Parameterization of Mottle Textures

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Abstract

Parameterization of textures can be useful for detection of textual similarities and matching. In this project we have developed a stochastic model to generate a set of parameters from the texture image domain and frequency domain. This model is aimed at quantification of textures for detection of similarities and differences. Our attention has been concentrated on the parameterization of mottled textures.

To test the model we have used it to generate texture images back from the parameters obtained from image analysis. The similarities and differences between the generated image and original images are used to refine and test the parametric model.

This work was supported in part by U.S. Army Engineering Topographic Laboratories research contract No. DACA76-85-C-0001, in part by an Eastman Kodak Company Grant/Fellowship and in part by the Air Force Systems Command, Rome Air Development Center, Griffiss Air Force base, New York 13441-5700, and the Air Force Office of Scientific Research, Bolling AFB, DC 20332 under Contract No.F30602-85-C-0008. This contract supports the Northeast Artificial Intelligence Consortium.

We thank the Xerox Corporation University Grants Program for providing equipment used in the preparation of this paper.
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**Source of Funding:**
DACA76-85-C-0001

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Parameterization of textures can be useful for detection of textual similarities and matching. In this project we have developed a stochastic model to generate a set of parameters from the texture image domain and frequency domain. This model is aimed at quantification of textures for detection of similarities and differences. Our attention has been concentrated on the parameterization of mottled textures. To test the model we have used it to generate texture images back from the parameters obtained from image analysis. The similarities and differences between the generated image and original.
20. ABSTRACT (Continued)

Images are used to refine and test the parametric model.
1 Introduction

The analysis of texture is often an important step in classifying and analyzing image fields. Yet texture measurement remains a highly nebulous subject. Many measures have been proposed, but their effectiveness is questionable because the structural characteristics of textural fields are still ill defined [8]. In a survey of the subject Haralick contended that "despite its importance and ubiquity in image data, a formal approach or precise definition of texture does not exist" [4]. Over the past few years various techniques have been developed for the analysis of textures. These techniques are basically of two types -

1) Structural methods

Textures can be described in terms of a set of primitives and placement rules. This point of view was first expounded by Rosenfield and Lipkin [10]. This sort of framework has similarity to language where the primitives can be seen as being symbols with the placement rule as the grammar for the language. The textures that can be described easily using this kind of framework have well defined primitives with obvious placement rules. For example in a chessboard the white and black squares can be viewed as the primitives with replacement rules to replace a white (black) square by a white (black) square with black (white) adjacent to it. The rules also ensure that the replacement rules can only terminate if the board is a square.

2) Statistical methods

Statistical methods though preferable would usually be hard to obtain from a given image unless the broad characteristics of the image is known beforehand. The various methods of statistical analysis of images studied include the consideration of auto-correlation, Markov processes and co-occurrence statistics [8]. The statistical measures are specially attractive due to the ease with which they can be obtained from the image. The disadvantage of the statistical methods is that in practice different textures may give the same set of parameters. If enough parameters are used so that different images give different sets of parameters then the amount of parameters required can be huge. However the statistical methods are useful if only the broad characteristics of the image is desired. Moreover if the range of images on which the parameterization is to be done is known and we are to disambiguate among them then the parameters can be selected to suit the application.

2 Parameters

It is convenient to parameterize an image in some way so that similarities in images can be detected. The way the image characteristics vary over some domain can be a useful criterion for detecting similarities. With a constrained domain it may be possible to parameterize a texture model in a useful way. In our case the goal is to quantify the informal notion of the mottle severity and to do it we employ a statistical model.

2.1 Frequency Domain
The frequency domain parameters are useful in expressing the spatial regularities in the image. The radial density in the frequency domain indicates the size distribution of image components (for example the size of the square in the checkerboard). Many textures show preferential density at only a few frequencies depending on the number of basic texel types in the image. The angular density indicates the orientation of the components of the image. If the texels of the image follow certain specific placement rules then the orientation of the image determines the angular density. On the other hand if the orientation of the texels are not at all constrained by the placement of the neighboring texels then the angular density is more or less uniform throughout the angular domain. The orientation distribution may characterize the image to certain extent. For example in a chess board image the angular density is concentrated in two perpendicular directions whereas an image of randomly oriented tiles shows uniform angular density.

2.2 Spatial Domain

Spatial domain parameters indicate the gray level distribution itself. For the textures that do not show selective frequency or angular preference the image domain parameters indicate how the image characteristics are distributed. The various statistical image domain parameters that can be considered useful include the mean, variance, skewness and kurtosis of the pixel gray level values. The parameters thus obtained are likely to be the same for similar images and different for dissimilar images.

2.3 Reconstruction

For textures that show regularity or controlled randomness in the frequency domain it may be possible to reconstruct the main characteristics of the images using the frequency domain parameters. Our model for reconstruction assumes such characteristics. The concentration has been on the frequency domain parameters because mottle is a large-scale phenomenon whose statistical characterization in the spatial domain seems impractical [5].

For regeneration of images from the frequency domain parameters we distribute a fixed number of points in the frequency domain using the parameters of radial and angular density obtained from the Fourier analysis of the image. This is done so that the frequency and angular properties of the original image are maintained in the generated image. This frequency domain reconstruction is used to generate the image back by using the inverse Fourier transform. In all that follows, "reconstruction" actually means generation of a texture that should be similar to the original according to the relevant quantitative and psychophysical criteria. The goal is to produce textures that are different instances of what a viewer would call "the same" texture.

3 A Mottle Model

Mottled textures arise in both industrial and in natural environments. An industrial mottle sample is shown below (figure 1) [3]. In appearance, mottle is coarsely irregular or blotchy, without sharp intensity changes. Neither pixel statistics nor structural texture description methods seem well adapted to characterizing mottled textures [4,8]. Fractal dimension was considered, but would be a better model for turbulence. Fourier methods appear to offer the best hope,
although they have been superceded in many domains. Both of the traditional fourier domain approaches are inadequate, though: human inspection of the power spectrum is not quantitative, and reduction to radial or annular bins provides too much unstructured detail.

Our model is a stochastic, terse description of the spatial frequency content of a sample and its contrast. Our hopes are that

1. the parameters of our model will decouple,
2. the parameters will correspond in an intuitive way with texture characteristics,
3. the parameters may be extracted from data samples, and
4. a simple combination of at most two measures will correlate well with industrial standard metrics for mottle severity [3].

If we are successful, most of the model parameters will be constant for a particular application, and the remaining simplified model will be useful in practice. However, there is significant leeway for extending this model (through more complex and interdependent probability density functions, for example.)

Our strategy is to develop and understand a generative model and then work on extracting its parameters from real images.

Phenomenologically it seems that mottle samples could be represented by distributions of points in frequency space. Each frequency space point can be represented by \( p \), its radial distance from the origin; \( \theta \), the angle of that radial vector and \( Z \), the complex vector representing the weight of this Fourier component. These parameters should be sufficient to generate mottle, but one important component of texture seems to be its "contrast". In the model so far, contrast depends in a complex way on the joint properties of the other three parameters. Thus our strategy is to characterize the geometric parameters as simply and independently as possible and to add a "contrast parameter". This decoupling is highly desirable from many points of view.

Assuming the parameters are independent, an MxM mottled image may be generated by the following simple model.

1. \( P(p) \), the probability distribution function of \( p \) (\( p \) varies from 0 to \( M/2 \) or substantially less.) This parameter is related to spatial extent of mottle.
2. \( P(\theta) \), the pdf of \( \theta \) (\( \theta \) varies from 0 to 180 degrees.) This parameter is related to the directionality of mottle.
3. \( P(Z_{\text{real}}), P(Z_{\text{imaginary}}) \) the pdfs of frequency components. These parameters are related to contrast, brightness, and appearance of mottle. We will refer to the two distributions by the shorthand, \( P(Z) \).
4. \( N \): the number of frequency components to use in texture synthesis.
Figure 1: Mottle sample, courtesy Eastman Kodak Company. This image is a photograph of an Ikonas video display, showing a) digitized vidicon image trimmed to eliminate the effects of an unevenly lit light table, and b) the same image scaled linearly for maximum displayable dynamic range.

(5) S, a function mapping Image intensities X parameters $\rightarrow$ image intensities, and acting as a scaling filter that provides variable contrast of the final output.

The process is to generate N random points $(p, \theta, Z)$ and place them along with their complex conjugate reflections in Fourier space, so that their inverse transform is real. $P(p), P(\theta), P(Z)$ are independent. Joint distributions are of course more general but harder to characterize, and we hope they are unnecessary. The inverse Fourier transform is applied, then S, the scaling function, is applied to adjust the "contrast" of the result. The computational steps in the model are diagrammed in fig. 2.

In our current version of the model we have chosen uniform distributions for the parameters $p, \theta, \text{and } Z$, and (rather arbitrarily) an exponential $x' = \gamma x$ scaling function. Another candidate is a linear $(x' = ax + b)$ scaling model. The S function finally chosen should relate in a simple way to the underlying physical causes of mottle or to the phenomenal characteristics of industry standards. Each parameter is thus characterized by two probability distribution parameters. In the table below we give the probability distribution parameters that we have used in texture generation experiments. The mean and standard deviation are easily related both to the other distributions (especially gaussian and poisson) and to measurements on real data.
4 Texture Generation Experiments

A program was written to generate textures according to the parameters given in the previous section. Representative samples of artificial textures are shown in figures 3, 4, 5, and 6.
Figure 3: Artificially generated textures: $0 \leq \rho \leq 4$, $0 \leq \theta \leq 180$. $Z_{\text{real}} = Z_{\text{imaginary}} = 10$, $\gamma = 0.9$. From left to right, the columns have $N = 8, 16, 32, 64, 128,$ and $256$. 
Figure 5: Artificially generated textures: $0 \leq \rho \leq 6$ for the first three rows and $0 \leq \rho \leq 4$ for the lower four. $0 \leq \theta \leq 180, 0 \leq \text{Real Imaginary} \leq 10$. $N = 64$. From left to right, the rows have $\gamma = 0.7, 0.8, 0.9, 1.0, 2.0, \text{and } 3.0$. 
5 Refinements

Generation of textures has verified that the fourier domain mottle model has some descriptive power. Computerized texture generation could also be a means to development of quantitative quality standards. A number of refinements could be made, however. These include:

1. Multiplicative (linear) scale filter. This issue can be based on the physics of mottle formation or the psychophysics of human detection of mottle. We plan to treat it as an experimental issue so that we can simplify the relation of the scaling function to industrial standards.

2. Normalized mean brightness. This parameter raises or lowers the overall brightness over which the mottle ranges. It can thus be used to ensure that the appearance of mottle reflects the differing effects of various film substrates. It is useful because it is psychophysically important: a dark image is hard to compare with a bright one.
(3) Addition of high frequency noise to simulate grain. This could be an independent parameter. It could alternatively be included by modifying the probability density functions $P(\theta)$, $P(p)$, and $P(Z)$.

(4) Joint probability distribution functions. If our variables cannot be adequately approximated by independent probability distribution functions, more sophisticated stochastic methods must be used.

(5) Poisson or Gaussian distributions. Uniform distributions will be useful, but other distributions could be even better.

6 The model parameters

For textures showing dominant directional or spatial frequency characteristics it should be possible to represent the sample using the radial distribution (denoting frequencies) and angular distribution (denoting angular direction preference) in the frequency domain. Moreover from the distribution of magnitude of the Fourier domain points we should be able to know how the concentration of angular and frequency preference is weighted.

To achieve our goal of texture quantification it is necessary to obtain some parameters from the image domain. The parameters that seem to be most useful are mean and contrast measurements with respect to the gray levels. Besides these it would be useful to know how the mean and contrasts vary when we look at different portions of the images. This quantifies certain basic variations between different portions of the image.

Thus the model consists of two type of parameters, one set in the frequency domain and another in the image domain.

6.1 Frequency domain parameters

The frequency domain parameters are useful for those images that show preferential spatial frequency or direction or both. The extraction of this type of characteristic is easier using the frequency domain directly. Imagine figuring out what the sizes of the texels are (given that the texels in the image have only a fixed number of different characteristic sizes) from the raw gray levels. On the other hand from the frequency domain the characteristic sizes may be found from the peaks in the radial spatial frequency distribution. Moreover if the image consists of many frequencies distributed in some random way (say normal, uniform or any such distribution) then studying the frequency domain radial density can give the parameters of the distribution easily. Moreover the distribution form can also be checked using various correlation techniques. The directional preference can also be easily found using the frequency domain. In such case we only have to look at the angular density. Uniform angular density indicates a random orientation of the texels whereas sharp peaks in the angular density histogram give the orientation of the underlying spatial frequencies and thus the texels themselves.

Thus the frequency domain parameters in the model consist of the distribution with respect to the radial and angular density. Besides this the distribution with respect to density at different magnitude of $Z$, $Z_{\text{real}}$ and $Z_{\text{imag}}$ in the frequency
domain was also considered. These distributions describing the frequency domain
representation of the image are the following:

1) The distribution of spatial frequency density with respect to radial distance

This distribution can be calculated by summing up the frequency domain
values of all the points at a given distance from the origin. If the probability
distribution of the frequency is needed then it can be calculated by dividing the
individual densities by the sum over the densities at all the frequencies. The
formula for calculating the individual density is given by

\[ P(k, \rho) = \sum_i \sum_j \left( \frac{Z_k(i,j)}{\text{Dist} \left[ (i,j), (X_c, Y_c) \right] = \rho} \right) \]

where

\[ Z(i,j) \] is the power spectrum value at \( i,j \)

\( (X_c, Y_c) \) is the center point

\( \text{Dist.} \) is the function to calculate distance

\( \rho \) is the radial distance.

\( k \) is for real, imaginary or magnitude.

In the above formula \( k \) is to denote the distribution with respect to real, imaginary
or the magnitude of the Fourier transform. Note that we should take the modulus for
the real and imaginary distributions. The average frequency (radial distance) can be
calculated by summing the product of radial distance and probability for that radial
distance. Similarly the variance can be calculated by taking the product of the
probability and the square of the difference of the radial distance and the average
radial distance. Note that in a similar way we can calculate skewness, kurtosis and
higher order terms by taking different powers of the difference from the mean.

2) The distribution with respect to angle

The distribution of the angular density can be calculated in the same way by
summing up the Fourier domain values of all points such that the angle formed
by the line joining the point to the origin with the X axis is \( \theta \). The probability
distribution with respect to angle can also be calculated in a similar way by
dividing the density by the sum of the densities at all the angles. The formula to
calculate the angular density is given by

\[ P(k, \theta) = \sum_i \sum_j \left( Z_k(i,j) \cdot \text{angle} \left[ (i,j), (X_c, Y_c) \right] = \theta \right) \]

where

\( \text{angle} \) is the function to calculate angle.

\( \theta \) is the angle

\( k \) is for real, imaginary or magnitude.
The average and variance of the angle can also be calculated in the same way as in 1. Again higher order terms can also be easily obtained from the probability distribution.

3) The distribution of Z the power spectrum value

The distribution of the power spectrum value can be calculated by calculating the number of points that have that power spectrum value. The probability of a particular point having a particular value of Z (the power spectrum value) can be calculated by dividing the density at Z by the total number of points. The average value of Z can be calculated by summing the product of Z and the probability of a point having Z as the power spectrum value. The variance can similarly be calculated by taking the sum over the product of the probability and the square of the difference from the mean. Again we can calculate the higher order terms also if required. The formula for calculating the density is given below.

\[ P(Z) = \sum_i \sum_j (1) \mid (Z(i,j) = Z) \]

where

\( Z(i,j) \) is the power spectrum value.

4) The distribution of real part of the Fourier transform

The distribution of density at the different values of real part of power spectrum can also be calculated in the same way as 3 above. Note however that the magnitude of the real part is considered. Again the average and variance can be calculated in the same way as above.

\[ P(\text{real}) = \sum_i \sum_j (1): \mid \text{Real}(Z(i,j)) \mid = \text{real} \]

where

\( \text{Real}(Z(i,j)) \) is the real part of Fourier transform

5) The distribution of the imaginary part of the Fourier transform

The distribution of density at different values of imaginary part of the power spectrum can also be calculated in the same way as 3 and 4 above. Note that in this case also we take the magnitude of the imaginary part of the power spectrum (otherwise the sum would be 0!). The average and variance can also be calculated similarly.

\[ P(\text{imag}) = \sum_i \sum_j (1): \mid \text{Imag}(Z(i,j)) \mid = \text{imag} \]

where

\( \text{Imag}(Z(i,j)) \) is the imaginary part of Fourier transform
In representing the distributions, a conservative approach is to keep the complete distribution. However, for characterization of the image, the simple statistical parameters such as average and variance of the distribution should be enough since most textures show highly preferential frequency and direction preference. If the distribution of the various values (such as frequency and direction) is expected to be non-uniform on the two sides of the mean, then the higher order terms can also be used. In general, the adequacy of characterization of a particular domain by a particular set of parameters is an empirical issue.

6.2 Spatial domain parameters

The frequency domain parameters are often not enough to characterize the image. This is especially so if the image does not show uniformity in the frequency distribution. In such cases, it is useful to go back to the image domain parameterization. Moreover, the actual image domain parameters are more likely to indicate the form of images to humans. The distribution of grey levels in the local neighborhood of the points gives an indication of what the image looks like. The overall distribution of the parameters obtained at local neighborhood of the image indicates the global characteristics of the image.

The image domain parameters consist of the parameters obtained from the calculation of mean, variance and ratio of dynamic range to the maximum in the local neighborhood of the image point varying over the image domain.

The mean denotes the average intensity at the local neighborhood. The variance and the ratio of dynamic range and maximum indicate the contrast at the local neighborhood of the point.

The overall mean and variance of the above desired distributions indicates certain global properties. For example, the variance of variance indicates how the contrast varies as we move to different positions in the image.

1) The distribution of mean

The mean at the local neighborhood of a point $I(i,j)$ indicates the average intensity around that point of the image. We can consider the local neighborhood as the points which are within a certain distance of $(i,j)$ in some convenient metric. Any of the local neighborhood criteria can be used. We have used the city block metric in our work. The formula for calculating the average intensity at a point $(i,j)$ can be given as follows

$$\mu(i,j) = \left(\frac{1}{m^2}\right) \left[ \sum_x \sum_y I(x,y) \right]$$

where

- $x$ and $y$ vary over the local neighborhood.
- $m$ is the size of the neighborhood.
- $I(x,y)$ is the gray level value at the point $(x,y)$
The mean of the above μ can be calculated by summing up the μ's and dividing by the total number of points for which μ was taken. The variance can also be calculated as it was calculated for the frequency domain parameters. The mean of mean gives the indication of global mean. The variance of the means is one measure of perceived contrast, "but contrast is a complex perceptual phenomenon".

2) The distribution of the variance

In some models of texture, variance in the local neighbourhood indicates the local turbulence in the grey level values. Moreover the variance in the local neighbourhood can also be used to determine if the size of the neighbourhood chosen is reasonable for the given image. The desirable size of the neighbourhood should not be much larger than the texel size. The size of the neighbourhood should similarly not be too small. The variance at different neighbourhood size can thus give indication of the texel size. The variance of a local neighbourhood of the point is calculated as follows

$$\sigma^2(i,j) = \frac{1}{m^2} \sum_x \sum_y \left[ I(x,y) - \mu(i,j) \right]^2$$

where x, y vary as above.

The mean and variance of the variance can be calculated in the same way as other means and variances. The mean of the variance indicates the average turbulence in the gray levels. The variance of variance would indicate how the contrast varies over the whole image.

3) The distribution of dynamic range-maximum ratio

The dynamic range of the gray level is defined as the difference between the maximum and the minimum values of the gray levels.

The ratio of the dynamic range and the maximum gray level value in the local neighbourhood of the point is another form of contrast measurement which indicates how steeply the gray level value is changing in a local neighbourhood. Again this can also be used as the guide for guessing the texel size. The formula to obtain this ratio is

$$R(i,j) = \frac{\max(I(x,y)) - \min(I(x,y))}{\max(I(x,y))}$$

where x, y vary as above.

The mean and variances of R(i,j) are calculated in the same way as we calculated the means and variances of other parameters. The mean of this ratio indicates the expected local steepness in the gray level values. The variance indicates how the steepness varies over the whole image domain.

Instead of the actual distribution, the number of parameters can be reduced by keeping only mean and variance of the above three distributions. The mean and variance would indicate the global average and variation in the local properties. These parameters can be used to quantify certain aspects of the image in spatial domain.
7 Experiments

We ran several experiments for three types of textures.

1) A digital image of a mottle standard from Eastman Kodak Company.

2) Unfocussed images of certain textures from Brodatz's book[2].

3) Synthetic mottle generated by running the model in a generative mode (see Figs. 3-5)[4].

Tables 1 and 2a&b show parameters derived from these textures and demonstrate a certain consistency with intuition about textural similarity. Figs. 7 and 8 show representative textures, a subset of those used in the study.

<table>
<thead>
<tr>
<th>Image</th>
<th>Avg μ</th>
<th>Avg V</th>
<th>Avg R</th>
<th>Var μ</th>
<th>Var V</th>
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Table 1: Spatial domain data
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<th>Var $Z$</th>
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Table 2a: Frequency domain data

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<th>Avg $\theta$</th>
<th>Avg $Z$</th>
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<td>264.54</td>
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<td>1284.4</td>
<td>800.51</td>
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</table>

Table 2b: Frequency domain data

8 Reconstruction

As observed before for textures showing special characteristics in the Fourier domain it should be possible to reconstruct an image similar to the original from the
parameters calculated in the frequency domain. For textures such as mottles it may also be possible to reconstruct using only the image domain parameters. For reconstruction from spatial domain parameters we can give the gray level values of the pixels according the the distribution in the neighbourhood if the distribution of the various parameters in the local neighbourhood of the point is kept. For textures that seem more or less uniform we can distribute the gray levels based only on the global characteristics.

For reconstruction from the frequency domain parameters we can distribute some points in the frequency domain according to the density in the radial and angular bins. For textures that show very concentrated radial and angular preference only the statistical parameters of the radial and angular distribution may be needed for generation of points.

8.1 Reconstruction from frequency domain parameters

For reconstruction it might be expected that we would require to keep a large number of parameters. However if the texture shows a special form of distribution in the radial and angular direction then by knowing the characteristics of the distribution we can generate the frequency domain points according to the characteristics of the distribution. For example suppose we know that the frequency domain points of the image show a uniform distribution with certain mean and variance for both radial and angular direction. Then we can distribute the points uniformly in the frequency domain with the corresponding mean and variances. Various mottles were generated using such distribution with certain modifications. The resemblance of the synthetic images with the actual mottles gave the motivation for this project. We can similarly generate textures for various kinds of distribution. If the distribution selected is one that is obtained from the parameters( or the whole distribution ) obtained from the actual image we should get an image similar to the original image. In our experiments so far we have used the full distribution of $p, \theta$ and real and imaginary $Z$ distributions for generation, rather than reconstructing the distributions from a few parameters.

The reconstruction from the distribution can be done as follows:

1) Select number of points $N$ that will be distributed in the frequency domain.

2) Generate $N$ points in the frequency domain according to the distribution (i.e. generate $X,Y,Z_r,Z_i$ for each point such that the distribution and density of the points follow the characteristics of the original distribution).

3) Take the inverse Fourier tranform to generate the image.

4) Scale the image gray levels appropriately.

The number of points selected in the Fourier domain should not be too small because in that case the distribution may not be able to capture the characteristics of the image. Step number 2 above can be performed in various ways since distributions are approximated by histograms and, in practice we can get similar histograms by distributing the points and weights at those points in a number of ways. This step is discussed in detail below. The scaling of the image is required since the inverse Fourier transform thus generated may have negative values (since the generated frequency domain points do not necessarily give an exact Fourier transform of an image with positive gray level values). Moreover since the average
intensity of the points may change we would have to do appropriate scaling of the image so that the image generated would have similar intensity to the original (since the frequency domain analysis is supposed to give only the frequency and angular distribution). This step can be done by calculating the average intensity of the original image.

8.2 Distributing points according to desired histograms

The distribution of points according to the model or derived (approximate) distribution parameters can be done in several ways. Some of the methods are discussed below.

When the whole distribution of the radial and angular density is kept:

a) We can just distribute the points according to the radial and angular probability distribution in the frequency domain. For this we can keep a cumulative distribution of probability of any radius or angle. A random value (uniform) is generated according to any of the standard random number generating functions. The radius (angle) in which this value falls in the cumulative distribution can be taken as the radius (angle) of the generated point. The magnitude of the real and imaginary part of the power spectrum can be chosen randomly according to their distribution. This is the simplest way to distribute the points. The sign of the values can again be taken randomly with probability half. Note that the points generated should have a complex conjugate partner at a position \((-x,-y)\) \((x,y)\) is the position of the generated point). This is done so that we can get a real image after taking the inverse transform.

b) Since a single high valued point at a higher frequency may generate high frequency components even though the sum of the values at that frequency is not considerable we would like to reduce the magnitude of the values at the points at high frequency. This can be done in two ways. The simplest way would be to reduce the intensity at any distance \(R\) by dividing it by \(R^2\) (i.e proportional to the number of points at that distance ). This method discourages the high frequency components, Alternatively we can blur the point at high frequency. This reduces the magnitude of points at high frequency while keeping the total sum of values at high frequency according to the distribution. Note that we would have to do superposition if the effect of more than one point blurring is felt at some other point. This blurring can be done in any form of distribution (gaussian or other template such as 1/2 at center and rest in the neighbourhood of size depending on the frequency). Other values can be found in the same way as in (a) above.

c) In this case we can distribute the points randomly but take the weight at those points according to the distribution. The points can be generated according to any uniform distribution. In this case we would not have the problem of high frequency having concentrated values. The total sum at any frequency follows the distribution since there will be more points generated at the high frequency. Note that the value generated at a frequency should have the dividing factor of \(R^2\).

d) Another variation would be to combine the two ideas above i.e to generate the points according to the probability distribution and then generate the
values according to the density at those points. This method would disfavour the high frequency components. This might be useful when high frequency components are to be suppressed.

In all the above methods we keep the full distribution of the probability density at any radius or angle. The strategy changes slightly when we have to generate according to the distribution of points that follow a distribution pattern such as normal, uniform or any known such distribution. Using distribution parameters would be useful in decreasing the amount of information required to characterize the texture. Not much work has been done on this kind of generation since the results using distribution information are still not fully understood. Work in the immediate future will concentrate on the regeneration issue. One idea is to distribute the points and density in much the same way as above but instead of using the uniform generation or the generation according to the distribution of density at a frequency and angle we would generate points and values at the points using a random generator which follows the characteristics of the known distribution. Random generators for non-uniform distribution are more complicated (Knuth discusses the generation of random numbers according to various types of distribution) [7]. Alternatively the characteristic distribution function of the desired distribution can be computed analytically and used with the existing software.

8.3 Reconstruction from image domain parameters

For images that appear random and only the contrast or mean intensity seems significant it may be possible to reconstruct the texture from the image domain parameters. Moreover certain textures may not have a well defined frequency preference nor have a reasonable pattern in which the frequencies are distributed. For such images the frequency domain reconstruction may not give good reconstruction. Thus it may be desirable to study the techniques to reconstruct the images from the image domain parameters. The reasons for this type of reconstruction are similar to the choice of using the image domain parameters. Some mottled textures seem to be a likely candidate for this type of reconstruction. However not much thought has been given to image domain models. The procedure followed to reconstruct the image are quite similar to the frequency domain reconstruction case.

In this case we take the parameters on the image domain. Then using the distribution we calculate the value of the gray level at each point to obtain the reconstructed image. If the statistical characteristics (such as mean, variance, dynamic range) are kept for a neighbourhood of points selected at appropriate distances (the appropriate distance being the size of the neighbourhood) then we can generate the gray level values at the local neighbourhood according to the statistical parameters at that distribution. The procedure for generating the gray level values is the following:

For each selected point for which the local parameters were kept:

- generate the gray level values of all the points in the local neighbourhood of the point such that it satisfies the mean, variance in that neighbourhood. This can be done by assuming normal or uniform distribution in the local neighbourhood. The resulting values can be appropriately scaled to satisfy the dynamic range constraint.
Note that due to normal or uniform distribution of gray level values in the local neighbourhood the size of the neighbourhood should not be large (as compared to the texel size). Moreover if we take the size to be too small then the number of parameters kept would increase as the inverse of the square of the neighbourhood size.

If the texture doesn't show any particular changes in mean, variance or dynamic range as we concentrate on different portions of the image then we can keep only the overall characteristics of the local characteristics. The local characteristics can then be obtained by randomly selecting the local characteristics according to the global values. The regeneration can then follow in the same way as if we had the local characteristics.

9 Experiments on regeneration

Most of our work on regeneration of images was on the regeneration from frequency domain parameters. We ran our model for reconstruction on various mottled images from the Brodatz's book, a checkers board image and some of the synthetic mottles of Hinkelman. Some of the images on which the model was tried along with their power spectrum, the generated power spectrum and the regenerated image are shown in Figs. 9-13. The schemes used for the regeneration were the following:

Fig. 9 (checkerboard image) -- schemes a and c mentioned in section 8.2.

Fig. 10 and 11 (Hin1 and Hin2) -- schemes a and b mentioned in section 8.2.

Fig. 12 and 13 (Br3 and Br) -- schemes b and c mentioned in section 8.2.

The results were good for the checkerboard image. The results were not so good for the mottles. The power spectrum of the various images were similar to the generated ones as expected. The results for the images obtained by taking the inverse Fourier transform were not as good as expected in the case of the mottles, even though in most cases some of the characteristics of the images were captured in the generated images.

10 Discussion

The parameters generated showed good similarities for similar type of images. The method thus shows some promise for parameterization of the mottles. The reconstruction for images which show a high degree of regularity such as checkerboard can also be done with some correspondence between the original and reconstructed image. The model thus could be used for reconstruction of textures showing high degree of regularity. The reconstruction of mottles from the parameters was not as successful. This may be due to the reason that there is a lesser degree of regularity in the mottled images. To reconstruct the mottles we can try variations of the idea so that the generated images resemble the original image to a greater extent. Various modifications of the above ideas can be tried so that a better fit for the mottles can be found. Future work will concentrate along these lines. The various models that can be tried are

1) Use the image domain parameters in the regeneration of the image.
2) Use the Fourier domain parameterization to reconstruct the image. Use the image domain parameters on the generated image to force some of the image domain characteristics on the image.

3) Use various contrast measures obtained from the image (the contrast measures can be the dynamic range, the variance etc.) and force them on the regenerated image. This is likely to ensure that the various contrast measures are preserved. Note that in forcing one of the characteristics we may violate other properties. The way would be to find a compromise between the various characteristics.

Another important topic is to extract the "contrast" parameter (the arguments to the function \( S \)). This is much more interesting, especially because it is our hope that the \( P(p) \), \( P(\theta) \), and \( P(Z) \) can be "frozen in" for a situation and the contrast parameters will provide the quantitative measure of mottle severity. In this case one way to extract the contrast parameters would be to choose a likely value (say \( y_0 \) for definiteness), "unscale" by \( y_0 \), (ie apply the inverse of the scaling using \( y_0 \)), then do steps 1, 2, 3 above and check to see the resulting \((\mu, \sigma)\) fit the mottle shape model. If they do not, change \( y_0 \) to \( y_1 \) and repeat. The hope is that a hill-climbing process on \( y \) will find that \( y \) which results in the best fit of the other mottle parameters to the data.

An alternative approach to quantitative analysis with the model does not assume that the \( P(p) \), \( P(\theta) \), and \( P(Z) \) can be "frozen in". In other words, it allows these parameters to be calculated as part of the image quality estimation. Rather than expending large amounts of effort on calculating a very accurate value for \( y \) based on the other parameters, we fall back on our independence assumption. The first two steps of the analysis process would be as in verifying the model, but rather than finding \( y \) by means of a complicated iteration (hill-climbing), use some simpler method:

1) Fourier transform the image,

2) Compute \( \mu, \sigma \) of \( P(p) \), \( P(\theta) \), and \( P(Z) \)

3) Apply a simple contrast measure.

In this case, the hope is that some carefully chosen but easily computable contrast measure will be adequate to distinguish relevant variations in images. Such a metric could address some of the psychobiological issues mentioned below, or it could be a simple rule of thumb that happens to be effective. The difference between the maximum and minimum image values is a very primitive measure: another is the variance of the gray levels. The Spatial Gray Level Dependence contrast measure is somewhat more sophisticated, and presupposes some estimate of the scale. None of these has much psychophysical value, though. It is worth noting that the limiting factor in performing the above analysis on line is the size of the area analyzed, which determines the time of the Fourier Transform. The approach mentioned above performs several Fourier Transforms and thus requires more time.
Fig. 7: Textures used in the experiment

upper left: mottle picture obtained from Kodak (mot1).
lower left: mottle picture obtained from Kodak (mot2).
upper middle: Brodatz's texture "Pressed cork" unfocussed (Br1).
lower middle: Brodatz's texture "Handmade paper" unfocussed (Br2).
upper right: Brodatz's texture "Pigskin" unfocussed (Br3)
lower right: Brodatz's texture "Pressed cork" unfocussed (Br4)
(different scaling than Br1).
Fig. 8: Textures used in the experiment

upper left: another mottle texture (mot3).
lower left: Brodatz's texture "Pigskin" unfocussed (Br5) (different portion of the same image as Br3).
upper right: Generated texture 1 (Hin1).
lower right: Generated texture 2 (Hin2).
Fig. 9: Texture Reconstruction

lower left: The original image (checkered from [2]).
upper left: The power spectrum of the original image.
lower middle and right: Reconstructed images using different generation schemes (see text).
upper middle and right: Power spectrum of the reconstructed images.

Comment: In reconstruction 1 (middle) the frequency has increased in vertical direction. In reconstruction 2 (right) there is a tilt in the direction and increase in the vertical frequency. In reconstruction 1 the sharp parallel horizontal lines in the Fourier transform have not been captured. In 2 due to the scheme chosen the lines have not been captured. Overall high amount of directional and frequency information has been captured.
Fig. 10: Texture Reconstruction.

lower left: The original image (Hin1).
upper left: The power spectrum of the original image.
lower middle and right: Reconstructed images using different generation schemes (see text).
upper middle and right: Power spectrum of the reconstructed images.

Comment: The power spectrum in the reconstruction resembles to a high extent the original power spectrum. The slight amount of high frequencies in the original spectrum has not been captured satisfactorily. The net frequency in the image seems to have been reduced.
Fig. 11: Texture Reconstruction.

lower left: The original image (Hin2).
upper left: The power spectrum of the original image.
lower middle and right: Reconstructed images using different generation schemes (see text).
upper middle and right: Power spectrum of the reconstructed images.

Comment: The power spectrum in the reconstruction resembles to a high extent the original power spectrum. The slight amount of high frequencies in the original spectrum has not been captured satisfactorily. However the frequency component is better than in the previous (Hin1) image. The generated image has high angular preference which is missing in the original image.
Fig. 12: Texture Reconstruction.

lower left: The original image (Br3).
upper left: The power spectrum of the original image.
lower middle and right: Reconstructed images using different generation schemes (see text).
upper middle and right: Power spectrum of the reconstructed images.

Comment: In this regeneration the brightness in the upper and lower middle portion of the power spectrum is missing in the reconstructed power spectrum. The image shows similarity in the frequency and randomness of the direction. The contrast is higher in the generated image.
Fig. 13: Texture Recognition

lower left: The original image (BR2)
upper left: The power spectrum of the original image.
lower middle and right: Reconstructed images using different generation schemes (see text).
upper middle and right: Power spectrum of the reconstructed images.

Comment: The frequency component of the reconstructed image has been reduced slightly. The contrast in the generated images has decreased. By increasing the contrast, a better similarity may be achieved.
11 References


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