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A SOLAR RECEIVER-REACTOR WITH SPECULARLY REFLECTING WALLS
FOR HIGH-TEMPERATURE
THERMOELECTROCHEMICAL AND THERMOCHEMICAL PROCESSES

by

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Submitted to

ENERGY

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A SOLAR RECEIVER-REACTOR WITH SPECULARLY REFLECTING WALLS
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THERMOELECTROCHEMICAL AND THERMOCHEMICAL PROCESSES

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Abstract—A new kind of receiver-reactor for high temperature solar furnaces is proposed. The main body of the receiver component is an ellipsoid of revolution with specularly reflecting inner walls. The reactor component, a crucible, is placed at one focal point and the aperture at the other. With this arrangement, substantially all of the incident radiation from the concentrator should reach the reactor directly or after one reflection from the cavity walls. An analysis of the radiative exchange among the surfaces is presented. The analysis provides a tool for a parametric study and optimization of the design. It is found that, in contrast to that of conventional well-insulated cavity receivers, its collection efficiency is not very sensitive to the size of its aperture.

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INTRODUCTION

A recent review of reactor/receiver concepts for solar thermochemical processes prompts us to observe that cavity receivers for solar furnaces are usually well insulated enclosures made of refractory materials. Our own experimental work with thermochemical processes has made use of heavily insulated cavity receivers lined with zirconia, alumina, and various commercial felts and insulating boards. The interior lining of the cavity is often the principal solar energy absorbing surface. From there, energy is transferred to the reactor. When such a perfectly insulated, windowed receiver is reradiating energy as fast as it is receiving it, it is at the highest temperature it is capable of achieving, [cf. Eq. (1)]:

\[ T_{\text{max}} = \left( \frac{I \alpha_{\text{r}} \epsilon_{\text{r}}}{a \sigma} \right)^{1/4}. \]  

(1)

The symbols used in this paper are defined in Appendix A.

The efficiency with which energy can be used by the process at this temperature is zero. To be energy efficient, a process must usually occur at an inner wall temperature that is substantially below \( T_{\text{max}} \). With real receivers, the efficiency is, of course, even lower. Since the inner wall of the receiver is at the highest temperature in the system, conductive losses to the outer shell of the receiver as well as convective losses through the aperture are significant.
Hunt\textsuperscript{10} and his coworkers have proposed a promising direct-absorption receiver that uses a suspension of small particles, which may or may not be catalytic, to serve the functions of energy absorption, heat transfer, catalysis, and possibly reaction as well. He has demonstrated the use of soot particles for heating air. The geometric arrangement proposed here may well be adapted to direct absorption processes.

However, many processes of interest, electrolysis or the reactions which approach them in thermodynamic equivalence as we operate at higher and higher temperatures,\textsuperscript{*} are most conveniently carried out in cells or reactors that are enclosed and may be physically separated from the walls of the cavity. It thus behooves us to seek alternatives to the kinds of cavity receivers we have used in the past. Especially as reactor temperatures go above 2000K, conduction losses may become excessive, and the list of unreactive ceramic materials of construction becomes small. Moreover, conventional cavity receivers operating at high temperatures prompt us to use small apertures to reduce reradiation losses. Contrariwise, larger apertures intercept more sunlight reflected from imperfect and imperfectly matched heliostats and concentrators. To some extent, the dispersion problem may be made tractable by the use of techniques which have been pioneered by Winston\textsuperscript{11} and his coworkers. Nevertheless, flexibility in
using larger apertures is likely to become advantageous when one uses, for either heliostats or concentrators, the superbly reflecting acrylic films now undergoing development, since these may reflect with greater dispersion than high-quality glass mirrors.

In the present work, we examine a new scheme for making high-temperature cavity receivers to contain reactors. This approach may obviate some of the problems of using ordinary cavity receivers.

**SYSTEM**

We substitute a radiation reflector for refractory insulating materials. Our system is shown schematically in Fig. 1 and consists of a cavity-receiver which contains a reactor, the crucible. Our objective is, with a given concentrator, to transfer as much energy as we can to the crucible at a temperature that is high enough so that we can effect the desired process. The inner walls of the receiver are good reflectors, and the outer wall of the crucible is a good absorber. Thus, an incident ray will lose a relatively small fraction of its power each time it is reflected from the cavity wall. If it strikes the crucible, it will be substantially absorbed. Eventually, all of its energy will have either been absorbed by the crucible and receiver walls or have left through the aperture.
Because all surfaces, even good mirrors, absorb radiation to some extent, we strive to reduce the number of internal reflections and thus reduce the amount of energy lost to the cavity walls. To accomplish this purpose, we make the inner wall of the receiver essentially an ellipsoid of revolution with specularly reflecting walls. We center the radiation source, the aperture, at one focal point \((F_2)\) and the radiation sink, the crucible, at the other \((F_1)\). Thus, if the concentrator characteristics and the crucible size are well matched, most of the radiation incident from the aperture should reach the crucible directly or after one reflection from the receiver wall. The crucible should thus absorb most of the incident radiation.

As its temperature rises, the crucible will emit more and more diffuse radiation. Some of the emitted radiation is reflected by the ellipsoid-of-revolution back to the crucible. Some escapes through the aperture directly or after reflection from the receiver wall. We can recover some of the emitted radiation by making that portion of the receiver wall which holds the aperture a specularly reflecting sphere whose center is also the center of the crucible \((F_1)\) as is shown in Fig 1.

Such an arrangement promises several intriguing advantages. The walls of the cavity may be kept cool without incurring a con-
comitant energy loss, thus eliminating the need for ceramic materials. In addition, as will be shown in the analysis that follows, the performance of this reactor is not adversely affected to an appreciable extent by reasonable enlargement of the aperture. This feature gives us freedom to use an aperture large enough to accommodate more radiation from the concentrator than we might have if we were subject to the constraint implied by Eq. (1). Finally, in some circumstances, energy loss by convection to the walls, which is not included in this analysis, may actually be substantially reduced by the use of a windowed aperture and evacuation of the space between the receiver and the crucible.

THE PROBLEM

The problem we wish to solve is the following. Given a general receiver-crucible configuration such as that shown in Fig. 1 and a particular energy-flux distribution through the aperture (such as that furnished by a real concentrator) for the steady state, calculate the net power to the crucible, i.e., the energy-absorption efficiency of the system. The parameters are the dimensions of the components, the optical characteristics of the surfaces in the cavity, specularity, absorptivity, reflectivity, and the temperature of the crucible.

A desirable corollary result would be the spatial distribution of the energy flux through the cavity walls, since that is
useful design information.

Figure 1 shows our literal representation of the pertinent objects and surfaces that we use in the analysis which follows. The crucible, whose area is $A_1$, is a sphere of radius $r_1$ with its center on $F_1$, one of the two focal points of the ellipsoid. The aperture, whose area is $A_2$, is a circle of radius $r_2$ which lies in the plane $A$ with its center on $F_2$, the other focal point of the ellipsoid. Plane $A$ is normal to the major axis of the ellipsoid through $F_2$. The major axis length of the ellipsoid is $2a$, and its minor axis length is $2b$. We divide its area into two areas, $A_3$ and $A_5$. The remainder of the cavity is formed by the area $A_4$, which is a segment of a sphere concentric with the crucible whose radius is $r_4 = (4c^2 + r_2^2)^{0.5}$. This part of the cavity contains the circular aperture whose area is $A_2$. Plane $B$ contains the circumference which is common to the ellipsoid and the sphere. It marks their junction to form the cavity. $A_6$ is the area of that portion of the ellipsoid that is eclipsed by the crucible.

ANALYSIS

For analysis of the radiative exchange among the surfaces, we assume that the space inside the cavity is filled with a non-participating medium, i.e., one that neither absorbs, emits, nor reflects. We have thus also neglected convection inside the
cavity. We also assume that each of the defined surfaces is isothermal and gray, the crucible emits and reflects in a diffuse manner, and the cavity walls reflect in a specular manner. Finally, we assume that the cavity walls do not emit. If the cavity walls were at 300K and the emissivity were 0.05, the emitted flux would amount to about 25 W/m². Neglecting this emitted radiation introduces little error and simplifies the problem considerably. Our general approach is to do an energy balance on each of the appropriate surfaces in the system. The energy fluxes we seek emerge naturally from the energy balances.

Algebraic solutions are usually elegant. They provide quicker answers and understanding of the behavior of systems than do numerical approaches. But algebraic methods are sometimes difficult to apply to problems such as these without significant simplification. We might, for example, assume a uniformly distributed incident power over all of each individual area. This assumption is, however, far from realistic. We cannot, a priori, be confident of our answers. An alternative that can give more precise design information is the Monte-Carlo method. It provides important information about the energy flux distribution over each surface and highlights the location of critical regions. Moreover, one may use a realistic incident flux distribution with this method, which requires the use of a time-consuming computer program. We have opted to study the problem using both methods.
This decision provided us with numerical answers to our present problem and, at the same time, permitted us to evaluate the use of the algebraic method for future applications.

Our algebraic method is based on the radiosity concept and was simply and quickly formulated when we assumed that the power distribution across each surface was uniform. Our Monte-Carlo method gave more precise answers using a realistic flux from the concentrator. As it turned out, using the simplified flux distribution provided easy algebraic solutions that were acceptable approximations to the Monte-Carlo solutions.

Algebraic solution

Energy balance on the crucible surface $A_1$

The net energy flow to the crucible is the difference between the incident power entering it and the power leaving it. The power per unit area leaving $A_1$, its radiosity $B_1$, consists of two components. The direct emission is $\varepsilon_1\sigma T_1^4$. The diffusely reflected portion of the incident energy is $\rho_1 H_1$. Thus,

$$B_1 = \varepsilon_1\sigma T_1^4 + \rho_1 H_1.$$  

The power per unit area incident on $A_1$ is $H_1$ and arrives in four possible ways, as is shown in Fig. 2. The power arriving directly from $A_2$ is $A_2 B_2 F_{21}$; that from $A_2$ after reflection on $A_3$
is $A_2 B_2 \rho_3 F_2 (3,1)$; that from $A_1$ after reflection on $A_4$ is $A_1 B_1 \rho_4 F_1 (4,1)$; and that from $A_1$ after reflection on $A_5$ is $A_1 B_1 \rho_5 F_1 (5,1)$.

Thus,

$$A_1 H_1 = A_2 B_2 (F_2 + \rho_3 F_2 (3,1)) + A_1 B_1 (\rho_4 F_1 (4,1) + \rho_5 F_1 (5,1)). \quad (3)$$

The net power from $A_1$, i.e., the process thermal power available in the crucible, is thus given by

$$q_{1A_1} = (\varepsilon_1 \sigma T_1^4 - \alpha_1 H_1) A_1. \quad (4)$$

**Energy balance on the aperture plane $A_2$**

The aperture plane is neither a source nor a sink. The power per unit area leaving $A_2$, the radiosity $B_2$, is the radiation power arriving at the aperture from the concentrator. It is given by

$$B_2 = P/A_2 = IA_0 h_c A_2, \quad (5)$$

where $P$ is the power from the concentrator. If the aperture is large enough to capture all the reflected energy coming from the concentrator, $P = IA_0 h_c A_2$, and the second of eqns. 5 can be
used.

The incident power per unit area on $A_2$, $H_2$, comes in two possible ways: directly from $A_1$, $A_1B_1F_{12}$; from $A_1$ after reflection on $A_3$, $A_1B_1\rho_3F_{1(3)}$. Thus,

$$A_2H_2 = A_1B_1(F_{12} + \rho_3F_{1(3)})$$  \hspace{1cm} (6)

The net power through the aperture is given by

$$q_2A_2 = (B_2 - H_2)A_2.$$  \hspace{1cm} (7)

This is the power that must ultimately be used as process heat or lost through the walls of the cavity.

**Energy balances on $A_2$, $A_4$, $A_5$.**

The net power lost through the walls of the cavity is the difference between the power incident on areas $A_3$, $A_4$, and $A_5$ and the power leaving them. The power per unit area incident on $A_3$, $H_3$, comes in two possible ways: directly from $A_1$, $A_1B_1F_{13}$; directly from $A_2$, $A_2B_2F_{23}$. Thus,

$$A_3H_3 = A_1B_1F_{13} + A_2B_2F_{23}.$$  \hspace{1cm} (8)
In a similar way it can be shown that

\[ A_4H_4 = A_1B_1F_{14} + A_2B_2F_{24}, \quad (9) \]

and

\[ A_5H_5 = A_1B_1F_{15} + A_2B_2F_{25}. \]  

(10)

If no portion of the incident sunlight entering through the opening strikes \( A_4 \), then the view factor \( F_{24} = 0 \). Also, since \( A_2 \) does not see \( A_5 \), \( F_{25} = 0 \). (Calculation of the remaining view factors is shown in Appendix B).

Since \( A_3 \), \( A_4 \), and \( A_5 \) were assumed to emit no radiation, their net power fluxes are given by

\[ q_i = -\alpha_i H_i, \quad (11) \]

where \( i = 3, 4, 5 \).

Overall energy conservation requires that

\[ \sum_{i=1}^{5} q_i A_i = 0. \]  

(12)
Equations (2-11) can be solved simultaneously to yield the dependent variables in terms of a set of independent variables, the parameters of the problem. For example, one may specify an energy flux through the aperture, emissivities, absorptivities, reflectivities, and the dimensions of all the internal surfaces. If one now imposes the constraint that the crucible must be at a particular temperature, the steady state solution will require that there be particular additional energy fluxes, i.e., heat transfer from the crucible and from the walls of the cavity. Energy transfer from the crucible is useful process heat. Energy transfer from the walls is that which is lost to the surroundings. The limiting temperature of the crucible, Eq. 13, is obtained by using the previous set and setting $q_z$ in Eq. 4 equal to zero. Thus,

$$T_{1,\text{max}} = \left[ \frac{IAe_{\text{cr}}(F_z + \rho_3 F_z x_{1,1})}{A\sigma (1-\rho_4 F_1 x_{1,1} - \rho_5 F_1 x_{1,1})} \right]^{1/4}. \quad (13)$$

As a consequence of Kirchhoff's identity, $\sigma = \varepsilon = 1 - \rho$, the emissivity and absorptivity have been eliminated and $T_{1,\text{max}}$ becomes independent of the crucible's absorptivity. With a given concentrator, however, it does depend on the size of the crucible. The smaller the crucible, the higher is the maximum achievable temperature. Nevertheless, to get any useful process heat we must operate at a crucible temperature lower than $T_{1,\text{max}}$. 
Monte Carlo method

This method has been widely used for the analysis of radiative transport\textsuperscript{13,14}. In using this method, we choose as our system the space bounded by the surfaces defined in Fig. 1, but not including the surfaces. The surfaces then become energy sources and sinks. We follow probable paths of discrete bundles of energy from sources outside the system, in this case the aperture and the crucible, until they finally leave the system through the sinks, in this case the cavity walls, the crucible, and the aperture. The bundles of energy coming into the cavity from the concentrator are assumed to go through the center of the aperture. This assumption gives an optimistic estimate of the energy absorption efficiency, but greatly simplifies the solution.

The number of bundles through the aperture in a particular direction is related to the incident-angle-dependent flux densities characteristic of the particular concentrator of interest. The characteristics of the University of Minnesota concentrator are described in Appendix C. The number of diffusely emitted bundles originating at the crucible is proportional to its emissive power. Their direction is chosen randomly from a set that is weighted according to a cosine distribution. Once we know the direction of an individual bundle we can determine the surface of incidence. Here a random choice, depending on the surface ab-
sorptivity, determines whether the bundle is absorbed or reflected. The aperture is treated as a surface whose absorptivity is one. When a bundle is absorbed, a count is recorded, and its history is terminated. The energy transferred to a surface is then easily computed as the number of bundles absorbed there. If the bundle is reflected, its direction is determined depending on whether the surface is a specular or diffusely reflecting one. This procedure is repeated for a large enough sample of energy bundles so that the results are statistically meaningful.

Because of the axial-symmetry of the geometry and power distribution, we found it convenient to divide the crucible and cavity walls into 18 rings each of which subtends a 10° angle in a plane of the receiver axis at the center of the crucible, as is shown in Fig. 3, and to assume that, over each of these rings, the incident power is uniformly distributed. The accuracy of the results depends on the number of subdivisions and the number of bundles of power considered. However, the computational cost of increasing these numbers is high. Sample sizes of 120,000 bundles of energy were found to be adequate for this study.

RESULTS

Our input parameters are the distribution of solar energy
incident on the receiver from the concentrator and the dimensions and radiative characteristics of the receiver-crucible surfaces. As our prototype concentrator we used the characteristics of our University of Minnesota furnace. On a typical sunny day in Minneapolis we may expect an insolation of 833 W/m², so we used that value for the insolation in this study, \( I \). The concentrator area, \( A_c \), is 13.18 m², the rim angle, \( \theta_{rim} \), is 45°, and the collector efficiency, \( \eta_c \), is 0.6. The incident power distribution is described in Appendix C. Our baseline design used: \( r_1 = 0.05 \) m; \( r_2 = 0.05 \) m; \( a = 0.25 \) m; eccentricity = \( c/a = 0.6 \); \( \epsilon_1 = 0.9 \); \( \epsilon_3 = \epsilon_4 = \epsilon_5 = 0.1 \). We then varied the radius of the crucible and its temperature to observe how they affect the thermal performance of the reactor.

We define the efficiency of the receiver-crucible as the ratio of the energy absorbed by the crucible, the process heat, to the energy incident on the cavity opening. Thus,

\[
\eta = \frac{q_1 A_1}{IA_c \eta_c}.
\]

The efficiency is plotted in Fig. 4 as a function of the crucible temperature, \( T_1 \), for different crucible radii. When the inner cavity walls (\( A_3, A_4, \) and \( A_5 \)) approach perfect mirrors in reflectivity, the energy loss through the walls approaches zero and the fraction of the incident energy being reradiated through the
aperture becomes \((1-n)\).

Figure 5, shows the power distribution on the cavity walls using our baseline parameters. It reveals the critical regions where cooling the walls may become necessary. Reference to Figs. 3 and 5 suggests that the region near the apse of the ellipsoid, the distal end which is most likely to be the site of a sting mount for a reactor, is also the part of the receiver which will have to carry the greatest cooling burden. That may present either a difficulty or an advantage. When one is working with a flow-through reactor, that region may be exploited for preheating reactants in a regenerative cooling process.

Figure 6 shows how the limiting temperature, given by Eq. 13, depends on the radius of the crucible. The eccentricity of the ellipsoid is the parameter. It is evident that higher temperatures can be achieved with smaller crucibles. With a 4 cm diameter crucible, the limiting temperature is 2200K; with a 2 cm diameter crucible, the limiting temperature is about 3100\(^\circ\)K. However, small crucibles require high precision in the geometry and optics of the cavity.

Insofar as the aperture size is concerned, \(T_{a,max}\) is almost independent of \(r_2\), varying by only 10\(^\circ\)K in the range \(r_2 = 0.03\) to 0.05 m. We can therefore make the aperture big enough to permit
the interception of all of the solar image without causing much degradation of the thermal performance of the reactor. Thus, we do not have to compromise to achieve a balance between radiation capture and reradiation losses. A parametric study of the dimensions of the crucible and the cavity may help us decide the optimum design according to the specifications of the problem.

CONCLUDING REMARKS

The analysis presented here can be extended to include free convection heat transfer between the crucible and the cavity walls through a non-radiation-participating medium. However, the principle of this design, all the incident radiation through the aperture will reach the crucible directly or after only one reflection on the cavity walls, fails if the cavity contains an absorbing, emitting, or scattering medium.

It does seem evident, however, that the idea is worthy of experimental investigation. Some good general rules to follow in the use of such receiver-reactors are evident. These are: (1) The crucible should be made of a good absorbing material, preferably with a long-wave emittance as low as possible to reduce losses. (2) The cavity walls must have high specular reflectance for radiation in the solar spectrum. They also must have a high thermal conductivity, so that the heat transferred by
conduction through the walls will flow fast enough to avoid a high gradient of temperatures between the inner and outer surfaces. (3) Energy loss to the walls can be reduced by the use of a windowed aperture and evacuation of the space between the receiver and the crucible. It is desirable that the window be transparent to radiation of wavelengths in the solar spectrum and have a high reflectivity to long-wave radiation, in order to recapture some of the infrared radiation being emitted by the crucible. Hemispheroidal windows would also minimize refraction of the sunlight passing through it from the concentrator.

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APPENDIX A, SYMBOLS

A surface area
a length of the semi-major axis of the ellipsoid
Ac nominal normal projected area of the concentrator
Al area of a concentrator mirror with rim angle \( \theta \)
Am projected area, in a plane normal to the furnace axis, of a concentrator mirror with rim angle \( \theta \)
ap aperture area
B  power leaving a surface per unit area, radiosity
b  length of the semi-minor axis of the ellipsoid
c  distance between the focal points and the center
    of the ellipsoid
F₁, F₂  focal points of the ellipsoid
Fᵢⱼ  view factor from surface i to surface j; fraction
    of the radiant energy leaving surface i which goes
directly to surface j.
Fᵢⱼ(k)  view factor from surface i to surface j after
        reflection from surface k
H  power incident on a surface per unit area
I  solar intensity
Pᵢⱼ  power contributed by a concentrating mirror hav-
    ing the rim angle Θᵢ to a receiver having an aper-
    ture of radius rᵢ
q  net power through a surface, per unit area; when
    q>0, energy is flowing into the cavity space
r  radius
T  Kelvin temperature
Vᵢⱼ  fraction of the incident power coming from a con-
    centrating mirror having the rim angle Θᵢ which is
captured by an aperture of radius rᵢ
α  absorptivity
αₑ  effective absorptivity of a conventional cavity-
    receiver
APPENDIX B, VIEW FACTORS

It can be shown by ray tracing that, for the baseline geometry, all the energy that comes from the aperture \( A_2 \) and is reflected in \( A_3 \) arrives at \( A_1 \). However, in some situations, depending on the eccentricity of the ellipsoid, dimensions of the aperture and crucible, and concentrator-rim-angle, some radiation will miss the crucible. We have neglected this loss, although there may be circumstances in which it could become significant. An analogous statement might be made about radiation from \( A_1 \) to \( A_2 \). Thus, \( F_{1,3,2} \approx F_{1,3} \), and \( F_{2,3,1} \approx F_{2,3} \). Moreover, all the energy that comes from \( A_1 \) and is reflected in \( A_4 \) or \( A_5 \), goes back to \( A_1 \). Thus, \( F_{1,4,1} = F_{1,4} \), and \( F_{1,5,1} = F_{1,5} \).

Because \( A_2, A_3, A_4, \) and \( A_5 \) form a closed space,
\[ F_{12} + F_{13} + F_{14} + F_{15} = 1. \]

Using the view factor from a sphere to a disk of radius \( r \) and at a distance \( h \), whose normal passes through the center of the sphere,

\[
F_{\text{sphere-disk}}(r,h) = 0.5 \left[ 1 - \frac{1}{(1+(r/h)^2)^{0.8}} \right],
\]

we get

\[
F_{12} = F_{\text{sphere-disk}}(r_2, 2c),
\]

\[
F_{14} = F_{\text{sphere-disk}}(y_1, c+x_1) - F_{12},
\]

\[
F_{15} = F_{\text{sphere-disk}}(|y_2|, |x_2| - c),
\]

and

\[
F_{13} = 1 - (F_{12} + F_{14} + F_{15}).
\]

\( F_{21} \) is not the geometrical view factor from \( A_2 \) to \( A_1 \), but the portion of the incident solar radiation entering through \( A_2 \) that strikes \( A_1 \). This incident radiation is a function of the geometry of the concentrating device. If the solar radiation is incident on the opening at a rim angle \( \theta_{r1m} \), then, use of the
reciprocity relation gives

\[ F_{21} = (A_1/A_2)F_{12}(1 - \cos \theta_{1m}). \]

As we noted before, we assume that no portion of the incident radiation strikes \( A_4 \) or \( A_5 \). Hence, \( F_{24} = F_{25} = 0 \), and \( F_{23} = 1 - F_{21} \).

APPENDIX C, INCIDENT POWER DISTRIBUTION

The flux distribution of the incoming power from the concentrator through the aperture can be determined from the geometry of the concentrator and the aperture. For the solar concentrator of the University of Minnesota, the calculation is done following the model developed in Ref. 9.

The power contributed by each concentrating mirror to a receiver having an aperture of radius \( r_j \) is \( P_{a_j} = V_{1j}IA_{n1} \), where \( V_{1j} \) is the fraction of the incident power captured by the aperture of radius \( r_j \), and \( A_{n1} = A_1 \cos (\theta_1/2) \) is the projected area of the concentrating mirror having the rim angle \( \theta_1 \), in the plane normal to the furnace axis.

Figure 7 shows the flux distribution for an aperture big
enough \((r_J > 3.2 \text{ cm})\) so that \(V_{s,3} = 1\), for a solar flux of 833 W/m² and a collection efficiency of 0.6.
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8. R.B. Diver, D.E. Carlson, F.J. MacDonald, and E.A. Fletcher,


FIGURE CAPTIONS

Fig. 1. Schematic diagram of the receiver-reactor, which shows our literal representation of the objects and surfaces which are pertinent in the analysis.

Fig. 2. Light paths of the incident, from the aperture, radiation and reradiation from the crucible.

Fig. 3. Subdivision of the surfaces of interest for the Monte-Carlo analysis.

Fig. 4. Variation of the energy collection efficiency with the crucible temperature for the baseline configuration. The parameter is the radius of the crucible. The data points were calculated by the Monte-Carlo method. The lines drawn in association with them are the results of the algebraic solutions.

Fig. 5. Variation of the power which must be conducted through the wall of the receiver with axial position according to the designation defined in Fig. 3. The Monte-Carlo method was used with our baseline configuration.

Fig. 6. Limiting surface temperature of the crucible as a function of its radius for our baseline device. The eccentricity of
the ellipsoid is the parameter. The algebraic solution was used.

Fig. 7. Distribution of the energy incident from the concentrator, taken from Ref. 9.
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| Dr. William Tolles                  | Naval Ocean Systems Center |
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