Resonant Level Lifetime In GaAs/AlGaAs Double-Barrier Structures

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The lifetime of the lowest quasi-bound state localized between the barriers of a GaAs/AIGaAs double-barrier structure is calculated as a function of barrier and well dimensions. The results are consistent with high-frequency experiments.
Recently there has been considerable interest in GaAs/AlGaAs double-barrier structures. These structures exhibit a number of interesting features, including negative differential resistance (NDR) [1], fast response times [2], and bistability in current-voltage response [3]. For applications purposes, these structures are particularly attractive because of their NDR and fast response times. The regions of NDR in the I-V characteristics are generally associated with the sharp peaks in the transmission coefficient at certain (resonant) energies, and a theory of I-V response based on this assumption was proposed [4].

The experiments of Sollner et al. [2] indicate that there is significant current response to applied fields at frequencies as high as f = 2.5 THz. This suggests that processes responsible for the NDR have characteristic times which are shorter than the period 1/f = 4 \times 10^{-13} \text{s}. The characteristic times involved in resonant transport are believed to be the lifetimes of the quasi-bound levels between the barriers [5]. At low frequencies, such that 1/f > > \tau, where \tau is the lifetime of the lowest energy quasi-bound state, current response is expected to follow the dc I-V curve. However, at high frequencies, where 1/f < < \tau, one expects negligible resonant response to the applied voltage.

A semiclassical estimate of the lifetime \tau has been made by Luryi [6]; however, when compared with experiments [2], his estimate is too large by almost three orders of magnitude. In order to reconcile his predictions with experiment, Luryi proposed a kinematic argument (sequential tunneling) to explain the observed NDR. Recently, Weil and Vinter [7] have shown that both explanations of the peak in the I-V curves lead to the same prediction for the I-V response. At this point it is not clear whether these two mechanisms are fundamentally distinct.

It is the purpose of this report to show that within a simple effective-mass picture, a quantum mechanical calculation of the resonant level lifetime is consistent with existing experimental observations [2]. Our results can be used to estimate the lifetimes of the resonant levels for a range of aluminum concentrations and barrier and well dimensions.

We solve the one-dimensional effective-mass Schrödinger equation with the double-barrier potential

\[
V(x) = \begin{cases} 
0 & 0 < x < a \\
V & a < x < a + b \\
0 & x > a + b 
\end{cases}
\]

where \psi(x) is an even function of x (the growth direction). In the half-space x > 0, we take the even wavefunctions to be

\[
\psi(x) = \begin{cases} 
A \cos(g_1 x) & 0 < x < a \\
B e^{g_1 x} + C e^{-g_1 x} & a < x < a + b \\
D e^{ig_1 x} & x > a + b 
\end{cases}
\]

where

\[
g_1 = \sqrt{\frac{2m_e}{\hbar^2} V - g^2}
\]

and the wavevectors, g and g_1, and energy eigenvalues, E = \hbar^2 G^2/2m_e, are complex.
numbers \[8\]. This wavefunction represents a particle in the well which can tunnel out to the left or right. Matching the wavefunction and its first derivative at \(x = a\) and \(x = a + b\) gives a system of four homogeneous equations for the constants \(A, B, C,\) and \(D\). These equations will have a nontrivial solution if we require the determinant of the coefficient matrix to vanish. For the wavefunctions of equation (2), this condition gives the following equation for the allowed wavevectors:

\[
\begin{align*}
    h(z) \cot(z) + & \\
    h(z)[1 + \exp(2bH(z))] + & it[1 - \exp(2bH(z))] = 0 , \quad (3)
\end{align*}
\]

where \(z = ga, \delta = b/a,\) and the dimensionless potential is defined as \(U = 2mA^2V/\hbar^2\). The complex function \(h(z)\) is chosen such that

\[
\begin{align*}
    h(z) = & [u - z^2]^{1/2} \quad \text{Re}(z^2) < U , \\
    h(z) = & i[z^2 - U]^{1/2} \quad \text{Re}(z^2) > U ,
\end{align*}
\]

where the principal branch is taken for the square root. When \(\text{Re}(z^2) < U\), the resonant level lies below the tops of the barriers; when \(\text{Re}(z^2) > U\), the resonant level lies above the tops of the barriers. Wavevectors for the odd eigenfunctions satisfy an equation identical to equation (3), but with \(\cot(z)\) replaced by \(-\tan(z)\).

The roots of equation (3) are located at \(z_n, n = \pm 1, \pm 2, \pm 3, \ldots\), such that \(z_n = -z^*_{-n}\). We have numerically solved for these roots as a function of \(U\) and \(\delta\). All roots have negative imaginary parts \[9\], and the magnitude of both the real and imaginary parts increases with increasing \(n\). This leads to wavefunctions with a time dependence given by \(\exp(-i\Omega_n t)\) \(\exp(-t/2\tau_n)\), where the energy of a resonant level, \(\hbar\Omega_n\), and the lifetime, \(\tau_n\), are functions of the roots \(z_n:\)

\[
\begin{align*}
    \Omega_n & = \omega_0 \Re\left[z_n^2\right] , \\
    \tau_n & = \frac{-1}{2\omega_0 \Im[z_n^2]} , \quad (5)
\end{align*}
\]

where \(\omega_0\) is defined by

\[
\omega_0 = \hbar/2mA^2 .
\]

If we order the roots with positive real parts according to \(\text{Re}(z_1) < \text{Re}(z_2) < \text{Re}(z_3)\ldots\), the increasing index \(n\) labels resonant levels of increasing energy and decreasing lifetime.

In figure 1 we plot the logarithm of the lifetime of the lowest resonant level as a function of \(\delta\), for several values of \(U\). The curves are straight lines, which indicates that the lifetime is an exponential function of \(\delta\), as one might expect. For a given well width, \(a\), the lifetime increases with both the barrier height and width. In figure 2 we show the energy of the lowest resonant level, \(\hbar\Omega_1\), as a function of \(\delta\). For a given barrier height, the energy has a strong dependence for narrow barriers (values of \(\delta \leq 1\)). Note also that for a given barrier width, the energy of the resonant state increases with barrier height, which is a simple consequence of electron confinement.

We apply our results to the experiments of Sollner et al. \[2\], which had the parameter values \(a = 24\ \text{Å}, \delta = 2,\) and \(V = 0.23\ \text{eV}.\) Using
the effective mass for conduction electrons in GaAs, \( m_c = 0.067 \, m_0 \), the frequency scale is \( \omega_0 = 1.38 \times 10^{14} \, \text{s}^{-1} \), and \( U = 2.5 \) is the dimensionless potential. For these parameter values we find the lifetime of the lowest resonance to be \( \tau_1 = 6.4 \times 10^{-13} \, \text{s} \). We believe this value for the lifetime is consistent with the high frequency experiments, considering the degree of approximation involved. We have ignored the effects of bias voltage on the potential energy shape and have used the effective mass theory in a rather cavalier manner, ignoring both the mixing of \( \Gamma \) - and \( X \)-point states for high aluminum concentration [10] and details of band structure [11]. We believe that a more careful treatment including these effects would not substantially alter our results.

Figure 1. The base 10 logarithm of the product \( \omega_0 \tau \) is plotted versus \( \delta = \hbar / a \), where \( \tau \) is the lifetime of the lowest resonant level (\( \omega_0 \tau \) in the text).

Figure 2. The energy of the lowest resonant state, \( \tilde{\mathcal{E}} \Omega \) (in \( \hbar \Omega \), in the text), is plotted (in units of \( \hbar \omega_0 \)) versus \( \delta \), for several values of \( U \).

References


8. See, for example, Quantum Mechanics, L. D. Landau and E. M. Lifshitz, Pergamon Press, New York (1977), section 134.

9. The apparent root when \( h(z) = 0 \) of equation (3) is spurious. At this root, \( g_1 = 0 \) and \( g^2 = 2m_e V/h^2 \), so the wavefunction in the region \( 0 < x < a + b \) is \( \psi(x) = Bx + C \). Proceeding as in the derivation of equation (3) gives \( \cot(z) - (\zeta + i) = 0 \), and \( z = \sqrt{U} \) is not a solution.

