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SOLAR ENERGY ABSORPTION EFFICIENCY OF AN ELLIPSOIDAL RECEIVER-REACTOR WITH SPECULARLY REFLECTING WALLS

by

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SOLAR ENERGY ABSORPTION EFFICIENCY OF AN ELLIPSOIDAL RECEIVER-REACTOR WITH SPECULARLY REFLECTING WALLS

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Abstract—An ellipsoidal cavity-receiver with specularly reflecting inner walls, in which the reactor component is positioned at one focal point and the aperture at the other, may be useful in solar applications. Most of the incident radiation from a solar concentrator should reach the reactor directly or after one reflection from the cavity walls. Because the source (aperture) and sink (reactor) have finite areas, the ellipsoidal reflector no longer conveys all of the entering radiation into the reactor; some radiation entering the cavity does not reach the target after one reflection and is eventually absorbed by the cavity walls after multiple reflections or escapes through the aperture. We have examined the conditions for which this radiation loss becomes significant and have estimated the effects on the energy-collection efficiency of the system.

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Introduction

In a previous paper, we examined a new kind of receiver-reactor for high-temperature solar furnaces. The main body of the receiver component is an ellipsoid of revolution with specularly reflecting inner walls. The reactor component, a crucible, is centered at one focal point and the aperture at the other. With this arrangement, one might intuitively expect that incoming radiation from the concentrator should reach the reactor directly or after one reflection from the cavity walls because rays which pass through one focal point of an ellipse must, after specular reflection, pass through the other. We then presented an analysis in which we assumed that all of the incident radiation that is reflected from the cavity walls arrives at the crucible. However, as we pointed out, in some situations, depending on the eccentricity of the ellipsoid, dimensions of the crucible and aperture, and geometry and rim angle of the concentrator, some radiation will miss the crucible. In the previous analysis, we neglected this loss in order to simplify the solution. In this paper, we examine the magnitude of the effect of the omission. We estimate that portion of the radiation that misses the crucible after one reflection from the cavity walls. Assuming that such radiation is ultimately lost, we calculate the energy-absorption efficiency of the system and the maximum crucible temperature it is capable of achieving. The result is compared with a previous result to show the magnitude of the effect.
A schematic diagram of the system components is shown in Fig. 1. The receiver is an ellipsoid-of-revolution with specularly reflecting inner walls. Its major axis length is $2a$, its minor axis length is $2b$, and the distance between its focal points $F_1$ and $F_e$ is $2c$. Its eccentricity is $c/a$. The crucible is a sphere of radius $r_c$, with its center on $F_1$. The aperture is a circle of radius $r_a$, with its center on $F_e$. It lies in a plane perpendicular to the major axis. The receiver-reactor system is coaxial with the solar concentrator-mirror array whose rim angle is $\phi_{rim}$. The focus $F_e$ is at the focal point of the concentrator. The size of the aperture is chosen to be equal to that of the image of the solar concentrator on the focal plane so that virtually all of the incoming energy from the concentrator enters the cavity. In practice the radius of the aperture would therefore be determined by the characteristics of the concentrator.

We assume that the power per unit area coming from the concentrator through the aperture is of uniform intensity over the entire aperture. By this we mean that the power flux is the same from every point in the aperture. The radiation is confined to a cone whose apex angle is the rim angle $\phi_{rim}$ of the concentrator, and within this cone the power flux entering the receiver is the same in every direction.

Our objective is to find the portion of radiation entering the cavity that: 1. strikes directly on the crucible, i.e. the view factor from the aperture to the crucible, $V_{a,r_c}$; 2. strikes the
crucible after one reflection on the cavity walls, i.e. the view factor from the aperture to the crucible after reflection from the ellipsoid, $V_{\text{indirect}}$. 3. misses the crucible after one reflection on the cavity walls, $V_{\text{miss}}$. Obviously,

$$V_{\text{direct}} + V_{\text{indirect}} + V_{\text{miss}} = 1.$$ 

**Analysis**

To solve this problem we used a Monte-Carlo ray-trace simulation. We followed the paths of a large number of rays which, within the constraints of the circumscribing cone, have a random angular distribution, counting the numbers of those that hit the crucible directly, hit it after one reflection, or miss it.

With Cartesian coordinates centered at the center of the ellipsoid, the equation of the ellipsoid is

$$F(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0. \quad (1)$$

Our incident ray passes through the aperture at a point $A(c, y_0, z_0)$ and has a direction parallel to the unit vector $u = u_i \mathbf{i} + u_j \mathbf{j} + u_k \mathbf{k}$. Since this ray comes from the concentrator,

$$|u| < \cos(\phi, \mathbf{m}). \quad (2)$$
The equation that gives the coordinates of a generic point \( P \), on the incident ray is, in vectorial notation,

\[
(P, -A) \times u = 0
\]

(3)

with

\[ |u| = 1. \]

(4)

Let \( E(x_1, y_1, z_1) \) be the point of intersection of the incident ray with the ellipsoid of revolution that gives \( x_1 < c \). The equation of the normal to the surface at point \( E \) is given by

\[
(P, -E) \times n = 0,
\]

(5)

where

\[ n = \nabla F. \]

(6)

The equation for the reflected ray with a direction parallel to unit vector \( r \) is

\[
(P, -E) \times r = 0
\]

(7)

with

\[ |r| = 1. \]

(8)

Inasmuch as the angle of incidence equals the angle of reflection and the rays are coplanar with the normal to the surface,

\[ u \times n = n \times r. \]

(9)
Equations (8) and (9) can be solved for \( r \). The distance \( \delta_1 \), between the reflected ray and the point \( F_1 \), is given by

\[
\delta_1 = |(F_1 - E) \times r|.
\]  

(10)

If \( \delta_1 < r_1 \), the ray hits the crucible; if \( \delta_1 > r_1 \), it misses it.

There is also the possibility of a direct strike on the crucible by the incident ray. The distance \( \delta_{\infty} \), between the incident ray and the point \( F_1 \), is

\[
\delta_{\infty} = |(F_1 - A) \times u|.
\]  

(11)

When \( \delta_{\infty} < r_1 \), the ray hits the crucible directly.

**Results**

Using a Monte-Carlo simulation with a sample of 50,000 rays, we have counted the number of direct hits, hits after a single reflection, and misses. The results are presented in Figs. 2 and 3. Figure 2 shows the variation of the view factors \( V_{\infty, \infty} \), \( V_{\infty, 1} \), \( V_{1, \infty} \) and \( V_{1, 1} \) as a function of the crucible radius. Figure 3 shows \( V_{\infty, \infty} \) as a function of the parameters \( r_1, r_2 \) and the eccentricity.

With a particular receiver, there is a crucible radius above which \( V_{\infty, \infty} = 0 \); all of the radiation will be captured directly or after a single reflection by a crucible whose radius is larger than
this radius. If the crucible is smaller, some of the radiation entering the cavity will never reach the crucible. It will eventually be absorbed by the cavity walls or escape through the aperture. Thus, although we may be able to obtain higher temperatures on small crucibles, their energy absorption efficiency, the fraction of the incident radiation which is usable as process heat in the crucible, will be somewhat lower. If the crucible is larger, all the energy that enters the receiver reaches the crucible directly or after one reflection. But the maximum temperature the crucible is capable of achieving will be lower because the surface from which the crucible is reradiating is larger. It is also apparent that the smaller the aperture the smaller is the radius for which $V_{\text{miss}}$ becomes 0. Because we have made the aperture size equal to that of the image of the concentrator at the focal plane, a smaller aperture implies a higher quality concentrator.

Energy-absorption efficiencies were calculated using a previously described method.¹ Their variations with the crucible temperature are shown in Fig. 4 for various crucible radii. The energy input to the cavity is 6590 W, the concentrator rim angle 45°, the aperture radius 5 cm, the ellipsoid semi-major axis length 25 cm, the eccentricity 0.6, the emissivity of the crucible 0.9, and the reflectivity of the cavity walls 0.9. The dashed lines refer to the optimistic estimation of our previous analysis. The full lines were obtained using the view factors from Fig. 2 assuming that: 1. The image of the solar concentrator at the focal plane is the size of the aperture. 2. Once an incident ray misses the crucible after one reflection, it never reaches it. We observe that for $r < 3\text{cm}$ the portion of
radiation that misses the crucible is no longer negligible and should be taken into account to give a realistic estimation of the thermal efficiency. For r,>3cm dotted lines and full lines coincide and no correction is necessary.

For a given crucible temperature, there is an optimum crucible size for which its energy absorption efficiency is maximum. This optimum size can be found with the help of Fig. 5. The efficiency is plotted as a function of the crucible radius for various crucible temperatures. As we go to higher temperatures, the optimum crucible size becomes smaller and the energy-absorption efficiency is lower. For example, at 1000K, the optimum crucible radius is approximately 3.2cm, which gives a maximum efficiency of about 0.71. At 1750K, the optimum crucible radius is 1.65cm and the maximum efficiency goes down to 0.33.

In the previous analysis, we found that the thermal performance of the reactor is not very sensitive to aperture size. This statement still holds as long as the aperture is big enough to intercept all the rays coming from the concentrator.

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References


FIGURE CAPTIONS

Fig. 1. Schematic diagram of the system components. The receiver is an ellipsoid-of-revolution with specularly reflecting inner walls. The crucible is a sphere centered on one focal point. The aperture is a circle centered on the other focal point. The receiver-reactor system is coaxial with the solar concentrator.

Fig. 2. Variation of the view factors with crucible radius. The view factors are defined as the portion of radiation entering the cavity that is subjected to all of the following effects: 1. strikes directly on the crucible, \( V_{1,\text{direct}} \); 2. strikes the crucible after one reflection from the cavity walls, \( V_{1,\text{reflect}} \); 3. misses the crucible after one reflection from the cavity walls, \( V_{1,\text{miss}} \). The semi-major axis length of the ellipsoid is 25 cm. The eccentricity is 0.6. The concentrator rim angle is 45°. The aperture radius is 5 cm.

Fig. 3. \( V_{1,\text{miss}} \), the portion of radiation that misses the crucible after one reflection from the cavity walls, is shown as a function of the crucible radius for various aperture radii and ellipsoid eccentricities. The semi-major axis length of the ellipsoid is 25 cm and the concentrator rim angle is 45°.

Fig. 4. The energy-absorption efficiency is shown as a function of the crucible temperature for various crucible radii. The energy input to the cavity is 6590 W. The concentrator rim angle is 45°. The aperture radius is 5 cm. The semi-major axis length of the ellipsoid is 25 cm. The eccentricity is 0.6. The emissivity of the crucible is...
0.9. The reflectivity of the cavity walls is 0.9. The dashed lines refer to our previous analysis, and the full lines were obtained by using the view factors from Fig. 2.

Fig. 5. The energy-absorption efficiency is shown as a function of the crucible radius for various crucible temperatures. The same parameters apply as in Fig. 4.
FIGURE 1
Figure 2

View Factors

\( V_{\text{miss}} \) \hspace{1cm} \( V_{\text{indirect}} \) \hspace{1cm} \( V_{\text{direct}} \)

radius of crucible [cm]

- \( a = 25 \text{ cm} \)
- \( c = 15 \text{ cm} \)
- \( \Phi_{\text{rim}} = 45 \)
- \( r_2 = 5 \text{ cm} \)
1. Crucible radius [cm] = 0.6

Crucible Temperature [K]

Efficiency

Crucible Temperature [K]

1. CRUG 4

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FIGURE 4
crucible radius [cm]

efficiency

- 750 K
- 1000 K
- 1250 K
- 1500 K
- 1750 K

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