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ON THE PAIRING PROCESS IN AN EXCITED, PLANE, TURBULENT MIXING LAYER

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Summary

The flow field in a plane, turbulent mixing layer which was disturbed by a small oscillating flap was investigated. Three experiments were carried out: one in which the flap oscillated sinusoidally at a single frequency $F_f$; a second in which the flap oscillated at two frequencies, a fundamental and a subharmonic, but the ensuing motion was dominated by the fundamental perturbation; and a third in which the amplitude of the subharmonic perturbation was increased until a distortion in the mean flow was noticeable. Two velocity components were measured at all phase angles relative to the subharmonic oscillation of the flap at densely spaced intervals. The data were used to map vorticity fields and the streak-line patterns for the purpose of assessing the relevance of the latter to the understanding of the dynamical process involved.

Introduction

Large coherent structures are intrinsic features of all free, turbulent shear flows in general and of the mixing layer in particular. They were discovered by using different techniques of flow visualization, such as Shlieren [1], shadowgraph [2], or dye injection [3]. Whereas photographs using Shlieren or shadowgraph are sensitive to the instantaneous gradients in the index of refraction existing at the time of exposure and thus outline the boundaries of the vortical fluid, photographs using dye or smoke represent streak lines. The latter depend on the location of the injector, the amount of the injected material (i.e., whether it marks the entire vortical region of the fluid at the point of injection), and the rate of diffusion between the injected material and the fluids being mixed. Flow visualization had a major impact on our outlook on turbulent shear flows, but it seldom provided the quantitative data needed to understand and control the dynamics of the processes involved. In the case of the mixing layer, concepts like rollup and pairing became

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a part of the common jargon, which is often being used without adequate consideration of the implied meanings.

Rollup implies the folding, rolling, and breakup of a continuous vortex sheet into discrete vortices, while pairing implies a sequential amalgamation of these vortices into larger vortical structures. Ho and Huerre [4] classify rollup as an inherently nonlinear effect occurring at a location at which the fundamental frequency attains its maximum amplitude and is accompanied by the generation of a subharmonic frequency. The nonlinear aspect of the rollup is never quantified but, since the process is identified with flow visualization, one assumes that it stems from a rollup of streak lines or species (see, for example, Roberts [5]). Hama [6] observed that a linear superposition of a weak traveling sine wave on a hyperbolic-tangent velocity profile results in a streak-line pattern which rolls up into an array of discrete vortices. This suggests that nonlinearities are not essential for the rollup process and that they do not represent a necessary condition for its occurrence, although they may exist in the flow.

Successive amalgamation of pairs of neighboring vortices marked by dye was visually observed by Winant and Browand [3], who attributed the divergence of the mixing layer to this process. They argued that the vorticity is continuously being redistributed into larger and larger eddies, whose size doubles at each interaction. The location at which pairing occurs coincides, according to Ho and Huang [7], with location at which the subharmonic frequency attains a maximum. Consequently, pairing is intimately associated with the existence of two frequencies in the flow: a fundamental—considered to be the most amplified frequency at the trailing edge of the splitter plate—and a subharmonic, which attains a maximum amplitude farther downstream. Ho and Huang showed that any number of eddies can be made to merge whenever the mixing layer is excited externally by a frequency corresponding to a rational fraction of the most amplified frequency at the initiation of mixing. A single vortex amalgamation may require the coexistence of two distinct waves in the flow, while more waves are necessary in order to repeat the process farther downstream. If the existence of distinct waves prior to the initiation of mixing is a necessary condition for amalgamation, then the entire observation may be the result, rather than
the cause, of the divergence of the mixing layer. One may observe from
the photographs taken by Brown and Roshko [2] that the mixing layer
diverges in spite of the absence of any visible amalgamations. The
digital image analysis of Hernan and Jimenez [8] concurs with this
observation and attributes most of the divergence of the mixing layer to
the growth of the large eddies and not to the pairing process.

In order to establish the cause for the divergence of the mixing layer
and the role of the subharmonic frequency in this process, one has to
first define pairing in terms of measurable quantities. For this
purpose, the mixing layer was excited at two synchronized frequencies,
F_f = 36 Hz and F_s = 18 Hz, which will be referred to in the text as the
fundamental and the subharmonic, although F_f is not the most amplified
frequency at the trailing edge of the splitter plate. The experiments
were conducted in a mixing layer facility described by Oster and
Wygnanski [9]: the velocity of the faster stream was 8 m/sec, while the
speed of the slower stream was 4.8 m/sec. Thus, the average convection
speed of the large eddies is U_c = 0.5(U_1 + U_2) = 6.4 m/sec and the ratio
of velocities described by the parameter R = (U_2 - U_1)/(U_2 + U_1) is 0.25.
Two components of velocity were measured at 80 x-stations separated by
20 mm in the direction of streaming, corresponding approximately to one-
tenth of the wave-length of the fundamental frequency. The
measurements were taken with a rake containing 7 x-wire probes which
was traversed across the flow. The velocities measured at each point
were sampled at 2.048 KHz, together with the signals activating the
flap, for a total duration exceeding 1000 cycles of the fundamental
frequency. The latter signals were used to obtain a phase reference for
averaging the data. The phase-locked data were used to compute the
coherent velocity fluctuations, as well as the coherent spanwise
component of vorticity. The high spatial and temporal resolution of the
data was required for the calculations of particle paths and streak
lines used to simulate flow visualization. Identical data, therefore,
were used to describe the dynamics of the flow in a laboratory
coordinate system and to describe the paths of particles in the
disturbed flow field. Consequently, any pattern observed in the
Lagrangian frame of reference has a precise experimental counterpart in
an Eulerian system. The purpose for this exercise was to define the
implied relationships between flow visualization and measurement in the
disturbed, two-dimensional mixing layer.
Results
The Mixing Layer Excited by a Single Traveling Wave

The experimental methods were first tested in a mixing layer excited by a single traveling wave train. A visual resemblance was established between streak lines calculated from the phase-locked velocity measurements and photographs obtained by stroboscopic illumination synchronized with the flap motion and using smoke as a means of flow visualization [10]. The photographs taken represented either a single realization or an ensemble average of many events, depending on the time of exposure; a visual comparison of such photographs did not reveal any significant large-scale feature which was not observed in the ensemble average.

A pattern of 7 streak lines originating from hypothetical sources located at $X = R*Ff*X/U_c = 0.278$ downstream of the splitter plate and displaced by a distance of $Y = AY*Ff/U_c = 0.011$ around the centerline of the mixing layer was calculated (for the particular choice of the dimensionless group, see Oster and Wygnanski [9]). Although data are available for the entire distance between $X = 0.278$ and $X = 2.5$ from the trailing edge of the splitter plate, the pattern shown in Figure 1a covers only one-half of this distance since, in this interval, the vortex sheet rolls up into the characteristic row of discrete lumps.

One may observe that particles emanating from the high-speed side of the mixing layer are convected downstream faster than particles emanating from the low-speed side, causing an accumulation of particles at the crest of the wave and resulting in the rollup of the streak lines. The streamwise location at which rollup is observed depends on the phase angle between the streak-line pattern and the flap; the rollup process is centered around $X = 0.625$. The effect of nonlinearities on the rollup process may be estimated by filtering out all the frequencies except for the fundamental from the time series, representing phase-locked ensemble-averaged velocities measured throughout the flow field, and recalculating the streak-line pattern. The results of these calculations are shown in Figure 1b. Although the rollup process is impeded somewhat by the absence of the nonlinearities, it is not eliminated. The vortex sheet keeps rolling up into lumps of concentrated vorticity and keeps stretching and thinning in between. The leading nonlinearity in
Fig. 1. Phase-locked streak lines in a mixing layer excited by a simple frequency: (a) unfiltered, (b) filtered, (c) calculated.

Fig. 2. Vorticity field corresponding to streak lines shown in Figure 1.

this experiment is the harmonic constituent of the spectrum, which contributes as much as 30% to the average (taken across the shear layer) amplitude of the phase-locked velocity signal at $\bar{x} = 0.7$.

The filtered and unfiltered contours of the spanwise component of vorticity corresponding to the streak lines shown in Figures 1a and 1b are plotted in Figures 2a and 2b. The nonlinearities have some effect on the intensity of the measured vorticity (i.e., the number of contours within a vortical structure) but not on its distribution inside the shear layer. Within the resolution of the experiment, the spanwise component of the coherent vorticity becomes concentrated in discrete lumps having dimensions which are commensurate with the dimensions of the eddies represented by the concentration of particles shown in Figure 1. It is interesting to note that vorticity is not uniformly distributed across the perturbed shear layer, even far upstream of the location at which rollup occurs. By observing the calculated vorticity contours generated at various time delays or phase angles relative to the flap, one can follow the process by which the concentrated vorticity on the high-speed side of the mixing layer catches up with the vorticity concentration on the low-speed side. At an $x$-location corresponding to the rollup of particles, the two concentrations of vorticity become displaced in the transverse direction only. Farther downstream (Figure 2), the boundary between the two concentrations of vorticity disappears and one is left with a single vortical structure, which is convected downstream. By overlaying the vorticity contours on the streak-line pattern, one may notice that, at larger downstream distances, there is some discrepancy.
in the overall dimension of the two patterns. This discrepancy might be partly reduced by increasing the number of streak lines calculated, as well as lowering the initial contour level plotted in Figure 2. Nevertheless, more particles are always concentrated at the trailing (upstream) part of the vortex, while the leading (downstream) portion of the vortical structure spans the gap between the lumpy concentration of particles and the thin braid preceding it. Thus, a lack of particles on the high-velocity side of the braid does not necessarily imply that the flow is entirely irrotational.

It had been shown by Gaster et al. [11] that linear stability theory, applied to a prescribed mean flow field, predicts quite well the transverse distribution of disturbances produced by a traveling wave. It is interesting, therefore, to compare the observations cited above with the linear model, in spite of the fact that the latter does not correctly predict the amplification in the direction of streaming. A streak-line pattern was calculated using a simple inviscid model for the perturbation which was superimposed on a divergent mean velocity profile identical to the one described in equation 5.2 of Gaster et al. The divergence angle $\alpha$ applicable to this experiment was 37.7, and the value of $x_0$ was 83 mm. The differences stem from the larger amplitude of the imposed excitation. Since the decaying modes could not have been calculated using the inviscid model, one had to assume that the perturbation retained its amplitude and shape downstream of the neutral point. One could easily modify this constraint, but it did not seem to warrant the additional computational effort. The results of these calculations are plotted in Figure 1c for an initial dimensionless amplitude of the perturbations (at $\hat{x} = 0.278$) of 5%. Although the detailed comparison between Figures 1b and 1c reveals some minor differences, it is evident that the linear model captures the main features of the rollup process. Hama [6], who computed the streak lines resulting from a linear superposition of a neutral sinusoidal disturbance on a parallel shear layer, concluded that a complex streak-line pattern is caused by the particles being trapped by the "cat's eye"-shaped stream lines computed in a reference frame moving at $U_C$. This also is the case in the present investigation.
The distribution of vorticity in the perturbed mixing layer was also computed by using the inviscid model, and the results are shown in contour form in Figure 2c. Once again, the main features of the vorticity distribution are captured by the simple model, in spite of the quantitative differences attributed, in part, to the inadequate prediction of the amplification rate in the direction of streaming. The most important feature shown in Figure 2c, which is also confirmed by experiment, is the rollup of vorticity from an undulating sheet to a lumpy structure. This rollup occurs whenever the imposed perturbation becomes neutrally stable as a result of the divergence of the mean flow. Somewhat similar vorticity contours were computed by Michalke [12] for a temporally evolving perturbation in a parallel shear layer. Whenever the perturbation is strongly amplified, the vorticity is concentrated in two cores which are displaced in time and in the transverse direction; at the neutral point, however, the vorticity is reorganized into a single core spanning the entire width of the mixing layer (see Figure 10 in [12]).

Browand and Weidman [13] drew vorticity contour maps based on conditionally sampled measurements in an unexcited mixing layer. The conditional sampling algorithm was supported by flow visualization in which dye was injected into the boundary layer generated on the splitter plate. The contours shown in Figure 6 of their manuscript are very similar to the contours shown here in Figure 2, provided one keeps in mind that the location of the high-velocity stream is inverted in the two sets of figures. Browand and Weidman refer to the lumpy vorticity distribution showing a single core in the center as the one generated during the final stages of pairing. They define an intermediate stage of pairing as the one being similar to the distribution shown on the left-hand side of Figure 2 (i.e., for $X < 0.5$).

One may adopt the view that the pairing process and the reorganization of vorticity, resulting from the divergence of the mean flow during the period in which a wavy disturbance is being amplified, are one and the same. Pairing is completed when the disturbance becomes neutrally stable to its local environment. Browand and Weidman [13] observed that the Reynolds stress generated during the intermediate stages of the pairing is twice as high as the Reynolds stress observed at the end of
this process. This can be explained by the fact that the Reynolds stress produced by large coherent structures vanishes when the pairing process is complete. The mean flow responds to the reduction in the Reynolds stress by a concomitant reduction in the angle of divergence which, in an externally excited flow, can even become negative. The new interpretation which lumps pairing and rollup together as representing the same process has to reconcile numerous difficulties, such as:

1. The streak lines observed in the present experiment show no sign of amalgamation, while the lumps of dyed fluid in the experiments of Browand and Weidman [13] merge in the process.
2. The amplitude of the subharmonic frequency in the present experiments was negligible, yet the subharmonic frequency is reported to control vortex pairing [4]. One may ask, therefore, if the presence of a subharmonic is a necessary condition for the pairing process.

These questions will be addressed in the next section.

The Mixing Layer Excited by Two Waves--A Fundamental and a Subharmonic

The mixing layer in this experiment was excited by the two synchronized perturbations described in the "Introduction." The initial amplitude of the fundamental frequency measured at $\hat{x} = 0.225$ was approximately twice as high as that of the subharmonic, while the harmonic frequency, indicating the initial degree of nonlinearity, was at least an order of magnitude lower than the fundamental. The fundamental frequency became saturated at $\hat{x} = 0.84$, while the rate of amplification of the subharmonic was enhanced at that location and, at $\hat{x} = 1.6$, the amplitude of the subharmonic surpassed the amplitude of the fundamental. The most rapid rate of amplification of the subharmonic started at $\hat{x} = 1.4$, where the first harmonic frequency $F_h$ and the fundamental frequency $F_f$ started to decay. The changes in the rate of amplification of $F_s$ suggest that the subharmonic frequency derives some of its growth from a nonlinear interaction with the fundamental; perhaps, it even resonates with the fundamental in a manner described by Kelly [14] and observed experimentally in a circular mixing layer by Cohen and Wygnanski [15].

Two patterns of streak lines, one originating at the first x-station at which measurements were made (i.e., at $\hat{x} = 0.225$) and the other
originating at the x-station at which the fundamental frequency attained its maximum amplitude (i.e., at $X = 0.84$), were calculated. The motion of the flap was subdivided into five equal segments; thus, the patterns shown in Figure 3 correspond to different phases of the excitation. One can clearly discern two lumps of tagged fluid undergoing amalgamation in Figure 3a. The process starts at $X = 0.56$, where one lump of fluid becomes displaced in the transverse direction, and proceeds slowly in time and space. The tagged fluid particles rotate around one another in a manner reminiscent of the observations of Winant and Browand [3] and Browand and Weidman [13]. The amalgamation is incomplete at $X = 1.68$.

The flow field which gave rise to the streak-line pattern shown in Figure 3b was identical in all its detail to the flow field responsible for the pattern shown in Figure 3a, only the location of the source of the imaginary dye or smoke injector was changed. Nevertheless, one cannot observe a pairing interaction in Figure 3b which is as clearly discernible as the one in Figure 3a. Cimballa [16], who examined the streak lines generated by smoke injected into a wake of a circular cylinder, pointed out that the pattern which he photographed depended on the location of his smoke generator. Both observations suggest that this type of flow visualization has to be treated with caution.
Since the eddies undergoing pairing in Figure 3a are sandwiched between other eddies which do not pair, one can assess the size of the ensuing vortical structure, and the concentration of vorticity within it, relative to its unpaired neighbors both upstream and downstream. The vorticity contours shown in Figure 4 during a rollup or during a pairing interaction are almost identical, regardless of the differences in the streak-line patterns, and resemble the vorticity distribution discussed in the previous section. Therefore, rollup and pairing appear to have the same significance as far as vorticity is concerned. At both instances, the reorganization of vorticity corresponds to the $\bar{x}$-location at which the amplitude of the dominant perturbation became saturated or, in the context of the linear stability theory, the flow became neutrally stable. Since rollup and pairing might be one and the same, the overall response of the mixing layer to either should also be identical. This observation was already made by Ho and Huerre [4], although it was not recognized as such. The only difference between the data presented by Ho and Huerre in Figure 19 and in Figure 3 stems from the fact that the eddies shown in Figure 19 underwent the rollup twice—once corresponding to $F_f$ and then corresponding to $F_h$. It is interesting to note that the vorticity concentration in the eddies undergoing "pairing" is weaker than in the simply rolled eddies. This stems from a nonlinear process (to be discussed).

The importance of the subharmonic frequency ($F_s$) in the amalgamation process observed in Figure 3a is considered by decomposing the signal into its Fourier constituents and reconstructing the streak-line pattern as discussed in the previous section. Spectral analysis reveals that, in addition to $F_f$ and $F_s$, which were externally introduced by the motion of the flap, two additional frequencies are present in the flow: $F_h$, which is generated by the finite amplitude of the fundamental, and $F_{\text{sum}} = F_f + F_s$, which is generated by the nonlinear interaction of the fundamental and the subharmonic or by a secondary interaction between $F_h$ and $F_s$. Taking one frequency at a time and superposing it on the mean velocity field results in a regular array of eddies which do not coalesce (Figure 5a); in fact, $F_s$ by itself does not even rollup within the distance considered. The streak-line pattern resulting from superposition of $F_{\text{sum}}$ on the mean velocity profile is somewhat distorted, presumably because $F_{\text{sum}}$ is generated by a combination of nonlinear interactions.
Fig. 5. Streak lines corresponding to data shown in Figure 3 which were generated by filtering and retaining only one frequency as marked: (a) $F_s$, (b) $F_f$, (c) $F_{sum}$, (d) $F_h$

A linear superposition of two or three frequencies on the mean velocity generates the streak-line patterns shown in Figure 6. It is obvious that the combination of the fundamental with either the subharmonic or the harmonic, or both together, does not result in the amalgamation of eddies shown in Figure 3a and replotted for convenience at the top of Figure 6. It is therefore the fundamental, in conjunction with $F_{sum}$, which is responsible for the "pairing" effect observed at $\bar{x} = 0.85$ in Figure 6. One should not infer that the subharmonic will not cause the amalgamation of streak lines farther downstream (the process is, in fact, detected near the end of the test section); it is simply preceded by $F_{sum}$, which rolled up somewhat upstream of the fundamental. The perturbation associated with $F_{sum}$ is out of phase with the fundamental every two wave lengths of $F_f$, which gives rise to the apparent pairing interaction and the concomitant reduction in the intensity of the vorticity concentration observed in Figure 4. One may also note that, at the completion of rollup, the vorticity contours are almost circular. During either amplification or decay, these contours have an elongated, elliptical shape and their inclination changes by $\pi/2$ when they pass from a state of growth to a state of decay. The generation of Reynolds stress and the transfer of momentum between the mean motion and the large structures depend on the tilt of these eddies, as observed by Browand and Ho [17].

In order to show that a "pairing interaction" can occur on the scale of the subharmonic wave, the initial excitation level of the subharmonic
frequency was increased. A vigorous "pairing" was observed in the streak-line pattern based on the unfiltered, phase-locked velocities shown in Figure 7a. When all frequencies other than $F_s$ and $F_f$ were removed from the time series, the pairing interaction remained virtually unchanged (Figure 7b). The pairing process disappeared from the streak-line pattern by the removal of $F_f$ (Figure 7c), showing a rollup of the subharmonic instead. The accompanying vorticity contours, shown in Figures 7d-f, were not restructured by the removal of all the nonlinearities (except for the mean flow distortion) nor were they reorganized by the removal of the fundamental. The concentration of vorticity within the large eddies was simply reduced somewhat by the selective removal of all but the subharmonic frequency, as indicated by the number of contours plotted in the respective figures. The fact that the disappearance of "streak-line pairing" at $\bar{x} = 1.5$ did not alter the distribution of vorticity produced by the subharmonic lends credence to the suggestion that pairing of vorticity signifies a switch from an amplification stage to a decay stage of a given disturbance.
Conclusions

A simple flow field generated by a linear superposition of sinusoidal disturbances on a two-dimensional shear layer results in a complex pattern of streak lines which roll, amalgamate, or even tear one another (see Wygnanski and Petersen [18], Figure 14). One should not rely, therefore, on flow visualization alone for the purpose of formulating concepts pertaining to the dynamics of the flow. It is obvious that simple dynamical systems may produce complex particle trajectories. One may define "pairing" and "rollup" as the reorganization of vorticity which occurs when a wave attains its neutrally amplified location. The only nonlinearity needed to predict this process is implied by the divergence of the mean flow. The experimental results discussed indicate that nonlinear models based on stability theory may predict quite well the interaction between the mean flow and the large-scale structures in two dimensions.

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References

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