A study of lateral pressures on basement walls due to Snelling soils (U) Texas Tech Univ Lubbock Dept of Civil Engineering W K Wray Sep 87 NES/CR/GL-87-3

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A STUDY OF LATERAL PRESSURES ON BASEMENT WALLS DUE TO SWELLING SOILS

by

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A study was made of the technical literature pertaining to the topic of lateral pressures transmitted to nonyielding, rigid basement walls by swelling expansive soils. The objective of the study was to determine if there was available a method whereby pressures transmitted to rigid basement walls could be practically and reasonably predicted for design purposes. An extensive bibliography of pertinent technical publications was developed. The publications which formed the bibliography were topically categorized as being either principally theoretical or principally experimental in their presentation or result. Selected publications of each type are discussed, and their principal findings are reported. Of these selected publications, three methods or procedures were concluded to have more potential for practical applications than the others and are presented in detail. These three methods or procedures are those of Skempton, Fredlund, and Katti. The study concluded that these three methods were each (Continued)
19. ABSTRACT (Continued).

relatively easy to apply and held more promise than others for use in the design of rigid basement walls constructed in expansive soils, but that none of the methods appeared to have been proven, validated, or verified by field measurements of horizontal pressures transmitted to nonyielding basement walls. The study concludes with a recommendation to develop a field experiment whereby horizontal pressures can be measured and used to validate, modify, or reject the three methods.
PREFACE

This study was accomplished under an Interagency Personnel Agreement (IPA) between the US Army Engineer Waterways Experiment Station (WES), Vicksburg, Mississippi, and Texas Tech University, Lubbock, Texas.

The study was performed under the general supervision of Dr. W. F. Marcuson III, Chief, Geotechnical Laboratory (GL), and Mr. C. L. McAnear, Chief, Soil Mechanics Division (SMD), GL, and under the direct supervision of Mr. G. B. Mitchell, Chief, Engineering Group, SMD, GL. Dr. W. K. Wray, Texas Tech University, prepared the report.

COL Dwayne G. Lee, CE, was Commander and Director of WES during the publication of this report. Dr. Robert W. Whalin was Technical Director.
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This study was accomplished under an Interagency Personnel Agreement (IPA) between the US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, and Texas Tech University, Lubbock, Texas. The writer gratefully acknowledges the financial support and encouragement provided to him by the US Army Engineer Waterways Experiment Station in conducting this study.
1. Estimating lateral earth pressures has been a problem to civil engineers since antiquity. The first to publish a rational approach by which lateral earth pressures could be estimated was C. A. Coulomb. He published his method in 1773 and 1776. His effort was followed by that of W. J. M. Rankine in 1857.

2. These two methods differ in the assumptions made to simplify the problem, but both are fairly simple in application and have subsequently come to be known as the "classical" methods of solution for lateral earth pressure. A number of other methods have been proposed since the presentation of the Rankine method. Some of these methods are mathematical in their approach and others are graphical solutions such as Poncelet Method (1840), Culmann Method (1866), Trial Wedge Method (Bowles, 1982), and Logarithmic Spiral Method (Bowles, 1982). The fact that so many other methods of solution have been proposed since the presentation of the Rankine method should alert the reader that none of the methods of predicting lateral earth pressure that have been introduced to date have been completely satisfactory in general practice.

3. Each of the methods cited above has essentially a common basis, i.e., either the structure supporting the soil moves away from the soil mass and permits the soil to fail along a specified shear plane, or the retaining structure is forced into the soil mass causing a shear plane to occur. In the first instance, the soil is permitted to relax and strain laterally as the soil moves outward, attempting to follow the structure as it moves away from the retained soil mass. This straining condition is known as the "active" state or condition. The opposite condition, i.e., when the retaining structure is forced into the soil mass, results in the
soil mass being compressed. If the force moving the retaining structure into the soil is sufficiently large, it will cause the soil to fail in shear. This straining condition is known as the "passive" state or condition.

4. The active and passive states describe the two conditions when the soil is being either stretched or compressed. When neither of these conditions are occurring, the condition of the soil is said to be "at rest." The methods of predicting the lateral earth pressure cited above are applicable to "conventional" soils. For the purpose of this report, "conventional" will describe those soils that experience negligible volume changes when the soil water content changes, i.e., nonexpansive soils. Although it is well known that most clay soils will swell when the soil water content increases and will shrink when the soil water content decreases, expansive soils shrink or swell excessively. However, for an expansive soil to swell, its boundaries must not be restrained. Thus, as expansive soils swell vertically, the ground surface experiences an increase in elevation; conversely, as expansive soils shrink, the ground surface experiences a decrease in elevation.

5. Expansive soils also swell or increase in volume laterally as well as vertically. If the ground surface is cracked and fissured, the lateral increase in volume is accommodated by the cracks or fissures closing as the soil mass expands into the voids of the cracks. But when there are no cracks or fissures, or when the cracks or fissures are very small and close up without accommodating all of the volume increase required by the swelling soil mass, the soil becomes restrained in the lateral directions. The result is the development of a lateral swelling pressure. The magnitude of this swelling pressure can be very great; pressures as great as 20 tons per
square foot have been measured (Chen, 1975). Thus, if a swelling soil were to be retained behind a vertical structure and if the retained soil were to become wetter, resulting in a swelling of the soil, the lateral pressures acting on the vertical structure would increase above that predicted by any of the lateral earth pressure methods cited above. If the soil swelling were large enough, it could cause the retaining structure to fail by either sliding or rotating. This failure would be the result if the structure were a retaining wall. However, if the retaining structure were a basement wall, the result would not likely be sliding or rotating of the wall in response to the increase in lateral earth pressure. Because a basement wall is typically restrained from both sliding and rotation, the likely result is an overstressed condition and a fractured or cracked CMU block or reinforced concrete wall.

6. Despite the many methods available to the design professional by which he can predict the lateral earth pressures expected to be acting on a basement wall from conventional or nonexpansive soils, there is no reliable method presently available that permits the designer to estimate the pressures that are likely to act on the wall should expansive soil adjacent to the wall begin to swell. Thus, the purpose of this study is to examine the state of the art and determine if there is a rational method by which this increase in lateral pressure due to swelling soil can be reasonably estimated. Additionally, the geometry of the backfill adjacent to the basement wall is to be investigated to determine if geometry influences the magnitude of the lateral pressure increase that is transmitted to the basement wall.
PART II: REVIEW OF THE TECHNICAL LITERATURE

7. Although there are a number of publications in the technical literature that address the subjects of swelling pressure, most can be divided into the categories of being either principally theoretical or principally experimental. Many of the "theoretical" papers used laboratory tests to evaluate certain factors or constants, or the laboratory data were used to develop equations that could be used to predict future results. Most reports of experimental work discussed below used remolded or compacted soils in the experimental program rather than in situ or undisturbed samples and did not result in predictive models or similar applications.

Theoretical Analyses

8. Skempton (1961). The author had an opportunity to investigate a failure in a vertical wall in a deep excavation in an overconsolidated soil where no field complications, such as underdrainage or adjacent structures, existed. In the process of the investigation, the author calculated the coefficient of at-rest earth pressures, $K_o$, and effective horizontal soil stress by three separate methods. At the conclusion of the investigation, he made two significant observations:

a. There is a general tendency for $K_0$ to decrease with depth (for this site, from a maximum occurring at a depth of about 20 ft over a total depth of 110 ft).

b. $K_0$ is reduced in the top 10 ft, at least at this site, due to weathering and softening prior to deposition of the top stratum of postglacial clay.

9. Some of the findings of the investigation as well as some applicable "theoretical" considerations reported by Skempton are worth enumerating:

a. Swelling pressure is equal to the soil suction.

b. Swelling pressure can also be deduced from undrained strength tests.
10. Skempton also evaluated the swelling pressure via oedometer tests and directly measured soil suction. His results are reported in Table 2-1. Skempton calculated (Table 2-2) and then compared \( K_0 \) and the coefficient of passive earth pressure, \( K_p \), with depth and, in the vicinity of the shear failure, \( K_0 \) was found to be approximately equal to \( K_p \), resulting in the failed slope. Skempton's method is discussed in more detail in Part III.

Table 2-1.

Comparison of capillary pressures calculated by three different methods. (After Skempton, 1961)

<table>
<thead>
<tr>
<th>Capillary Pressure, ( p_k ) (lb/ft(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample</strong></td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>5 &amp; 6</td>
</tr>
<tr>
<td>7 &amp; 8</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

11. Brooker and Ireland (1965). As the result of a laboratory investigation, the authors found some relationships between at-rest earth pressure and some soil properties. Their testing program was conducted using remolded specimens at a water content corresponding to a liquidity index of about 0.5, which was well above the optimum water content for all of the test soils. The significant findings reported include:

a. They concluded that the \( K_0 = 1 - \sin \phi' \) equation for estimating the at-rest coefficient of lateral earth pressure proposed by Jaky (1948) is probably more representative of sands, whereas the relationship \( K_0 = 0.95 - \sin \phi' \) is more representative of normally consolidated clays. The validity of either of these equations with respect to overconsolidated clays was not addressed.
Table 2-2.
Soil Properties (After Skempton, 1961)

<table>
<thead>
<tr>
<th>Total Depth (ft)</th>
<th>P (lb/ft²)</th>
<th>Over-consolidation ratio</th>
<th>Water content (%)</th>
<th>Soil cohesion (lb/ft²)</th>
<th>p_k (lb/ft²)</th>
<th>p_k/p</th>
<th>K_o (lb/ft²)</th>
<th>o_h' pK_o (lb/ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>690</td>
<td>44</td>
<td>36.0</td>
<td>950</td>
<td>1,260</td>
<td>1.82</td>
<td>2.17</td>
<td>1,500</td>
</tr>
<tr>
<td>15</td>
<td>970</td>
<td>32</td>
<td>34.3</td>
<td>1,290</td>
<td>2,080</td>
<td>2.15</td>
<td>2.64</td>
<td>2,560</td>
</tr>
<tr>
<td>20</td>
<td>1,260</td>
<td>25</td>
<td>33.0</td>
<td>1,610</td>
<td>2,850</td>
<td>2.26</td>
<td>2.80</td>
<td>3,530</td>
</tr>
<tr>
<td>30</td>
<td>1,830</td>
<td>17</td>
<td>32.0</td>
<td>2,100</td>
<td>4,050</td>
<td>2.22</td>
<td>2.74</td>
<td>5,000</td>
</tr>
<tr>
<td>40</td>
<td>2,420</td>
<td>13</td>
<td>31.1</td>
<td>2,500</td>
<td>5,010</td>
<td>2.07</td>
<td>2.53</td>
<td>6,120</td>
</tr>
<tr>
<td>50</td>
<td>3,030</td>
<td>11</td>
<td>30.4</td>
<td>2,800</td>
<td>5,750</td>
<td>1.90</td>
<td>2.29</td>
<td>6,950</td>
</tr>
<tr>
<td>60</td>
<td>3,610</td>
<td>9</td>
<td>29.9</td>
<td>3,070</td>
<td>6,390</td>
<td>1.77</td>
<td>2.10</td>
<td>7,580</td>
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<tr>
<td>70</td>
<td>4,210</td>
<td>8</td>
<td>29.6</td>
<td>3,300</td>
<td>6,960</td>
<td>1.65</td>
<td>1.93</td>
<td>8,100</td>
</tr>
<tr>
<td>80</td>
<td>4,800</td>
<td>7</td>
<td>29.3</td>
<td>3,500</td>
<td>7,450</td>
<td>1.55</td>
<td>1.79</td>
<td>8,600</td>
</tr>
<tr>
<td>90</td>
<td>5,410</td>
<td>6.5</td>
<td>29.0</td>
<td>3,700</td>
<td>7,920</td>
<td>1.47</td>
<td>1.67</td>
<td>9,050</td>
</tr>
<tr>
<td>100</td>
<td>6,020</td>
<td>6</td>
<td>28.8</td>
<td>3,860</td>
<td>8,320</td>
<td>1.38</td>
<td>1.54</td>
<td>9,280</td>
</tr>
<tr>
<td>110</td>
<td>6,620</td>
<td>5.5</td>
<td>28.7</td>
<td>4,020</td>
<td>8,700</td>
<td>1.32</td>
<td>1.46</td>
<td>9,700</td>
</tr>
</tbody>
</table>

LL = 95 PL = 30 c' = 380 lb/ft² φ' = 20°
b. In plots of $K_0$ vs overconsolidation ratio (OCR), it could be seen that $K_0$ approaches $K$ at overconsolidation ratios greater than 20. Thus, OCR is a factor in evaluating $K_0$.

c. From plotting $K_0$ as a function of effective angle of internal friction, $\phi'$, it was seen that $K_0$ initially increased as $\phi'$ increased, reaching a maximum at an OCR of approximately 20. As OCR increased beyond 20, $K_0$ then began to decrease. Thus, the effective angle of internal friction is also a factor in determining the value of $K_0$.

d. Although also later reported by others, Brooker and Ireland found that as the effective angle of internal friction decreased, plasticity, in the form of the plasticity index, increased.

12. Although not concluded by the authors, it appears that two additional conclusions can be drawn from their observations:

a. There appears to be an optimum condition at which the combination of cohesion and friction retain the greatest radial stress and, consequently, the greatest $K_0$ value.

b. Soils of low to medium plasticity appear to develop higher values of $K_0$ than do either cohesionless or high to very high plasticity soils, especially at overconsolidation ratios greater than about 16.

13. Ranganatham and Satyanarayana (1965). A method of predicting swelling potential according to the subjective classifications of low, medium, high, and very high is presented. A regression equation is developed using shrinkage index, defined as the difference between the liquid limit and the shrinkage limit, and swell activity, defined as the ratio between the change in shrinkage index and the corresponding change in clay fraction, as independent variables to predict the dependent variable swelling potential. Their predictive equation is:

$$SP = (SI)^{2.67}N$$  \hspace{1cm} (2-1)

where

$SP =$ swelling potential

$SI =$ shrinkage index, $w_l - w_s$

$N =$ a factor defined as $C^{3.44}/(C - n)^{2.67}$
C = clay fraction

n = intercept of the horizontal axis of the shrinkage index vs. clay fraction experimental curve, between 4 and 22

\[ \alpha = a \text{ dimensionless constant} = 4.57 \times 10^{-5} \]

The value of the factor N could actually be determined from a set of curves presented in the paper rather than calculated; however the authors determined that for natural soils, Equation (2-1) could be expressed more simply as

\[ SP = (41.13 \times 10^{-5}) \times (SI)^{2.67} \quad (2-2) \]

14. Although the authors combined the results reported by Seed, et al. (1962) with the laboratory results of tests they conducted to develop their relationships, a discussion of their results was included in the "theoretical" category because their equation can be used to predict a quantity (or in this case a "quality"). Additionally, they found the horizontal swelling pressure to be greater than the vertical swelling pressure in the case of retaining walls with expansive soil backfills.

15. Livneh, Kassiff, and Wiseman (1969). Using soils native to Israel, the authors developed regression equations that permit a user to estimate some soil properties. Plasticity index, PI, can be estimated from

\[ PI = 0.75 LL - 8.9 \quad (2-3) \]

where LL represents liquid limit in percent. A coefficient of correlation of \( r = 0.95 \) was found for this relationship. They also found that the maximum dry density as measured in the modified AASHTO (modified Procter) test could be estimated from

\[ \gamma_{ma} = 2113 - 7.06 LL \quad (2-4) \]

where \( \gamma_{ma} \) is maximum dry density in kg/m\(^3\). The optimum moisture content, \( w_{ma} \), may be estimated as well as
\[
&w_{ma} = 0.98PL - 3.78
\]  
(2-5)

where \(w_{ma}\) is in percent and PL represents plastic limit, also in percent. They also note that the results of their testing program showed that swelling pressure decreased as initial porosity and moisture content increased and that swelling pressure increased as plasticity increased.

16. Komornik and David (1969). Plasticity, initial moisture content, and density are shown to be indicative parameters of swelling behavior—all three together only, not in other combinations of individual or dual combinations. A regression equation relating swelling pressure, \(P_s\), in terms of liquid limit, LL, natural water content, \(w_n\), and dry density, \(\gamma_d\) was developed:

\[
\log P_s = 2.132 + 0.0208(\text{LL}) + 6.65 \times 10^{-4}(\text{\(\gamma_d\)}) - 0.0269(\text{\(w_n\})}
\]  
(2-6)

The coefficient of correlation for this equation was \(r = 0.60\). However, they found negligible correlation between the shrinkage index and either plasticity index or amount of free swell. The correlation between swelling pressure and the two parameters, liquid limit and dry density, was found to be \(r = 0.51\). By contrast, the liquid limit was found to be the best single parameter indicator, but it had a correlation coefficient of only 0.16.

17. Bolt (1956) obtained a quantitative equation for swelling pressure as a function of ion concentration, valency of adsorbed cation, temperature, charge density of the clay mineral, void ratio, specific gravity of the clay, and specific surface area of the clay mineral. The authors present an empirical relationship between Bolt's theoretical swelling pressure equation and Terzaghi and Peck's equation for estimating the consolidation test compression index, \(C_c = 0.009(\text{LL} - 10)\) (Terzaghi and Peck, 1967).

18. As other investigators have discovered, the authors note that as plasticity (as represented by the plasticity index) increases, free swell
increases. Greater initial density resulted in greater free swell, implying more particles oriented perpendicular to the external load (direction of swell).

19. Aitchison and Richards (1969). This is a discussion of the fundamental principles of soil physics of unsaturated soils as they apply to engineering. The authors emphasize that the effective stress principle is applicable to the problem of unsaturated soils just as it is to saturated soils with the exception that in addition to the total applied stress (mechanical loading or stress), soil suction in the form of \((u_a - u_w)\) must also be considered. In fact, they suggest that it may be necessary in some instances to include the two components of total suction, matrix and osmotic suction, in the effective stress equation for unsaturated soils rather than total suction only. They also point out that for unsaturated soils, the chi-factor, \(X\), is generally not quantifiable. However, they also suggest that it is not necessary to know the value of \(X\) if quantification of physical properties is accomplished with the correct ambient values of total stress and total suction. They also point out that changes in soil suction result in soil volume change, and these changes, usually resulting from changes in climate or environment, must be considered before the expected magnitude of volume change can be reasonably predicted.

20. Komornik (1969). The author enumerates and then discusses some of the factors affecting damage due to movements of expansive clays in the field. His observations are primarily based on his experience with swelling soils in Israel, is experience suggests that an equilibrium soil suction value of 1 kg/cm² (approximately 1 atmosphere) may be assumed to reside in the soil, and he permits vertical swelling to occur from this condition. His experience also indicates that lateral earth pressures acting against
vertical retaining walls and other buried structures are "far in excess" of active and at-rest pressures for swelling clay backfills.

21. Nayak and Christensen (1971). The authors developed two equations to predict swelling pressure and swelling potential using results of an experimental program conducted by them using compacted swelling clays soils. The basic forms of the relationships are derived from theoretical considerations of the diffuse double layer and the osmotic pressure for parallel clay plates. Their general regression equation for swelling pressure, \( P_p \), in \( \text{lb/in}^2 \) is

\[
P_p = (3.5817 \times 10^{-2})(PI)^{1.12} \left( C^2/w_i^2 \right) + 3.7912
\]

(2-7)

and their general regression equation for swelling potential, \( S_p \), a percentage, is

\[
S_p = (2.29 \times 10^{-2})(PI)^{1.45} \left( C/w_i \right) + 6.38
\]

(2-8)

where

\[
PI = \text{plasticity index, in percent}
\]

\[
C = \text{clay content, by weight, as a percentage}
\]

\[
w_i = \text{initial moisture content, by weight, as a percentage}
\]

The regression equations appear to represent the experimental data very well with correlation coefficients of \( r = 0.92 \) for Equation (2-3) and \( r = 0.97 \) for Equation (2-4).

22. Wroth and Vaughan (1973). The authors present and discuss a number of methods of measuring lateral stresses in situ, methods of deducing in situ stresses from laboratory tests on "undisturbed" samples, and some indirect observations of in situ stresses. Although this paper is more of a review of the subject rather than presenting new methods or study results, it does report three rather important observations that are also reported by
others in some form or fashion.

a. As a normally consolidated soil is unloaded and becomes overconsolidated, $K_0$ increases, reaching a value of about unity at an overconsolidation ratio of approximately 5. [Ladd (1965) first made this observation.]

b. The value of $K_0$ is only limited by the state of passive failure which is reached at very high values of OCR.

c. As overconsolidated soil is reloaded from an overconsolidated state, the value of $K_0$ rapidly falls below unity, approaching the minimum value associated with normal consolidation once the preconsolidation pressure is exceeded.

23. Abdelhamid and Krizek (1976). Although the authors report the results of a laboratory investigation into evaluating $K_0$, their paper is included in this section because the investigation is used to test or validate some work by others that is considered to be theoretical in nature. The authors consolidated a clay from a slurry and measured $K_0$ continuously. The value of $K_0$ for a dispersed specimen was found experimentally to be 0.69 in one trial and 0.68 in a second trial. Jaky's formula ($1 - \sin \phi$') for predicting $K_0$ (1948) was found to predict $K_0$ to be 0.72, the Brooker and Ireland method predicted a value of 0.67, and Rowe's method (1957) predicted 0.73. For a flocculated specimen, $K_0$ was experimentally measured by the authors to be 0.75 in the first trial and 0.66 in a second trial. Jaky's formula predicted $K_0$ to be 0.69, the Brooker and Ireland method predicted a value of 0.64, and Rowe's method estimated $K_0$ to be 0.71. Considering an accuracy of ±0.05 to be applicable to $K_0$, the authors conclude that:

a. Jaky's formula provides the most consistent results. [Wroth (1972) also concluded that Jaky's formula was "sufficiently accurate for engineering purposes."]

b. The Brooker and Ireland method provides acceptable but low results. They also express some reservation about Rowe's method.

24. Additionally, they note that the relationship between vertical and horizontal effective stresses during unloading is nonlinear and $K_0$ is not
constant. $K_0$ slowly increases until it reaches a value of 1.0 at an OCR of approximately 2, after which it increases more rapidly until the OCR reaches approximately 8, whereupon it approaches the coefficient of passive earth pressure, $K_p$.

25. Schmertmann (1983, 1984). The author poses the question: "Does the effective lateral stress in one-dimensional compression of a normally consolidated cohesive soil, such as in the oedometer test, increase, remain the same, or decrease during secondary compression aging?" Although the question is addressed to normally consolidated cohesive soils while expansive soils are typically overconsolidated, the question may also be applicable to overconsolidated soils and quite pertinent to the discussion of lateral earth pressures on restrained vertical walls since $K_0$ values are the appropriate ones to use rather than coefficient of active pressure, $K_a$, in estimating restrained lateral pressures. The author posed this question to a total of 40 geotechnical engineers whom he considered to be "prominent for their research and work with soil consolidation or related soil behavior." The results of the survey showed that of the 32 responses, 16 believed $K_0$ would increase, 9 believed that $K_0$ would remain the same, 4 suggested that it would decrease, and 3 admitted that they either did not know or did not have an opinion. The author categorizes the response by a number of factors: residence (USA, Canada, Europe), employment (research/teaching or consulting/practice), and age of respondent (50 or less, over 50 years old). The conclusion reached by the author was that the geotechnical engineering profession does not have a common understanding of the soil behavior involved. In the subsequent published discussions of this paper, four discussers (Allam and Sridharan, 1984; McRoberts, 1984; Nagaraj, 1984; Soydemir, 1983) argue that $K_0$ increases (1 opinion), remains the same
(1 opinion), and decreases (2 opinions).

26. Pufahl, Fredlund, and Rahardjo (1983). The authors consider lateral earth pressures produced by saturated clays with negative porewater pressures and unsaturated expansive clays with positive matrix suctions from a theoretical, limit analysis standpoint. Simple earth pressure equations are formulated in terms of total stresses using the Mohr-Coulomb failure criteria and the assumptions consistent with the Rankine earth pressure theory. Since two stress state variables are required to describe the behavior of unsaturated soils, the conventional practice of separating the pressure that the soil exerts on the wall into the pressure produced by the soil structure (effective stress) and that produced by the water (neutral stress) cannot be applied to unsaturated clays because the conventional practice equations cannot be separated into two distinct components. The authors show the change in lateral earth pressures resulting from a decrease in pore water pressure or an increase in matrix suction. They also address the change in lateral pressures resulting from a change in matrix suction under conditions where walls are restrained from moving, and this change depends upon the ratio $K_t$ of horizontal to vertical stress and the matrix suction of the backfill at the time that it is placed behind the wall. When structural members are cast directly against undisturbed clays, similar criteria are shown to govern the magnitude of the lateral pressures that may be generated due to changes in matrix suction. They show that the maximum lateral pressure that can be developed in some cases is equal to the passive pressure of the soil when it is saturated. The effect of vertical surface tension cracks are also analyzed and shown to have little effect on the design conditions. This paper and the method of analysis which it presents are discussed in more detail in Part III.
Experimental Results

27. Although a number of the papers discussed above in the Theoretical Analyses section included the results of some laboratory or field experiment or measurements, they were included in the theoretical section because the investigators used those results to predict some property or parameter, such as swelling potential or swelling pressure. The papers that are discussed below present the results of some laboratory or field investigation wherein some trend is identified, some general observation is noted or verified, or some other result or conclusion pertinent to this study is reported.

28. Parcher and Liu (1965). Using a laboratory testing program of compacted expansive soils, the authors found that the unit swelling in the horizontal direction almost invariably exceeded that in the vertical direction, regardless of how the compaction was accomplished. They then attempted to relate the results to soil structure and double-layer phenomena. They cited eight factors influencing the magnitude of the heave and the swelling pressure:

a. Composition of the soil. The mineralogical makeup of the soil constituents are of primary importance in determining the potential of soil to shrink/swell. (See also Grim, 1958 and 1959; Rosenqvist, 1959; Michaels, 1959.)

b. Initial water content. Expansive soils that are initially dry and then permitted to imbibe water will swell more than when they are initially wet and then permitted to imbibe water. The reverse is, of course, true with respect to shrinking. (See also Holtz and Gibbs, 1956; Lambe, 1960; Seed, Woodward, and Lundgren, 1962.)

c. Soil structure. Soils in which the particles or clay platelets are arranged parallel to each other and perpendicular to the direction of swelling will exhibit greater amounts of swelling and larger swelling pressures than those soils with particles arranged differently. (See also Mitchell, 1956; Pacey, 1956; Lambe, 1960; Seed and Chan, 1961.)

d. Availability and properties of water. Obviously, a source of free water must be available to the soil to result in an
increase in volume. Additionally, the dissolved exchangeable cations in the pore water influences the amount of water demanded by the swelling soil. (See also Low, 1959; Taylor, 1959.)

e. Confining pressure. This actually refers to the degree of restraint to swelling imposed on the swelling soil. Even very small movements by the restraining device or structure can result in a significant dissipation of the swelling pressure. Additionally, even a perfectly restrained specimen can still imbibe water and attempt to swell. (See also Seed, Mitchell, and Chan, 1961.)

f. Curing period. This factor is applicable to specimens prepared in the laboratory. Often, air dry soils are prepared for testing, and free water of a desired amount is added to the soil so that it has a particular initial water content. Allowing the added water sufficient time to become thoroughly dispersed throughout the specimen is required to ensure uniform soil water conditions before testing begins. (See also Barber, 1956.)

g. Time permitted for swelling. The soil structure and the mineralogical constituents of the specimen influence the rate at which the added water is distributed throughout the specimen. Thus, some initial swelling may occur as a result of water distribution through slickensides or fissures followed by additional swell days or even weeks later from water moving through the tighter soil matrix bounded by the slickensides or fissures. (See also Lambe, 1960.)

h. Temperature. The thickness of the double layer as well as the rate of permeability is influenced by temperature. Increased temperatures permit water to permeate at a faster rate; however, increased temperatures also result in thinner double layers. (See also Lambe, 1960.)

29. Blight (1967). Citing a number of references to the technical literature, Blight notes that $K_0$ is dependent on the stress history of the clay and that as the overconsolidation ratio increases, $K_0$ increases, becoming greater than 1.0 when the OCR ratio exceeds about 5 or 6. He also summarized Skempton's conclusion (1961) that the values of $K_0$ for in situ clays can be deduced by comparing the in situ effective overburden stress with the isotropic effective stress in the soil after undisturbed sampling, which is to say that the horizontal effective stress approaches the passive pressure resistance of the clay. This observation was supported by Terzaghi
who also concluded that the fissures and slickensides which usually form in heavily overconsolidated clays are associated with horizontal stress equal to the passive pressure resistance of the clay. Using this method of Skempton, Blight measured $K_0$ on two South African soils. He formed three conclusions from his investigation:

a. In situ horizontal effective stresses in saturated lacustrine clay that has been overconsolidated by desiccation and then rewetted may approach the passive pressure resistance of the fissured clay.

b. If the clay becomes desiccated owing to a lowering of the water table, and if the shrinkage is large as the clay dries out, the horizontal effective stress will decrease.

c. Consequently, horizontal stresses at rest in expansive clays with a lacustrine origin will generally be lower than the minimum passive pressure resistance of the clay, even when the clay has fully heaved beneath a covered surface.

30. Ahmed (1967). The author performed laboratory experiments with a retaining wall model. Clean sand was used as the soil behind the wall and the wall was permitted to move; thus, $K_a$ was measured rather than $K_0$. Although sand was used as the retained soil and although active conditions were measured rather than at-rest conditions, Ahmed reported one finding from his model testing that may have application to the problem being studied presently. A relationship between lateral force and the ratio of fill thickness to wall height was found to exist. These results are reported in Fig. 2-1. This figure shows that at a fill thickness to wall height ratio of approximately 0.50, there is no further increase in lateral force transmitted to the wall with increasing thickness of fill. A similar result is implied in the Katti, Bhangale, and Moza (1983) report (presented below) of studies made with select cohesive nonswelling clay backfill between the wall and the swelling soil.
Figure 2-1. Relationship between measured lateral pressure and thickness of backfill. (After Ahmed, 1967)
31. Komornik and Livneh (1968). A laboratory investigation of anisotropy of swelling was accomplished using a swelling pressure ring constructed by the investigators. They reported that the amount of swell was greater parallel to the direction of compaction; this was attributed to the plate structure of the montmorillonitic clay which had been aligned perpendicular to the direction of compaction. They also found that for the same amount of vertical swell, the lateral swelling pressure measured was smaller with a predominantly parallel orientation than with a predominantly perpendicular orientation. In their opinion, the significance of this finding is that if a compacted backfill is used behind a retaining structure, it will likely experience a lesser lateral pressure than what was predicted from laboratory testing using an undisturbed sample. However, they also concluded that the effect of anisotropy on swelling pressure and amount of swell is relatively small and is dependent on the degree of saturation of the sample. They reported that anisotropy was found to affect the lateral swelling pressures between 40 and 50 percent in the case of low vertical pressures permitting vertical swell exceeding 1 percent. The overall conclusion by Komornik and Livneh was that when testing clay in the vicinity of a structure, the orientation effect should be taken into consideration.

32. Saito and Yanai (1969). The authors report the results of laboratory testing conducted on compacted expansive soil samples. The compacted specimens were then soaked under laterally confined conditions. Swelling pressures were measured during a program of progressive loading. For measuring the percent of swell, the specimens were preloaded under a uniform surcharge of 60 g/cm² and then permitted to imbibe water from all surfaces of the specimen (circumferentially as well as from the top and
bottom specimen surfaces). Some of the conclusions reported included:

a. As plasticity index increased, maximum swelling pressure and percent of swelling at optimum moisture content increased.

b. The greater the plastic ratio (defined as PI/PL), the greater the rate of change of swelling pressure as a function of change in dry density. This relationship was observed to be consistent between swelling percent and swelling pressure.

c. Soil compacted with a greater number of blows resulted in lesser swelling when the molding water content was on the wet side of the optimum water content.

d. The greater the plasticity index, the greater the difference in undrained strength before and after soaking.

33. Satyanarayana (1969). The results of a laboratory investigation on the effect of sand either mixed with the expansive soil or using a sand layer to reduce the swelling pressure of the expansive soil are described. The tests were accomplished by placing statically compacted Indian black cotton soil specimens in an oedometer and permitting the soil to freely imbibe water and swell. The swelling pressure was measured by applied loading from the consolidation device. The author formed four conclusions from the results of this experiment:

a. The percentage of reduction in swelling pressure consequent on mixing expansive clay with small quantities of sand is greater than the percentage of sand added.

b. At lower percentages of sand, the clay acts as a filler in the voids formed by the sand particles, and the net swollen volume of the clay particles is responsible for the swelling pressure developed. At sand contents of about 50 percent and more, clay particles tend to collect in pockets, giving rise to some swelling pressure greater than that predicted by the net swollen volume of clay particles.

c. Providing sand cushions reduces the swelling pressure in proportion to their capacity to absorb the swell of the expansive clay. This capacity depends on the facility with which the sand layer can deform by compression and by lateral movement.

d. Attention to test details is essential if interpretable measurements of swelling pressure are to be made.
34. Katzir and David (1969). The authors discuss construction in or over expansive marls and provide recommendations for such construction. They also point out that the shrinkage limit was found to have a poor correlation with other swelling properties.

35. Komornik, Wiseman, and Ben-Yaacob (1969). Studies were performed at a number of sites in Israel to investigate the potential swelling behavior with depth of the clay strata during different seasons. Undisturbed samples were taken and subsequently tested in the laboratory, while moisture and temperature variations with depth were measured. The authors concluded that the depth of the active zone is influenced by (1) soil type, (2) climate, (3) soil profile or stratigraphy, (4) the depth to the groundwater table, and (5) surface vegetation. They view heave as a rebound process; i.e., the effective stress decreases when the soil wets up, resulting in heave. They also conclude that if an increase in external loading is equal to an associated decrease in soil suction, no heaving will occur. They show theoretically that the effect of temperature on soil suction is small, and for partially saturated soils it is assumed that most moisture movement due to thermal gradient is in the vapor phase. They reported that both liquid limit and plastic limit were decreased by about 10 percent for an increase in temperature from 20°C to 30°C (68°F to 86°F). They suggested that the coefficient of linear expansion for a soil with a volumetric water content of "about 50 percent" was approximately $4 \times 10^{-5}$ per degree Centigrade. As a result, the authors estimate the thermal expansion between winter and summer accounts for approximately 3 mm (0.12 in., 0.01 ft) of heave at Lod Airport in Israel; this can also account for lateral surface movements as well. They also suggest that the equilibrium soil suction can be estimated by plotting (swelling pressure minus $\gamma z$) vs. $w_n/PL$ ratio. They
also reported measuring lateral swelling pressures approximately equal to the vertical swelling pressures.

36. Komornik and Zeitlen (1970). A laboratory investigation is reported in which studies were made of both the lateral and vertical pressures developed by a compacted clay, under different placement conditions, when various amounts of swell were permitted during saturation. Some observations noted and reported included:

a. At constant density, the amount of swell was larger for those specimens which were compacted at a lower water content.

b. At constant water content, the amount of swell was larger for those which were compacted to a high density.

c. The swelling pressures associated with no vertical movement did not show large differences with changes in moisture content for specimens prepared to the same density.

d. The higher the density, the higher the vertical swelling pressures, regardless of the water content of the sample.

e. The higher the density, the lower the compressibility.

37. Gould (1970). In an invited paper, Gould offered the observation that "there seems to be no question that effective lateral pressures exceeding overburden actually exist in situ in overconsolidated clays or clay-shales . . . In fact, at shallow depths, the pressure ratio can actually approach the passive state." He also presented a case study in which "stress meters" (pressure cells) were emplaced in the basement walls of a structure which were subjected to lateral pressures from a compacted backfill. The backfill was compacted to 98 percent of the standard Proctor maximum density at a moisture content 1 percent above the optimum. The soil had Atterberg limit properties of a liquid limit of 37 and a plasticity index of 21. Triaxial tests indicated $K_0$ to be between 0.55 and 0.60. Plotting "stress meter" results vs depth indicate the measured pressures coincide very closely to the straight line produced by using $K_0$. 27
calculations.

38. Gokhale and Jain (1972). The purpose of the experiment was to study the influence of initial moisture content and the nature of granular material on the anisotropic swell behavior of expansive soil. The authors describe a laboratory testing procedure using compacted samples of Indian black cotton soil to study anisotropic swelling. Two specimens were taken from each compacted sample; one specimen taken in the vertical direction and one taken in the horizontal direction. Each specimen was placed in a consolidometer and preloaded to a pressure of 1 psi. A continuous supply of water was made available to the specimen during swelling.

39. The authors observed that the amount and rate of swelling were guided by the initial particle orientation. For samples compacted at a water content below the optimum value, the variation in swelling trended in vertical and horizontal directions as a result of the particle arrangement, the pressure exerted by the air in the voids, and the subsequent expulsion of the same. For samples compacted at a water content above the optimum, the initial dispersed state of the particles guides the anisotropy. The swelling in such samples is considerable in directions transverse to the particle orientation. At optimum moisture levels, because of the minimal differences in the particle arrangement for samples in vertical and horizontal directions, the swelling was similar in both directions and, consequently, no anisotropic behavior was noted. Increasing proportions of granular components in the soil reduced the rate of swelling, protracted the time for completion of swelling, and reduced the anisotropic behavior. Reduction in the grain size of the granular material facilitated quick swelling. The variations in such samples for vertical and horizontal directions are attributed to "packing" in the system and the constraints
imposed by sand grains on osmotic swelling.

40. Kassiff, Baker, and Ovadia (1973). A laboratory test was devised to measure the effect of known pore water solute concentration on swelling. Among other findings, the investigators conclude that at small suction changes there seems to exist a threshold of suction changes above which the amount of swell percentage and swell pressure are linear functions of the suction change. They also develop theoretical relationships for predicting swell as a function of initial moisture content, initial dry density, change in (mechanical) swelling pressure, and change in soil suction.

41. Didier (1973). The author describes a laboratory test to measure changes in swelling pressure with time. The experiment used oven-dried expansive soil statically compacted in an oedometer-type device subjected to triaxial pressures in a temperature-controlled cell. Didier found that a reduction in swelling pressure occurred, reaching a minimum after about 400 minutes, followed by a subsequent rather rapid increase in pressure, reaching a maximum at 16 to 24 hours after the initiation of the test. Didier then placed a thin layer of sand particles between the expansive clay and the loading piston and then permitted the soil to swell. He found that the sand layer in direct contact with the swelling soil resulted in a lower final swelling pressure. He also found that the swelling clay did not penetrate the sand during swelling beyond the first "row" of sand grains.

42. Satyanarayana (1973). The results of a laboratory testing program conducted with compacted soil samples is reported. The investigator compared the results obtained on similar specimens compacted by either static or dynamic means and found:

a. The magnitude of swelling and the swelling pressure of specimens prepared by static compaction are larger than those prepared by dynamic compaction. This was attributed to static compaction techniques producing a more random soil
structure than that obtained from dynamic compaction.

b. The amount of swelling and swelling pressure decreased with increasing initial water content regardless of method of compaction (static or dynamic).

c. Larger amounts of swell and swelling pressure resulted when higher compaction efforts were employed.

d. Both the amount of swell and the swelling pressure increased with increasing dry density regardless of the method of compaction or the direction of sampling.

43. Brackley (1973). Results of a laboratory test program conducted on dynamically compacted South African expansive clay soils are reported. Brackley concludes that swell pressure is a function of void ratio only and suggests that swell pressure may be estimated from the suction vs water content curve or, although less accurately, from the virgin consolidation line. He also proposes that the percentage of free swell, FS, may be estimated from

\[
FS = \left( m_s - m_o \right) \left( 0.36 + m_o \right) \times 100
\]  

(2-9)

where

- \( m_o \) = original water content
- \( m_s \) = compaction water content at which no swell occurs when the sample is placed in water

44. Snethan and Halliburton (1973). The results of a laboratory investigation program using compacted clay samples is reported. The authors found that at initial water contents below that of optimum, the trend toward decreasing lateral swelling pressure with increasing initial moisture content is offset by the tendency toward increased lateral pressure with increasing dry density. At initial water contents slightly below and above optimum, the dry density was found not to change rapidly. Thus, the effect of water content and compacted soil structure was concluded to determine the swelling behavior. At initial water contents above optimum, the influence
of increasing water content and decreasing dry density combine to reduce the lateral swelling pressure.

45. It was also found that the vertical swelling pressure exceeded the lateral swelling pressure for nearly all initial conditions. The swelling ratio, defined as the lateral swelling pressure/vertical swelling pressure, was found to be approximately equal to 1.09 at water contents on the dry side of optimum. At water contents above optimum, however, the swelling ratio was found to be relatively constant, ranging between the values of 0.50 and 0.65.

46. Katti and Kate (1975). Large-scale model laboratory tests showed that the heave of underlying Indian black cotton expansive soil reduced rapidly with the increase in thickness of an overlying cohesive nonswelling soil and reached a value of no heave when the depth of the overlying cohesive nonswelling soil became approximately 1.0 to 1.2 m thick. The authors also postulated that the internal characteristics of the cohesive nonswelling soil layer were responsible to a large extent for counteracting the heave and swelling pressure of the underlying expansive soil media. Their studies indicated that the lateral pressure of the underlying expansive soil below an adequate thickness of the cohesive nonswelling layer is equal to the lateral pressure in the no-volume-change depths in expansive soils.

47. Katti, et al. (1980). Results from large-scale model laboratory tests indicated that a cohesive nonswelling soil layer was effective in resisting vertical heaving and swelling pressure.

48. Joshi and Katti (1980). Large-scale laboratory model tests were conducted using Indian black cotton soil to measure lateral swelling pressures. A number of observations or conclusions were reported:
a. In triaxial testing, lateral and vertical swelling pressures after saturation were about the same.

b. Lateral swelling pressure increased from the surface to a depth of about 3 ft and then remained approximately constant.

c. Lateral swelling pressures increased with time after saturating, reached a maximum (at approximately 20 to 30 days), then decreased slightly, and remained approximately constant (after approximately 40 days) until the conclusion of the test.

d. Development of lateral pressure was fairly linear and rapid under increasing surcharge to approximately 2 tsf, then continues to increase but not as rapidly.

e. Lateral swelling pressures measured during surcharge reduction were always higher (within the swelling pressure range) than were the initial wetting equilibrium pressures.

f. K decreased as surcharge increased but tended to reach a minimum of approximately 2.0.

g. Beyond the swelling pressure range, the increase in lateral pressure appears to be similar to that of "conventional" (nonexpansive) soil.

h. For samples saturated under surcharges greater than the swelling pressure, "locked lateral pressures" were absent for surcharge releases up to a point where the surcharge magnitude was approximately equal to the swelling pressure. Then for further surcharge release, "locked lateral pressures" existed.

49. The authors also proposed the "Developed Cohesion Concept," whereby adsorbed water around adjacent clay particles helps develop cohesion which resists particle swelling. The authors presented theoretical calculations which suggest swelling pressures should be in the range of 1.5 to 5.0 times the value of the soil cohesion for their experimental soils, a range within which their experimental values fall.

50. Katti, Bhangale, and Moza (1983). The results of a very comprehensive large-scale model laboratory testing program using sand, a nonswelling cohesive soil, and an Indian black cotton expansive soil are reported by the investigators. In this 102-page report, a number of
observations, findings, and conclusions are reported. In this investigation, the work is confined to a "no movement of wall" condition and, thus, the values from the tests represent the conditions of earth pressure at rest, i.e., $K_0$ conditions. Preliminary results indicated that the expansive black cotton soil required approximately 45 days with free access to water to become completely saturated; however, to ensure saturation was being reached before testing commenced, the investigators permitted the soil to imbibe water for a period of 70 days. The results of this testing program are discussed in more detail in Part III. Some of the principal findings, observations, and conclusions include:

a. Calculations using Jaky's $K_0 = 1 - \sin \phi'$ equation (Jaky, 1948) showed it to be valid for dry, loosely placed (in the terminology of the investigators, "filled up") soil but the equation was not found to be accurate for compacted soils.

(1) The results of the lateral pressures measured for loosely placed air-dry sand, loosely placed air-dry cohesive nonswelling soil, and loosely placed black cotton soil showed the at-rest earth pressures, $K_0$, for each of these soils to be, respectively, 0.63, 0.48, and 0.26. Using Jaky's equation, $K_0$ was calculated to be 0.625, 0.48, and 0.59 for the same soils, respectively.

(2) When the air-dry soils were compacted, $K_0$ values were measured to be in excess of 1.0 for each soil type: 2.33 for sand, 1.16 for the cohesive nonswelling soil, and 1.1 for the expansive black cotton soil.

(3) When the compacted soils were saturated, the $K_0$ values for the sand and the cohesive nonswelling soil increased slightly as would be expected: from 2.33 to 2.58 for the saturated sand, and from 1.16 to 1.50 for the saturated cohesive nonswelling soil.

(4) The relationship of lateral pressure with depth was found to be linear for eight of the nine test conditions, i.e., loosely placed air-dry sand, loosely placed air-dry cohesive nonswelling soil, loosely placed air-dry expansive soil, compacted air-dry sand, compacted air-dry cohesive nonswelling soil, compacted air-dry expansive soil, saturated compacted sand, and saturated compacted cohesive nonswelling soil. When plotted as a function of depth, the measured lateral pressure curves all plotted as a straight line.
b. The relationship between depth and measured lateral pressure for compacted expansive soil which is permitted to swell was found to be nonlinear, increasing very rapidly from the surface to a depth of approximately 145 cm. At this depth, the lateral pressure continues to increase but not nearly as rapidly.

c. The greatest lateral swelling pressure was found to occur not at 100 percent saturation but at some degree of saturation less than 100 percent.

d. Three different test series were accomplished to determine the influence of varying thicknesses of the cohesive non-swelling soil placed: on top of the expansive soil (called "cover"), between the wall and the expansive soil (called "backing"), and as a combination of cover and backing thicknesses. In summary, the results showed:

(1) As the thickness of the backing increased from zero to 1 m, the magnitude of the measured lateral pressure decreased and approached that of the saturated, compacted cohesive nonswelling soil by itself.

(2) As the thickness of the cover increased from zero to 1 m, the magnitude of the lateral pressure did not measurably decrease, but the magnitude of the vertical heave decreased and approached a condition of no heave.

(3) As the thickness of both the backing and cover increased toward 1 m, both the measured lateral pressure and the amount of vertical heave decreased in a manner similar to that observed when only either condition was included by itself in the test.

e. "Locked in" lateral pressures were observed in all cases. The term "locked in" refers to lateral pressures in excess of overburden and surcharge vertical loadings. Although the source of these larger lateral pressures was attributed to the energy input into the soil during compaction, these same conditions have been observed by other investigators.

51. Kavazanjian and Mitchell (1984, 1985). The authors report on an investigation conducted to determine if $K_o$ changes with time. They cite two experiments that showed $K_o$ for normally consolidated soils decreased with time. They then performed a theoretical analysis that resulted in the conclusion that, ultimately, $K_o$ should approach a value of 1.0, implying a condition in which the soil reaches a minimum energy state due to the absence of any global deviator stress. Thus, normally consolidated soils
with a measured value of $K_0$ of less than 1.0 would tend to have $K_0$ increase with time toward 1.0, and overconsolidated soils with a $K_0$ greater than 1.0 should tend to have $K_0$ decrease with time, approaching 1.0. (See also Holtz and Jamiolkowski, 1985; Lacerda and Martins, 1985; Leonards, 1985.)

52. Sridharan, Sreepada Rao, and Sivapullaiah (1986). The authors report the results of a laboratory experimental program to determine the effect of testing method on swelling pressure. The testing program employed three different test methods and used Indian black cotton soil at various initial water contents, including specimens made from oven-dried soil. In other parts of the testing program, initial water content and dry density were varied. Their findings showed that the method whereby the specimen is permitted to fully swell against a nominal seating pressure followed by loading to bring the specimen confined in an oedometer back to its initial height yields the greatest swelling pressure.

53. A second method which employs three or more specimens initially loaded to different pressures bracketing the expected swelling pressure was also used. The specimens were then allowed to swell against the applied pressure resulting in a straight line $e$-$\log p'$ plot from which the swelling pressure can be determined by entering the plot with the initial void ratio yielding the least swelling pressure.

54. The third method involves applying additional loading as necessary to maintain the confined specimen at the initial height. This method is often termed the "confined volume test," and it was found to yield swelling pressures intermediate between the first two methods.

55. The authors also reported that the third method was quick to perform and had an advantage over the other methods because it only required one test specimen. However, the results of this method are sensitive to
both loading increment and rate of loading. They found that slower rates of loading or smaller loading increments resulted in higher maximum swell magnitudes. The second method can be performed quicker than the first method but has the disadvantage of requiring three "identical" specimens.

56. The authors also reported:

a. The effect of stress path is significant in determining the swelling pressure.

b. Swelling pressure is primarily dependent on the initial dry unit weight or void ratio of the soil.

c. The effect of initial water content has less influence on swelling pressure than the other two factors.

d. Time vs. swelling magnitude and time vs. swelling pressure (constant volume) could be reasonably represented by a rectangular hyperbola.
PART III: METHODS FOR PREDICTING LATERAL PRESSURES IN EXPANSIVE SOILS

57. The principal objective of the literature search reported in Part II was to search for a method whereby the lateral pressure exerted on a basement wall by expansive soils could be estimated. After reviewing and analyzing the available literature, it appears that three methods have sufficient potential for providing the desired earth pressures to warrant detailed presentation and consideration. These three methods are those of Skempton (1965), Fredlund [(1979 and 1987); Pufahl, Fredlund, and Rahardjo (1983); Rahardjo (1982); Rahardjo and Fredlund (1983); Rahardjo and Fredlund (1984)], and Katti [Katti and Kate (1975); Katti, et al. (1980); Joshi and Katti (1980); Katti, Bhangale, and Moza (1983)]. The Skempton and Fredlund methods are based on soil mechanics theory, while the Katti method is the empirical result of an extensive laboratory experimental program.

Skempton Method

58. Referring to the Mohr's circle depicted in Fig. 3-1, Skempton defines the term $p_k$ as the capillary pressure in a soil specimen before it is tested and sheared. The difference between the minor principal stress, $\sigma_3$, and $p_k$ is the change in porewater pressure, $\Delta u_f$; $\Delta u_f$ will represent the porewater pressure during shearing. Thus,

$$ p_k = \sigma'_3 + \Delta u_f $$  \hspace{1cm} (3-1)

But if tested in the unconfined compression test, $\sigma'_3 = 0$. Therefore,

$$ \Delta u_f = A_f |\Delta \sigma_1 - \Delta \sigma_3|_f = A_f (2c) $$  \hspace{1cm} (3-2)

and substituting Equation (3-2) into Equation (3-1) yields

$$ p_k = \sigma'_3 + A_f (2c) $$  \hspace{1cm} (3-3)

The pore pressure parameter $A_f$ can be evaluated from laboratory tests. The other two parameters can also be evaluated from laboratory tests, permitting
Figure 3-1. Mohr's circle relationship for Skempton's method showing capillary pressure and the effective stress in the specimen before shearing. (After Skempton, 1961)
Equation (3-3) to be solved for $p_k$. The effective stress value $p_k$ can also be evaluated from oedometer testing as well as from strength testing. However, a third method of evaluating $p_k$ is probably easier to accomplish and more straightforward than either the strength or the oedometer methods. This method is to directly measure $p_k$ using soil suction techniques. As Skempton shows, the swelling pressure is equal to the total soil suction. Therefore, any method that measures total suction will permit $p_k$ to be directly evaluated. If vertical soil stress $\sigma_v$, is calculated as

$$\sigma_v = \gamma z \quad (3-4)$$

and recalling that the coefficient of at-rest earth pressure, $K_0$, is defined as the ratio of horizontal soil stress to vertical soil stress

$$\sigma_h = (\sigma_v - u_o)K_0 + u_o \quad (3-5)$$

or

$$\sigma_h = pK_0 + u_o \quad (3-6)$$

where for Equations (3-4) - (3-6)

$\gamma$ = soil unit weight

$z$ = height of soil having a unit weight of

$u_o$ = piezometric pore water pressure

$p$ = effective vertical stress in situ

$K_0$ = coefficient of at-rest earth pressure

After sampling and extracting

$$p = 0 \quad (3-7)$$

and from Equation (3-1)

$$p_k = 0 - u = -u \quad (3-8)$$

If $\Delta u$ is the change in porewater pressure due to sampling, then

$$u = u_o + \Delta u \quad (3-9)$$
or
\[ u = u_0 + (1/3)(\Delta \sigma_1 + 2\Delta \sigma_2) + \]
\[ [A_s - (1/3)]|\Delta \sigma_1 - \Delta \sigma_3| \] (3-10)

If the sample is extracted, then
\[ \Delta \sigma_1 = -\sigma_v \text{ and } \Delta \sigma_3 = -\sigma_h \] (3-11)

If the soil is overconsolidated, as are most expansive soils, and \( K_o > 1 \), then from Equation (3-11)
\[ |\Delta \sigma_1 - \Delta \sigma_3| = \sigma_h - \sigma_v \] (3-12)

59. Substituting Equation (3-12) into Equation (3-10), where \( u_0 = 0 \) and \( K_o = \sigma_h/\sigma_v \);
\[ u = -p[K_o - A_s(K_o - 1)] \] (3-13)

Then substituting Equation (3-13) into Equation (3-8)
\[ p_k = p[K_o - A_s(K_o - 1)] \] (3-14)

If the pore pressure parameter \( A_s \) is evaluated from triaxial testing, \( p \) calculated from site investigation results, and \( p_k \) evaluated using any of the three methods described above, then \( K_o \) can be calculated by rearranging Equation (3-14):
\[ K_o = \frac{[(p_k/p) - A_s]}{(1 - A_s)} \] (3-15)

The \( K_o \) values reported in Table 2-2 were calculated using Equation (3-15).

60. The passive earth pressure, \( p_p \), represents an upper bound to the lateral earth pressure. This pressure can be calculated from
\[ p_p = c'[(2 \cos \phi') / (1 - \sin \phi')] + \\
p[(1 + \sin \phi') / (1 - \sin \phi')] \] (3-16)

and since the coefficient of passive earth pressure, \( K_p \), is equal to \( \sigma_h/\sigma_v \) or \( p_p/p \), then
\[ K_p = p_p/p = (c'/p)[(2 - \cos \phi') / (1 - \sin \phi')] + \\
[(1 + \sin \phi') / (1 - \sin \phi')] \] (3-17)
61. Since Skempton's method considers the in situ soil suction in calculating \( K_0 \) and the resulting lateral earth pressure, it appears that this method is predicting the expected pressure. If the soil should be found to be wetter (low soil suction) at one time and drier (higher soil suction) at another, the method considers the swelling potential and a larger \( K_0 \) value is calculated for the drier condition than for the wetter one. Thus, in summary, to apply Skempton's method, what is needed is: (1) soil unit weight with depth; (2) piezometric porewater pressures with depth; (3) in situ soil suction measurements; and (4) two pore pressure parameters obtained from laboratory triaxial testing.

Fredlund Method

62. Although the limit analysis approach to the problem was developed and presented in several separate papers (Fredlund, 1979; Fredlund, 1987; Rahardjo, 1982; Pufahl, Fredlund, and Rahardjo, 1983; Rahardjo and Fredlund, 1983; Rahardjo and Fredlund, 1984), there appears to be little doubt that this concept is that of Fredlund. Thus, the following development is described as "Fredlund's Method."

63. In retrospect, the concept of transitioning from limit analysis applied to slope stability problems to that of lateral earth pressure problems should have been a natural extension to earth pressure problems in unsaturated soils. However, in this writer's opinion, this extension was most likely not all that was obvious at the time it was initially conceived.

64. In its application, the limit analysis as applied to slope stability analysis assumes a hypothetical failure plane, divides the soil mass above the failure plane into a number of "slices," and then, through equilibrium analysis, arrives at a force required to resist the movement of the soil mass down the failure plan (active failure) or resist the movement
of the soil mass up the failure plane (passive failure). In applying the limit analysis to lateral earth pressures, the assumption is made that the retaining wall is located at the edge of the last slice and the force required to prevent slope stability failure now becomes the lateral earth force acting on the retaining wall.

65. When applying most of the methods of calculating lateral earth pressures that are currently in common use, the forces or pressures assumed to be acting on the wall are typically divided into two components: those forces or pressures created by the soil structure (effective stresses) and those created by the water (neutral stresses). If the soil has an apparent cohesion intercept on the Mohr-Coulomb failure envelope, the upper part of the retained soil mass in active failure conditions is apparently in tension. Often this part of the effective pressure distribution is neglected because it is assumed that the soil structure cannot adhere to the wall despite the total pressure applied to the wall is compressive. This procedure is difficult to apply to unsaturated soils because, as Fredlund points out, a single effective stress equation cannot be used to describe the behavior of the unsaturated soil mass. As explained by Fredlund, the two stress state variables that appear to be most satisfactory for most soil mechanics problems are \((\sigma - u_a)\) and \((u_a - u_w)\), where \(\sigma\) is the total applied stress, \(u_a\) is the pore air pressure, and \(u_w\) is the porewater pressure. Using these two stress state variables, where the \((\sigma - u_a)\) term represents the net total stress and the \((u_a - u_w)\) term represents the matrix suction, the pressures acting on the wall can be computed in terms of total stress, pore air pressures, and matrix suction.

66. As suggested by Fredlund, et al. (1978), the effect of matrix suction is to increase the apparent cohesion of the soil. Thus, the
shearing resistance, \( \tau \), of an unsaturated soil is represented as
\[
\tau = (\sigma - u_a) \tan \phi' + [c' + (u_a - u) \tan \phi^b]
\]
where
\[
\phi' = \text{the effective angle of shearing resistance of a saturated soil}
\]
\[
\phi^b = \text{the angle of shearing resistance with respect to changes in matrix suction}
\]
\[
c' = \text{the effective cohesion intercept}
\]
Although \( \phi^b \) may not be constant over an extremely large range of soil suction, Ho and Fredlund (1982) report that it appears to be constant over suction ranges commonly encountered in practice. Equation (3-18) is illustrated in Fig. 3-2 where matrix suction forms the third axis of the Mohr-Coulomb failure envelope.

In the paper in which the active and passive pressure equations are presented [Pufahl, Fredlund, and Rahardjo (1983)], simplified limit equations are developed for several different conditions, including:

- a. Saturated soil.
- b. Saturated soil with a groundwater table located below the ground surface.
- c. Intact unsaturated soil.
- d. Unsaturated soil with tension cracks extending from the soil surface.
- e. Changes in lateral earth pressures due to changes in the water table.
- f. Unsupported excavations in intact soil.
- g. Unsupported excavations with surface cracks in the soil.

Earth Pressure Equations for Saturated Soils

Although the principal thrust of this study is directed toward unsaturated expansive soils, it is believed that initially developing the
Figure 3-2. Active and passive earth pressure for saturated and unsaturated soils. (After Pufahl, et al., 1983)
Fredlund equations for a saturated soil condition will be beneficial to the reader. In addition, the derivation of some of the fundamental equations used in the development below is included in Appendix A.

69. As the unsaturated soil represented by Circle 1 of Fig. 3-2 moves to the saturated condition represented by Circle 2, the porewater pressure approaches that of the pore air pressure and Equation (3-18) reduces to the more familiar form of the Coulomb shear strength equation:

\[ \tau = (\sigma - u_w) \tan \phi' + c' \] (3-19)

From Mohr's circle analysis, it can be shown

\[ \sigma_1 = \sigma_3 \tan^2(45^\circ + \phi/2) + 2c'\tan(45^\circ + \phi/2) \] (3-20)

If

\[ N_\phi = \tan^2(45^\circ + \phi/2) \] (3-21)

which was first introduced by Terzaghi & Peck (1967) as the "flow value," then Equation (3-20) can be simplified to

\[ \sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \] (3-22)

or rearranging Equation (3-22) to permit solving for \( \sigma_3 \):

\[ \sigma_3 = (\sigma_1/N_\phi) - (2c\sqrt{N_\phi}) \] (3-23)

Since for active failure conditions, \( \sigma_1 > \sigma_3 \), Equations (3-22) and (3-23) can be rewritten as

\[ \sigma_v = \sigma_h N_\phi + 2c\sqrt{N_\phi} \] (3-24)

and

\[ \sigma_h = (\sigma_v/N_\phi) - (2c\sqrt{N_\phi}) \] (3-25)

The pressure distribution in the soil during active failure is illustrated in Fig. 3-3. If \( \sigma_v \) is defined by Equation (3-4), then the depth \( Z_t \) is found to be
where \( Y \) is the total unit weight of the soil.

70. Then, if soil cohesion is assumed to be constant and if the tension zone depicted in Fig. 3-3c is disregarded, the pressure distribution on the wall will be area ABC of Fig. 3-3c and the active lateral force will be

\[
P_A = \left[ (YH/N_\phi) - (2c'/\sqrt{N_\phi}) \right] \left[ (H - Z_t)/2 \right]
\]

(3-27)

71. The passive earth pressures for this condition of saturation are illustrated in Fig. 3-4. The total passive pressure at any depth is

\[
\sigma_h = \sigma_v N_\phi + 2c' \sqrt{N_\phi}
\]

(3-28)

and the total passive lateral force will be

\[
P_p = (YH^2/2)N_\phi + 2c'H \sqrt{N_\phi}
\]

(3-29)

Earth Pressure Equations for Unsaturated Soils

72. From Circle 1 of Fig. 3-2, the total active pressure will be

\[
\sigma_h = \left( \sigma_v / N_\phi \right) - \left( 2/\sqrt{N_\phi} \right) \times \left[ c' + (u_a - u_w) \tan \phi - u_a \tan \phi' \right]
\]

(3-30)

In Equation (3-30), the soil suction contribution is represented by the term \((u_a - u_w)\tan \phi\). The pore air pressure, \(u_a \tan \phi'\), is included in Equation (3-30) to make it technically correct, but practically, \(u_a \approx 0\) (relative to the atmosphere) and the term can be neglected without important effect on the result.

73. Equation (3-30) indicates that the soil matrix suction is a constant with depth. This is not typically found to be true in practice. Therefore, Fredlund recommends the following linear relationship be used to estimate the variation of suction with depth when \(z < D\):

\[
(u_a - u_w)_z = (u_a - u_w)_s [1 - (z/D)]
\]

(3-31)
Figure 3-3. Active earth pressure for saturated soil conditions with zero porewater pressure. (After Pufahl, et al., 1983)

Figure 3-4. Passive earth pressure for saturated soil conditions with zero porewater pressure. (After Pufahl, et al., 1983)
Thus, for a depth less than D below the surface, the total active pressure is estimated as

\[ \sigma_h = \left( \sigma_v / N_\phi \right) \left[ c' + (u_a - u_w)_S \right] \left( 1 - z/D \right) \tan \phi^b \]  
(3-32)

The total active pressure below a depth D is given by

\[ \sigma_h = \left( \sigma_v / N_\phi \right) - 2\sqrt{\sigma_v / N_\phi} \left[ c' - (z - D)Y_w \tan \phi' \right] \]  
(3-33)

where \( Y_w \) is the unit weight of water. These relationships are illustrated in Fig. 3-5. The depth \( Z_{tu} \) over which tension apparently exists between the soil and the wall when \( Z_{tu} > D \) is estimated as

\[ Z_{tu} = 2\sqrt{\sigma_v / N_\phi} \left[ c' + (u_a - u_w)_S \tan \phi^b \right] / \right) \]  
(3-34)

where \( Y_u \) is the unsaturated unit weight of the soil. If \( Z_{tu} > D \), then

\[ Z_{tu} = \left( 2\sqrt{\sigma_v / N_\phi} \left[ c' + D Y_w \tan \phi' \right] + \right) \]  
(3-35)

where \( Y_S \) is the saturated unit weight of the soil.

74. The total active lateral force on the wall is represented by area CDEF in Fig. 3-5. In order to simplify the calculations, Fredlund approximates area CDEF to that of a triangle with CFE assumed to be linear and the total force is then estimated to be

\[ P_A = f (Y_u D / N_\phi) + \left[ Y_S (H - D) / N_\phi \right] - \left( 2\sqrt{\sigma_v / N_\phi} \right) \left[ c' - (H-D)Y_w \tan \phi' \right] \times \left( (H-Z_{tu})/2 \right) \]  
(3-36)

75. In a similar manner, the total passive lateral pressure above depth D is found from

\[ \sigma_h = \left( \sigma_v / N_\phi \right) + \left( 2\sqrt{\sigma_v / N_\phi} \right) \left[ c' + (u_a - u_w)_S \right] \left[ 1 - (z/D) \tan \phi^b \right] \]  
(3-37)

and the total passive lateral pressure below depth D is

\[ \sigma_h = \left( \sigma_v / N_\phi \right) + \left( 2\sqrt{\sigma_v / N_\phi} \right) \left[ c' - (z - D) Y_w \tan \phi' \right] \]  
(3-38)
Figure 3.5. Active earth pressure for unsaturated soil conditions.

\( \frac{(2/\sqrt{N})\mu_{0}(\phi_{0}-\phi_{w})}{(1-z/D)\tan \phi} \)

\( \sigma_y \)

\( \sigma_n \)

\( H \)

\( Z \)

\( D \)

(After Pufahl et al., 1983)
The total passive lateral force is estimated from

\[ P_p = N [ (\gamma_u D^2/2) + \gamma_u (H - D) + \gamma_s (H - D)^2 ] + (2\sqrt{N\phi}) [(c\prime H) + [(u_a - u_w)_s (\tan \phi')]D/2] - \]

\[ [\gamma_w \tan \phi' (H - D)^2]/2 \]  

(3-39)

and is illustrated in Fig. 3-6.

76. In developing the Fredlund equations, several simplifying assumptions were made, i.e., the failure plane is assumed to be planar, the surface of the soil behind the wall is horizontal, and wall friction is neglected. These assumptions are similar to those made by Rankine in developing his classical theory of lateral earth pressure (1857). It was also assumed that only drained loadings are considered; i.e., all porewater pressures are due only to existing groundwater levels and environmental conditions and not to changes in total applied stresses. It was also assumed when computing total active lateral pressures and forces that any portion of the wall apparently in tension could be neglected without important effect on the result.

77. In order to apply the Fredlund equations, several common soil properties are required: (1) saturated and unsaturated unit weights of soil; (2) depth to the groundwater table; (3) wall height; (4) effective value of soil cohesion; (5) matrix suction; and (6) saturated and unsaturated effective angles of shearing resistance. At first impression, the Fredlund equations are algebraically complex and formidable. However, once used, the user should find them to be easy to use thereafter. As with the Skempton equations, the Fredlund equations appear to be able to account for wetter or drier initial conditions in predicting the effect on the pressure likely to be exerted on the wall when the soil becomes wetter and expands. However, the equations predict only active and passive pressures.
\[ (2(N - N_{\phi}))(u_a - u_w) (1 - z/D) \tan \phi \]

Figure 3.6. Passive earth pressure for unsaturated soil conditions. (After Poulos, et al., 1983)
It is expected that the in situ pressures will be greater than those of the active case but should not be as great as the passive pressures. Thus, the principal shortcoming in using the Fredlund equations is that the passive pressure equations must be used in design applications in order to ensure that the lateral pressures are not underpredicted.

**Katti Method**

78. Katti has apparently adopted the position that despite steps that might be taken to prevent it from doing so, expansive soils will invariably swell. Therefore, instead of trying to prevent the soil from experiencing any heave, he has accepted the idea that the soil will swell; his approach, then, has been to attempt to mitigate the consequences of the unavoidable swell. Much of his work has been experimental in nature, in which he and his associates attempted to discover systems that permitted the soil to swell but which accommodated the resulting heave and did not allow but a nominal amount of the heave or the pressure resulting from the swelling to actually impact on the structure being protected.

79. In work with Kate (1975) preceding the work reported with Bhangale and Moza (1983), which forms the principal basis of the following discussion, Katti found that pressures transmitted to structures due to swelling soils could be reduced by placing a nonswelling clay soil between the structure and the expansive soil. This concept was subsequently studied in detail in an extensive laboratory experimental program (Katti, Bhangale, and Moza, 1983). This experimental program had three major objectives which pertained to the problem of lateral earth pressures due to swelling soils:

a. Determine the lateral pressure distribution due to expansive soil when varying thicknesses of cohesive nonswelling soil (abbreviated as "CNS") are placed between a rigid wall and the expansive soil.
b. Attempt to determine an adequate thickness of CNS material placed on top of the expansive soil to "counteract" the swelling and swelling pressure.

c. Determine the lateral pressure distribution due to expansive soil when varying thicknesses of CNS material are placed between a rigid wall and the expansive soil with an adequate thickness of CNS material also placed on top of the expansive soil.

80. The experiments were conducted in stiffened, rigid tanks ranging from 3.0 to 4.2 m (9.8 to 13.8 ft) in height, 0.9 to 1.35 m (3.0 to 4.4 ft) in width, and 1.25 to 2.45 m (4.1 to 8.0 ft) in depth. Thus, the tests were conducted under conditions of no lateral wall movement which resulted in at rest earth pressure conditions being duplicated in the experiments. Three different soils were used in the experimental program. The properties of each of these soils are summarized in Table 3-1. Each of the soils were tested under different conditions of placement: "filled up" (air-dry soil placed under specified standardized free fall conditions without any mechanical compaction), "air-dry compacted," and "compacted saturated." The experimental program was conducted in four different series of test soils and conditions. These series are described in Table 3-2. The soils were placed in the test tanks under very strict procedures with measurements taken every 7.5 cm (3 in.) to ensure uniformity in the test soils. The lateral pressures were measured using reaction jacks and proving rings placed at 60 cm (24 in.) vertical intervals. Detailed drawings of the tanks, lateral pressure measuring units, and the assembled testing system are provided in the 1983 report.

81. The experimental measurements, calculations, and results are reported in detail in this 102-page written report. However, the pertinent measurements and results can be summarized in a few tables and graphs. Table 3-3 (Col. 3) and Fig. 3-7 report the results of testing the expansive
### Table 3-1.

<table>
<thead>
<tr>
<th>Soil Properties</th>
<th>Expansive Soil</th>
<th>Cohesive Nonswelling Soil</th>
<th>Sand</th>
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<tr>
<td><strong>Physical Properties</strong></td>
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<td></td>
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<tr>
<td>Liquid Limit, %</td>
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<tr>
<td>Differential Free Swell, %</td>
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<tr>
<td>Swelling Pressure of Oven Dry Soil at e=1.0, kg/cm²</td>
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<td>Cohesive Nonswelling</td>
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<td>Expansive Soil</td>
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<td>Expansive Soil</td>
<td>Compacted and Saturated</td>
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<td>Compacted and Saturated</td>
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<td>Nonswelling Backfill</td>
<td>With Select Cohesive Non-Swelling</td>
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<td></td>
<td></td>
<td>Backfill of:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 cm Thickness</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>40 cm Thickness</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>60 cm Thickness</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100 cm Thickness</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Expansive Soil With No Select Cohesive</td>
<td>Compacted and Saturated</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nonswelling Backfill</td>
<td>With Select Cohesive Non-Swelling</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cover Material of:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0 cm Thickness</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<td>60 cm Thickness</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100 cm Thickness</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>Expansive Soil With 100 cm Thickness of</td>
<td>Compacted and Saturated</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Select Cohesive Non-Swelling Backfill of:</td>
<td>With Select Cohesive Non-Swelling</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0 cm Thickness</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 cm Thickness</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>60 cm Thickness</td>
<td></td>
</tr>
</tbody>
</table>
Table 3-3.

Measured lateral pressures with various combinations of cohesive nonswelling soil (CNS) used either as cover or as backfill between the wall and the swelling expansive soil. (After Katti, et al., 1983.)

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Saturated Compacted Cohesive Non-Swelling Soil</th>
<th>Saturated Compacted Swelling Soil</th>
<th>Measured Lateral Pressure, Kg/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>No CNS Cover and:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20 cm CNS Backing</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
<td>0.04</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>25</td>
<td>-</td>
<td>0.17</td>
<td>-</td>
</tr>
<tr>
<td>40</td>
<td>0.125</td>
<td>-</td>
<td>0.18</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>0.47</td>
<td>-</td>
</tr>
<tr>
<td>75</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>85</td>
<td>-</td>
<td>1.32</td>
<td>-</td>
</tr>
<tr>
<td>100</td>
<td>0.33</td>
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<td>-</td>
</tr>
<tr>
<td>130</td>
<td>-</td>
<td>1.12</td>
<td>-</td>
</tr>
<tr>
<td>135</td>
<td>-</td>
<td>2.28</td>
<td>-</td>
</tr>
<tr>
<td>145</td>
<td>-</td>
<td>2.53</td>
<td>-</td>
</tr>
<tr>
<td>160</td>
<td>0.52</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>190</td>
<td>-</td>
<td>1.74</td>
<td>0.97</td>
</tr>
<tr>
<td>195</td>
<td>-</td>
<td>-</td>
<td>2.52</td>
</tr>
<tr>
<td>205</td>
<td>-</td>
<td>2.76</td>
<td>-</td>
</tr>
<tr>
<td>220</td>
<td>0.77</td>
<td>-</td>
<td>1.10</td>
</tr>
<tr>
<td>250</td>
<td>-</td>
<td>1.69</td>
<td>1.07</td>
</tr>
<tr>
<td>255</td>
<td>-</td>
<td>-</td>
<td>2.73</td>
</tr>
<tr>
<td>265</td>
<td>-</td>
<td>2.76</td>
<td>1.92</td>
</tr>
<tr>
<td>310</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 3-7. Measured lateral pressures with depth for cohesive non-swelling soil (CNS) only, swelling expansive soil only, and swelling expansive soil with various thicknesses of CNS between the wall and the expansive soil. (After Katti, et al., 1983)
soil by itself, i.e., no CNS material on top of the expansive soil nor any CNS material between the wall and the expansive soil. This situation is represented in Fig. 3-7 by the curve labeled "0 CNS BACKING." ("Backing" is the term used by Katti to describe the CNS material placed between the wall and the expansive soil. Perhaps a more appropriate description might be to refer to this material as "select cohesive backfill." Additionally, Katti adopted the term "cover" to refer to the material used to place on top of the expansive soil.) Fig. 3-7 also illustrates the effect on the measured lateral pressures due to increasing the thickness of the "backing." The actual pressures at the measurement depths are reported in Table 3-3 (Cols. 4-7). As can be seen, with thicknesses of backing greater than 40 cm (16 in.), the lateral pressures became increasingly less curvilinear and approached the linear pressures measured in the CNS material when it was tested in the tank by itself (Table 3-3, Col. 2).

82. The effect on the lateral pressures due to increasing the depth of CNS covering is illustrated in Fig. 3-8. These results are also reported in Table 3-3 (Cols. 8-10). Fig. 3-8 also includes the pressure distribution curve of the CNS soil by itself and the expansive soil without any backing or cover which were also shown in Fig. 3-7. The results of this test series show that regardless of the depth of cover (at least up to a maximum of 1 m), there is little, if any, reduction in lateral pressure on the wall below the depth of the cover. The results indicate that the pressure imposed on the wall above the interface between the CNS cover material and the expansive soil is equal to that measured in the CNS soil by itself, while below this interface, the pressures are not reduced from what they were measured to be when the expansive soil was tested by itself.

83. Fig. 3-9 illustrates the combined effect of 1 m of CNS material
Figure 3-8. Measured lateral pressures with depth for cohesive nonswelling soil (CNS) only, swelling expansive soil only, and swelling expansive soil with various thicknesses of CNS on top of the expansive soil (cover). (After Katti, et al., 1983)
Figure 3-9. Measured lateral pressures with depth for cohesive nonswelling soil (CNS) only, swelling expansive soil only, and swelling expansive soil with various thicknesses of CNS both as cover and between the wall and the swelling expansive soil. (After Katti, et al., 1983)
cover with varying thicknesses of CNS select backfill. The measured pressures at the various depths are reported in Table 3-3 (Cols. 11-13). Fig. 3-9 illustrates that the combination of a 1 m thick CNS cover with an increasing thickness of CNS select backfill has the effect of reducing the lateral pressures acting on the wall. However, comparing the results of this test series to those obtained in the first test series (expansive soil with no CNS cover and increasing thicknesses of select backfill), it is obvious that the combination is not as successful in reducing the pressures acting on the wall as using only increasing thicknesses of CNS select backfill.

84. This same conclusion is illustrated in Fig. 3-10 which compares the percentage of reduction in lateral pressure achieved by each of the systems with an increasing thickness of select cohesive backfill. The effect of a CNS material cover in reducing the lateral pressure on the wall is also shown in Fig. 3-10. This curve confirms what Fig. 3-8 illustrated, i.e., essentially no reduction in lateral pressure on the wall below the depth of the CNS cover regardless of the thickness of the cover. The data used to develop Fig. 3-10 is reported in Table 3-4.

85. Despite its failure to reduce the lateral pressures exerted on the wall by the swelling expansive soil, the cover was found to still have a benefit. It was found that as the select cohesive nonswelling cover material increased in depth, the heave measured at the surface of the test soil decreased. As reported in Table 3-5 and illustrated in Fig. 3-11, the thickness of the select backfill had no significant effect on the total heave measured at the surface of the test soil. However, as the select CNS cover material increased in depth, the measured surface heave decreased to essentially zero with 1 m of cover thickness. This reduction of surface
Figure 3-10. Relationship between the reduction in measured lateral pressures and thicknesses of cohesive nonswelling soil (CNS) either as cover or backfill between the wall and the swelling expansive soil. (After Katti, et al., 1983)
Table 3-4.

Reduction in measured lateral pressures as a result of various thicknesses of cohesive nonswelling soil (CNS) between the wall and the swelling expansive soil with and without CNS cover.

*(After Katti, et al., 1983)*

<table>
<thead>
<tr>
<th>Condition</th>
<th>Depth (cm)</th>
<th>Lateral Pressure, Kg/cm²</th>
<th>Percent Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Expansive Soil Only</td>
<td>Exp. Soil With Backing or Cover</td>
</tr>
<tr>
<td>20 cm CNS Backing w/o cover</td>
<td>250</td>
<td>2.71</td>
<td>1.89</td>
</tr>
<tr>
<td>40 cm CNS Backing w/o cover</td>
<td>220</td>
<td>2.63</td>
<td>1.10</td>
</tr>
<tr>
<td>60 cm CNS Backing w/o cover</td>
<td>250</td>
<td>2.71</td>
<td>1.07</td>
</tr>
<tr>
<td>100 cm CNS Backing w/o cover</td>
<td>220</td>
<td>2.63</td>
<td>0.94</td>
</tr>
<tr>
<td>20 cm CNS Backing with 100 cm CNS cover</td>
<td>250</td>
<td>2.71</td>
<td>2.00</td>
</tr>
<tr>
<td>20 cm CNS Cover with backing</td>
<td>255</td>
<td>2.75</td>
<td>2.73</td>
</tr>
<tr>
<td>60 cm CNS Cover with backing</td>
<td>220</td>
<td>2.63</td>
<td>2.79</td>
</tr>
<tr>
<td>100 cm CNS Cover with backing</td>
<td>250</td>
<td>2.71</td>
<td>2.64</td>
</tr>
</tbody>
</table>

*Extrapolated from $P_s = (\text{depth} - 3.47)/288.05$*
Table 3-5.

Measured heave resulting from various thicknesses of cohesive nonswelling soil (CNS) used in various combinations of cover and backfill between the wall and the swelling expansive soil. (After Katti, et al. 1983)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Measurement Depth (cm)</th>
<th>Measured Heave (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Expansive Soil</td>
</tr>
<tr>
<td>20 cm CNS Backing w/o CNS cover</td>
<td>0</td>
<td>11.1</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>8.9</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>40 cm CNS Backing w/o CNS cover</td>
<td>0</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>160</td>
<td>0</td>
</tr>
<tr>
<td>60 cm CNS Backing w/o CNS cover</td>
<td>0</td>
<td>10.7</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>9.0</td>
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<tr>
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<tr>
<td></td>
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<tr>
<td></td>
<td>130</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>100 cm CNS Backing w/o CNS cover</td>
<td>0</td>
<td>10.3</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>9.0</td>
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<td>2.6</td>
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<tr>
<td></td>
<td>100</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>160</td>
<td>0</td>
</tr>
<tr>
<td>20 cm CNS Cover w/o CNS backing</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>-</td>
</tr>
<tr>
<td>60 cm CNS Cover w/o CNS backing</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>70</td>
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</tr>
<tr>
<td></td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>160</td>
<td>-</td>
</tr>
<tr>
<td>100 cm CNS Cover w/o CNS backing</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>130</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>190</td>
<td>-</td>
</tr>
<tr>
<td>20 cm CNS Backing w/ 100 cm CNS cover</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>-</td>
</tr>
<tr>
<td></td>
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<tr>
<td>60 cm CNS Backing w/ 100 cm CNS cover</td>
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<tr>
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<td>70</td>
<td>-</td>
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<td>130</td>
<td>-</td>
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<td></td>
<td>190</td>
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<tr>
<td>100 cm CNS Backing w/ 100 cm CNS cover</td>
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<tr>
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<td>70</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td></td>
<td>190</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 3-11. Measured surface heave resulting from various thicknesses of cohesive nonswelling cover soil (CNS) with and without any CNS backfill between the wall and the swelling expansive soil. (After Katti, et al., 1983)
heave to essentially zero was shown to also hold when the thickness of the
cover material was held at a constant thickness of 1-m while the select
backfill thickness increased from 20 cm (8 in.) to 1-m (39 in.). Thus, the
CNS cover was beneficial in reducing the amount of surface heave but has no
apparent benefit in reducing the magnitude of the lateral swelling pressures
imposed on the wall by the swelling expansive soil.

86. Finally, the coefficients of at rest earth pressure and passive
earth pressure were calculated for several depths for the various
combinations of cover thickness and select backfill thickness. These
results are shown in Table 3-6, and some of the values of \( K_o \) are compared
with depth and thickness of select backfill in Fig. 3-12. These comparisons
show the at-rest earth pressure coefficient decreased significantly with
just 20 cm (8 in.) of select backfill between the wall and the swelling
expansive soil. A lesser and nearly linear decrease in the value of \( K_o \)--but
still a significant decrease--occurred until the thickness of the select
backfill reached 60 cm (24 in.). With no CNS cover, \( K_o \) increased slightly
(approximately 12 to 14 percent) as the thickness of the select backfill
increased from 60 to 100 cm (24 to 39 in.). However, with 1-m of select
cover thickness, \( K_o \) continued to decrease as the thickness of the select
backfill increased from 60 to 100 cm, but the \( K_o \) values of the soil covered
with 1 m of select CNS material all were significantly greater than those
with the same thickness of select backfill but without CNS cover. Thus,
with respect to reducing the at-rest earth pressure coefficient, the CNS
material was not as successful when used as either a covering only or in
combination with the select backfill.

87. Some observations are reported regarding the \( K_o \) values obtained
from the various test series. \( K_o \) values calculated for air-dry loose sand
Table 3-6.

Calculated values of the coefficient of at-rest earth pressure, $K_o$, at selected depths resulting from various combinations of cohesive nonswelling soil (CNS) used as cover or backfill between the wall and the swelling expansive soil. (After Katti, et al., 1983)

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Expansive Soil Only</th>
<th>CNS Soil Only</th>
<th>No CNS Cover and CNS Backing Thickness of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_o$</td>
<td>$K_p$</td>
<td>$K_o$</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>65</td>
<td>-</td>
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<td>70</td>
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<td>-</td>
</tr>
<tr>
<td>85</td>
<td>9.43</td>
<td>13.00</td>
<td>1.50</td>
</tr>
<tr>
<td>100</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>125</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>130</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>145</td>
<td>9.12</td>
<td>10.6</td>
<td>-</td>
</tr>
<tr>
<td>160</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>190</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*In CNS soil
Table 3-6. (Continued)

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>No CNS Backing and CNS Cover Thickness of:</th>
<th>100 cm CNS Cover and CNS Backing Thickness of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 cm</td>
<td>60 cm</td>
</tr>
<tr>
<td></td>
<td>$K_0$</td>
<td>$K_0$</td>
</tr>
<tr>
<td></td>
<td>$K_p$</td>
<td>$K_p$</td>
</tr>
<tr>
<td>5</td>
<td>1.67*</td>
<td>30.00*</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>40</td>
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<td>65</td>
<td>10.44</td>
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<td>130</td>
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<td>145</td>
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<tr>
<td>160</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>190</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 3-12. Reduction of the calculated coefficient of at-rest earth pressure, $K_0$, at selected depths with various thicknesses of cohesive nonswelling soil (CNS) between the wall and the swelling expansive soil with and without CNS cover. (After Katti, et al., 1983)
and air dry loose CNS material were 0.63 and 0.46, respectively. In comparison, the $K_0$ values calculated by Jaky's equation (1948) were 0.63 and 0.48, respectively. However, the $K_0$ values calculated in the experiment for the compacted samples of each of the soils were well in excess of 1.0, much greater than that estimated from the Jaky equation.

88. The $K_0$ values for the compacted sand prior to saturation were somewhat smaller than the values after saturation. It was also noted that the change in $K_0$ with depth was linear. Similar results were observed for the cohesive nonswelling soil, i.e., a slight increase in $K_0$ following saturation and a linear increase in the magnitude of $K_0$ with depth. However, this was not the observation with respect to the expansive soil. Prior to saturation, $K_0$ was found to increase linearly with depth as was the case with the other two soils. But when permitted to imbibe water, the value of $K_0$ increased rapidly and nonlinearly with depth as was illustrated in Fig. 3-7. However, after a depth of approximately 150 cm (60 in.), the rate of increase in $K_0$ begins to decrease as the overburden pressure becomes greater.

89. It was also noted in the report that the peak value of lateral pressure takes place at percentages of saturation less than 100 percent. Each of the compacted expansive soil test samples was permitted to imbibe water for a period of 70 days to ensure saturation conditions. In each instance, the maximum heave was observed to occur at approximately 30 to 40 days following the introduction of free water to the sample. The peak heave was followed by a small reduction which then remained essentially constant for the remainder of the imbibition period.

90. Finally, Katti, et al., introduced an equation that permits the user to estimate a design lateral pressure:
\[ p_L = p_{CNS} + (0.2)(q_{sw} - p_{CNS}) \]  

where

\( p_L \) = design lateral pressure, kg/cm\(^2\)

\( p_{CNS} \) = lateral pressure of CNS material for the corresponding depth, kg/cm\(^2\)

\( q_{sw} \) = swelling pressure of oven-dry expansive soil at no volume change condition, kg/cm\(^2\)

91. Thus, this very extensive laboratory testing program has shown that inserting another cohesive soil material of a specified minimum thickness between the expansive soil and the wall can effectively reduce the lateral pressure transmitted to the wall from the swelling expansive soil. The testing program showed that the thickness of this cohesive soil material should be at least 40 cm (16 in.) to begin to reduce the transmitted pressure and most probably the minimum thickness should be approximately 1 m. Fig. 3-10 shows that approximately 90 percent of the maximum expected swelling pressure in the expansive soil can be prevented from reaching the wall with a 1 m thick layer of select cohesive nonswelling backfill material. Additionally, the experimental program showed that placing a covering comprised of cohesive nonswelling material over the expansive soil will not reduce the lateral pressures transmitted to the wall from the swelling expansive soil, but the CNS cover material will reduce the vertical heave of the surface. A cover thickness of 1 m was found to eliminate effectively all surface heave. The magnitude of the coefficient of at-rest earth pressure was also shown to be reduced significantly until at least approximately 60 cm (24 in.) of select cohesive nonswelling backfill material had been placed between the wall and the swelling expansive soil. With greater thickness, \( K_0 \) was found to remain essentially constant or
perhaps to increase slightly in magnitude. Finally, an equation for predicting design lateral earth pressure was presented which required knowing the lateral earth pressure in the CNS material at a particular depth and the swelling pressure of oven-dry expansive soil at no volume change conditions, which could presumably be determined from oedometer testing.
PART IV: CONCLUSIONS

92. This study was limited to a search of the technical literature. Nonetheless, some conclusions can be drawn from this effort.

93. Although not exhaustive, the search of the technical literature being reported here is as complete as time and budgetary limitations permitted. There are numerous publications on topics that are supplementary, peripheral, or supportive of this study topic of lateral earth pressures resulting from swelling expansive soil. A large number of these publications were reviewed but not included in the bibliography that accompanies this report because they were considered not to be either primarily or secondarily contributing to the specific problem being studied. Additionally, there are some papers published in foreign regional conference proceedings that from their titles appear to discuss some aspect directly related to the study topic, but, they could not be obtained for one or more of a number of reasons. Thus, despite these enumerated limitations, this study most likely comprises the most complete literature search of this particular topic to date and will provide an excellent departure point for further study of this problem.

94. There are many publications that report on studies made of the swelling pressures developed by swelling expansive soils. However, it was found that many of these results did not have any direct relationship or application to the problem being studied. Often, the experiment produced results obtained from compacted expansive soils under conditions that simply did not conform to what is typically encountered in the field. Consequently, although these papers and reports were duly read and considered, they were not included in the discussion presented in Part II.

95. From the literature search conducted in this study, three methods
or procedures appear to possess potential for predicting the lateral pressures that might be expected to occur in the field when structures with rigid basement walls are constructed in expansive soils.

a. Skempton's Method. Skempton introduced a method whereby the coefficient of at rest earth pressure and the effective horizontal pressure might be calculated using data obtained from a field investigation and a subsequent laboratory testing program. Soil properties required to apply this method include the effective vertical stress, the in situ capillary pressure or soil suction, and the pore pressure parameter, \( A \). Using these soil properties for each point below the surface to be investigated, the at rest coefficient of earth pressure and the effective horizontal earth pressure can be calculated.

b. Fredlund's Method. Using some simplifying assumptions, Fredlund applied limit analysis theory to the problem of lateral earth pressures in unsaturated soils. Although application of the equations developed from limit analysis is somewhat limited because of the simplifying assumptions, they nonetheless provide an analytical solution to a very complex problem. As with Skempton's method, the soil properties needed to apply these equations are routinely available from typically conducted site investigations and laboratory tests: unit weights of the soil for both saturated and unsaturated conditions, the effective angles of internal friction representing both saturated and unsaturated conditions, the effective value of the soil cohesion, the matrix soil suction, and the depth to the groundwater table. However, the Fredlund equations are only applicable to the active and passive failure conditions and do not address at rest conditions. Thus, to use these equations to attempt to predict the design lateral pressures expected to be transmitted to a basement wall, the passive condition equation must be used to estimate an upper limit to the expected lateral pressure, which is a shortcoming in attempting to use this method in design applications.

c. Katti's Method or the Select CNS Backfill Method. The results of an extensive laboratory investigation showed that placing a minimum thickness of a select cohesive nonswellling soil (CNS) between the basement wall and the swelling expansive soil effectively reduced the swelling pressures transmitted to the wall. The results obtained from the test program for the two soils used in the investigation showed that at least 40 cm of the select backfill are required to realize any significant reduction in the lateral pressures and that 100 cm are necessary to reduce the lateral pressures developed in the restrained expansive soil by 90 percent. Additionally, the tests showed that as the thickness of the select backfill increased, the pressures transmitted to the
wall began to approach those transmitted to the wall by the select backfill soil by itself in $K_0$ conditions. Other results from the study showed that covering the expansive soil with as much as 100 cm of the select backfill soil produced little reduction in the lateral pressures transmitted to the wall. Therefore, these results suggest that the lateral pressures expected to be exerted against a basement wall constructed in expansive soil may be mitigated by placing a suitable thickness of nonswelling cohesive soil between the wall and the naturally occurring expansive soil of the construction site. Extending the results of this test, it is likely that if the select backfill is thick enough, the lateral pressures generated by the swelling expansive soil could be sufficiently diminished so that the wall design could be accomplished based upon $K_0$ conditions of the select backfill soil.

96. Based on the results reported by both Ahmed and Katti, et al., it appears that the geometry of the thickness of the backfill placed against the wall has an effect on the magnitude of the lateral pressure transmitted to the wall. However, the results reported by both investigators imply that there is a minimum thickness for a particular wall height, whereby the transmitted pressure is reduced to a minimum. Further increase in backfill thickness beyond this minimum value does not result in any significant further decrease in horizontal pressure on the wall.

97. The three methods presented above have considerable potential for application to practical problems. However, none have apparently actually been applied to a field problem, or at least applied to a field problem where measurements were taken to assess the validity of the pressures predicted by any of the methods. Therefore, if any of these methods should be used to attempt to estimate lateral pressures for design of a nonyielding basement wall, appropriate caution should be applied in using any of the results obtained.
98. Because this study was limited to a search of the technical literature and no laboratory or field testing was accomplished to validate the conclusions presented in Part IV, it is recommended that an experiment be devised and conducted that will provide field measurements that can be used to compare the predicted lateral pressures to the lateral pressures actually occurring under field conditions. It is believed that only through such comparative means can the three methods or procedures that appear to have potential be evaluated and either verified, modified, or rejected.
REFERENCES


Ahmed, H. I., 1967, "Theoretical and Experimental Analysis of the Bin Effect on Lateral Earth Pressure," Thesis presented in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering, University of Washington, Seattle, WA.


Culmann, K., 1866, "Die Graphische Statik," (Section 8, Theorie Der Stutz und Futtermauern), Zurich, Meyer und Zeller.


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APPENDIX A: DERIVATION OF SELECTED FREDLUND LIMIT EQUILIBRIUM EQUATIONS

1. The derivations shown in this Appendix are included primarily to assist the reader in understanding the basis for the equations presented in a series of papers written by Fredlund either as the sole author or as a co-author. Most of the equations developed here, however, will be more directly related to the Pufahl, Fredlund, and Rehardjo 1983 paper.

Saturated Soils

2. First, an understanding of the relationship between the various components comprising cohesive saturated and unsaturated soils and their contributive effect on the active and passive earth pressures exerted by these soils on a vertical plane (e.g., a wall) should be clearly in hand.

3. A saturated cohesive soil derives the lateral earth pressure that it exerts from two components: that contributed by the mass of the retained soil and the effect of soil cohesion. The effect of the friction component of the soil mass can be represented as a hydrostatic pressure distribution and is considered to be a "positive" term. The effect of the soil cohesion is to counteract the effect of the "hydrostatic" component of lateral earth pressure by tending to restrain the soil from relaxing and exerting a lateral pressure on a vertical wall, e.g. Thus, in the active state soil cohesion is considered to be a "negative" term. It is considered to be a constant value over the full depth of a given soil stratum. Therefore, each of these components can be diagrammatically represented as in Figure 3-3.

The net effect of the two components can be determined using superposition:

\[ \sigma_h = (\sigma_v/N_\phi) - 2c' \sqrt{N_\phi} \]  

(3-25)

Thus, at any depth \( z \), the "hydrostatic" component (Figure 3-3a) is equal to \( \sigma_v/N_\phi \) and the cohesive component (Figure 3-3b) is a constant value equal to \(-2c' \sqrt{N_\phi}\).
4. Near the surface, the lateral earth pressure exerted by the soil on the wall is zero, although mathematically the value is a negative number. At some depth \( z_t \), the "hydrostatic" component just balances the cohesive component. This depth can be found by equating the two terms:

\[
\frac{\sigma_v}{N_\phi} = 2c' \sqrt{N_\phi} \\
\gamma z_t / N = 2c' \sqrt{N_\phi} \\
z_t = \frac{(2c' \sqrt{N_\phi})}{\gamma}
\]

where \( \gamma = \rho g \) = unit weight of soil

\( \rho = \) total mass density of the soil

\( g = \) acceleration due to gravity

5. The total active earth pressure can be determined by finding the area of the cross-hatched portion of Figure 3-3c:

\[
P_A = \frac{1}{2} \times \text{base} \times \text{height} \\
= \frac{1}{2} \left[ \left( \frac{\sigma_v}{N_\phi} \right) - \left( 2c' \sqrt{N_\phi} \right) \right] (H - z_t) \\
P_A = \frac{\left( (\gamma H/N_\phi) - (2c' \sqrt{N_\phi}) \right) [(H - z_t)/2]}{(A-27)}
\]

6. Passive earth pressures are found in a similar manner. The difference between the two lateral earth pressures is illustrated in Figure 3-4 where the cohesive component is now resisting the soil compression.

7. The soil is experiencing passive earth pressure conditions. Thus, the cohesion term reinforces the "hydrostatic" component, i.e., both are "positive" terms:

\[
\sigma_h = \left( \frac{\sigma_v}{N_\phi} \right) + \left( 2c' \sqrt{N_\phi} \right) 
\]

(A-3)

8. The passive earth pressure at \( z = 0 \) (top of the trapezoidal cross-hatched area depicted in Figure 3-4c) is

\[
\sigma_v = 2c' \sqrt{N_\phi}
\]

(A-4)

and the passive earth pressure at \( z = H \) is

\[
\sigma_h = (\gamma H)N_\phi + 2c' \sqrt{N_\phi}
\]

(A-5)
The total earth pressure, $P_p$, is found from the area of the trapezoidal Figure 3-4c:

$$P_p = 1/2 \times (\text{base}_1 + \text{base}_2) \times \text{(height)}$$

$$= 1/2[(2c' \sqrt{N_\phi}) + (H \gamma_N + (2c' \sqrt{N_\phi})]H$$

$$P_p = 2c' \sqrt{N_\phi} + (H^2 \gamma_N) / 2 \quad (3-29)$$

9. The relationship between matrix soil suction and depth is typically curvilinear rather than directly proportional. Figure A-1 illustrates the typical relationship (solid line Curve a-b-c). Curve c-b-d represents the hydrostatic condition. If a depth to constant suction can be defined, such as depth $z = D$ [where the groundwater table (GWT) is encountered], then Fredlund recommends approximating the curvilinear relationship between points a and b with a straight-line relationship, e.g., dashed line curve a-b. Thus, the matrix suction $(u_a - u_w)_z$ at any depth $0 \leq z \leq D$ can be estimated using the slope of the straight-line curve:

$$(u_a - u_w)_z + (u_a - u_w)_s[1 - (z/D)] \quad (3-31)$$

10. Unsaturated soils must include an additional earth pressure term besides the hydrostatic (friction) and soil cohesion terms of the saturated soil condition. As Fredlund has pointed out (e.g., Pufahl, Fredlund, and Rahardjo, 1983) the soil cohesion is effectively increased by the influence of soil suction in unsaturated soils. This effect is termed "total cohesion" and is illustrated in Figure 3-2 and stated algebraically in Equation (3-30). It should be recognized by the reader that the $u_a \tan \phi'$ term in Equation (3-30) is zero for practical purposes since it is unlikely that the pore air pressure, $u_a$, is appreciably different from that of atmospheric pressure. Thus, neglecting the pore air pressure term in Equation (3-30) and replacing the constant suction term in Equation (3-30)
Figure A-1. A typical soil suction profile with Fredlund's linear relationship simplification shown (Line dbc).  
(After Pufahl, et al., 1983)
with Equation (3-31), the result is the total active pressure above depth 
\( z = D \):

\[
\sigma_h = \left( \frac{\sigma_v}{N_\phi} \right) - \left( \frac{2}{\sqrt{N_\phi}} \right) \times \\
\left\{ c' + (u_a - u_w)_s [1 - (z/D) \tan \phi \] \times \right.
\]

\[ (3-32) \]

The three terms that comprise the active lateral earth pressure in unsaturated soils are illustrated in Figure 3-5: Figure 3-5a depicts the "hydrostatic" representation of the friction component, Figure 3-5b represents the soil cohesion component, and Figure 3-5c (similar in shape to the approximated relationship shown in Figure A-1) represents the soil matrix suction. Both the soil cohesion and soil suction terms will act to restrain the retained soil mass from relaxing. Thus, these terms are presented as "negative" values for active conditions and "positive" values for passive conditions. At depth \( z = 0 \), the total active pressure would be the sum of each of the three components at that depth:

\[
P_a = 0 + (-2c'/\sqrt{N_\phi}) + (-2c'\sqrt{N_\phi})(u_a - u_w)_s \\
\times [(1 - (z/D)) \tan \phi \] \times \]

\[ (A-6) \]

At some deeper depth, \( z = z_{tu} \), the sum of the three components will equal zero. However, if the GWT is present, this will affect the magnitude of \( z_{tu} \). If \( z_{tu} \leq D \),

\[
0 = [(\gamma_u z_{tu})/N_\phi] - (2c'/\sqrt{N_\phi}) - \\
(2/\sqrt{N_\phi})(u_a - u_w)_s [1 - (z_{tu}/D)] \tan \phi \]

\[ (A-7) \]

Expanding terms and collecting variables:

\[
(2c'/\sqrt{N_\phi}) + (2/\sqrt{N_\phi})(u_a - u_w)_s \tan \phi = \\
z_{tu} [\gamma_u + (2\sqrt{N_\phi}/D)(u_a - u_w)_s \tan \phi] \]

\[ (A-8) \]

Finally resulting in

\[
z_{tu} = \left\{ 2\sqrt{N_\phi} [c' + (u_a - u_w)_s \tan \phi] \right\} / \\
\left\{ \gamma_u + [(2\sqrt{N_\phi}/D) \times (u_a - u_w)_s \tan \phi] \right\} \]

\[ (3-34) \]
In the above equations, \( Y_u \) is the unsaturated unit weight of the soil. When \( z_{tu} > D \), part of the "tension" zone will be above the GWT (unsaturated) and part below the GWT (saturated). Thus

\[
0 = (\sigma_v/N_\phi - 2c'/\sqrt{N_\phi} - (2\sqrt{N_\phi})(u_a - u_w)_s [1 - (z/D)]) x \\
\tan \phi^b + (2\sqrt{N_\phi})(z_{tu} - D) Y_w \tan \phi' \tag{A-9}
\]

However, if \( z_{tu} > D \), the \((u_a - u_w)\) term has approached zero and the unsaturated soil makes no contribution below the GWT. Thus, if \((u_a - u_w)\) approaches zero and if both sides of Equation (A-9) are multiplied by \( N_\phi \), the equation becomes

\[
0 = D\gamma_u + (z_{tu} - D)\gamma_s - 2c'\sqrt{N_\phi} + 2\sqrt{N_\phi} z_{tu} Y_w \tan \phi' - \\
2\sqrt{N_\phi} Y_w D \tan \phi'
\]

Expanding and collecting terms:

\[
0 = D (\gamma_u - \gamma_s) + z_{tu} (\gamma_s + 2\sqrt{N_\phi} Y_w \tan \phi') - \\
2\sqrt{N_\phi} [c' + D Y_w \tan \phi']
\]

And solving for the tension depth:

\[
z_{tu} = \left[D(\gamma_s - \gamma_u) + 2\sqrt{N_\phi} (c' + D Y_w \tan \phi')\right]/ \\
[\gamma_s + 2\sqrt{N_\phi} (Y_w \tan \phi')] \tag{A-10}
\]

11. The total active lateral pressure is represented by the cross-hatched area CDEF in Figure 3-5d. Although the cross-hatched area is an irregular geometrical shape, the area could be found from superposition (e.g., a trapezoidal area plus that of a small triangle). However, only a small error is introduced if CFE is assumed to be a straight line, thus resulting in a triangular shape, CDE, and the total active earth pressure becomes

\[
P_A = 1/2 \times \text{(base)} \times \text{(height)}
\]

base = \((\sigma_v/N_\phi) - (2c')/\sqrt{N_\phi} + (2/\sqrt{N_\phi})(H - D) Y_w \tan \phi'
\]

height = \(H - z_{tu}\)
\[ P_A = \{ (\gamma_u D \phi / \phi) + [\gamma_s (H-D) / \phi] - (2 / \sqrt{N_\phi}) [c' - (H-D) y_w \tan \phi'] \} \times [(H - z_{tu}) / 2] \] (3-36)

12. Passive pressures are illustrated in Figure 3-6 and calculated as:

a. At \( z = 0 \)

\[ \sigma_h = \sigma_N \phi + 2c' \sqrt{N_\phi} + 2c' \sqrt{N_\phi} (u_a - u_w) \phi \{ 1 - (z/D) \} \tan \phi \]

\[ \sigma_h = 2 \sqrt{N_\phi} [c' + (u_a - u_w) \phi \tan \phi] \] (A-11)

b. At \( z = D \)

\[ \sigma_h = \sigma_N \phi + 2c' \sqrt{N_\phi} + 2c' \sqrt{N_\phi} (u_a - u_w) \phi \{ 1 - (z/D) \} \tan \phi \]

\[ \sigma_h = \gamma u \phi \phi \phi + 2c' \sqrt{N_\phi} \] (A-12)

c. At \( z = H \)

\[ \sigma_h = \sigma_N \phi + 2c' \sqrt{N_\phi} + 2c' \sqrt{N_\phi} (u_a - u_w) \phi \{ 1 - (z/D) \} \tan \phi - 2 \sqrt{N_\phi} (H-D) y_w \tan \phi' \]

\[ \sigma_h = N_\phi \phi (\gamma u \phi \phi + \gamma_s (H-D)) \phi + 2 \sqrt{N_\phi} [c' - (H-D) y_w \tan \phi'] \] (A-13)

13. The total passive force is calculated as the sum of the areas of two trapezoidal areas (Figure 3-6d). The area of the upper trapezoid is

\[ A_1 = (1/2 \times D) \{ 2 \sqrt{N_\phi} [c' + (u_a - u_w) \phi \tan \phi \] + \[ \gamma u \phi \phi \phi + 2c' \sqrt{N_\phi} \} \]

\[ A_1 = 2c'D \sqrt{N_\phi} + D \sqrt{N_\phi} (u_a - u_w) \phi \tan \phi \] + \[ (\gamma u \phi \phi + 2c' \sqrt{N_\phi}) / 2 \]

\[ A_1 = N_\phi \phi (\gamma u \phi \phi \phi \phi \phi + 2 \sqrt{N_\phi} [c' D + \[ D(u_a - u_w) \phi \tan \phi \] / 2] \} \] (A-14)

The area of the lower trapezoid is

\[ A_2 = [1/2 \times (H-D) \times \{ [\gamma u \phi \phi \phi + 2c' \sqrt{N_\phi}] + \[ \gamma u \phi \phi \phi + \gamma_s (H-D) \phi + 2c' \sqrt{N_\phi} - 2 \sqrt{N_\phi} (H-D) y_w \tan \phi' \] \]

\[ A_2 = \gamma u \phi \phi \phi - \gamma u \phi \phi \phi \phi + 2c'H \sqrt{N_\phi} - 2c'D \sqrt{N_\phi} \] + \[ [\gamma_s (H-D) \phi / 2] - (H-D)^2 \sqrt{N_\phi} \tan \phi' \]
\[ A_2 = N_\phi [\gamma_u D(H-D) + \gamma_s (H-D)^2/2] + 2 \sqrt{N_\phi} [c'(H-D) - [\gamma_w (H-D)^2 \tan \phi']/2} \]  

(A-15)

The total passive force, \( P_p \), is
\[
P_p = A_1 + A_2 \\
= N_\phi [\gamma_u D^2/2] + 2 \sqrt{N_\phi} [c'D + [D(u_a - u_w)_s \tan \phi^b]/2} \\
+ N_\phi [\gamma_u D(H-D) + \gamma_s (H-D)^2/2] + 2 \sqrt{N_\phi} [c'(H-D) - [\gamma_w (H-D)^2 \tan \phi']/2} \\
P_p = N_\phi [(\gamma_u D^2/2)] + \gamma_u D(H-D) + \gamma_s (H-D)^2/2] + 2 \sqrt{N_\phi} [c'H + [D(u_a - u_w)_s \tan \phi^b]/2 - [\gamma_w (H-D)^2 \tan \phi']/2} \\
(3-39)\]