**Title:** Nonlinear Waves: Coherence, Chaos, Pattern Formation, and Geometry

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**Supplementary Note:** The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

**Abstract:**

Coherence and chaos in partial differential equations was studied, with particular emphasis on (1) the damped-driven sine Gordon equation and (2) an optically bistable laser cavity. In addition, the propagation of rapidly oscillating nonlinear integrable waves was investigated.

The principal results about propagation in an optically bistable ring cavity may be summarized as the identification of the interplay between coherent transverse spatial structures and temporal chaos in the characteristics of the laser beam.
Principal mathematical results on the damped-driven sine-Gordon equation include (1) a numerical study of low dimensional chaotic attractors with coherent spatial structures, including dynamical system diagnostics of their time series, and direct numerical measurements establishing that the attractor is well co-ordinatized by a few nonlinear normal modes; (2) complete analytical identification of all homoclinic structures for the integrable sine-Gordon equation; (3) direct numerical detection of homoclinic crossings along the chaotic attractor of the full system.

Principal mathematical results about the propagation of rapidly oscillating integrable waves include (1) the identification and derivation of a Hamiltonian structure for the modulation equations and (2) a study of the process by which singularities are smoothed by dispersion through the injection of additional degrees of freedom into the field.
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Statement of the Problems studied

1) The structure of integrable and nonintegrable dynamical systems in the complex domain was investigated. The object is to relate singularities of solutions in the complex domain to the behavior of solutions for real time.

2) The Poisson geometry associated with various dynamical systems was studied. One wants to understand the connection between integrability and the existence of multiple Poisson structures.

3) Work continued on the analysis of the phase-space geometry of a particular dynamical system, the Neumann system. The goal is to understand the symmetries that underlie integrability.

These are all long-term projects. The research is continuing (with students and collaborators) and while the results have been presented in lectures, the final papers are not yet written.

Results

1) a) The Painlevé analysis for Nahm's equations was carried out. This is a rather intricate system arising in the theory of monopoles. The computation relies on representation theory of \( \mathfrak{sl}(2, \mathbb{C}) \). (To be published)

b) In collaboration with Y. Zeng, the Painlevé analysis for generalized integrable Toda lattices was carried out. The Painlevé data are given Lie-algebraic interpretation. (This work will be completed in China in May 1988.)

c) A student, M. Zou, has used complex domain techniques to prove nonexistence of analytic integrals of perturbed Toda lattices. He has also found various new examples relevant to Painlevé analysis: nonintegrable systems with the "weak Painlevé" property, integrable systems with polynomial Hamiltonian but nonalgebraic first integral, etc. (This work will form part of his Ph.D. thesis, expected by Spring 1989.)

d) a) A student, P. Damianou, has constructed an infinite set of Poisson structures on the Toda lattice phase space. This is a new example, which does not fit the hypotheses used in other work on bi-Hamiltonian structures. (A paper is being written.)

b) In collaboration with T. Ratiu, it was shown that the simultaneous resolution of simple singularities is a momentum map. It turned out that this result was known in representation theory; a
detailed study of the Poisson-geometric implications (not known) is to be part of Damianou's thesis.

3) In collaboration with N. Ercolani, the Neumann system was related to the geometry of Kummer varieties. This was published. Further connections between this geometry and the formalism of Hirota equations are still under investigation.

Publications and papers in progress


H. Flaschka, "Remarks on integrable Hamiltonian systems," submitted for publication.


Also, Ph.D. theses of Pantelis Damianou and Maorong Zou, in progress.

Scientific personnel

N. M. Ercolani, Assoc. Prof., Dept. of Mathematics, University of Arizona;

Y. B. Zeng, Assoc. Prof., Dept. of Mathematics, University of Science and Technology of China (visiting scholar at Univ. of Arizona, 1985-1987);

P. Damianou Ph.D. thesis student, Dept. of Mathematics, University of Arizona (partial support on this contract);

M. R. Zou Ph.D. thesis student, Dept. of Mathematics University of Arizona (partial support on this contract);

H. Nadelhoffer First-year graduate student, Dept. of Mathematics University of Arizona (partial support on this contract).
Statement of the Problem Studied.

Coherence and chaos in partial differential equations was studied, with particular emphasis on (1) the damped-driven sine Gordon equation and (2) an optically bistable laser cavity. In addition, the propagation of rapidly oscillating nonlinear integrable waves was investigated.

The principal results about propagation in an optically bistable ring cavity may be summarized as the identification of the interplay between coherent transverse spatial structures and temporal chaos in the characteristics of the laser beam. We were the first to study transverse effects in an optically bistable laser cavity with the natural nonlinear evolution equation of the system. Solitary wave profiles were shown to be the fixed points of an infinite dimensional map which describes the system; these solitary waves form the basis of a projection method which reduced the infinite dimensional map to a two dimensional one, from which the physical characteristics of the fixed points were deduced with extreme accuracy; two competing instabilities (propagational through the nonlinear cavity versus feedback) were identified and analyzed; a chaotic response was identified and its features connected to these instabilities. The above study was carried out in one transverse dimension. At the end of the grant period, a two dimensional study was initiated which is still in progress.


Principal mathematical results on the damped-driven sine-Gordon equation include (1) a numerical study of low dimensional chaotic attractors with coherent spatial structures, including dynamical system diagnostics of their time series, and direct numerical measurements establishing that the attractor is well co-ordinatized by a few nonlinear normal modes; (2) complete analytical identification of all homoclinic structures for the integrable sine-Gordon equation; (3) direct numerical detection of homoclinic crossings along the chaotic attractor of the full system. This study is a first in two respects: the first complete mathematical classification of all homoclinic structures for an integrable pde and the first direct numerical correlation of these objects with the chaotic attractors of the perturbed pde. At present we are using the mathematical expressions for these homoclinic structures in analytical (as opposed to
numerical) studies. References [2, 4, 5, 6, 8, 9, 10, 11, 12, 16, 18, 19, 22].

Principal mathematical results about the propagation of rapidly oscillating integrable waves which were obtained during the duration of this grant include (1) the identification and derivation of a Hamiltonian structure for the modulation equations and (2) a study of the process by which singularities are smoothed by dispersion through the injection of additional degrees of freedom into the field. Current work is in progress with N. Ercolani and D. Levermore. [References 7, 14, 15, 18, 20, 21, Ph.D. thesis, Jin Shan].
References


CONFERENCE PROCEEDINGS


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PAPERS FROM THE PERIOD 1984-1987 FOR ARMY GRANT

The main contributions during this period were:

i. A series of papers developing a method for obtaining macroscopic equations for describing the dynamics of patterns. # 1, 7, 12, 13, 19, 20, 21

ii. A series of papers on nonlinear optics. # 2, 4, 5, 8, 9, 18, 22, 23, 24

iii. Some new ideas on Turbulent transport. # 6

9. Two-dimensional spatial patterns in ring cavities (with D. McLaughlin and J. Moloney), to be submitted.


Curriculum Vitae
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PERSONAL DATA: Born in Council Bluffs, Iowa, October 11, 1944

EDUCATION:
1966 B.S. (Physics and Mathematics), Creighton University
1969 M.S. (Theoretical Physics), Indiana University
1971 Ph.D. (Theoretical Physics), Indiana University

EMPLOYMENT:
1966-67 (Summers) Physicist, Naval Ord. Lab., Silver Spring, MD
1970-72 Assistant Professor, Mathematics, New York University
1972-74 Assistant Professor, Mathematics, Iowa State University
1974-79 Associate Professor, Mathematics, University of Arizona
1979-present Professor, Mathematics, University of Arizona
1978-present Consultant, Los Alamos National Lab, Los Alamos, NM
1980-82 Consultant, Exxon Research Corp., Linden, NJ
1980-82 Visiting Member, Courant Institute, New York University
1986-present Chairman, Program in Applied Mathematics, University of Arizona

SPECIALIZATION:
Nonlinear wave equations and mathematical physics

AWARDS, DISTINCTION, AND HONORS:
1966-70 NDEA Title IV Fellow
1969-70 NSF National Fellowship
1976 Lester Ford Award by American Mathematics Association for the article "The Feynman Integral" (with J. B. Keller)
1978 "The Soliton - A New Concept in Applied Science" (with A. C. Scott and F. Y. F. Chu). Acknowledged by Citation Index as a "Citation Classic"
1978 Member, US-Japan Scientific Exchange
1979 Member, US-USSR Academy Exchange on Solitons

GRANTS:
1972-present National Science Foundation, Mathematics
1975- National Science Foundation, Conf. Solitons
1977-78 National Science Foundation (Japan Prog.)
1978-79 Army Research Office
1983-84 National Science Foundation, Conf. Inverse Methods
1983-present Air Force Office of Scientific Research
1985-present Office of Naval Research, Engineering
1985-present Army Research Office.
SERVICE:

1977-present Member, Applied Mathematics Program, University of Arizona.
1978-80     Member, Organizational Committee, Center for Nonlinear Studies, Los Alamos.
1984        Member, Organization Committee, Nonlinear Equations Conference, Santa Fe.

THESIS AND DISSERTATION COMMITTEES:

1975-78     E. A. Overman, Ph.D. (Co-direction with F. A. Hopf)
1975-79     M. G. Forest, Ph.D.
1984-        H. Adachihiara (Co-direction with A. C. Newell)
PUBLICATIONS


32. "Homoclinic Orbits for the Periodic Sine Gordon Equation" (with N. Ercolani and M. G. Forest), to appear in Physica D.


CONFERENCE PROCEEDINGS


OTHER PUBLICATIONS


INVITED LECTURES

1982 International Conference on Soliton Perturbation Theory, Nice.
1982 International Conference on Structure and Dynamics of Proteins, La Jolla.
1983 Workshop on Coherence and Chaos, Los Alamos.
1983 International Conference on Nonlinear Biophysics, Loma Linda.
1983 Analysis Colloquium, Duke University.
1984 Western States Mathematical Physics Meeting, Cal. Tech.
1984 Lectured at Bucharest Institute for Physics, Romania.
1984 Conference on Raman Scattering, Los Alamos.
1984 Applied Mathematics Colloquium, Stanford University.
1984 AMS-SIAM Conference on Nonlinear Evolution Eq's, Santa Fe.
1984 Mathematics Colloquium, Penn State University.
1984 Inaugural Conference of Nonlinear Research, Berkeley.
1985 Conference on Oscillation Theory, Mathematics Institute, Minnesota.
1986 Seminar, College of France.
1986 Lecture on Dynamical Systems at INRIA Workshop, Paris.
1986 Mathematics Seminar, Universite de Paris, XIII.
1986 Seminar, Observatoire de Nice.
1986 Physics Seminar, University of Montpellier.
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1986 Lecture at Inaugural MIDIT Workshop, Lyngby, Denmark.
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