COMPUTER PROGRAM TO CALCULATE THE PHYSICAL PROPERTIES OF ONE SABOT PETAL(U) ARMY BALLISTIC RESEARCH LAB ABERDEEN PROVING GROUND MD R A PENNEKAMP JUN 87 UNCLASSIFIED BRL-MR-3598 F/G 19/10 NL
APPENDIX B

AN APPROACH TO CALCULATE
THE FIRST N REACTION OF GNE
AND ITS POLARIZATION

S. S. PARK \& S. S. CHOI

JUNE 1967

J. LENNY BALISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

87 12 17 004
A FORTRAN computer program was developed to calculate the following physical properties of one sabot petal: the mass, the center of mass, and the complete inertia tensor about the center of mass. Only uniform density sabot petals are permitted. Any sabot segment angle is allowed up to 360 degrees, which is a complete body of revolution. This report includes the theory, the inertia tensor transformation equations, and listing of the program, and an example problem.
# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>v</td>
</tr>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Theory</td>
<td>1</td>
</tr>
<tr>
<td>III. Inertia Tensor Transformations</td>
<td>8</td>
</tr>
<tr>
<td>IV. Test Cases</td>
<td>12</td>
</tr>
<tr>
<td>V. Conclusions</td>
<td>15</td>
</tr>
<tr>
<td>List of References</td>
<td>21</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>23</td>
</tr>
<tr>
<td>Appendix A. Program Listing</td>
<td>29</td>
</tr>
<tr>
<td>Appendix B. Example Problem</td>
<td>35</td>
</tr>
<tr>
<td>Distribution List</td>
<td>37</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>1</td>
<td>Global coordinate system</td>
</tr>
<tr>
<td>2</td>
<td>Local coordinate system</td>
</tr>
<tr>
<td>3</td>
<td>Integration nomenclature</td>
</tr>
<tr>
<td>4</td>
<td>Lower limit of r explanation</td>
</tr>
<tr>
<td>5</td>
<td>Rotation transformation relationships</td>
</tr>
<tr>
<td>6</td>
<td>Machine vs program test case</td>
</tr>
<tr>
<td>7</td>
<td>Inertia tensor transformation example</td>
</tr>
<tr>
<td>B-1</td>
<td>Example problem contour</td>
</tr>
<tr>
<td>B-2</td>
<td>Example problem data file</td>
</tr>
<tr>
<td>B-3</td>
<td>Example problem solution</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

A FORTRAN program was written to find the physical properties of one sabot petal. The program was written to provide a simple analytical model to determine these properties. This analytical model is an easy alternative to making actual physical measurements. Though the program was developed for analyzing a sabot petal, it can be used for any homogeneous partial or complete body of revolution. The only restriction is that the body of revolution must be defined by an angle $\theta_T$ as shown in Figure 1.

Section II provides the theory for finding the physical properties of one sabot petal. The program takes user-provided contour points of the sabot's RZ plane profile (Figure 1) and uses them to divide the sabot petal into a finite number of partial pieces. The program then finds the physical properties of each partial piece and by using the correct addition formulas, adds all the partial piece results to get the solution for the total sabot petal. Other than the coordinates of the profile, the only information the user provides is the material density of the sabot and the angle of the sabot petal, $\theta_T$.

In Section III, the procedure for transforming the inertia tensor to any Cartesian coordinate is described. All the equations are provided, but the user must be careful of two things. First, the user must not confuse skew lines for parallel lines in the translation transformation equations. Second, the user must follow the "right-handed rule" when determining the sign of the rotation angle in the rotation transformation equations.

Test cases are found in Section IV. The theory is tested on a 25mm aeroballistic model by comparing the results found by physical measurement machines with the answers predicted by the program. The transformation equations are shown to work on a uniform rod.

In Appendix A there is a listing of the program, and in Appendix B an example problem is solved following step-by-step instructions on the execution of the program.

II. THEORY

The sabot petal is divided into partial pieces as required to describe its particular geometry. This is done by describing the profile of the sabot petal in the RZ plane (Figure 1) using contour points. The program uses the contour points to construct the partial pieces and then finds their physical properties. After each partial piece's physical properties are found, the physical properties of the total sabot petal are found by adding the partial piece's properties together using the appropriate addition formulas.

The first physical property found is the volume. The volume for any partial piece $j$ of the total sabot petal is:

\[ V(j) = \iiint dV \]  \hspace{1cm} (1)
If a local cylindrical coordinate system is used (Figure 2), Equation (1) becomes:

\[ v(j) = \int_{0}^{\theta_1} \int_{0}^{r_{mz+b}} \int_{0}^{Z_t} r \, dr \, dz \, d\theta \]  

(2)

See Figure 3 for integration nomenclature and Figure 2 for the definition of \( \theta_1 \). The resulting equation for the volume of partial piece "j" is:

\[ v(j) = \theta_1 \left( m^2 z_t^3/3 + z_t^2 m b + b^2 z_t \right) \]  

(3)

Note that the upper limit of \( r \) is restricted to straight line segments. This means that any nonlinear curve in the ZR plane must be approximated by straight line segments. If this is necessary, the approximating error can be minimized by making the number of approximating segments sufficiently large. This is the only approximation used in the program.

The lower limit of \( r \) in Equation (2) is always taken as zero. The sabot petal, which is a body with cavities, is handled by the program as follows: The outer surfaces of the sabot define solid bodies of calculable volume; the inner surfaces, which are the boundaries of the cavities, define other solid bodies whose volumes are flagged to be negative. The volume for the total sabot petal is then computed as the sum of the positive and negative volumes. This approach made the programming easier and, more importantly, kept the required input data to a minimum. Figure 4 illustrates the above procedure. This procedure is used for all the physical properties found by this program.

The equation for the volume of the total sabot petal is:

\[ V = \sum_{j=1}^{n} v(j) \]  

(4)

where \( v(j) \) is the volume of partial piece "j"

\( n \) is the number of partial pieces.

Because only a uniform density is allowed, the total mass of the sabot petal is:

\[ M = V \rho \]  

(5)

where \( \rho \) is the density of the material.

With the total mass found, the center of mass can be calculated. The \( x \) location for the center of mass of any body is defined as:

\[ x_c = \frac{\iiint \rho x \, dv}{\iiint \rho \, dv} \]  

(6)
Since the density is assumed to be uniform, it can be taken out of the integrals and canceled out. This leaves a denominator equal to the volume of the mass in question. Since the volume of any partial piece "j" has been determined, the only integral that needs to be determined to calculate the x center of mass for any partial piece "j" is:

$$\iiiint x \, dv$$  \hspace{1cm} (7)

In a local cylindrical coordinate system, Expression (7) becomes

$$\int_0^{z_t} \int_0^{mz+b} \int_0^{\theta_1} r^2 \cos \theta \, dr \, d\theta \, dz$$  \hspace{1cm} (8)

Expression (8) is evaluated to be:

$$\frac{2}{3} \sin \theta_1 \left\{ \frac{1}{4} m^3 z_t^4 + m^2 z_t^3 b + \frac{3}{2} b^2 m z_t^2 + b^3 z_t \right\}$$  \hspace{1cm} (9)

The x location of the center of mass for partial piece "j" in the global coordinate system is

$$xc(j) = \frac{(2/3)\sin \theta_1 \left\{ \frac{1}{4} m^3 z_t^4 + m^2 z_t^3 b + \frac{3}{2} b^2 m z_t^2 + b^3 z_t \right\}}{v(j)}$$  \hspace{1cm} (10)

The x location of the center of mass for the total sabot petal in the global coordinate system is the sum of the volumes v(j) multiplied by the center of mass xc(j). This value is then divided by the total volume of the sabot petal to give XC, the x location of the center of mass of the total sabot petal in the global coordinate system. This is done in Equation (11).

$$XC = \frac{\sum_{j=1}^{n} v(j) xc(j)}{\sum_{j=1}^{n} v(j)}$$  \hspace{1cm} (11)

The definition of the y location of the center of mass is similar to the definition of the x location of the center of mass. The only integration that needs to be performed to determine the y center of mass for any partial piece "j" is:
However,
\[ \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta = 0 \] (13)

This means the y location of the center of mass of all the "j" partial pieces is 0. This result was expected by the argument of symmetry (Figure 2). By replacing all the "x"s with "y"s in Equation (11) and using the result from Equation (13), the y location of the center of mass for the total sabot petal, \( YC \), is 0.

The definition of the z location of the center of mass is the same as the definition of the x and y locations of the center of mass. The integral, which is similar to Expressions (8) and (12), is:
\[ \int_{0}^{Z_t} \int_{0}^{mz+b} \int_{\theta_1}^{\theta_2} r^2 \sin \theta \, d\theta \, dr \, dz. \] (14)

Expression (14) is evaluated to be:
\[ \theta_1 \{ (1/4)Z_t^4m^2 + (2/3)Z_t^3mb + (1/2)b^2Z_t^2 \} \] (15)

The z location for the center of mass of partial piece "j" in the global coordinate system is:
\[ zc(j) = Z_0 + \frac{\theta_1 \{ (1/4)Z_t^4m^2 + (2/3)Z_t^3mb + (1/2)b^2Z_t^2 \}}{v(j)} \] (16)

where \( Z_0 \) is the location of the local coordinate system's origin in the global coordinate system (Figure 2). The z location for the center of mass for the total sabot petal in the global coordinate system is found the same way the x location was found.
\[ ZC = \frac{\sum_{j=1}^{n} v(j)zc(j)}{\sum_{j=1}^{n} v(j)} \] (17)
The mass moments of inertia are defined as

\[ I_{xx} = \iiint \rho(y^2 + z^2) \, dV \quad (18) \]
\[ I_{yy} = \iiint \rho(x^2 + z^2) \, dV \quad (19) \]
\[ I_{zz} = \iiint \rho(x^2 + y^2) \, dV \quad (20) \]

Equations (18), (19) and (20) for partial piece "j" in its local cylindrical coordinate system are:

\[ i_{xx}(j) = \rho \int_0^{Z_t} \int_0^{mz+b} \int_{\theta_1}^{\theta_1} (z^2r + r^3\sin^2\theta) \, d\theta \, dr \, dz \quad (21) \]
\[ i_{yy}(j) = \rho \int_0^{Z_t} \int_0^{mz+b} \int_{\theta_1}^{\theta_1} (z^2r + r^3\cos^2\theta) \, d\theta \, dr \, dz \quad (22) \]
\[ i_{zz}(j) = \rho \int_0^{Z_t} \int_0^{mz+b} \int_{\theta_1}^{\theta_1} r^3 \, d\theta \, dr \, dz \quad (23) \]

Equations (21), (22), and (23) are evaluated to be:

\[ i_{xx}(j) = \rho \left( \theta_1 + \frac{\sin2\theta_1}{2} \right) \left( \frac{m^4z_t^5}{20} + \frac{b^2m^2z_t^3}{2} + \frac{b^3mz_t^2}{2} + \frac{b^4z_t}{4} \right) \]
\[ + \frac{\theta_1z_t^5m^2}{5} + \frac{\theta_1z_t^4mb}{2} + \frac{\theta_1z_t^3b^2}{3} \quad (24) \]

\[ i_{yy}(j) = \rho \left( \theta_1 + \frac{\sin2\theta_1}{2} \right) \left( \frac{m^4z_t^5}{20} + \frac{b^2m^2z_t^3}{2} + \frac{b^3mz_t^2}{2} + \frac{b^4z_t}{4} \right) \]
\[ + \frac{\theta_1z_t^5m^2}{5} + \frac{\theta_1z_t^4mb}{2} + \frac{\theta_1z_t^3b^2}{3} \quad (25) \]
Equations (24), (25), and (26) are partial piece "j"'s mass moments of inertia about its own local coordinate axes. In order to find the mass moments of inertia for the total body, all the partial piece results must be found about axes through the centers of mass of the partial pieces. This is accomplished by the use of the parallel axis theorem:

\[ I_{xx} = I_{x'x'} + M d^2 \]  

where \( I_{xx} \) is the moment of inertia about an axis through the center of mass

\( I_{x'x'} \) is the moment of inertia about an axis that is parallel to an axis through the center of mass

\( M \) is the mass of the object

\( d \) is the distance between the \( xx \) and \( x'x' \) axes

The mass moments of inertia of partial piece "j" about axes through its center of mass and parallel to its local system are derived from Equation (27) and the results from Equations (24), (25), and (26). They are:

\[ i_{x'x'}(j) = i_{xx}(j) - \rho v(j)[zc(j) - Z0]^2 \]  

\[ i_{y'y'}(j) = i_{yy}(j) - \rho v(j)[xc(j)^2 + (zc(j) - Z0)^2] \]  

\[ i_{z'z'}(j) = i_{zz}(j) - \rho v(j)xc(j)^2 \]  

The mass moments of inertia for the total sabot petal about axes through its center of mass and parallel to the global coordinate system are

\[ IX'X' = \sum_{j=1}^{n} i_{x'x'}(j) + \rho v(j)(ZC - zc(j))^2 \]  

\[ IY'Y' = \sum_{j=1}^{n} i_{y'y'}(j) + \rho v(j)[(ZC - zc(j))^2 + (XC - xc(j))^2] \]  

\[ IZ'Z' = \sum_{j=1}^{n} i_{z'z'}(j) + \rho v(j)[(ZC - zc(j))^2 + (XC - xc(j))^2] \]  

\[ i_{zz}(j) = \frac{\rho_1 b}{2} \left\{ \frac{m^4 z_t^5}{5} + m^3 z_t b + 2b^2 m^2 z_t^3 + 2b^3 m z_t^2 + b^4 z_t \right\} . \]  

(26)
IZ'Z' = \sum_{j=1}^{n} iz'z'(j) + \rho v(j)(xc - xc(j))^2 . \quad (33)

Equations (31), (32), and (33) were derived from the parallel axis theorem for mass moments of inertia and Equations (28), (29), and (30).

The mass product of inertia terms are defined as

\[ I_{xy} = \iiint \rho XY \, dV \quad (34) \]
\[ I_{xz} = \iiint \rho XZ \, dV \quad (35) \]
\[ I_{yz} = \iiint \rho YZ \, dV . \quad (36) \]

For partial piece "j" in its local cylindrical coordinate system, Equations (34), (35) and (36) reduce to

\[ i_{xy}(j) = \rho \int_{0}^{z_{2}} \int_{0}^{mz+b} \int_{\theta_{1}}^{\theta_{2}} r^3 \cos \theta \sin \theta \, d\theta \, dr \, dz \quad (37) \]
\[ i_{xz}(j) = \rho \int_{0}^{z_{2}} \int_{0}^{mz+b} \int_{\theta_{1}}^{\theta_{2}} zr^2 \cos \theta \, d\theta \, dr \, dz \quad (38) \]
\[ i_{yz}(j) = \rho \int_{0}^{z_{2}} \int_{0}^{mz+b} \int_{\theta_{1}}^{\theta_{2}} zr^2 \sin \theta \, d\theta \, dr \, dz . \quad (39) \]

Equations (37), (38) and (39) are evaluated to be

\[ i_{xy}(j) = 0 \quad (40) \]

\[ i_{xz}(j) = (2/3)\rho \sin \theta_1 \left[ \frac{z_t^5m^3}{5} + \frac{3}{4} z_t^4m^2b + z_t^3mb^2 + \frac{z_t^2b^3}{2} \right] \quad (41) \]
\[ i_{yz}(j) = 0 . \quad (42) \]
Equations (40), (41) and (42) are the mass product of inertia expressions for partial piece "j" about its local coordinate system. Like the mass moments of inertia, the product of inertia terms need to be found about axes through the center of mass of the partial piece "j". This is accomplished by using the parallel axis theorem for products of inertia.²

\[
I_{xy} = I_{x'y'} + M X_c Y_c \tag{43}
\]

\[
I_{xz} = I_{x'z'} + M X_c Z_c \tag{44}
\]

\[
I_{yz} = I_{y'z'} + M Y_c Z_c \tag{45}
\]

where \( I_{x'y'}, I_{x'z'} \) and \( I_{y'z'} \) are the products of inertia about the center of mass.

\( I_{xy}, I_{xz} \) and \( I_{yz} \) are the products of inertia about a set of axes parallel to the center of mass.

\( X_c, Y_c \) and \( Z_c \) are the \( x, y \) and \( z \) locations of the center of mass in an \( XYZ \) coordinate system.

Equations (44) and (41) are used to generate the equation for \( I_{xz} \) for partial piece "j" about its own center of mass:

\[
i_{x'z'}(j) = i_{xz}(j) - \rho v(j)x_c(j)(z_c(j) - Z_0). \tag{46}
\]

\( I_{xz} \) for the total sabot petal about axes through its center of mass and parallel to the global coordinate system, is

\[
I_{X'Z'} = \sum_{j=1}^{n} i_{x'z'}(j) + \rho v(j)(x_c(j) - X)(z_c(j) - Z). \tag{47}
\]

Equation (47) was arrived at by the use of the parallel axis theorem for products of inertia and Equation (46). Because the \( y \) center of mass is zero for every "j" partial piece (Equation (13)) and \( i_{xy}(j) \) is zero for every "j" partial piece (Equation (40)), by Equation (43), \( i_{x'y'}(j) \) for every "j" partial piece is zero. The total sabot petal's \( I_{x'y'}, I_{X'Y'} \), is zero by the same argument. The same argument holds for \( I_{y'z'} \) for the total sabot petal; therefore \( I_{Y'Z'} \) is zero.

III. INERTIA TENSOR TRANSFORMATIONS

The program calculates the inertia tensor about a set of axes that go through the center of mass of the sabot petal. This section explains how to
transform the inertia tensor properly from this coordinate system to any other Cartesian coordinate system. There are two transformation procedures: translation and rotation. Either procedure can be done independently or they can be done in successive steps. The equations of this section make use of the results found by the program as much as possible for ease of use. Care should be taken if these equations are to be used for anything but this purpose.

Translation of the inertia tensor requires the use of both types of parallel axis theorems used in Section II of this report. Remember to use consistent units and make sure the axes are parallel. The mass moment of inertia terms are translated by Equation (27). With the results found by the program, the general Equation (27) is rewritten as the following equations.

\[ I'_{XX} = I_{XX} + M[(ZC - Z'O)^2 + (-Y'O)^2] \]  
\[ I'_{YY} = I_{YY} + M[(XC - X'O)^2 + (ZC - Z'O)^2] \]  
\[ I'_{ZZ} = I_{ZZ} + M[(XC - X'O)^2 + (-Y'O)^2] \]  

where \( X'O, Y'O \) and \( Z'O \) are the \( X, Y \) and \( Z \) locations of the desired coordinate system's origin in the global coordinate system.

For the product of inertia terms, Equations (43), (44) and (45) are rewritten as:

\[ I'_{XY} = M(XC - X'O)(-Y'O) \]  
\[ I'_{XZ} = I_{XZ} + M(XC - X'O)(ZC - Z'O) \]  
\[ I'_{YZ} = M(-Y'O)(ZC - Z'O) \]  

Note that all the values in Equations (48-53) are found by the program except \( X'O, Y'O \) and \( Z'O \), which are user-defined.

Only one translation is required to translate the coordinate system of the inertia tensor found by the program to any other coordinate system in space. This is important for the user to know because Equations (48-53) are only valid for the case where the moments and products of inertia are known about axes that go through the center of mass of the body. This means the user cannot take the results from one translation and use Equations (48-53) to make another translation. In general, only one translation is allowed.

Rotation of the coordinate system is allowed and one proper way to do this is presented. The equation to rotate the inertia tensor is:

\[ [T'] = [R][T][B]^{-1} \]  

9
where: 

- $[T']$ is the inertia tensor about the new coordinate system
- $[T]$ is the inertia tensor about the old coordinate system
- $[B]$ is the transformation rotation tensor
- $[B]^{-1}$ is the inverse of the transformation rotation tensor.

Equation (54) allows rotation about a point, but for simplicity only the rotation about any of the coordinate axes will be found. This may require more than one rotation to get to the desired coordinate system, but this is allowed as long as the procedure is followed as directed. Before Equation (54) can be solved, the $B$ tensor must be determined. Figure 5 and Equations (55-63) show how to transform the coordinates of an EFG coordinate system to coordinates in an E'F'G' coordinate system. The transformation is a rotation about the G axis; therefore $G' = G$.

\[
\begin{align*}
    d_1 &= ftan\theta \\
    L_1 &= e + d_1 \\
    e' &= L_1cos\theta \\
    e' &= ecos\theta + fsin\theta \\
    d_2 &= etan\theta \\
    L_2 &= f - d_2 \\
    f' &= L_2cos\theta \\
    f' &= -esin\theta + fcos\theta \\
    g' &= g 
\end{align*}
\]

In matrix form, the rotation transformation from the EFG coordinate system to the E'F'G' coordinate system is:

\[
\begin{bmatrix}
    e' \\
    f' \\
    g' 
\end{bmatrix} =
\begin{bmatrix}
    cos\theta & sin\theta & 0 \\
    -sin\theta & cos\theta & 0 \\
    0 & 0 & 1 
\end{bmatrix}
\begin{bmatrix}
    e \\
    f \\
    g 
\end{bmatrix}
\]

(64)
Therefore
\[
[B] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}.
\] (65)

A property of a transformation tensor that rotates one Cartesian coordinate system to another is:

\[
[B]^{-1} = [B]^T
\] (66)

where the "T" superscript denotes a transposed matrix.

Equations (65) and (66) allow Equation (54) to be written in matrix form as:

\[
\begin{bmatrix}
IEE' & IEF' & IEG' \\
IFE' & IFF' & IFG' \\
IGE' & IGF' & IGG'
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
IEE & IEF & IEG \\
IFE & IFF & IFG \\
IGE & IGF & IGG
\end{bmatrix} \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (67)

where: \( \theta \) is the angle of rotation shown in Figure 5.
- EFG is the coordinate system shown in Figure 5.

The equations that result from the solution of Equation (67) are

\[ IEE' = IEE \cos^2 \theta + IFF \sin^2 \theta + IEF \sin \theta \] (68)
\[ IFF' = IEE \sin^2 \theta + IFF \cos^2 \theta - IEF \sin \theta \] (69)
\[ IGG' = IGG \] (70)
\[ IEF' = IEF' - \frac{1}{2}(IFF - IEE) \sin \theta + IEF(\cos \theta - \sin \theta) \] (71)
\[ IEG' = IGE' = IEG \cos \theta + IGF \sin \theta \] (72)
\[ IFG' = IGF' = IFG \cos \theta - IEG \sin \theta \] (73)

Equations (68-73) are generalized equations and to use them in the XYZ coordinate system make the following substitutions. For a rotation about the

<table>
<thead>
<tr>
<th>X axis</th>
<th>Y axis</th>
<th>Z axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let ( Y = E )</td>
<td>Let ( Z = E )</td>
<td>Let ( X = E )</td>
</tr>
<tr>
<td>( Z = F )</td>
<td>( X = F )</td>
<td>( Y = F )</td>
</tr>
<tr>
<td>( X = G )</td>
<td>( Y = G )</td>
<td>( Z = G )</td>
</tr>
</tbody>
</table>
in Equations (68-73).

The above instructions must be followed to insure a proper transformation from one right-handed Cartesian coordinate system to another. As many rotations as required are allowed using Equations (68-73). If a translation and rotation of the inertia tensor are to be performed, translate by Equations (48-53), then rotate as many times as necessary by Equations (68-73). If a rotation of the axis system is performed, Equations (51) and (53) are in general no longer valid.

IV. TEST CASES

The purpose of the program was to provide an alternative to making actual physical measurements to determine the physical properties of one sabot petal. The test of the program was to compare the physically measured properties with properties found by the program. The sabot petal used for the test is from a 25mm aeroballistic model. The petal is made from aluminum and is one quarter of a complete sabot package. It is shown in Figure 6.

The sabot petal mass, center of mass and mass moments of inertia were found by physical measurement machines located at the Transonic Range facility of Aberdeen Proving Ground. These machines use precision air bearings and have an accuracy of 0.03%. The program determined the properties using 22 contour points. The results for this particular case have a maximum difference of 4.8% when compared with the results from physical measurement equipment. If 203 points are used to more accurately describe the buttress grooves and circular arcs, the maximum difference drops to 2.9%. The comparison of the three cases is shown in Figure 6. This shows a trend towards smaller percentage differences when the number of contour points is increased. If the theory is correct, this trend should occur.

An example of the use of the transformation equations is provided. A uniform rod is divided into six pieces as shown in Figure 7. The inertia properties of each piece are determined by the program. If the transformation equations are correct, the total rod's inertia tensor about its center of mass can be found by transforming all the partial piece results to the location of the center of mass of the total rod and adding them together.

TABLE 1 shows the physical properties calculated by the program for each partial piece. These results are then translated using Equations (48-53). The translated results are shown in TABLE 2. The translated results are then rotated using Equations (68-73) and the results are shown in TABLE 3. With these transformations completed, all the partial pieces' inertia tensors are known about the same set of axes, a set whose origin is the center of mass of the total rod. This allows the results found in TABLE 3 to be added and the sums should equal the inertia terms for the total rod.
TABLE 1. Program-determined Properties.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>MASS</td>
<td>.3173</td>
<td>.0529</td>
<td>.4760</td>
<td>.3173</td>
<td>1.481</td>
<td>.5288</td>
</tr>
<tr>
<td>XC</td>
<td>.4244</td>
<td>.6591</td>
<td>.6002</td>
<td>.6366</td>
<td>.3514</td>
<td>.4919</td>
</tr>
<tr>
<td>ZC</td>
<td>1.000</td>
<td>1.000</td>
<td>3.000</td>
<td>3.000</td>
<td>4.000</td>
<td>2.000</td>
</tr>
<tr>
<td>IX'X'</td>
<td>.1851</td>
<td>.0182</td>
<td>1.471</td>
<td>.9656</td>
<td>8.318</td>
<td>.8121</td>
</tr>
<tr>
<td>IY'Y'</td>
<td>.1279</td>
<td>.0205</td>
<td>1.451</td>
<td>.9682</td>
<td>8.034</td>
<td>.7346</td>
</tr>
<tr>
<td>IZ'Z'</td>
<td>.1015</td>
<td>.0035</td>
<td>.0665</td>
<td>.0301</td>
<td>.5575</td>
<td>.1364</td>
</tr>
<tr>
<td>IX'Z'</td>
<td>-103</td>
<td>-103</td>
<td>-103</td>
<td>-103</td>
<td>-103</td>
<td>-103</td>
</tr>
<tr>
<td>X 10^7</td>
<td>-0.149</td>
<td>-0.037</td>
<td>-0.596</td>
<td>-0.000</td>
<td>-2.38</td>
<td>0.000</td>
</tr>
</tbody>
</table>

TABLE 2. Tensor Translations.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ixx</td>
<td>5.257</td>
<td>.866</td>
<td>3.373</td>
<td>2.234</td>
<td>9.799</td>
<td>5.573</td>
</tr>
<tr>
<td>Iyy</td>
<td>5.257</td>
<td>.892</td>
<td>3.526</td>
<td>2.365</td>
<td>9.697</td>
<td>5.624</td>
</tr>
<tr>
<td>Izz</td>
<td>.158</td>
<td>.026</td>
<td>.238</td>
<td>.159</td>
<td>.740</td>
<td>.264</td>
</tr>
<tr>
<td>Ixy</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Ixz</td>
<td>-0.538</td>
<td>-1.39</td>
<td>-0.571</td>
<td>-0.403</td>
<td>-0.520</td>
<td>-0.781</td>
</tr>
<tr>
<td>Iyz</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
TABLE 3. Tensor Rotations.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Sum 1-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ixx</td>
<td>5.257</td>
<td>.890</td>
<td>3.384</td>
<td>2.332</td>
<td>9.792</td>
<td>5.576</td>
<td>27.231</td>
</tr>
<tr>
<td>Iyy</td>
<td>5.257</td>
<td>.868</td>
<td>3.516</td>
<td>2.267</td>
<td>9.704</td>
<td>5.621</td>
<td>27.233</td>
</tr>
<tr>
<td>Izz</td>
<td>.158</td>
<td>.026</td>
<td>.238</td>
<td>.159</td>
<td>.740</td>
<td>.264</td>
<td>1.585</td>
</tr>
<tr>
<td>Ixy</td>
<td>0.000</td>
<td>.007</td>
<td>.038</td>
<td>-.057</td>
<td>.026</td>
<td>-.013</td>
<td>.001</td>
</tr>
<tr>
<td>Ixz</td>
<td>-.538</td>
<td>.036</td>
<td>.552</td>
<td>.202</td>
<td>.502</td>
<td>-.754</td>
<td>0.000</td>
</tr>
<tr>
<td>Iyz</td>
<td>0.000</td>
<td>-.134</td>
<td>-.148</td>
<td>.349</td>
<td>.135</td>
<td>-.202</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The inertia terms for a homogeneous right circular cylinder are found by the following equations:\(^2\)

\[
\begin{align*}
I_{xx} &= I_{yy} = \frac{1}{12}M(3r^2 + h^2) \\
I_{zz} &= \frac{1}{2}Mr^2 \\
I_{xy} &= I_{xz} = I_{yz} = 0
\end{align*}
\]

where \(M\) is the mass of the rod, \(r\) is the radius of the circular cross section and \(h\) is the height of the cylinder.

The inertia terms for the total rod are

\[
\begin{align*}
I_{xx}' &= I_{yy}' = 27.235 \text{ lb-in}^2 \\
I_{zz}' &= 1.587 \text{ lb-in}^2 \\
I_{xy}' &= I_{xz}' = I_{yz}' = 0
\end{align*}
\]
If the transformation equations are correct, the expected results found by Equations (77-79) should equal the results found in the sum row in TABLE 3. It has been shown that for this particular problem, the transformation equations work. The small differences in the expected results are due to round-off error.

V. CONCLUSIONS

Inertia properties are sometimes important to know. Shapes like sabot petals usually make it difficult to calculate inertia terms analytically. The alternative to an analytical calculation is to make actual physical measurements. Access to physical measurement equipment is sometimes difficult and getting the necessary information can take some time. The intention of this program is to provide this information quickly and accurately.

The inertia properties of an object are defined about coordinate axes. Because the inertia information may be needed about axes other than the defined axes, transformation equations were derived. The report includes all the equations needed to translate and rotate the inertia tensor to the required coordinate system. The equations were shown to work in a selected example. The equations should make the process of coordinate transformation easier for the user.
Figure 1. Global coordinate system.

Figure 2. Local coordinate system.
\[ (Z, R) \text{ GLOBAL COORDINATE SYSTEM} \]

\[ r \text{ LOCAL COORDINATE SYSTEM} \]

\[ m = \frac{(r_2 - r_1)}{Z_t} \]

\[ b = r_1 \]

\[ \square \text{ PARTIAL PIECE 'j'} \]

---

**Figure 3.** Integration nomenclature.

---

**Figure 4.** Lower limit of \( r \) explanation.
ROTATION ABOUT THE G AXIS

Figure 5. Rotation transformation relationships.

MACHINE

<table>
<thead>
<tr>
<th>MACHINE</th>
<th>MASS</th>
<th>XC</th>
<th>ZC</th>
<th>IXX</th>
<th>IYY</th>
<th>IZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>25MM SABOT</td>
<td>.03550</td>
<td>.2606</td>
<td>2.027</td>
<td>.02895</td>
<td>.02865</td>
<td>.0000659</td>
</tr>
<tr>
<td>22 PT.</td>
<td>.03493</td>
<td>.2609</td>
<td>2.049</td>
<td>.02847</td>
<td>.02820</td>
<td>.0000240</td>
</tr>
<tr>
<td>203 PT.</td>
<td>.03537</td>
<td>.2512</td>
<td>2.045</td>
<td>.02882</td>
<td>.02864</td>
<td>.0000807</td>
</tr>
</tbody>
</table>

UNITS - INCHES & lb (f)

Figure 6. Machine vs program test case.
Figure 7. Inertia tensor transformation example.
LIST OF REFERENCES


LIST OF SYMBOLS

b  local variable; r intercept; used with m to define the upper bound of integration on r

d  distance

B  rotation transformation tensor

dr  local r differential length

dV  differential volume

dz  local z differential length

e  Cartesian coordinate used to define the rotation transformation tensor

e'  Cartesian coordinate that results after a rotation about the G axis

E  Cartesian axis used to define the rotation transformation tensor

E'  Cartesian axis that results after a rotation about G axis

f  Cartesian coordinate used to define the rotation transformation tensor

f'  Cartesian coordinate that results after a rotation about the G axis

F  Cartesian axis used to define the rotation transformation tensor

F'  Cartesian axis that results after a rotation about G axis

G  Cartesian axis used to define the rotation transformation tensor

IEE  mass moment of inertia about the E axis

IEE'  mass moment of inertia about the E' axis

IEF  mass product of inertia about the E and F axes

IEF'  mass product of inertia about the E' and F' axes

IEG  mass product of inertia about the E and G axes

IEG'  mass product of inertia about the E' and G' axes

IFF  mass moment of inertia about the F axis

IFF'  mass moment of inertia about the F' axis

IFG  mass product of inertia about the F and G axes
LIST OF SYMBOLS (Continued)

IFG' mass product of inertia about the F' and G' axes

IGG mass moment of inertia about the G axis

IGG' mass moment of inertia about the G' axis

i_{xx}(j) partial piece "j"'s mass moment of inertia about its own local x axis

i_{xx}'(j) partial piece "j"'s mass moment of inertia about an axis through its center of mass and parallel to its own local x axis

I_{xx} a body's mass moment of inertia about the X axis in an XYZ coordinate system

I_{xx}' a body's mass moment of inertia about an axis through its center of mass and parallel to the X axis in an XYZ coordinate system

I_{XX} the total sabot's mass moment of inertia about an axis through its center of mass and parallel to the X global axis

I_{XX}' mass moment of inertia of the total sabot about a user-defined X axis

i_{xy}(j) partial piece "j"'s mass product of inertia about its own local x and y axes

i_{xy}'(j) partial piece "j"'s mass product of inertia about axes through its center of mass and parallel to its own local x and y axes

I_{xy} a body's mass product of inertia about the X and Y axes in an XYZ coordinate system

I_{xy}' a body's mass product of inertia about axes through its center of mass and parallel to the X and Y axes in an XYZ coordinate system

I_{XY} the sabot's total mass product of inertia about axes through its center of mass and parallel to the global X and Y axes

I_{XY}' mass product of inertia of the total sabot about user-defined X and Y axes

i_{xz}(j) partial piece "j"'s mass product of inertia about its own local x and z axes

i_{xz}'(j) partial piece "j"'s mass product of inertia about axes through its center of mass and parallel to its own local x and z axes

I_{xz} a body's mass product of inertia about the X and Z axes in an XYZ coordinate system
LIST OF SYMBOLS (Continued)

$I_x'z'$ a body's mass product of inertia about axes through its center of mass and parallel to the X and Z axes in an XYZ coordinate system

$IX'Z'$ the sabot's total mass product of inertia about axes through its center of mass and parallel to the global X and Z axes

$I'XZ$ mass product of inertia of the total sabot about user-defined X and Z axes

$iyy(j)$ partial piece "j"'s mass moment of inertia about its own local y axis

$i'y'y'(j)$ partial piece "j"'s mass moment of inertia about an axis through its center of mass and parallel to its own local y axis

$I_y$ a body's mass moment of inertia about the Y axis in an XYZ coordinate system

$I'y'$ a body's mass moment of inertia about an axis through its center of mass and parallel to the Y axis in an XYZ coordinate system

$IY'Y'$ the total sabot's mass moment of inertia about an axis through its center of mass and parallel to the Y global axis

$I'YY$ mass moment of inertia of the total sabot about a user-defined Y axis

$iyz(j)$ partial piece "j"'s mass product of inertia about its own local y and z axes

$i'y'z'(j)$ partial piece "j"'s mass product of inertia about axes through its center of mass and parallel to its own local y and z axes

$I_yz$ a body's mass product of inertia about the Y and Z axes in an XYZ coordinate system

$I'y'z'$ a body's mass product of inertia about axes through its center of mass and parallel to the Y and Z axes in an XYZ coordinate system

$IY'Z'$ the sabot's total mass product of inertia about axes through its center of mass and parallel to the global Y and Z axes

$I'YZ$ mass product of inertia of the total sabot about user-defined Y and Z axes

$izz(j)$ partial piece "j"'s mass moment of inertia about its own local z axis

$iz'z'(j)$ partial piece "j"'s mass moment of inertia about an axis through its center of mass and parallel to its own local z axis
LIST OF SYMBOLS (Continued)

Izz  a body's mass moment of inertia about the Z axis in an XYZ coordinate system

Iz'z' a body's mass moment of inertia about an axis through its center of mass and parallel to the Z axis in an XYZ coordinate system

IZ'Z' the total sabot's mass moment of inertia about an axis through its center of mass and parallel to the Z global axis

IZZ mass moment of inertia of the total sabot about a user defined Z axis

j used as an index to denote any partial piece used to describe the total body

m local variable; dr/dz; used with b to define the upper bound of integration on r

M mass

n number of partial pieces

r local cylindrical coordinate

T inertia tensor

T' inertia tensor after a coordinate transformation

v(j) partial piece "j"'s volume

V the sabot's total volume

x local Cartesian coordinate

X Cartesian coordinate

X'O X location of a user defined system's origin in the global coordinate system

xc(j) partial piece "j"'s X value of the center of mass in the global coordinate system

Xc a body's X value of center of mass

XC the total sabot's X value of the center of mass in the global coordinate system

y local Cartesian coordinate

Y Cartesian coordinate
LIST OF SYMBOLS (Continued)

\( Y'0 \) Y location of a user defined system's origin in the global coordinate system

\( yc(j) \) partial piece "j"'s Y value of the center of mass in the global coordinate system

\( Yc \) a body's Y value of center of mass

\( YC \) the total sabot's Y value of the center of mass in the global coordinate system

\( z \) local Cartesian and cylindrical coordinate

\( Z \) Cartesian and cyclindrical coordinate

\( zc(j) \) partial piece "j"'s Z value of the center of mass in the global coordinate system

\( ZO \) Z location of the partial piece "j"'s origin in the global coordinate system

\( Z'O \) Z location of a user defined system's origin in the global coordinate system

\( Zc \) a body's Z value of center of mass

\( ZC \) the total sabot's Z value of the center of mass in the global coordinate system

\( z_t \) local variable; z length of partial piece "j" and limit of integration on z

**Greek Symbols**

\( d\theta \) angular differential

\( \theta \) cylindrical angular coordinate

\( \theta_1 \) limit of integration in local coordinate system; see Figure 2

\( \theta_T \) total angular measurement on one sabot petal; see Figure 1; note that \( 2\theta_1 = \theta_T \)

\( \rho \) density

\( \Sigma \) summation sign
APPENDIX A. LISTING OF THE PROGRAM

General Notes

1. Program is written in FORTRAN 77

2. Arrays Z and R hold the body contour points. The dimension of the arrays is set at 100. This allows for 100 contours to describe the body. This can be increased if necessary.

3. Array D(j,k) holds all the information about each partial piece.
   
   j=number of the partial piece

   k=1 holds the volume of partial piece "j"
   k=2 holds the XC in the global system of partial piece "j"
   k=3 holds the ZC in the global system of partial piece "j"
   k=4 holds the IX'X' of partial piece "j"
   k=5 holds the IYY' of partial piece "j"
   k=6 holds the IZ'Z' of partial piece "j"
   k=7 holds the IX'Z' of partial piece "j".

   The dimension of the D array is set at (100,7). If more than 100 partial pieces are needed to describe the body, increase "j" only. Leave "k" set at 7.

4. The computer for which the FORTRAN program was written uses I/O unit "5" to read information from the screen and "6" to write information to the screen. If this is not the case on the system where this program is going to be placed, the correct read and write numbers will have to be exchanged for "5" and "6" in the program listing.

5. The important variable names in the program are described.

   All "TOT" variables like "VOLTOT" and "IXZTOT" are the values for the total body.

   All "PAR" variables like "VOLPAR" and "XCPAR" are partial values for the total body.

   NUMREG = number of partial pieces.
   NUMPTS = number of contour points to describe the body.
   TDHEG = \theta_T in degrees. See Figure 1.
   TH = \theta_1 in radians. See Figure 2.
   RHO = density of the material. Units are defined by user's entries.
   B = r intercept of the top boundary of a partial piece in its own coordinate system. Units are defined by user's entries. It is b in Figure 3.
M = slope \( \frac{dr}{dz} \) of the top boundary of a partial piece in its own coordinate system. It is \( m \) in Figure 3.

\( Z \) = length of a partial piece. Units are defined by user's entries. It is \( z_c \) in Figure 3.

"T" variables like \( T_1 \) or \( T_5 \) are used for clarity. They represent terms in the large physical property equations.

* PROGRAM TO COMPUTE THE PHYSICAL PROPERTIES OF ONE SABOT PETAL

* DIMENSION AND DATA ENTRY

* DIMENSION D(100,7),R(100),Z(100)
REAL IXXTOT,IYYTOT,IZZTOT,IXZTOT
CHARACTER FNAME*45
PI=3.1415927
WRITE(6,*)'ENTER FILE NAME'
READ(5,101)FNAME
OPEN(10,FILE=FNAME,STATUS='OLD')
I=1
4 READ(10,*)(Z(I),R(I))
IF(I.EQ.1)GOTO8
IF((Z(I).EQ.Z(1)).AND.(R(I).EQ.R(1)))GOTO9
8 I=I+1
GOTO4
9 READ(10,*)RHO,THDEG
CLOSE(10)
TH=THDEG*PI/360.
3 NUMPTS=I-1
NUMREG=0

* FINDS THE PROPERTIES OF ALL THE PARTIAL PIECES AND STORES THEM IN
* THE "D" ARRAY

* DO 5 K=1,NUMPTS
IF(Z(K).EQ.Z(K+1))GOTO5
NUMREG=NUMREG+1
L=NUMREG
CALL VOLCEN(R(K+1),R(K),Z(K+1),Z(K),D(L,1),D(L,2),D(L,3),TH)
IF(D(L,1).EQ.0.)GOTO5
CALL RMXANY(R(K+1),R(K),Z(K+1),Z(K),D(L,1),D(L,2),D(L,3),
:O(L,4),D(L,5),TH,RHO)
CALL RMIIZZ(R(K+1),R(K),Z(K+1),Z(K),D(L,1),D(L,2),D(L,6),TH,RHO)
CALL RMOIXZ(R(K+1),R(K),Z(K+1),Z(K),D(L,1),D(L,2),D(L,3),
:O(L,7),TH,RHO)
5 CONTINUE

* FINDS THE VOLUME, X AND Z LOCATION OF THE CENTER OF MASS FOR THE SABOT PETAL

VOLPAR=0.
XCPAR=0.
ZCPAR=0.
DO 6 I=1,NUMREG
   VOLPAR=VOLPAR+D(I,1)
   XCPAR=XCPAR+D(I,1)*D(I,2)
6   ZCPAR=ZCPAR+D(I,1)*D(I,3)
* (EQ.4)
   VOLTOT=VOLPAR
* (EQ.11)
   XCTOT=XCPAR/VOLTOT
* (EQ.17)
   ZCTOT=ZCPAR/VOLTOT
* CALCULATES IX'X', IY'Y', IZ'Z', AND IX'Z' FOR THE SABOT PETAL
* DO 7 I=1,NUMREG
* (EQ.31)
   IXXTOT=IXXTOT+D(I,4)+(ZCTOT-D(I,3))*(ZCTOT-D(I,3))*RHO*D(I,1)
* (EQ.32)
   IYYTOT=IYYTOT+D(I,5)+((XCTOT-D(I,2))*(XCTOT-D(I,2)) +
          (ZCTOT-D(I,3))*(ZCTOT-D(I,3)))*RHO*D(I,1)
* (EQ.33)
   IZZTOT=IZZTOT+D(I,6)+(XCTOT-D(I,2))*(XCTOT-D(I,2))*RHO*D(I,1)
* (EQ.47)
   IXZTOT=IXZTOT+D(I,7)+(D(I,2)-XCTOT)*(D(I,3)-ZCTOT)*RHO*D(I,1)
* DATA OUTPUT AND FORMAT STATEMENTS
* WRITE(6,102)FNAME
WRITE(6,103)RHO
WRITE(6,104)THDEG
WRITE(6,105)VOLTOT*RHO
WRITE(6,106)XCTOT
WRITE(6,107)ZCTOT
WRITE(6,108)IXXTOT
WRITE(6,109)IYYTOT
WRITE(6,110)IZZTOT
WRITE(6,111)IXZTOT
WRITE(6,112)
WRITE(6,113)
WRITE(6,114)
* 101 FORMAT(A45)
102 FORMAT( 'FILE NAME: ',A45)
103 FORMAT( 'DENSITY: ',G11.4E2)
104 FORMAT( 'THETA T: ',F6.2,' DEGREES',/)
105 FORMAT( 'MASS: ',G12.6E2,/)
106 FORMAT( 'XC: ',G12.6E2)
107 FORMAT( 'YC: 0.,/)
108 FORMAT( 'ZC: ',G12.6E2,/)
109 FORMAT( 'IX'X': ',G12.6E2)
110 FORMAT( 'IY'Y': ',G12.6E2)
111 FORMAT( 'IZ'Z': ',G12.6E2,/)
112 FORMAT( 'IX'Y': 0.,)
113 FORMAT( 'IX'Z': ',G12.6E2)
114 FORMAT( 'IY'Z': 0.,)
* VOLCEN CALCULATES THE VOLUME AND CENTER OF MASS FOR ANY PARTIAL PIECE

SUBROUTINE VOLCEN(R2,R1,Z2,Z1,VOL,XC,ZC,TH)
REAL M
B=R1
M=(R2-R1)/(Z2-Z1)
Z=Z2-Z1
T1=M*Z*Z*Z*Z/3.
T2=B*M*Z
T3=B*B*Z
* (EQ.3)
VOL=TH*(T1+T2+T3)
IF(VOL.EQ.0)RETURN
T5=B*M*M*Z*Z*Z.
T6=B*B*Z*Z*M*3./2.
T7=B*B*B*Z
* (EQ.10)
XC=2./3.*SIN(TH)*(T4+T5+T6+T7)/VOL
T10=B*B*Z*Z/2.
* (EQ.16)
ZC=Z1+(TH*(T8+T9+T10))/VOL
RETURN
END

* RMXANY CALCULATES IX'X' AND IY'Y' FOR ANY PARTIAL PIECE

SUBROUTINE RMXANY(R2,R1,Z2,Z1,VOL,XC,ZC,IXX,IYY,TH,RHO)
REAL M,IXX,IYY
B=R1
M=(R2-R1)/(Z2-Z1)
Z=Z2-Z1
T4=B*B*M*M*Z*Z/2.
T5=B*B*B*M*Z*Z/2.
T6=Z*Z*Z*M*Z*Z/M*5.
T7=Z*Z*Z*Z*Z*Z/2.
T8=Z*Z*Z*B*Z/3.
X=TH-.5*SIN(2*TH)
* (EQ.24)
IXX=RHO*(X*(T1+T2+T3+T4+T5)+TH*(T6+T7+T8))
* (EQ.28)
IXX=IXX-(ZC-Z1)*(ZC-Z1)*VOL*RHO
X=TH+.5*SIN(2*TH)
* (EQ.25)
IYY=RHO*(X*(T1+T2+T3+T4+T5)+TH*(T6+T7+T8))
* (EQ.29)
IYY=IYY-(XC*XC+(ZC-Z1)*(ZC-Z1))*VOL*RHO

32
* RMOIZZ CALCULATES $I_Z'$ FOR ANY PARTIAL PIECE

SUBROUTINE RMOIZZ(R2, R1, Z2, Z1, VOL, XC, IZZ, TH, RHO)
REAL M, IZZ
B = R1
M = (R2 - R1) / (Z2 - Z1)
Z = Z2 - Z1
T1 = M * M * M * Z * Z * Z * Z * Z / 5.
T2 = B * M * M * Z * Z * Z
T3 = 2. * B * B * M * M * Z * Z
T4 = B * B * B * M * Z * Z
T5 = B * B * B * B
* (EQ. 26)
IZZ = RHO * TH * (T1 + T2 + T3 + T4 + T5)
* (EQ. 30)
IZZ = IZZ - VOL * RHO * XC * XC
RETURN

* CALCULATES $IX'_Z$ FOR ANY PARTIAL PIECE

SUBROUTINE RMDIXZ(R2, R1, Z2, Z1, VOL, XC, ZC, IXZ, TH, RHO)
REAL M, IXZ
B = R1
M = (R2 - R1) / (Z2 - Z1)
Z = Z2 - Z1
T1 = Z * Z * Z * Z * M * M * M / 5.
T2 = Z * Z * Z * Z * M * B * .75
T3 = Z * Z * M * B * B
T4 = Z * Z * B * B / 2.
* (EQ. 41)
IXZ = RHO * Z / 3. * SIN(TH) * (T1 + T2 + T3 + T4)
* (EQ. 46)
IXZ = IXZ - RHO * VOL * XC * (ZC - Z1)
RETURN
END
APPENDIX B. EXAMPLE PROBLEM

The physical properties of a sabot with the a ZR profile shown in Figure B-1 are to be determined. The sabot petal is made of aluminum and is one-third of a complete sabot package.

STEP 1 FIND THE COORDINATES OF THE CONTOUR POINTS IN THE ZR PLANE. SEE FIGURE B-1.

STEP 2 CREATE A DATA FILE OF THE FORM OF FIGURE B-2.

This file contains the coordinates found in STEP 1. The data file entries must follow the contour of the body in a clockwise manner until the first point is reached. The first point is then entered again. This is necessary because this signals to the computer that the profile is completely entered. The choice of the first point is arbitrary.

The last record of the data file has two values. The first value is the density and the second value is the angle $\theta_T$. See Figure 1. For the example problem, $\theta_T = 120$. Aluminum, the material of the example problem, has a density of $0.00314$ slug/inch. Please note that the length units of density are the same as the length units used to describe the ZR profile.

STEP 3 EXECUTE THE PROGRAM

The only entry required is the name of the file created inSTEP 2.

STEP 4 SOLUTION

The solution is written to the screen of the terminal. If a terminal is not used, the program will have to be modified by the user. The example problem's solution is shown in Figure B-3. The units are consistent with the units entered in STEP 2. For the example problem the mass is in slugs, the X and Z locations of the center of mass are in inches from the origin of the global coordinate system (Figure 1) and the moments and product of inertia are in slugs inches. Remember the moments of inertia are determined about axes through the center of mass and parallel to the global coordinate system.
**Figure B-1.** Example problem contour.

**Figure B-2.** Example problem data file.

**Figure B-3.** Example problem solution.

---

**FILE NAME:** EPOF  
**DENSITY:** 0.3140E-02  
**THETA T:** 120.00 DEGREES  
**MASS:** 0.749573E-02  
**XC:** 0.826570  
**YC:** 0.  
**ZC:** 1.71011  
**IX'X':** 0.791766E-02  
**IY'Y':** 0.620526E-02  
**IZ'Z':** 0.312264E-02  
**IX'Y':** 0.  
**IX'Z':** -.657281E-03  
**IY'Z':** 0.  

---

**CONTOUR POINTS**

<table>
<thead>
<tr>
<th>Z</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td>2</td>
<td>0.515</td>
</tr>
<tr>
<td>3</td>
<td>1.75</td>
</tr>
<tr>
<td>4</td>
<td>2.75</td>
</tr>
<tr>
<td>5</td>
<td>2.15</td>
</tr>
<tr>
<td>6</td>
<td>2.515</td>
</tr>
<tr>
<td>7</td>
<td>2.59</td>
</tr>
<tr>
<td>8</td>
<td>3.55</td>
</tr>
<tr>
<td>9</td>
<td>1.6</td>
</tr>
<tr>
<td>10</td>
<td>0.511.25</td>
</tr>
<tr>
<td>11</td>
<td>0.125</td>
</tr>
</tbody>
</table>

**UNITS - INCHES**
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
</table>
| 12           | Administrator
              Defense Technical Info Center
 ATTN: DTIC-FDAC
 Cameron Station, Bldg. 5
 Alexandria, VA 22304-6145            | 1             | Commander
  US AMCCOM ARDEC CCAC
 Benet Weapons Laboratory
 ATTN: SMCAR-CCB-TL
 Watervliet, NY 12189-4050       |
| 1            | HQDA
 DAMA-ART-M
 Washington, DC 20310          | 1             | Commander
 US Army Armament, Munitions and
 Chemical Command
 Rock Island, IL 61299-7300  |
| 1            | Commander
 US Army Materiel Command
 ATTN: AMCDRA-ST
 5001 Eisenhower Avenue
 Alexandria, VA 22333-0001   | 1             | Commander
 US Army Aviation Systems Command
 ATTN: AMSAV-ES
 4300 Good fellow Blvd
 St. Louis, MO 63120-1798   |
| 1            | Commander
 US Army Armament Research,
 Development and Engineering
 Center
 ATTN: SMCAR-MSI
 Dover, NJ 07801-5001         | 1             | Commander
 US Army Aviation Research and
 Technology Activity
 Ames Research Center
 Moffett Field, CA 94035-1099 |
| 1            | Commander
 US Army Armament Research
 Development and Engineering
 Center
 ATTN: SMCAR-TDC
 Dover, NJ 07801-5001        | 10            | C.I.A.
 OIR/DB/Standard
 GE47 HQ
 Washington, DC 20505      |
| 2            | Commander
 US Army Armament Research
 Development and Engineering
 Center
 ATTN: SMCAR-AET
 Mr. A. Loeb
 Mr. D. Mertz
 Dover, NJ 07801-5001   | 1             | Commander
 US Army Communications -
 Electronics Command
 ATTN: AMSEL-ED
 Fort Monmouth, NJ 07703-5301 |
| 1            | Commander
 US Army Armament Research
 Development and Engineering
 Center
 ATTN: SMCAR-LCU-CT
 Mr. Lee Whitmore
 Dover, NJ 07801-5001     | 1             | Commander
 CECOM R&D Technical Library
 ATTN: AMSEL-IM-L
 (Reports Section) B.2700
 Fort Monmouth, NJ 07703-5000 |
| 1            | Commander
 US Army Missle Command
 Research, Development, and
 Engineering Center
 ATTN: AMSMI-RD
 Redstone Arsenal, AL 35898-5230 |
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Director</td>
<td>Aberdeen Proving Ground</td>
<td>Dir, USAMSAA</td>
</tr>
<tr>
<td></td>
<td>US Army Missile and Space Intelligence Center</td>
<td></td>
<td>ATTN: AMXSY-D</td>
</tr>
<tr>
<td></td>
<td>ATTN: AIAMS-YDL</td>
<td></td>
<td>AMXSY-MP, H. Cohen</td>
</tr>
<tr>
<td>1</td>
<td>Commander</td>
<td>Cdr, USATECOM</td>
<td>AMST-SI-F</td>
</tr>
<tr>
<td></td>
<td>US Army Tank Automotive Command</td>
<td></td>
<td>Cdr, CRDC, AMCCOM</td>
</tr>
<tr>
<td></td>
<td>ATTN: AMSTA-TSL</td>
<td></td>
<td>ATTN: SMCCR-RSP-A</td>
</tr>
<tr>
<td></td>
<td>Warren, MI 48397-5000</td>
<td></td>
<td>SMCCR-MU</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SMCCR-SPS-IL</td>
</tr>
<tr>
<td>1</td>
<td>Director</td>
<td>Cdr, CRDC, AMCCOM</td>
<td>SMCCR-SPS-IL</td>
</tr>
<tr>
<td></td>
<td>US Army TRADOC Analysis Center</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: ATOR-TSL</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>White Sands Missile Range, NM 88002-5502</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Commandant</td>
<td>Cdr, USATECOM</td>
<td>AMST-SI-F</td>
</tr>
<tr>
<td></td>
<td>US Army Infantry School</td>
<td></td>
<td>Cdr, CRDC, AMCCOM</td>
</tr>
<tr>
<td></td>
<td>ATTN: ATSH-CD-CS-OR</td>
<td></td>
<td>ATTN: SMCCR-RSP-A</td>
</tr>
<tr>
<td></td>
<td>Fort Benning, GA 31905-5400</td>
<td></td>
<td>SMCCR-MU</td>
</tr>
<tr>
<td>1</td>
<td>Commander</td>
<td>Cdr, CRDC, AMCCOM</td>
<td>SMCCR-SPS-IL</td>
</tr>
<tr>
<td></td>
<td>US Army Development and Employment Agency</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: MODE-ORO</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fort Lewis, WA 98433-5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>AFWL/SUL</td>
<td>AFWL/SUL</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kirtland AFB, NM 87117</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Air Force Armament Laboratory</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: AFATL/DLODL</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Tech Info Center)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Eglin AFB, FL 32542-5000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. BRL Report Number ____________________________ Date of Report ________________

2. Date Report Received ____________________________

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.)

4. How specifically, is the report being used? (Information source, design data, procedure, source of ideas, etc.)

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided or efficiencies achieved, etc? If so, please elaborate.

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.)

7. If indicating a Change of Address or Address Correction, please provide the New or Correct Address in Block 6 above and the Old or Incorrect address below.

(Remove this sheet, fold as indicated, staple or tape closed, and mail.)
END
DATE
FILMED
FEB.
1988