STRAIN BASED COMPLIANCE METHOD
FOR DETERMINING CRACK LENGTH FOR
A C(T) SPECIMEN

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Copies of this report should not be returned unless is required by security considerations, contractual obligations, or notice on a specific document.
This report describes the estimation of crack length values based on back-surface strain measurements in a compact-type, C(T), specimen. The method of crack length determination is well suited for use with closed-loop computer controlled test systems.

A finite element analysis is used to determine the theoretical back-face strain values for crack length (a) to width (W) ratios from 0.2 to 0.8 and a mathematical relationship for back-face strain compliance as a function of a/W is derived. The a/W values calculated from measured strain compliance are compared with those calculated from crack-mouth-opening-displacement, CMOD, compliance. The variation in a/W predicted from measured strain compliance as compared to a/W predicted from CMOD compliance is approximately +2 percent at a/W = 0.2 to -2 percent at a/W = 0.8. A second strain measurement location is determined which enhances the sensitivity of the back-face strain compliance for a/W values from 0.2 to 0.5 and a mathematical relationship of dual-location strain compliance to crack length ratios is derived. The a/W values...
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calculated from measured dual-location strain compliance and CMOD compliance are compared and the variation derived from the two different methods is approximately ± 1 percent of a/W values ranging from 0.2 to 0.8. A test is conducted at 1200°F to confirm the applicability of the procedure for back-face strain compliance at elevated temperatures.
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This report describes one of the work activities covered under Task 2 of the contract. The work described herein was conducted between August 1983 and May 1985.
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SECTION 1
INTRODUCTION

The recent introduction of closed-loop computer controlled test systems and computer data acquisition systems has resulted in different methods being employed to generate fatigue crack length data. One of the present methods of nonvisual crack length determination is based on a crack-mouth-opening-displacement, CMOD, compliance method. The CMOD method has proven to be an effective nonvisual means of determining crack length at room temperature, but is somewhat limited for use at elevated temperatures.

The University of Dayton Research Institute (UDRI) is presently using a standard clip gage as specified in ASTM E399-83, "Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials," [1], for crack length determination at room temperature. This method of crack length determination at room temperature has been shown to yield results that are as accurate as the visual method [2] but is usually restricted to near room temperature conditions by the materials used in the clip gage construction.

Nonvisual crack length determination at elevated temperature generally employs a device that resides outside the elevated temperature zone and is connected to the specimen through a temperature resistant material such as quartz.
Although this type of device is used for high temperature CMOD measurements, it generally restricts test frequencies to 3 Hz or lower.

The UDRI has investigated a number of nonvisual crack length determination methods at elevated temperatures and high frequencies for threshold determination and major-minor fatigue. One promising method of obtaining nonvisual crack lengths at elevated temperature/high frequency is specimen strain. When a compact type, C(T), specimen is subjected to a tensile load, it deforms as shown in Figure 1. The resulting negative elastic strain per unit load, measured on the back-face of the specimen, can be correlated to crack length as shown previously by Richards and Deans [3].

The purposes of this report are: (a) to evaluate the use of specimen strain for determining crack length at elevated temperature/high frequency and (b) to compare strain determined crack lengths with CMOD and visually determined crack lengths.
Figure 1. Deformation of a C(T) Specimen Under Tension Loading.
SECTION 2
ANALYTICAL AND EXPERIMENTAL RESULTS

2.1 ANALYTICAL BACK-FACE STRAIN COMPLIANCE

A two-dimensional finite element analysis [4] was used to determine the strain profile on the back-face of a compact type specimen for a/W ranging from 0.2 to 0.8. The strain profile is shown in Figure 2, where the analytical strain is plotted as a function of location as measured by the specimen half height ratio (Y/H). From the strain analysis, an average back-face strain value for a 1/4 inch (6.4 mm) gage length was calculated. The back-face strain values were then used to calculate a positive non-dimensional strain compliance, -EBCW, at various a/W values ranging from 0.2 to 0.8.

The average strain compliance values were calculated for a 1/4 inch (6.4 mm) gage length on a C(T) specimen with W = 40 mm. For a change of ±25 percent in the ratio of gage length to specimen width, changes in compliance values range from approximately ±1/2 percent at a/W = 0.2 to approximately ±3 percent at a/W = 0.8. The maximum change in calculated crack length is 0.003 inch (0.08 mm).

A mathematical expression was derived to express -EBCW as a function of a/W. This mathematical expression was in the form of a sixth degree polynomial.
Figure 2. Analytical Back-Face Strain Profile from Finite Element Analysis.
\[-EBCW = 40.7730 - 673.330\alpha + 4648.77\alpha^2 - 16372.8\alpha^3 + 31712.6\alpha^4 - 31853.6\alpha^5 + 13172.9\alpha^6, \quad (1)\]

where

\begin{align*}
E &= \text{Elastic Modulus}, \\
B &= \text{Specimen Thickness}, \\
C &= \varepsilon/P = \text{Compliance}, \\
W &= \text{Specimen Width}, \\
\alpha &= \alpha/W = \text{Crack Length Ratio},
\end{align*}

and where

\begin{align*}
\varepsilon &= \text{Strain}, \\
P &= \text{Load}, \\
\alpha &= \text{Crack Length},
\end{align*}

Figure 3 shows the analytically determined \(-EBCW \) versus \(\alpha/W\) data points and the polynomial fit to those points.

For computer applications, the crack length ratio can be expressed as a function of strain compliance as follows:

\[a/W = 0.99999 - 2.00085U - 0.75959U^2 + 10.01565U^3 - 18.39149U^4 + 14.23767U^5 - 4.05333U^6\quad (2)\]
Figure 3. Analytical Back-Face Strain Compliance Versus Crack Length Ratio.
where

\[ U = \frac{1}{\sqrt{-EBCW}}. \] (2a)

The function used for \( U \) is based on the CMOD calibration expression formulated in Reference 5.

The flatness of the curve at \( a/W \) values below 0.4 causes the strain compliance to be less sensitive to crack growth extension than for values above 0.4. The sensitivity is approximately 30 times greater at \( a/W \) values near 0.8 than at \( a/W \) values near 0.2.

2.2 EXPERIMENTAL BACK-FACE STRAIN DATA

Four IN100 metric C(T) specimens with a width, \( W \), of 40 mm and a thickness, \( B \), of 10 mm were strain-gaged with 1/4 inch, 350 ohm gages placed on the back-face at the notch tip plane. Figure 4 shows a specimen mounted in the loading clevises with strain gages on the back-face and the top-face. For the present, the discussion will be concerned only with the back-face strain gage. The back-face gage was wired in a quarter bridge configuration using an excitation voltage of 10 VDC and an amplification of 1000. The specimens were tested under constant stress intensity factor conditions and data were collected at \( a/W \) values approximately equal to the analytical data points. The load versus strain data were recorded on an X-Y plotter and were also collected with an analog-to-digital data acquisition system. The
Figure 4. Strain Gaged C(T) Specimen.
computer calculated the strain compliance based on a linear regression fit to a specified window of the unloading portion of the data. A strain compliance value was calculated from the autographic data plots by visually determining a best fit line through the unloading trace, using the same window as used by the computer. The strain compliance determined by the two methods generally varied by less than 0.2 percent.

To evaluate -EBCW, the experimental values of B and W were measured to within 0.5 percent and the experimental value for C could be determined within 0.5 percent but the value for modulus (E) must be determined based on a calibration using an initial visual crack length plus an estimated curvature correction [2] and Equation 1.

A second crack length determination was done using CMOD compliance [6]. The CMOD compliance method of crack length determination has been shown to yield results that are comparable in accuracy to visual measurements [2]. The strain determined a/W value is plotted against the CMOD determined a/W value in Figure 5. The error in strain determined crack length is less than 2 percent of the CMOD determined crack length.

One IN718 C(T) specimen with W = 40 mm was instrumented with a free filament strain gage using ceramic cement. The specimen was tested at 1200°F and the strain determined a/W value
Figure 5. Relationship of Back-Face Strain Determined Crack Length and CMOD Determined Crack Length at Room Temperature.
was plotted against the CMOD determined a/W value (Figure 6). The results are very similar to the room temperature results.

2.3 ANALYTICAL DUAL-LOCATION STRAIN COMPLIANCE

The insensitivity of the back-face gage at small a/W values led to the decision to consider the use of more than one gage. Specimen thickness precluded the use of dual back-face gages, therefore a gage on the top-face of the specimen was considered. To increase the sensitivity in the range of $0.2 < \frac{a}{W} < 0.5$, it was decided to place the strain gage at a position that would produce the greatest change in strain as $\frac{a}{W}$ varies from 0.2 and 0.5. An analytical top-face strain profile, shown in Figure 7, was determined from a finite element analysis. The greatest strain differential between $0.2 < \frac{a}{W} < 0.5$ occurs with a gage placement of $X/W = 0.5$. From the strain analysis, an analytical top-face strain value for 1/4 inch (6.4 mm) gage length was calculated. The sum of the top-face and back-face strain values were then used to calculate the dual-location non-dimensional strain compliance at a/W values ranging from 0.2 to 0.8.

A mathematical expression was derived to express $-EBCW$ as a function of $a/W$. This mathematical expression was in the form of a sixth degree polynomial

$$
-EBCW = 22.5980 - 365.802a + 2874.93a^2 - 10954.2a^3 + 22855.1a^4 - 24477.2a^5 + 10704.9a^6.
$$

(3)
Figure 6. Relationship of Back-Face Strain Determined Crack Length and CMOD Determined Crack Length at 1200°F.
Figure 7. Analytical Top-Face Strain Profile from Finite Element Analysis.
Figure 8 shows the analytically determined -EBCW versus a/W data points and the polynomial fit to those points. The polynomial expression fits the data points with an error of less than 1 percent.

The crack length ratio can be expressed as a function of dual-location strain compliance as follows:

\[
a/W = 0.99996 - 2.03989U + 1.83981U^2 - 36.0058U^3 + 154.478U^4 - 234.244U^5 + 114.972U^6
\]

(4)

where

\[
U = 1/\sqrt{-EBCW}.
\]

(4a)

2.4 EXPERIMENTAL DUAL-LOCATION STRAIN DATA

Two metric compact type specimens with a width of 40 mm and a thickness of 10 mm were strain-gaged with 1/4 inch, 350 ohm gages placed on the back-face at the notch tip plane and on the top-face at X/W = 0.5. The strain gages were wired in a Wheatstone bridge configuration with the gages in opposite legs of the bridge and precision resistors in the other legs.

The specimens were tested under constant stress intensity factor conditions; strain-load and CMOD-load data were collected and stored using a computer and the strain-load data were also
Figure 8. Analytical Dual-Location Strain Compliance Versus Crack Length Ratio.

\[ a/W = 0.99996 - 2.03989U + 1.83981U^2 - 36.0058U^3 + 154.478U^4 - 234.244U^5 + 114.972U^6 \]

where

\[ U = 1/\sqrt{-EBCW} \]
directly recorded using an X-Y plotter. The computer calculated the CMOD and strain compliance based on a linear regression fit to a specified window on the unloading portion of the data. A strain compliance value was calculated from the autographic data plots by visually determining a best fit line through the unloading trace, using the same window as used by the computer. The strain compliance determined by the two methods generally varied by less than 0.2 percent.

The dual-location strain determined a/W values are plotted against the CMOD determined a/W values in Figure 9. The variance in a/W values determined by the two methods is generally less than 1 percent.
Figure 9. Relationship of Dual-Location Strain Determined Crack Length and CMOD Determined Crack Length at Room Temperature.
SECTION 3
DISCUSSION AND RECOMMENDATIONS

3.1 DISCUSSION

A comparison of back-face and dual-location strain compliance curves is given in Figure 10. To illustrate the differences in sensitivity between the two compliance curves at values of $a/W$ less than 0.5, the slopes of each curve as a function of $a/W$ were plotted in Figure 11. The dual-location gages exhibit an increase in sensitivity over the back-face gage that ranges from 100 percent at $a/W = 0.2$ to less than 25 percent at $a/W = 0.5$.

There are no currently available guidelines for the acceptability of nonvisual crack length measurements versus visual measurements. The nonvisual determined crack lengths were compared to the visual measurements to determine the accuracy of the nonvisual methods. Dual-location strain compliance and CMOD compliance determined $a/W$ values are plotted against the visual surface measurement $a/W$ values (Figure 12). The nonvisual methods show a consistently longer crack length. When the visual crack lengths were corrected for crack curvature in accordance with the requirements of ASTM E647-83 [7] and the results were replotted in Figure 13, there was very good correlation with a maximum variation of ±1 percent between visual and nonvisual determined $a/W$ values.
Figure 10. Compliance of Analytical Strain Compliance Data.
Figure 11. Comparison of the Sensitivities of Analytical Strain Compliance Relationships as Determined by the Derivative of These Relationships.
Figure 12. Comparison of Visual (Surface Measurement) and Nonvisual Determined Crack Length.
Figure 13. Relationship of Visual (Curvature Corrected) and Nonvisual Determined Crack Length.
3.2 RECOMMENDATIONS

Compliance based on specimen strain has been shown to provide crack length values that are comparable to visual measurements (at room temperature). An elevated temperature test that was performed on a back-face gaged specimen indicates that the method can be used at 1200°F with appropriate calibration. All tests were conducted at a frequency of 20 Hz, but the frequency response of strain gages should allow for considerably higher test frequencies.

The back-face strain compliance and a/W relationships derived in this investigation are valid for a C(T) specimen having a ratio of active strain gage length to specimen width of approximately 1/6. These expressions are applicable for ratios within ±25 percent of 1/6. Outside of this range other expressions would have to be developed.

Additional tests are needed to validate the higher frequency capabilities and the dual-location strain compliance method at other temperatures.
REFERENCES


