where $G(k) = \text{Kalman Filter gain at time } k$

$P(k|k-1) = \text{Covariance of predicted estimate}$

$R(k) = \text{measurement covariance matrix, } E[VV^T]$

$Q(k) = \text{maneuver covariance matrix, } E[UU^T]$

$P(k|k) = \text{Covariance of filtered estimate}$

$\delta = \text{maneuvering weighting matrix}$

A constant gain matrix can also be used in the Kalman filter. Instead solving the full Riccati equations for each

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change of variables, a constant value is used throughout the problem. A constant gain matrix will simplify implementation of the Kalman filter.

One implementation of the third order model, as discussed in the previous section, is to model the system as linear, time invariant, given by the space state model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
\dot{r} \\
\dot{\phi} \\
\dot{\theta}
\end{bmatrix}
\] (49)

The Kalman Filter equations for this third order model are:

\[
\hat{X}(k \mid k) = \hat{X}(k \mid k-1) + G(k) \cdot \left[ Y(k) - Y(k \mid k-1) \right] 
\] (50)

\[
\hat{X}(k+1 \mid k) = \Phi \cdot \hat{X}(k \mid k) 
\] (51)

\[
\hat{Y}(k+1 \mid k) = H \cdot \hat{X}(k+1 \mid k) 
\] (52)

where \( X = \begin{bmatrix} r \\ \dot{r} \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \end{bmatrix} \) and

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Using a Kalman Filter on this third order model is very simple and requires few on line calculations. The gain matrix can be considered either constant or time varying. If time varying gains are used, they can be computed off-line and stored in memory. The Filter then utilizes the precomputed gain schedule and can select a gain depending on
the accuracy of the filter at that time. If a maneuver causes the filter to lose accuracy then a higher gain term can be utilized. If constant gains are used then they must be high enough to compensate for any maneuver the target might make. A high gain matrix will make the missile more responsive to any unwanted noise terms in the system since the missile cannot distinguish a noise input from a target maneuver.

As discussed previously, a Kalman Filter is not required for the range channel. The radar can measure range and range rate directly. Since the actual values of the range channel are not used by any other elements of the guidance subsystem, the radar is able to maintain its own tracking of the target in the center of the range gate, which has no consequences on the rest of the missile guidance. Some noise information can be gained when estimating the range channel with a Kalman Filter. A Kalman Filter is used for the range channel in the simulation of Section VI for completeness of simulation and practical experience.

A second implementation of the third order model is by using the equations obtained through the coriolis equations. The disassociated state space model is given as:

\[
\begin{bmatrix}
\dot{r} \\
\dot{r}' \\
\dot{r}'' \\
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
3\dot{\theta}(\dot{r}') & 3\dot{\theta}' & 0 \\
\end{bmatrix}
\begin{bmatrix}
r \\
r' \\
r'' \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}
\dot{a}_r
\] (53)

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\theta}' \\
\dot{\theta}'' \\
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -3\dot{r}' & -3\dot{r} \\
\end{bmatrix}
\begin{bmatrix}
\theta \\
\theta' \\
\theta'' \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}
\dot{a}_r
\] (54)

Two possible ways to implement the Kalman filter are to create an extended Kalman filter by linearizing the \(\Phi\) matrix, reducing the time dependence and cross coupling of the range and bearing channel, or keep the cross coupling
components and have the Kalman Filter maintain the values of the
time varying $\Phi$. If $\Phi$ is linearized, some of the cross
coupling and time dependence lost by linearization can be
compensated for by the maneuver covariance matrix $Q$. Given
the target acceleration probability model, Figure III-1, the
$Q$ matrix can be calculated, as derived in Reference 3, as
time varying and relates the cross coupling of the bearing
and range channel as:

$$QR = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^{2}_{m} \end{bmatrix} \quad \text{and} \quad QS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^{2}_{m} \end{bmatrix}$$ (55)

where $\sigma^{2}_{m} =$ acceleration variance

As discussed in the reference the $Q(3,3)$ element can be
increased to make the missile gain matrix put more weight on
any target acceleration elements.

If the $\Phi$ matrix is maintained as time varying and
nonlinear then the $Q$ matrix can be constant. The constant $Q$
matrix can be calculated as:

$$Q = K I$$ (56)

where $K =$ matrix gain
$I =$ identity matrix

Since the reference deals extensively with the time
invariant $\Phi$ and the time varying $Q$ matrix, this thesis will
deal with the time varying $\Phi$ and constant $Q$.

If Figure III-4 is used to define the maneuver
probability then a non-zero mean is established. The $Q$
matrix maintains the same properties just discussed with a
different calculation for $\sigma^{2}_{m}$. The non-zero mean can be
implemented by increasing $Q(3,3)$, increasing the weighting
matrix $\delta$, or by not assuming the $U(k)$ term is zero.
Kalman filter equations for this third order model are:

\[ \hat{X}(k | k) = X(k | k-1) + G(k) \cdot [ Y(k) - \hat{Y}(k | k-1) ] \]  \hspace{1cm} (57)

\[ \hat{X}(k+1 | k) = \Phi \cdot \hat{X}(k | k) + \Gamma \cdot U(k) \]  \hspace{1cm} (58)

\[ \hat{Y}(k+1 | k) = H \cdot \hat{X}(k+1 | k) \]  \hspace{1cm} (59)

\[ P(k | k-1) = \Phi \cdot P(k | k-1) \cdot \Phi^T + \Delta \cdot Q \cdot \Delta^T \]  \hspace{1cm} (60)

\[ G(k) = P(k | k-1) \cdot H^T \cdot \left[ H \cdot P(k | k-1) \cdot H^T + R(k) \right]^{-1} \]  \hspace{1cm} (61)

\[ P(k | k) = P(k | k-1) - G(k) \cdot H \cdot P(k | k-1) \]  \hspace{1cm} (62)
VI. SIMULATION

To aid in the efforts of simulation, the Dynamic Simulation Language (DSL) was used to integrate the equations of motion for the target and missiles, as well as the antenna angle channel. Two programs were written and are listed in Appendix B. The first program is the time varying third order model. The second program is the time invariant third order model and the second order model.

The output of the simulation is a set of graphs to compare the three missile models and their effectiveness in tracking the target.

The Kalman Filter is implemented in a Fortran subroutine at the end of the DSL main program. The basic filter equations used were described in the previous section.

A. ASSUMPTIONS

The following assumptions are made to simplify the implementation of the Kalman Filter and determine the effects of the time varying third order model.

- "Ideal" missile autopilot.
- Inertial cartesian reference frame for angle measurements.
- Final portion of intercept only.
- Cross coupled effects of missile motion on antenna stabilization system disregarded.
- Missile initialized to collision course.
- Missile constant speed of 1500 kts (Mach 1.5) or 2500 ft/sec.
- Target constant speed of 400 kts (Mach .75) or 667 ft/sec.
- Target lateral acceleration applied perpendicular to target velocity vector.
- Angle of attack not accounted for in velocity vector calculations.
- Missile located at the center of the cartesian reference frame.
- Target located to the right of the origin of the reference frame.
- No noise.

B. INITIALIZATION

The user is asked at the beginning of the program to establish the geometry by giving the target initial position, heading, speed and acceleration. The target heading is oriented relative to a vertical line, parallel to the Y axis, defining the North or 000 heading. The initial missile headings are calculated for constant velocity, zero acceleration collision course with a time to go estimate of range/missile velocity. Antenna parameters are initialized to initial line of sight and zero angular rate.

C. SECOND ORDER MODEL

The second order model is implemented using a proportional navigation constant of four and two s-plane poles at $s=-10$. This gives the equation for angular acceleration as:

$$\dot{\beta} = -20 \beta + 100 \left( \text{LOS} - \beta \right)$$

where
- $\dot{\beta}$ = the antenna angular acceleration
- $\beta$ = the antenna angle rate
- $\beta$ = the antenna angle position
- LOS = the actual angle to the target

D. THIRD ORDER MODEL

The Kalman filter is used to implement the third order model. The DSL main program calls the Kalman Filter subroutine at a sampling time of $h=0.01$ seconds. The Kalman Filter is executed, then control is passed back to the DSL program. A proportional navigation constant of four is used as discussed in Section II.
1. **Time Invariant, Constant Phi Model**

   The discrete time invariant third order model divided into two Kalman filters for range and bearing is:

   \[
   \begin{align*}
   \text{RNG}(k+1 | k) &= \text{RPHI} \cdot \text{RNG}(k | k) \\
   \text{RNG}(k | k) &= \text{RNG}(k | k-1) + \text{GR}(k) \cdot \begin{bmatrix} \text{DELR} \end{bmatrix} \\
   \text{S}(k+1 | k) &= \text{SPHI} \cdot \text{S}(k | k) \\
   \text{S}(k | k) &= \text{S}(k | k-1) + \text{GS}(k) \cdot \begin{bmatrix} \text{SDEL} \end{bmatrix}
   \end{align*}
   \]

   where

   \[
   \begin{align*}
   \text{RPHI} &= \text{SPHI} = \begin{bmatrix} 1 & 0.01 & 0.0005 \\ 0 & 1 & 0.01 \\ 0 & 0 & 1 \end{bmatrix} \\
   \text{DELR} &= \begin{bmatrix} \text{RM} - \text{RKP1} \\ \text{RDM} - \text{RDKP1} \end{bmatrix} \\
   \text{SDEL} &= \begin{bmatrix} \text{LOS} - \text{SKP1} \\ \text{LOS} - \text{SKP1} \end{bmatrix}
   \end{align*}
   \]

   Using Matlab functions of Aker and Place, with eigenvalues of 0.5, 0.5 and 0.5 constant gain matrices were obtained for range and bearing, given by \text{GR} and \text{GS}, respectively.

   \[
   \begin{align*}
   \text{GR} &= \begin{bmatrix} 0.5 & 0.0125 & 0.0025 \\ 0.0125 & 1.0 & 0.0125 \\ 0.0125 & 0.125 & 24.9 \end{bmatrix} \\
   \text{GS} &= \begin{bmatrix} 1.5 \\ 12.5 \\ 125.0 \end{bmatrix}
   \end{align*}
   \]

2. **Time Variant, Variable Phi Model**

   The discrete, time variant third order model divided into two Kalman filters for range and bearing is:

   \[
   \begin{align*}
   \text{RNG}(k | k) &= \text{RNG}(k | k-1) + \text{GR}(k) \cdot \begin{bmatrix} \text{DELR} \end{bmatrix} \\
   \text{RNG}(k+1 | k) &= \text{RPHI} \cdot \text{RNG}(k | k)
   \end{align*}
   \]

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\[ PR(k | k-1) = RPHI \cdot PR(k | k-1) \cdot RPHI^T + QR \] (70)

\[ GR(k) = PR(k | k-1) \cdot HR^T \cdot [HR \cdot PR(k | k-1) \cdot HR^T + RMCOV(k)]^{-1} \] (71)

\[ PR(k | k) = PR(k | k-1) - GR(k) \cdot HR \cdot PR(k | k-1) \] (72)

\[ S(k | k) = S(k | k-1) + GS(k) \cdot [SDEL] \] (73)

\[ S(k+1 | k) = SPHI \cdot S(k | k) \] (74)

\[ PS(k | k-1) = SPHI \cdot PS(k | k-1) \cdot SPHI^T + QS \] (75)

\[ GS(k) = PS(k | k-1) \cdot HS^T \cdot [HS \cdot PS(k | k-1) \cdot HS^T + RSCOV(k)]^{-1} \] (76)

\[ PS(k | k) = PS(k | k-1) - GS(k) \cdot HS \cdot PS(k | k-1) \] (77)

The time variant model uses two different matrices, one for range and the other for bearing. Two other matrices must be specified for each filter, the initial error covariance matrix and the target maneuvering covariance matrix. The two matrices are:

\[ RPHI = \begin{bmatrix} 1 & 0.01 & 0.00005 \\ 0.01*A & 1+0.00005*B & 0.01 \\ 0.01*A & 0.01*B & 1+0.00005*B \end{bmatrix} \] (78)

\[ SPHI = \begin{bmatrix} 1 & 0.01 \\ 0 & 1+0.00005*C & 0.00005 \\ 0 & 0.01*C+0.00005*D & 1+0.01*D+0.00005*(C+D) \end{bmatrix} \] (79)

where \( A = -3\hat{b} (\hat{b}^T) \)

\( B = -3\hat{b}^2 \)

\( C = -3\vec{r}^2/r \)

\( D = -3\vec{r}/r \)

The initial error covariance matrices are given as:

\[ PR(0 | 0) = \begin{bmatrix} 500 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 500 \end{bmatrix} \]
\[
\begin{bmatrix}
1e4 & 0 & 0 \\
0 & 1e4 & 0 \\
0 & 0 & 1e4
\end{bmatrix}
\]

The maneuvering covariance matrix can either be time varying or constant as discussed previously. The maneuvering covariance matrix accounts for the capabilities of the target as discussed in Section IV. Reference 3 gives the derivation for the time varying matrix for a constant \( \Phi \) matrix. The \( \Phi \) matrix also accounts for any time variance of the target model so the maneuvering covariance matrix can be constant. A constant matrix is assumed since \( \Phi \) is time varying. The maneuvering covariance matrix (QR AND QS) are given as:

\[
QR = \begin{bmatrix}
500 & 0 & 0 \\
0 & 500 & 0 \\
0 & 0 & 500
\end{bmatrix}
\]

\[
QS = \begin{bmatrix}
.01 & 0 & 0 \\
0 & .01 & 0 \\
0 & 0 & .01
\end{bmatrix}
\]

The simulation calculates the range model then the bearing model. The results of each filter are used in the other filter to calculate the \( \Phi \) values.
E. RESULTS

Similar simulations of the ideal missile cases, Section II, were used to evaluate the missiles. The simulation consists of head-on, tail and beam aspects with 0G, 3G and 6G target acceleration. The results of the computer simulation is shown in graph form in Figure VI-1 through VI-31. The computer program listings are contained in Appendix B.

1. Gains

Gain comparison plots are given in Figure VI-1 through Figure VI-4. The resultant gains from the time variant, varying Phi third order missile are the same for each scenario.

In predicting $R(k^*|k)$, the varying Phi model weights the range error by .5 and the range rate error by 0. The constant Phi model uses weights of .5 and .0125, approximately the same, Figure VI-1.

Gains for predicting $RD(k^*|k)$, for varying Phi, are .001 and .6 while those of the constant Phi are .0025 and 1.0, Figure VI-2. Little emphasis is placed on the range error, because the radar is measuring range rate, with a higher weighting factor.

In predicting $RDD(k^*|k)$, small gains are calculated by the varying Phi while the constant Phi model places a high emphasis on range rate error. The noise of the system will be noticed more in the prediction of $RDD(k^*|k)$ than the other parameters, because of the higher weighting factor.

Bearing channel gains give unusual curves for the varying Phi model, Figure VI-4. There is little weight placed on non-observed parameters as in SG2 and SG3, during most of the intercept, except in the initial stage and final stage. The gains are highest during the critical stages of the intercept. SG1 is a constant 1.0 giving equal weighting to the current estimate and the error. The constant Phi model has higher gains giving more weight to any errors.
Figure VI-1 Range Covariance Gains RG11 and RG12
Figure VI-2  Range Covariance Gains RG21 and RG22
Figure VI-3 Range Covariance Gains RG31 and RG32
BEAM OF TGT BEARING COVARIANCE GAINS

LEGEND

- VARYING PHI
- CONSTANT PHI

Figure VI-4 Bearing Covariance Gains SG1, SG2 and SG3
Figure VI-5: Position Plot for Head-on

LEGEND

- TARGET
- VARYING PHI
- CONSTANT PHI
  + 2ND ORDER REFERENCE
Figure VI-6 Line of Sight Angle Plot for Head-on
Aspect No Target Turn
Figure VI-7  Missile Commanded Acceleration Plot for Head-on
Aspect No Target Turn
Figure VI-8 Position Plot for Head-on Aspect 3G Target Turn
Figure VI-9  Line of Sight Angle Plot for Head-on Aspect 3G Target Turn
Figure VI-10  Missile Commanded Acceleration Plot for Head-on Aspect 3G Target Turn
Figure VI-11 Position Plot for Head-on 6G Target Turn
Figure VI-12 Line of Sight Plot for Head-on Aspect 6G Target Turn
Figure VI-13  Missile Commanded Acceleration Plot for Head-on Aspect 3G Target Turn
Figure VI-14 Position Plot for Tail Aspect No Target Turn
Figure VI-15 Line of Sight Angle Plot for Tail Aspect No Target Turn
Figure VI-16  Missile Commanded Acceleration Plot for Tail Aspect No Target Turn
Figure VI-17 Position Plot for Tail Aspect 3G Target Turn
Figure VI-18 Line of Sight Angle Plot for Tail Aspect 3G Target Turn
Figure VI-19  Missile Commanded Acceleration Plot for Tail Aspect 3G Target Turn
Figure VI-20 Position Plot for Tail Aspect 60 Target Turn
Figure VI-21 Line of Sight Angle Plot for Tail
Aspect 6G Target Turn
Figure VI-22 Missile Commanded Acceleration Plot for Tail Aspect 6G Target Turn
Figure VI-22 Position Plot for Beam

Legend
- Target
- Varying Phi
- Constant Phi
- 2nd Order Reference

X (ft) vs Y (ft)

0.0 2.0 4.0 6.0 8.0 10.0 12.0 14.0 16.0 18.0 x10^3
Figure VI-24  Line of Sight Angle Plot for Beam Aspect No Target Turn
Figure VI-25  Missile Commanded Acceleration Plot for Beam Aspect No Target Turn.
Figure VI-36  Position Plot for Beam Aspect 3G Target Turn
Figure VI-27 Line of Sight Angle Plot for Beam Aspect 3G Target Turn
Figure VI-28  Missile Commanded Acceleration Plot for Beam
Aspect 3G Target Turn

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Figure VI-29  Position Plot for Beam Aspect 6G Target Turn
Figure VI-30  Line of Sight Angle Plot for Beam Aspect 6G Target Turn
BEAM 6G TGT
MISSILE COMMANDED ACCELERATION

Figure VI-31  Missile Commanded Acceleration Plot for Beam Aspect 6G Target Turn
2. **Head-on Aspect**

Figures VI-5 through VI-16 are the results of missile guidance comparisons for head-on aspect initial condition with 0G, 3G and 6G constant target acceleration. The missile begins at the origin of the graph. The target initial position is at \( x = 10000 \) ft, \( y = 1000 \) ft. Applied lateral target acceleration is away from the missile.

**a. 0G Target Acceleration**

Flight paths for the three models are shown in Figure VI-5. All three paths appear to be the same, within the accuracy of the plotter. The line of sight angle, Figure VI-6 remains relatively constant for all three models, at the initial value, giving a constant bearing decreasing range, no maneuver intercept.

Commanded Missile Acceleration, Figure VI-7, shows that the second order model pulls more lateral acceleration than third order models. Higher gain terms make the constant \( \Phi \) third order model erratic when compensating for initialization error.

**b. 3G Target Acceleration**

In the position plot, Figure VI-8, the second order model begins to lag the third order model. The lagging means the missile is slower to compensate for line of sight rates. This is further illustrated by Figure VI-9, the line of sight angle plot. The change in line of sight is greater for the second order model.

Commanded acceleration, Figure VI-10 shows a larger increase for the second order model. The second order model requires approximately 7.5G to intercept a 3G target while the third order model require 4.66G.
c. 6G Target Acceleration

Increased target acceleration increases the lag of the second order model, Figure VI-11. Magnitude of line of sight is larger for the second order model, Figure VI-12, but the missile is compensating for the errors. The two third order models are fairly close. The difference is only fractions of radians.

The constant Phi third order model has less initial oscillations in commanded acceleration when the target acceleration increases. The second order model requires approximately 11G for a 6G target, 7.5G is required for the third order models.

3. Tail Aspect

Figures VI-13 through Figure VI-21 are the results of missile guidance comparisons for tail aspect initial condition with 0G, 3G and 6G constant target acceleration. The missile begins at the origin of the graph. The target initial position is at \(x = 15000\) ft, \(y = -500\) ft. Applied lateral target acceleration is into the missile.

a. 0G Target Acceleration

As shown in Figure VI-13, there is little difference in flight paths for the three missiles in the no target acceleration case. Line of sight angles remain constant, Figure VI-14, throughout the intercept.

There is some slight difference in commanded accelerations, Figure VI-15, with the second order model being the smaller of the three models.

b. 3G Target Acceleration

As the target applies acceleration, the lag of the second order missile gives a shorter flight path than the third order missiles, Figure VI-16. Line of sight angles are smaller for the second order model, Figure VI-17.

Acceleration required for the second order model is 3.4G and 5.3G for the third order models, Figure VI-18. By lagging the other missiles, the second order model allows
the target to complete part of the intercept, which allows
the missile to pull less acceleration.

c. 6G Target Acceleration

With higher target acceleration, the flight
paths, Figure VI-19 have approximately the same differences
as the 3G case. The second order model maintains a better
flight path throughout the intercept. The slope of the line
of sight curves, Figure VI-20, are higher with the second
order model maintaining a smaller angle difference.

Commanded acceleration is much smaller for the
second order model than the third order models, Figure VI-
21. To intercept the 6G target, the second order model
requires 6.2G and the third order models require 8.1G.

4. Beam Aspect

Figures VI-22 through VI-30 are the results of
missile guidance comparisons for head-on aspect initial
condition with 0G, 3G and 6G constant target acceleration.
The missile begins at the origin of the graph. The target
initial position is at \( x = 15000 \) ft, \( y = 0 \) ft. Applied
lateral target acceleration is away from the missile.

a. 0G Target Acceleration

Figure VI-22 through VI-24, show the three
missiles are practically the same for a no target
acceleration, beam aspect intercept. All three missiles
maintain a constant bearing decreasing range, small
acceleration intercept.

b. 3G Target Acceleration

Only slight differences are noticed when target
acceleration is applied. Flight paths, Figure VI-25, shows
very little deviation. The line of sight angle difference
of \( 0.01 \) rad between the models is approximately \( 0.575 \).
Acceleration is higher for the second order missile to allow
it to fly the same path as the third order models, Figure
VI-27.
c. 6G Target Acceleration

Flight paths have a pronounced difference with a higher target acceleration, Figure VI-28. Line of sight angles, Figure VI-29, have larger magnitudes and higher slopes. Commanded acceleration increases dramatically, now 25G is required of the second order model and 9.3G for the third order models.
VII. CONCLUSIONS

In analyzing second and third order missile tracking and guidance subsystems, the following conclusions are made:

- Proportional navigation guidance is the optimum method for missiles, given current design tradeoffs.

- Target modeling is very difficult and requires the analysis of many factors. Acceleration probabilities make modeling easier, but the proper acceleration model must be chosen.

- Cross coupling between coordinate reference axis components does exist and gives errors if not accounted for in the system model.

- Kalman filters are the best predictors for airborne missiles, if one is required.

- Complete time varying third order models give better results than approximated linear, time-invariant third order models.

- Only small differences are noticed in parameter values between second and third order models. Higher accelerations are required for the second order model.

- Second order missiles are better than third order missiles in tail aspect, constant acceleration intercepts.

- Implementation of a Kalman filter requires considerable amounts of computer resources, with limited time to complete the calculations.

- Some parameter terms are of the approximate order as system noise or non-significant calculations.

Some recommendations for future study and consideration are:

- A study of miss distance analysis for second and third order models.

- Analysis of the effects of the Q and P matrix initialization.

- Analyze the target acceleration probability model to find optimum values to assign to the probability model.

- Determine missile cross coupling effect of heading changes and autopilot torques on the sensor subsystem.

- Add noise to the system to determine the effects of the high gain terms on a noisy system.
REFERENCES


APPENDIX A. IDEAL MISSILE PROGRAM LISTING

MISSILE PROGRAM FOR FLIGHT PATH COMPARISON IN THE
THESIS. THIS PROGRAM IS DESIGNED TO FOLLOW THREE IDEAL
MISSILES USING DIFFERENT GUIDANCE TECHNIQUES TO TARGET
INTERCEPT. THE GUIDANCE TECHNIQUES ARE: PROPORTIONAL
NAVIGATION, PURE PURSUIT AND LEAD PURSUIT. PARAMETERS
ARE ALSO OBTAINED FOR A FOURTH MISSILE USING A DIRECT
FLIGHT PATH.

INITIAL CONST
G=32.2, D2R=.0175, K2F=1.66667, PITCH=2.7, PI=3.14159, RM1=20000
K=0
NN=0
MISSX0=0.0
MISSY0=0.0
VM = 2500.0
READ (2,5) VT, AT, THDG, TGTX0, TGY0
FORMAT (F6.1,2X,F5.1,2X,F6.1,2X,F6.1,2X,F10.2)
TGTV=VT*K2F
TGTA=-AT*G
THDG=THDG*D2R
TGTVX0=TGTV*SIN(THDG)
TGTVY0=TGTV*COS(THDG)

C INITIALIZATION OF DIRECT INTERCEPT MISSILE
DXT=MISSX0
DYT=MISSY0

C INITIALIZATION OF PROP NAV MISSILE CONSTANT VELOCITY, ZERO ACCEL
RO=((TGTX0 - MISSX0)**2 + (TGY0 - MISSY0)**2)**.5
TTGO= RO/VM
LOS = ATAN2(TGTY0-MISSYO, TGTX0-MISSX0)
PHDG=ATAN2(TGTY0+TGTVYO*TTGO-MISSYO,-TGTX0+TGTVX0*TTGO-MISSX0)
BD0 = TGTV*COS(THDG)/(COS(LOS) * RO)
B0 = LOS

C INITIALIZATION OF LEAD PURSUIT MISSILE
G0 = B0
GD0 = 0
LPHDG = LOS

METHOD RKSF
DERIVATIVE
C TARGET POSITION UPDATING
TGTHDG=INTGRL(THDG, (-1*AT) *PITCH*D2R)
TGTX0=TGTA*COS(TGTHDG)
TGTY0=-TGTA*SIN(TGTHDG)
TVELX=INTGRL(TGTVX0, TGTX0)
TVELY=INTGRL(TGTVY0, TGTY0)
XT=INTGRL(TGTX0, TVELX)
YT=INTGRL(TGTY0, TVELY)
TGTHDG = ATAN2(TVELX, TVELY)

C PROP NAV MISSILE POSITION UPDATING
BDDOT = -20*BDOT + 100*(PNLOS-B)
BDC'T = INTGRL(B0, BDDOT)
B = INTGRL(B0, BDOT)
PNHDG = INTGRL(PHDG, 4*BDOT)
PXN=INTGRL(MISSX0, VM*COS(PNHDG))
PYM=INTGRL(MISSY0, VM*SIN(PNHDG))

C PURE PURSUIT MISSILE
PPHDG = ATAN2(YT-PPYM,XT-PPXM)
PPXM = INTGRL(MISSX0,VM*COS(PPHDG))
PPYM = INTGRL(MISSY0,VM*SIN(PPHDG))

C LEAD PURSUIT MISSILE
LPHDG = ATAN2(YT+TVELY*.5*TTG-LPYM,XT+TVELX*0.5*TTG-LPXM)
LPXM = INTGRL(MISSX0,VM*COS(LPHDG))
LPYM = INTGRL(MISSY0,VM*SIN(LPHDG))

C DYNAMIC

C PROP NAV MISSILE GEOMETRY UPDATE
PNAM = 4*BDOT*PNRD
PNR=((XT-PPXM)**2 + (YT-PPYM)**2)**.5
PNLOS = ATAN2(YT-PPYM,XT-PPXM)
PNRD=TVELX*COS(PNLOS) + VM*COS(PNHDG-PNLOS)
PNLOSD = (-TVELX*SIN(PNLOS) - VM*SIN(PNHDG-PNLOS))/PNR

C PURE PURSUIT GEOMETRY UPDATE
PFR = ((XT-PPXM)**2 + (YT-PPYM)**2)**.5
PFLOS = ATAN2(YT-PPYM,XT-PPXM)
PPRD = TVELX*COS(PFLOS) - VM*COS(PPHDG-PFLOS)
PPLOSD = (-TVELX*SIN(PFLOS))/PFR

C LEAD PURSUIT GEOMETRY UPDATE
LPR = ((XT-LPXM)**2 + (YT-LPYM)**2)**.5
LPLOS = ATAN2(YT-LPYM,XT-LPXM)
LPFD = TVELX*COS(LPLOS) - VM*COS(LPHDG-LPLOS)
LPLOSD = (-TVELX*SIN(LPLOS))/LPR

VM*SIN(LPHDG-LPLOS)/LPR

C DIRECT INTERCEPT MISSILE GEOMETRY UPDATE
RD = ((YT-MISSYO)**2 + (XT-MISSXO)**2)**.5
DXT, DYT, FON= CHECK(RD,TIME,VM,XT,YT)
IF (PXM.GT.XT) CALL ENDRUN

C IF (K .LE. 0.0) THEN
NN=NN+1
WRITE (31,50) XT,YT,PXM..PYM
WRITE (32,50) PPXM,PPYM,LPXM,LPYM
WRITE (33,51) TIME,PNLOS,PPLOS,LPLCS
WRITE (34,52) TIME,PNLOSD,PPLOSD,LPLSOD
WRITE (35,53) TIME,PNHDG,PPHDG,LPHDG
50 FORMAT(4(F10.2,2X))
51 FORMAT(F5.2,3(2X,F10.5))
52 FORMAT(F5.2,3(2X,F10.5))
53 FORMAT(F5.3,2X,3(F10.6,2X))
R=10
C IF (PNR.LT.0.1*RO) THEN
R=1
ENDIF
ENDIF
K=K-1

C SAMPLE
C SAVE (A) 0.1,XT,YT,FXM,PYM,PPXM,PPYM,LPXM,LPYM
C PRINT 1.0,XT,YT,LPXM,LPYM,DDOT,LPLOS,LPHDG
CONTROL. FINTIM= 8.0,DELT=.01.
TERMINAL
DHEG = ATAN2(DYT-MISSYO,DXT-MISSX0)
GRAPH (A/A,DE=TEK618) XT
(SC=1600,LO=0.00),YT(SC=750,LO=0.0,PO=16000)
GRAPH (A/A,OV)
FXM(SC=1600,LO=0.0,AX=OMIT),PYM(SC=750,LO=0)
C GRAPH \( (A/A, OV) \)
PPXM(SC=1600, LO=0.0, AX=OMIT), PPYM(SC=750, LO=0, AX=OMIT)
C GRAPH \( (A/A, OV) \)
LPXM(SC=1600, LO=0.0, AX=OMIT), LPYM(SC=750, LO=0, AX=OMIT)
WRITE (2,15) DHDG, FON
15 FORMAT (F10.7, 2X, F5.2)
WRITE (116) NN
16 FORMAT (F4.0)
END
STOP
FORTRAN
SUBROUTINE CHECK (RD, TIME, VM, XT, YT, DXT, DYT,FON)
REAL*8 RD, TIME, VM, XT, YT, DXT, DYT, FON, DMISS
DMISS = VM*TIME
IF (DMISS .LT. RD) THEN
DXT = XT
DTY = YT
FON = TIME
ENDIF
RETURN
END
THIS IS A DSL PROGRAM TO FIND THE LOS AND RANGE OF A TARGET IN A CONSTANT G TURN FROM A MISSILE ON A DIRECT PATH.

INITIAL CONST
G=32.2,D2R=.0175,PITCH=2.7,K2F=1.66667,1=1.0,PI=3.14159,K=0
READ (2,10) VT,AT,THDG,TGTX0,TGY0
READ (2,11) MISHDG,DONE
10 FORMAT (F6.1,2X,F5.2,2X,F6.2,2(2X,F10.2))
11 FORMAT (F10.7,2X,F5.2)

MISSILE PARAMETERS
MISSX0 = 0.0
MISSY0 = 0.0
VM = 2500.0

TARGET PARAMETERS
TGTHDG=THDG*D2R
TGTV=VT*K2F
TGTV=(AT*G)
TGTVELY=VT*CO(S(TGTHDG))

EQUATIONS OF MOTION

MISSILE POSITION UPDATE
XE=INTGRL(MISSX0,VM*COS(MISHDG))

DYNAMIC
R=(XT-XM)**2 + (YT-YM)**2
LOS=ATAT2(YT-YM,XT-XM)
RD=TVELX*COS(LOS) = VM*COS(MISHDG-LOS)
LOS=(TVELY*COS(LOS) - VM*SIN(MISHDG-LOS))/R
IF(K.EQ.10) THEN
MM=MM+1
WRITE (37,15) XM,YM
WRITE (38,16) TIME,LOS,LOSD,MISHDG
10 FORMAT (2(F9.2,3X,-),F7.4)
K=K+1
END IF
IF ('TIME .GT. DONE) CALL ENDRUN

SAMPLE
CONTROL FTIM=10.0,DELT=.01
C PRINT 1.0,XT,YT,XM,YM,LOS,R
C SAVE (D) 0.1,XT,YT,XM,YM,R,LOS

END
STOP
APPENDIX B. THIRD ORDER SIMULATION PROGRAM LISTING

PROP NAV MISSILE PROGRAM FOR THESIS

DIMENSION PS(3,3), PR(3,3), RMCOV(2,2), RNG(3), S(3), DELR(3), SG(3)

DIMENSION RG(3,3)

K=0
MM=0
HH=0

METHOD RKSFX

G=32.2, D2R=.0175, K2F=1.66667

TE=.01

MISSX0=0.0

MISSY0=0.0

VM = 2500.0

AM0 = 0.0

READ (2,10) VT, AT, THDG, TGTX0, TGTY0

10 FORMAT (F6.1, 2X, F6.1, 2X, F6.1, 2X, F6.1, 2X, F10.2, 2X, FIO.2)

THDG=THDG*D2R

TGTVX0=TGTV* SIN (THDG)

TGTVY0=TGTV*COS (THDG)

INITIAL PS(0/0) MATRIX

PS(1,1)=1.0E+4

PS(1,2)=0.0

PS(1,3)=0.0

PS(2,1)=PS(1,2)

PS(2,2)=1.0E+4

PS(2,3)=0.0

PS(3,1)=PS(1,3)

PS(3,2)=PS(2,3)

PS(3,3)=1.0E+4

INITIAL PR(0/0) MATRIX

PR(1,1) = 500

PR(1,2) = 0

PR(1,3) = 0

PR(2,1) = PR(1,2)

PR(2,2) = 500

PR(2,3) = 0.0

PR(3,1) = PR(1,3)

PR(3,2) = PR(2,3)

PR(3,3) = 500

INITIALIZE THE RANGE MEASUREMENT COVARIANCE MATRIX

RMCOV(1,1) = 0.0

RMCOV(1,2) = 0.0

RMCOV(2,1) = RMCOV(1,2)

RMCOV(2,2) = 0.0

INITIALIZE THE BEARING MEASUREMENT NOISE COVARIANCE MATRIX

SMCOV = 0.0

INITIALIZATION OF PROP NAV MISSILE CONSTANT VELOCITY, ZERO ACCEL

LOS = ATAN2(TGTY0 - MISSY0, TGTX0 - MISSX0)

R = ((TGTX0 - MISSX0)**2 + (TGTY0 - MISSY0)**2)**.5

TGTG0 = R/VM

PHDG = ATAN2(TGTY0+TGTVO*TTGO-MISSY0, TGTX0+ TGTVO*TTGO-MISSX0)

VMX0 = VM*COS(PHDG)
VMY0 = VM*SIN(PHDG)
RKPI = R
RDKP1 = -VM*COS(PHDG-LOS) + TGTV*SIN(THDG)/COS(LOS)
RDDKP1 = 0
SKP1 = LOS
SDKP1 = (TGTVYO/COS(LOS) - VM*SIN(PHDG-LOS))/R
SDDKP1 = 0
B0 = LOS
B0 = 0

C
RNG (1) = RKP1
RNG (2) = RDKP1
RNG (3) = RDDKP1
S(1) = SKP1
S(2) = SDKP1
S(3) = SDDKP1

C DERIVATIVE
C TARGET POSITION UPDATING
TGTHDG = ATAN2(TVELX,TVELY)
TGTA = TGTA*COS(TGTHDG)
TGTA = (1-TGTA)*SIN(TGTHDG)
TVELX = INTGRL(TGTVO,TGTA)
TVELY = INTGRL(TGTVO,TGTA)
XT = INTGRL(TGTVO,TVELX)
YT = INTGRL(TGTVO,TVELY)
GR31=RG(3,1)
GR32=RG(3,2)
GS1=SG(1)
GS2=SG(2)
GS3=SG(3)

IF ( PXM .GT. XT ) THEN
   CALL ENDRUN
ENDIF

C

IF ( K .LE. 0 ) THEN
  MM=MM+1
  WRITE (40,20) XT,YT,PXM,PYM
  WRITE (42,21) TIME,GR11,GR12,GR21,GR22,GR31,GR32
  WRITE (44,22) TIME,GS1,GS2,GS3
  WRITE (46,23) TIME,LOS,LOS,D,LOS,DD,U
  IF ( RH .LT. .1*R ) THEN
    K=K-3
  ENDIF
ENDIF

K=K-1

20 FORMAT(4(2X,F10.2))
21 FORMAT (F5.2,6(1X,F10.4))
22 FORMAT (F5.2,3(2X,E12.5))
23 FORMAT (F5.2,3(2X,E11.4),2X,E14.6)

SAMPLE

C STATEMENTS TO SAVE DATA FOR USE WITH GRAFAEL

SAVE (A) 0.1,XT,YT,PXM,PYM
SAVE (B) 0.1,LOS,B,SK
SAVE (C) 0.1,LOS,D,DDOT,SDK
SAVE (D) 0.1,SDEL
SAVE (E) 0.1,RM,RK,RDDOTM,RDDK
SAVE (F) 0.1,U,DDOT
SAVE (G) 0.1,GS1,GS2,GS3
SAVE (H) 0.1,GR11,GR12,GR21,GR22

C PRINT 0.1,RM,RKP1,RK,RKDP1,RKDP2,RDDOTM,RDDK
C PRINT 0.1,PNHDG,LOS,B,SK,LOSD,BDOT,SDK,LOS,D,DDOT,SDDK

CONTROL FINTIM=10.0,DELT=.01
TERMINAL
WRITE (1,30) MM
30 FORMAT (F6.1)

C STATEMENTS FOR PLOTTING WITH GRAFAEL

GRAPH (A/A,DE=TEK616) XT
(SC=1600,LO=0.0),YT(SC=500,PO=16000)
GRAPH (A/A,OV) PXM (SC=1600,AX=OMIT),PYM(SC=500)
GRAPH (B/B,DE=TEK618) TIME,LOS(SC=.025,LO=-.1)
GRAPH (B/B,OV) TIME(AX=OMIT),B(PO=7.5,SC=.025,LO=-.1)
GRAPH (B/B,OV) TIME(AX=OMIT),SK(AX=OMIT,SC=.025,LO=-.1)
GRAPH (C/C,DE=TEK618) TIME,LOS,D
GRAPH (C/C,OV) TIME(AX=OMIT),BDOT(PO=7.5)
GRAPH (C/C,OV) TIME(AX=OMIT),SDK(AX=OMIT)
GRAPH (D/D,DE=TEK618) TIME,SDEL
GRAPH (E/E,DE=TEK618) TIME,RM(SC=2000.0,LO=0.0)
GRAPH (E/E,OV) TIME(AX=OMIT),RK(SC=2000.0,LO=0.0)
GRAPH (F/E,DE=TEK618) TIME,FDDOTM(SC=500.0)
GRAPH (F/E,OV) TIME(AX=OMIT),RDDK(AX=OMIT)
GRAPH (G/F,DE=TEK618) TIME,U
GRAPH (H/G,DE=TEK618) TIME,GS1
GRAPH (I/G,DE=TEK618) TIME,GS2
GRAPH (J/G,DE=TEK618) TIME,GS3
GRAPH (K/H,DE=TEK618) TIME,GR11
GRAPH (L/H,DE=TEK618) TIME,GR12
GRAPH (M/H,DE=TEK618) TIME,GR21
GRAPH (N/H,DE=TEK618) TIME,GR22
GRAPH (O/F,DE=TEK618) TIME,BDDOT

END
STOP
FORTTRAN

119
SUBROUTINE KALMAN (RNG, RM, RDOTM, RDDOTM, RK, RDK, RDDK, RK1, RKDP1, RDDK1, K, DELR, PR, RMCOV, S, LOS, LOSD, LOSDD, SK, SDK, SDDK, TIME, SKP1, SDKP1, SDDKP1, SDEL, SMCOV, PS, TK, RG, SG, HH)
C SUBROUTINE TO ITERATE A KALMAN FILTER FOR RANGE VARIABES
C GIVEN THE COVARIANCE MATRIX AND OBSERVATIONS
REAL*8 RNG(3), RM, RDOTM, RDDOTM, RK, RDK, RDDK, RK1,
* RDKP1, RDDK1, DELR(3), PR(3,3), RMCOV(3,3), S(3),
* LOS, LOSD, LOSDD, SK, SDK, SDDK, SKP1, SDKP1, SDDKP1, SDEL,
* SMCOV, TIME, PS(3,3), TK, TKSQ, A, B, C, D,
* TEMP2(2,2), TEMP3(3,3), RPHI(3,3), COVR(3,3),
* DET, SCOV, SPHI(3,3), RG(3,3), SG(3), QR(3,3), QS(3,3)
* TKSQ = TK*TK
C FIND THE NEW VALUES OF RPHI FROM THE PREVIOUS VALUES OF SIGMA
C MATRIX
A = -3*SDKP1*SDKP1
B = -3*SDKP1*SDKP1
RPHI(1,1) = 1
RPHI(1,2) = TK
RPHI(1,3) = 5*TKSQ
RPHI(2,1) = 5*A*TKSQ
RPHI(2,2) = 1 + 5*B*TKSQ
RPHI(2,3) = TK
RPHI(3,1) = A*TK
RPHI(3,2) = B*TK + 5*A*TKSQ
RPHI(3,3) = 1 + 5*B*TKSQ
C FIND THE PROJECTED COVARIANCE PR(K/K-1) = RPHI*PR(K-1/K-1)*RPHI
CALL ZERO(TEMP3,3)
C
DO 104 L=1,3
DO 103 M=1,3
DO 102 N=1,3
TEMP3(L,M) = TEMP3(L,M) + RPHI(L,N)*PR(N,M)
102 CONTINUE
103 CONTINUE
104 CONTINUE
C CLEAR THE OLD COVARIANCE MATRIX
CALL ZERO(PR,3)
C MULTIPLY BY RPHI TRANSPOSE
DO 107 L=1,3
DO 106 M=1,3
DO 105 N=1,3
PR(L,M) = PR(L,M) + TEMP3(L,N)*RPHI(M,N)
105 CONTINUE
106 CONTINUE
107 CONTINUE
C DEFINE THE Q MATRIX OF MANEUVER COVARIANCE
QR(1,1) = 500
QR(1,2) = 0.0
QR(1,3) = 0.0
QR(2,1) = 0.0
QR(2,2) = 500
QR(2,3) = 0.0
QR(3,1) = QR(1,3)
QR(3,2) = QR(2,3)
QR(3,3) = 500
C NOW ADD TO THE COVARIANCE MATRIX
DO 111 L=1,3
DO 110 M=1,3
PR(L,M) = PR(L,M) + QR(L,M)
110 CONTINUE
111 CONTINUE
NOW PR IS THE COVARIANCE OF THE PREDICTED ESTIMATE.

FIND THE ESTIMATE OF RANGE MATRIX AT STEP K

ZERO A TEMPORARY MATRIX
CALL ZERO (TEMP2, 2)
DO 121 L=1, 2
DO 120 M=1, 2
TEMP2(L,M) = PR(L,M) + RMCOV(L,M)
120 CONTINUE
121 CONTINUE
DET = TEMPP2(1, 1) * TEMPP2(2, 2) - TEMPP2(1, 2) * TEMPP2(2, 1)
COVR(1, 1) = TEMPP2(2, 2) / DET
COVR(1, 2) = (-1) * TEMPP2(1, 2) / DET
COVR(2, 2) = TEMPP2(1, 1) / DET
HERE COVR = (HPH + R) INVERSE

ZERO A TEMPORARY MATRIX
CALL ZERO (RG, 3)
DO 132 L=1, 3
DO 131 M=1, 2
DO 130 N=1, 2
RG(L,M) = RG(L,M) + PR(L,N) * COVR(N,M)
130 CONTINUE
131 CONTINUE
RG(L, 3) = 0
132 CONTINUE
RG = PH(HPH + R) (INVERSED) H

ZERO A TEMP MATRIX FOR THE RNG MATRIX
DO 140 L=1, 3
TEMP1(L) = 0
140 CONTINUE
DO 142 L=1, 3
DO 141 M=1, 2
TEMP1(L) = TEMP1(L) + RG(L,M) * DLR(M)
141 CONTINUE
142 CONTINUE
DO 143 N=1, 3
RNG(N) = TEMP1(N) + RNG(N)
143 CONTINUE

SAVE THE VALUES OF RANGE MATRIX AT STEP K
RK = RNG(1)
RDK = RNG(2)
RDDK = RNG(3)

ZERO THE OLD RANGE TEMPORARY MATRIX
DO 150 N=1, 3
TEMP1(N) = 0
150 CONTINUE
FIND THE ESTIMATE OF THE STEP K+1 FOR THE RANGE MATRIX
DO 152 L=1, 3
DO 151 M=1, 3
TEMP1(L) = TEMP1(L) + RPHI(L,M) * RNG(M)
151 CONTINUE
152 CONTINUE
SAVE THE VALUES OF RNG(K+1/K)
DO 153 N=1, 3
RNG(N) = TEMP1(N)
153 CONTINUE
RK1 = RNG(1)
RDK1 = RNG(2)
RDDK1 = RNG(3)
FIND THE COVARIANCE OF FILTERED ESTIMATE
RG = PH(HPH + R) INVERSED H

THEREFORE P(K/K) = P(K/K-1) + RG*P(K/K-1)
CALL ZERO(TEMP3, 3)
DO 163 L=1, 3
DO 162 M=1, 3
DO 161 N=1, 3
TEMP3(L, M) = TEMP3(L, M) + RG(L, N) * PR(N, M)
161 CONTINUE
162 CONTINUE
163 CONTINUE
C
DO 165 L=1, 3
DO 164 M=1, 3
PR(L, M) = PR(L, M) - TEMP3(L, M)
164 CONTINUE
165 CONTINUE
NOW PR IS THE COVARIANCE OF FILTERED ESTIMATE P(K/K)
C
SUBROUTINE TO ITERATE A KALMAN FILTER FOR SIGMA VARIABLES
C
WHERE H = (1 0 0)
C
CALCULATE THE NEW SPHI MATRIX
C
\begin{align*}
    C &= -3 \cdot RDDK/RK \\
    D &= -3 \cdot RDK/RK \\
    SPHI(1, 1) &= 1 \\
    SPHI(1, 2) &= TK \\
    SPHI(1, 3) &= .5 \cdot TK^2 \\
    SPHI(2, 1) &= 0 \\
    SPHI(2, 2) &= 1 + .5 \cdot C \cdot TK^2 \\
    SPHI(2, 3) &= TK + .5 \cdot D \cdot TK^2 \\
    SPHI(3, 1) &= 0 \\
    SPHI(3, 2) &= C \cdot TK + .5 \cdot C \cdot D \cdot TK^2 \\
    SPHI(3, 3) &= 1 + D \cdot TK + .5 \cdot (C + D) \cdot TK^2
\end{align*}
C
CALCULATE THE NEXT PROJECTED P MATRIX FOR NEXT STEP
T
PS(K, K-1) = SPHI * PS * SPHI
C
CLEAN A TEMPORARY MATRIX
CALL ZERO(TEMP3, 3)
DO 202 L=1, 3
DO 201 M=1, 3
DO 200 N=1, 3
TEMP3(L, M) = TEMP3(L, M) + SPHI(L, N) * PS(N, M)
200 CONTINUE
201 CONTINUE
202 CONTINUE
C
ZERO THE OLD PS MATRIX
CALL ZERO(PS, 3)
DO 205 L=1, 3
DO 204 M=1, 3
DO 203 N=1, 3
PS(L, M) = PS(L, M) + TEMP3(L, N) * SPHI(M, N)
203 CONTINUE
204 CONTINUE
205 CONTINUE
C
DEFINE THE Q MATRIX OF MANEUVER COVARIANCE
\begin{align*}
    QS(1, 1) &= 0.01 \\
    QS(1, 2) &= 0.0 \\
    QS(1, 3) &= 0.0 \\
    QS(2, 1) &= QS(1, 2) \\
    QS(2, 2) &= 0.01 \\
    QS(2, 3) &= 0.0 \\
    QS(3, 1) &= QS(1, 3) \\
    QS(3, 2) &= QS(2, 3) \\
    QS(3, 3) &= 0.01
\end{align*}
DO 211 L=1, 3
DO 210 M=1, 3
PS(L, M) = PS(L, M) + QS(L, M)
210 CONTINUE
122
CONTINUE

CALCULATE \((HPH + R)^{-1}\)

\[ \text{SCOV} = PS(1,1) + SMCOV \]

\[ SG(1) = PS(1,1)/\text{SCOV} \]
\[ SG(2) = PS(2,1)/\text{SCOV} \]
\[ SG(3) = PS(3,1)/\text{SCOV} \]

NOW FIND THE CURRENT VALUES OF THE SIGMA MATRIX

\[ S(1) = S(1) + SG(1) \cdot SDEL \]
\[ S(2) = S(2) + SG(2) \cdot SDEL \]
\[ S(3) = S(3) + SG(3) \cdot SDEL \]

STORE THE SIGMA MATRIX FOR USE IN THE PROGRAM

\( SK = S(1) \)
\( SDK = S(2) \)
\( SDDK = S(3) \)

FIND THE NEXT VALUES OF THE SIGMA MATRIX

\[ S(K+1) = SPHI \cdot S(K) \]

ZERO A TEMPORARY MATRIX

DO 220 L=1,3
   TEMP1(L) = 0
CONTINUE

DO 222 L=1,3
   DO 221 M=1,3
      TEMP1(L) = TEMP1(L) + SPHI(L,M) \cdot S(M)
   CONTINUE

CONTINUE

INPUT BACK INTO SIGMA MATRIX

DO 223 N = 1,3
   S(N) = TEMP1(N)
CONTINUE

STORE THE VALUE OF THE S MATRIX

\( SKP1 = S(1) \)
\( SDKP1 = S(2) \)
\( SDDKP1 = S(3) \)

NOW FIND THE P MATRIX AT STEP K

\[ PS(3,3) = PS(3,3) - PS(1,3) \cdot SG(3) \]
\[ PS(3,2) = PS(3,2) - PS(1,2) \cdot SG(3) \]
\[ PS(3,1) = PS(3,1) - PS(1,1) \cdot SG(3) \]
\[ PS(2,3) = PS(2,3) - PS(1,3) \cdot SG(2) \]
\[ PS(2,2) = PS(2,2) - PS(1,2) \cdot SG(2) \]
\[ PS(2,1) = PS(2,1) - PS(1,1) \cdot SG(2) \]
\[ PS(1,3) = PS(1,3) - PS(1,3) \cdot SG(1) \]
\[ PS(1,2) = PS(1,2) - PS(1,2) \cdot SG(1) \]
\[ PS(1,1) = PS(1,1) - PS(1,1) \cdot SG(1) \]

IF \((HH \leq 0.01)\) THEN

\( HH = 50 \)
WRITE (9,*) 'TIME IS ', TIME
WRITE (9, A) A, B, C, D
WRITE (9, *) A, B
WRITE (9, *) RPHI (L, M), M = 1, 3
450 CONTINUE
WRITE (9, *) 'SIGMA PHI MATRIX'
DO 451 L = 1, 3
   WRITE (9, *) SPHI (L, M), M = 1, 3
451 CONTINUE

OUTPUT THE COVARIANCE MATRICES AT STEP K

123
WRITE (9,*) 'THE RANGE COVARIANCE MATRIX IS:'
DO 401 M=1,3
   WRITE (9,*) (PR(M,N), N=1,3)
401 CONTINUE
WRITE (9,*) 'THE BEARING COVARIANCE MATRIX IS:'
DO 402 M=1,3
   WRITE (9,*) (PS(M,N), N=1,3)
402 CONTINUE
ENDIF
HH=HH-1
RETURN
END
SUBROUTINE ZERO (A,N)
C CLEAR A TEMPORARY MATRIX
C REAL*8 A(3,3)
   DO 301 L=1,N
      DO 300 M=1,N
         A(L,M)=0
300 CONTINUE
301 CONTINUE
RETURN
END
THIRD ORDER MISSILE SIMULATION USING CONSTANT GAINS
FOR THE KALMAN FILTER. A SECOND ORDER PROP NAV REFERENCE
MODEL IS SIMULATED FOR PLOTTING AND PARAMETER
COMPARISONS.

INITIAL

DIMENSION RNG(3), S(3), DELR(2), GS(3), GR(3,3), RPHI(3,3), SPHI(3,3)
K=0
NN=0

C
METHOD RKSFX
CONST G=32.2,D2R=.0175,K2F=1.66667
TK = 0.01
MISSX0=0.0
MISSY0=0.0
VM = 2500.0
AM0 = 0.0
READ (2,10) VT, AT, THDG, TGTXO, TGTYO
10 FORMAT (F6.1, 2X, F5.1, 2X, F6.1, 2X, F10.2)
TGTV=VT*K2F
TGTA=-AT*K2F
THDG=THDG*D2R
TGTVXO=TGTV*SIN(THDG)
TGTVYO=TGTV*COS(THDG)

C INITIALIZE THE RANGE PHI MATRIX
RPHI(1,1) = 1
RPHI(1,2) = TK
RPHI(1,3) = .5*TK*TK
RPHI(2,1) = 0
RPHI(2,2) = 1
RPHI(2,3) = TK
RPHI(3,1) = 0
RPHI(3,2) = 0
RPHI(3,3) = 1

C INITIALIZE THE BEARING PHI MATRIX
SPHI(1,1) = 1
SPHI(1,2) = TK
SPHI(1,3) = .5*TK*TK
SPHI(2,1) = 0
SPHI(2,2) = 1
SPHI(2,3) = TK
SPHI(3,1) = 0
SPHI(3,2) = 0
SPHI(3,3) = 1

C CONSTANT STEADY STATE GAIN VALUES RANGE
GR(1,1) = .5
GR(1,2) = 0.0125
GR(1,3) = 0
GR(2,1) = 0.0025
GR(2,2) = 1.0
GR(2,3) = 0
GR(3,1) = 0.1250
GR(3,2) = 24.9991
GR(3,3) = 0

C CONSTANT STEADY STATE GAIN VALUES BEARING
GS(1) = 1.5
GS(2) = 12.5
GS(3) = 1250.0

C INITIALIZATION OF PROP NAV MISSILE CONSTANT VELOCITY,
ZERO ACCEL
LOS = ATAN2(TGTV0 - MISSY0, TGTX0 - MISSX0)
R= ( (TGTX0 - MISSX0)**2 + (TGTV0 - MISSY0)**2 )**.5
TTGO= R/VM

125
PHDG = ATAN2(TGY0 + TGTVY0 * TTG0 - MISSYO, TGTX0 + TGTVX0 * TTG0 - MISSX0)

VMX0 = VM*COS(PHDG)
VMY0 = VM*SIN(PHDG)
RK1 = R
RDKP1 = -VM*COS(PHDG - LOS) + TGTV*SIN(TGTHDG)/COS(LOS)

RDDKP1 = 0
SKP1 = LOS
SDKP1 = (TGTVYO/COS(LOS) - VM*SIN(PHDG - LOS))/R
SDDKP1 = 0
B0 = LOS
BD0 = 0

RNG(1) = RK1
RNG(2) = RDDKP1
S(1) = SKP1
S(2) = SDKP1
S(3) = SDDKP1

INITIALIZATION OF SECOND ORDER PROP NAV REFERENCE MODEL
GO = LOS
GD0 = 0

TARGET POSITION UPDATING
TGTHDG = ATAN2(TVELX, TVELY)
TGTA = TGTA*COS(TGTHDG)
TGTA = (-1*TGTA) * SIN(TGTHDG)
TVELX = INTGRL(TGTVX0, TGTA)
TVELY = INTGRL(TGTVYO, TGTA)
XT = INTGRL(TGTX0, TVELX)
YT = INTGRL(TGY0, TVELY)

THIRD ORDER PROP NAV MISSILE POSITION UPDATING
BDOT = INTGRL(B, BDOT)
B = INTGRL(B0, BDOT)
AM = U
MVELX = INTGRL(VMX0, -AM*SIN(PNHDG))
MVELY = INTGRL(VMY0, AM*COS(PNHDG))
PNHDG = ATAN2(MVELY, MVELX)
PXM = INTGRL(MISSX0, MVELX)
PYM = INTGRL(MISSY0, MVELY)

SECOND ORDER PROP NAV MISSILE
GDDOT = -20*GDOT + (SOLOS-GAMMA)*100
GDOT = INTGRL(GD0, GDDOT)
GAMMA = INTGRL(G0, GDOT)
SOHDG = INTGRL(PHDG, 4*GDOT)
SOXM = INTGRL(MISSX0, VM*COS(SOHDG))
SOYM = INTGRL(MISSY0, VM*SIN(SOHDG))

DYNAMIC
THIRD ORDER PROP NAV MISSILE GEOMETRY UPDATE
RM = ((XT-PXM)**2 + (YT-PYM)**2)**.5
LOS = ATAN2(YT-PYM, XT-PXM)
RDOTM = TGTV*SIN(TGTHDG)/COS(LOS) - VM*COS(PNHDG - LOS)

COMPUTE THE ERROR TERMS
DELR(1) = RM - RK1
DELR(2) = RDOTM - RDDKP1
SDEL = LOS - SKP1

126
CALL HALMAN(RNG, RM, RDOTM, RDDOTM, RK, RDK, RDDK, RKP1, 
+RDKP1, RDDKPI, DELR, S, LOS, LOSD, LOSD, SK, SDK, 
+SDDK, TIME, SKP1, SDKP1, SDDKP1, SDEL, GS, GR, RPHI, SPHI)
C
BDDOT=SDDKP1+10*(SDK-BDOT)+33.33333*(LOS-B)
U = -4*BDOT*RDOTM
C
SECOND ORDER PROG NAV MISSILE GEOMTR. UPDATE
SOR = ((XT-SOXM)**2 + (YT-SOYM)**2)**.5
SOLOS = ATAN2(YT-SOYM,XT-SOXM)
SOU = -4*GDOT*SORD
C
IF (PM GT. XT) THEN
CALL ENDRUN
ENDIF
C
STATEMENTS TO SAVE DATA FOR PLOTTING WITH DISSPLA
IF (K LE. 0) THEN
WRITE (39,20) PM, PYM, PM, PYM
WRITE (47,30) TIME, LOS, U
WRITE (48,30) TIME, SOLOS, SOU
K=10
ENDIF
KENN=NN+1
ENDIF
K=K-1
20 FORMAT (4(2X,FI0.2))
30 FORMAT (F5.2,2X,Ell.3,2X,E14.6)
SAMPLE
C
STATEMENTS TO SAVE DATA FOR GRAFAEL
C
SAVE (A) 0.1, XT, YT, PM, PM
SAVE (B) 0.1, LOS, B, SK
SAVE (C) 0.1, LOSD, BDOT, SDK
SAVE (D) 0.1, SDEL
SAVE (E) 0.1, RDK, RDKP1, RDDK, RDDKPI
SAVE (G) 0.1, GS1, GS2, GS3
SAVE (H) 0.1, GR11, GR12, GR21, GR22
0.1, RM, RKP1, RK, RDOTM, RDDKPI, RDDK, RDDOTM, RDDK, RDDKPI, RDDK
0.1, FINTIM, LOS, B, SK, LOSD, BDOT, SDK, LOSD, BDDOT, SDDK
CONTROL FINTIM=10,0, DELL=.01
C
TERMINAL
READ (1, 40) NN
WRITE (1, 40) NN
40 FORMAT (F6.1)
C
STATEMENTS FOR PLOTTING USING GRAFAEL
C
GRAPH (A/A, DE=TEK618) XT
(SC=1600, LO=0.0), YT(SC=500, PO=16000)
C
GRAPH (A/A, OV) PM (SC=1600, AX=OMIT), PYM(SC=500)
C
GRAPH (B/B, DE=TEK618) TIME, LOS (SC=.025, LO=-.1)
C
GRAPH (B/B, OV) TIME (AX=OMIT), B(P0=7.5, SC=.025, LO=-.1)
C
GRAPH (B/B, OV) TIME (AX=OMIT), SDK (AX=OMIT, SC=.025, LO=-.1)
C
GRAPH (C/C, DE=TEK618) TIME, LOS
C
GRAPH (C/C, OV) TIME (AX=OMIT), BDOT(P0=7.5)
C
GRAPH (C/C, OV) TIME (AX=OMIT), SDK(AX=OMIT)
C
GRAPH (D/D, DE=TEK618) TIME, SDEL
C
GRAPH (E/E, DE=TEK618) TIME, RM(SC=2000.0, LO=0.0)
C
GRAPH (E/E, OV) TIME (AX=OMIT), RK(SC=2000.0, LO=0.0)
C
GRAPH (F/F, DE=TEK618) TIME, RDDOTM(SC=500.0)
C
GRAPH (F/F, OV) TIME (AX=OMIT), RDDK(AX=OMIT)
C
GRAPH (G/F, DE=TEK618) TIME, U
C
GRAPH (H/G, DE=TEK618) TIME, GS1
C
GRAPH (I/G, DE=TEK618) TIME, GS2
C
GRAPH (J/G, DE=TEK618) TIME, GS3
C
GRAPH (K/H, DE=TEK618) TIME, GR11

127
FORTRAN

SUBROUTINE KALMAN(RNG,RM,RDOTM,RDDOTM,RK,RDK,RDDK,RKP1,RDKP1,
RDDKP1,DELR,S,LOS,LOSDD,SK,SDK,SDKDD,TIME,SKP1,SDKP1,
* SDDKP1,SDEL,GS,GR,RPHI,SPHI)
C
SUBROUTINE TO ITERATE A KALMAN FILTER FOR RANGE
VARIABLES
C GIVEN THE COVARIANCE MATRIX, OBSERVATIONS
REAL*8 RNG(3),RM,RDOTM,RDDOTM,RK,RDK,RDDK,RKP1,RDKP1,RDDKP1,
* DELR(2),S(3),LOS,LOSDD,RPHI(3,3),A,B,C,D,TEMPI(3),SPHI(3,3),
* GR(3,3),GS(3)
C Find the new values of RPHI from the previous values
C of sigma
C Matrix
DO 110 N = 1,3
   TEMP1(N) = 0.0
   CONTINUE
C Constant gain inputs
DO 121 L = 1,3
   DO 120 M = 1,2
      TEMP1(L) = TEMP1(L) + GR(L,M) * DELR(M)
   120 CONTINUE
   CONTINUE
DO 125 N = 1,3
   RNG(N) = TEMP1(N) + RNG(N)
   CONTINUE
C Save the values of range matrix at step K
K-' = RNG(1)
R:' = RNG(2)
P:'DK = RNG(3)
C Zero the old range temporary matrix
C Find the estimate of the step K+1 for the range matrix
DO 131 L = 1,3
   DO 130 M = 1,3
      TEMP1(L) = TEMP1(L) + RPHI(L,M) * RNG(M)
   130 CONTINUE
   CONTINUE
DO 132 N = 1,3
   RNG(N) = TEMP1(N)
132 CONTINUE
C RKP1 = RNG(1)
K:'KP1 = RNG(2)
P:'DKP1 = RNG(3)
C
SUBROUTINE TO ITERATE A KALMAN FILTER FOR SIGMA
VARIABLES
C Given the covariance matrix, observation.
C Where H= (1 0 0)
C Now find the current values of the sigma matrix
S(1) = S(1) + GS(1)*SDEL
S(2) = S(2) + GS(2)*SDEL
S(3) = S(3) + GS(3)*SDEL
C
128
C STORE THE SIGMA MATRIX FOR USE IN THE PROGRAM
SK = S(1)
SDK = S(2)
SDDK = S(3)

FIND THE NEXT VALUES OF THE SIGMA MATRIX
S(K+1) = SPHI * S(K)

ZERO A TEMPORARY MATRIX
DO 140 L=1,3
  TEMP1(L) = 0
140 CONTINUE
DO 142 L=1,3
  DO 141 M=1,3
    TEMP1(L) = TEMP1(L) + SPHI(L,M) * S(M)
 141 CONTINUE
142 CONTINUE

INPUT BACK INTO SIGMA MATRIX
DO 143 N = 1,3
  S(N) = TEMP1(N)
143 CONTINUE

STORE THE VALUE OF THE S MATRIX
SKPI = S(1)
SDKI = S(2)
SDDKI = S(3)

RETURN
END
SUBROUTINE ZERO(A,N)
C CLEAR A TEMPORARY MATRIX
C REAL*6 A(3,3)
  DO 201 L=1,N
    DO 200 M=1,N
      A(L,M) = 0
  200 CONTINUE
201 CONTINUE
RETURN
END
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