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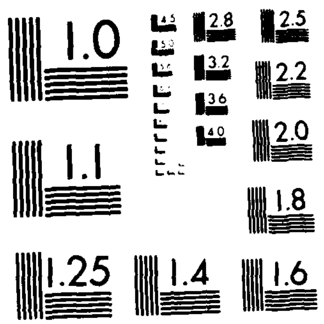
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TARGET MOTION EFFECTS ON PASSIVE SONAR: AN APPRAISAL OF BASIC FACTORS

by

Ezio PUSONE

DECEMBER 1986

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TARGET MOTION EFFECTS ON PASSIVE SONAR: AN APPRAISAL OF BASIC FACTORS

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Ezio Pusone

December 1986

This memorandum has been prepared within the SACLANTCEN Systems Research Division as part of Project 02.

John MARCHMENT
Division Chief

Abstract

Detection and tracking by passive sonar involves the observation of the target-radiated noise in bearing-cells and time-cells that have their dimensions predetermined by a combination of physical limitations, e.g. the size of the acoustic array, and general factors related to the geometry of the target motion. This present study considers the idealized situation of a plane wavefront being received by a linear array. Conclusions are given relating to the effects of target motion on the allowable observation times for constant speed and heading conditions.

Keywords

DETECTION
FREQUENCY DOMAIN
MONTECARLO TECHNIQUE
PASSIVE SONAR
PROBABILITY DENSITY FUNCTION
RANDOM ENCOUNTER
STEADY-STATE CASE
TARGET MOTION
TARGET-RADIATED NOISE
TIME DOMAIN
TRACKING

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1. Introduction

The normal passive detection system involves, in general, the assumption that the received wavefront of target radiated noise is a plane wave. The signal processing is based on this assumption and provides a broad-band 'bearing versus time' display or a narrow-band 'frequency versus time' display. Sometimes the latter data are presented as 'frequency versus bearing and time'.

For a 'bearing versus time' display (broadband) the bearing data for a given pass-band are usually obtained from the outputs of beams formed from the whole array by integrating them, that is to say averaging the outputs over some appropriate time — and thus the average represents beam output power. The beams are made to overlap at the 3-dB down points so that the width of the bearing cells can then taken to be equal to the 3-dB beamwidth of the beams.

Alternatively, the bearing data may be obtained from the display of the correlogram formed by applying the split-beam process* to a given pass-band. The cell width is then taken as the effective 3-dB width of this correlogram.

The time duration τ_i of the cells for the 'bearing versus time' display is the coherent integration time, and is determined more by the cycle time of the display system than any other consideration.

For a 'frequency versus time' display (narrow band) the frequency information in a given beam is the power output of the frequency analysis device applied to the beam signals. The effective coherent integration time, τ_a, is then given by the inverse of the frequency resolution, i.e. $1/\Delta f$ (the frequency cell-width). Thus, for narrow-band processing, the cell size is taken to be Δf (in frequency) by $1/\Delta f$ (in time), where Δf is the 3-dB width of the frequency analysis window.

In the above it is revealed that in the broad-band processing the size of the 'observation cells' is fixed; in consequence information can be lost if the signal moves through the cells in a time less than the coherent integration time. On the other hand, in the narrow-band processing some flexibility is usually available. However the minimum integration time is linked to the desired frequency resolution, and so if the target signal moves through the frequency cell in a time less than this integration time then once again information can be lost.

This paper examines the range of values of coherent integration times likely to be available in ASW encounters based on the assumption that a single plane wave is received from the target, and that the receiving vessel and target are moving on fixed courses at constant speeds.

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*Horton, C. "Signal processing for underwater acoustic waves", Washington, G.P.O., McGraw Hill, 1969.

2 Random Encounter

2.1 GEOMETRY

The situation to be studied is shown in Fig. 1. The assumptions are that the noise radiated by the target is received by the observer as a simple plane wave and that the effects of the motion of the observer and target can be described by geometrical changes. This is effectively converting the situation to a two-dimensional far-field problem and assuming that the phase fluctuations or geometric variations of the wavefront, which occur due to its passage through the sea, are negligible.

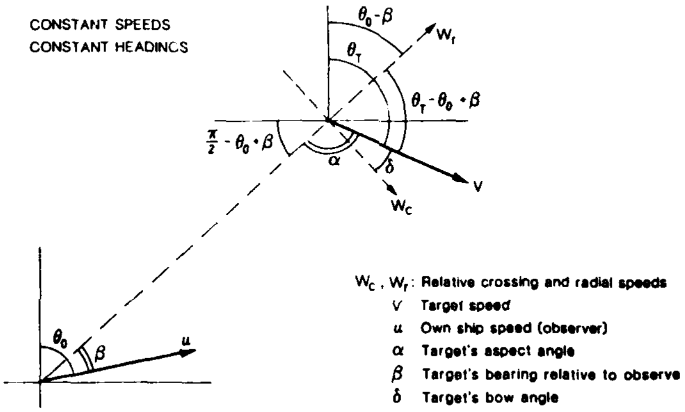


Fig. 1. Geometrical relationship of observer and target.

2.1.1 Kinematic relationships

From Fig. 1 we have the relative crossing speed w_c and the radial (or opening) speed w_r given by

$$w_c = v\cos\delta - u\sin\beta,$$

$$w_r = v\sin\delta - u\cos\beta.$$

And since $\cos\delta = \sin(\beta + \theta_T - \theta_0)$ and $\sin\delta = \cos(\beta + \theta_T - \theta_0)$, where $\pi/2 - (\beta + \theta_T - \theta_0)$ is the angle on the target's bow and β is the bearing of the target relative to the observer's heading, we have therefore

$$w_c = v \sin(\beta + \theta_T - \theta_0) - u \sin\beta, \qquad \text{(Eq. 1)}$$

$$w_r = v \cos(\beta + \theta_T - \theta_0) - u \cos\beta, \qquad \text{(Eq. 2)}$$

and the rate of change of β is given by

$$\dot{\beta} = -\frac{w_c}{r} + \dot{\theta}_0 . \qquad \text{(Eq. 3)}$$

Now the Doppler shift of a frequency f radiated by the target, as seen by the observer, is

$$\Delta f = -\frac{w_r}{c} \times f ,$$

where c = speed of sound, i.e.

$$\Delta f = -6.66 \times 10^{-4} f \times w_r . \qquad \text{(Eq. 4)}$$

With f in Hz, w_r in m/s, and the speed of sound taken to be 1500 m/s, the rate of change of frequency f caused by an outward acceleration along radial r is

$$\dot{f} = \frac{\partial}{\partial t}(\Delta f) = -6.66 \times 10^{-4} f \dot{w}_r . \qquad \text{(Eq. 5)}$$

From Eqs. 2 and 3 we have

$$\dot{w}_r = \left(\frac{w_c}{r} - \dot{\theta}_0\right) v \sin(\beta + \theta_T - \theta_0) - (\dot{\theta}_T - \dot{\theta}_0) v \sin(\beta + \theta_T - \theta_0)$$

$$-\left(\frac{w_c}{r} - \dot{\theta}_0\right) u \sin\beta + \dot{v} \cos(\beta + \theta_T - \theta_0) - \dot{u} \cos\beta,$$

which gives

$$\dot{w}_r = \frac{w_c^2}{r} + \dot{\theta}_0 u \sin\beta - \dot{u} \cos\beta - \dot{\theta}_T v \sin(\beta + \theta_T - \theta_0)$$

$$+ \dot{v} \cos(\beta + \theta_T - \theta_0). \qquad \text{(Eq. 6)}$$

Thus

$$\dot{f} = -\,6.66 \times 10^{-4}\, f \left\{ \frac{w_c^2}{r} + \dot{\theta}_O\, u \sin\beta - \dot{u} \cos\beta \right.$$

$$\left. -\, \dot{\theta}_T\, v \sin(\beta + \theta_T - \theta_O) + \dot{v} \cos(\beta + \theta_T - \theta_O) \right\}. \qquad \text{(Eq. 7)}$$

From Eq. 3,

$$\ddot{\beta} = -\frac{\dot{w}_c}{r} + \frac{w_c w_r}{r^2} + \ddot{\theta}_O .$$

But from Eq. 1,

$$\dot{w}_c = -\frac{w_r w_c}{r} + (\dot{\theta}_T - \theta_O)\, v \cos(\beta + \theta_T - \theta_O) + \dot{\theta}_O\, w_r$$

$$+\, \dot{v} \sin(\beta + \theta_T - \theta_O) - \dot{u} \sin\beta ,$$

Thus

$$\ddot{\beta} = \frac{2 w_c w_r}{r^2} + \ddot{\theta}_O - \frac{(\dot{\theta}_T - \dot{\theta}_O)}{r}\, v \cos(\beta + \theta_T - \theta_O) - \frac{\dot{v}}{r} \sin(\beta + \theta_T - \theta_O)$$

$$+\, \frac{\dot{u}}{r} \sin\beta - \frac{\dot{\theta}_O w_r}{r} , \qquad \text{(Eq. 8)}$$

and

$$\ddot{f} = 6.66 \times 10^{-4}\, f \left[3\, \frac{w_c^2 w_r}{r^2} + (\dot{\theta}_O)^2\, u \cos\beta - (\dot{\theta}_T)^2\, v \cos(\beta + \theta_T - \theta_O) \right.$$

$$-\, \frac{\dot{\theta}_O w_c}{r} \cos\beta + \frac{\dot{\theta}_T}{r}\, w_c \cos(\beta + \theta_T - \theta_O)$$

$$\left. +\, 3\, \frac{w_c}{r}\, \dot{v} \sin(\beta + \theta_T - \theta_O) + \frac{w_c}{r}\, \dot{u} \sin\beta \right]. \qquad \text{(Eq. 9)}$$

2.2 STEADY-STATE CASE

The steady state is the situation usually treated in target motion analysis namely that in which the observer and the target maintain fixed courses at constant velocities.

2.2.1 Steady-state kinematics

Under the steady-state conditions, Eqs. 3 and 7, for bearing rate and frequency rate respectively, reduce to

$$\dot{\beta} = -\frac{w_c}{r} , \qquad \text{(Eq. 10)}$$

$$\dot{f} = -6.66 \times 10^{-4}\ f \times \frac{w_c^2}{r} , \qquad \text{(Eq. 11)}$$

and Eqs. 8 and 9 become

$$\ddot{\beta} = \frac{2 w_c w_r}{r^2} , \qquad \text{(Eq. 12)}$$

$$\ddot{f} = 2 \times 10^{-3}\ f \times \frac{w_c^2 w_r}{r^2} . \qquad \text{(Eq. 13)}$$

2.2.2 Effects of target motion

The mathematical expressions for the integration times τ_i and τ_a are required for the time-domain analysis and the frequency domain analysis respectively. They represent the optimum observation times for a moving target.

Time-domain analysis

$$\text{Time in beam} = \frac{\text{3-dB beamwidth}}{\text{bearing rate}} ,$$

$$\tau_i = \frac{\theta}{\dot{\beta}} .$$

Frequency domain analysis

$$\text{Time in frequency bin} = \frac{\text{bin width}}{\text{frequency rate}} ,$$

$$\tau_a = \frac{\Delta f_o}{\dot{f}} .$$

To calculate τ we use the probability density functions (PDFs) in Eqs. 10 and 11, i.e. the respective functions for β (bearing rate) and f (rate of change of frequency) in their steady-state conditions, which involves an estimate of the spread of values of w_c likely to be observed in an ASW encounter. This is done by combining the PDFs of likely speeds and directions of target and tracker.

With the PDFs calculated, and the frequency-cell width and the spatial beamwidth of the towed array known, the PDFs of the the integration times can then be obtained.

The mathematics for the calculation of the PDF of τ_i in the time domain is developed in Sect. A.1 of Appendix A for a random encounter, i.e. for an encounter in which there are no constraints on the direction of either the target or the tracker.

The same procedure is followed for the PDF of τ_a in the frequency domain. This is shown in Sect. A.2 of Appendix A where the PDF of available observation time (τ_a) is calculated for the frequency versus time display.

2.2.3 Computer simulation results

From the technique outlined in Sect. A.1 of Appendix A, the PDF of the crossing speed w_c (shown in Figs. 2 and 3) is obtained using the assumed PDFs of speed for the ASW frigate, for the conventional submarine, and for the nuclear submarine (see p. 5). In this time domain analysis we derive the PDF of the bearing rate β (see Figs. 4 and 5) and finally the integration times (see Figs. 6 and 7) for both conventional and nuclear submarines.

The PDF of the available observation times derived in the frequency domain analysis were calculated with the method described in Sect. A.2 of Appendix A and the results are shown in Figs. 8 and 9.

Results obtained using a Montecarlo technique are shown in Figs. 10 and 11; the associated calculation is presented in Appendix B.

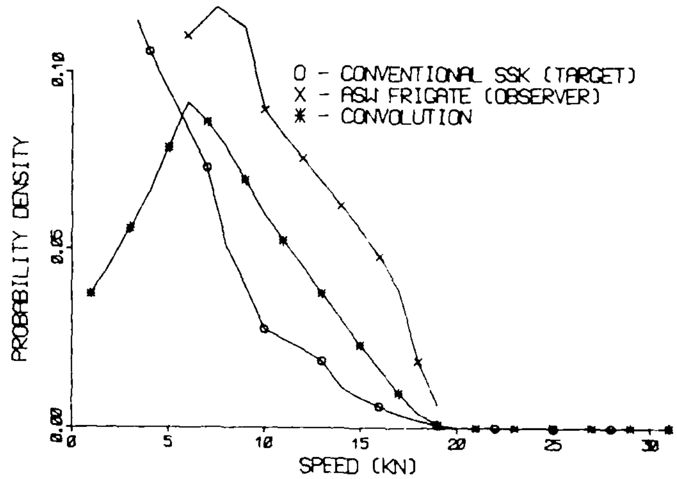


Fig. 2. Time domain analysis: probability density as a function of crossing speed for a conventional submarine target.

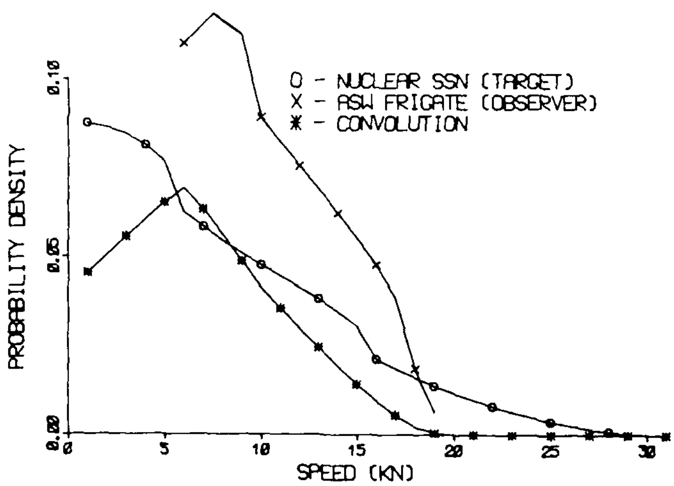


Fig. 3. Time domain analysis: probability density as a function of crossing speed for a nuclear submarine target.

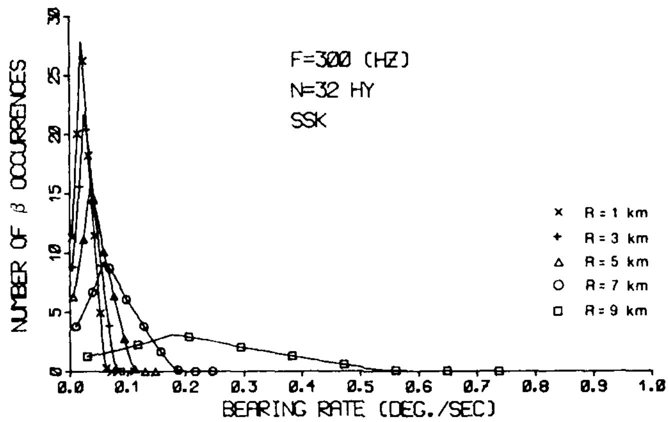


Fig. 4. Time domain analysis: probability density function of bearing rate (β) for a conventional submarine target.

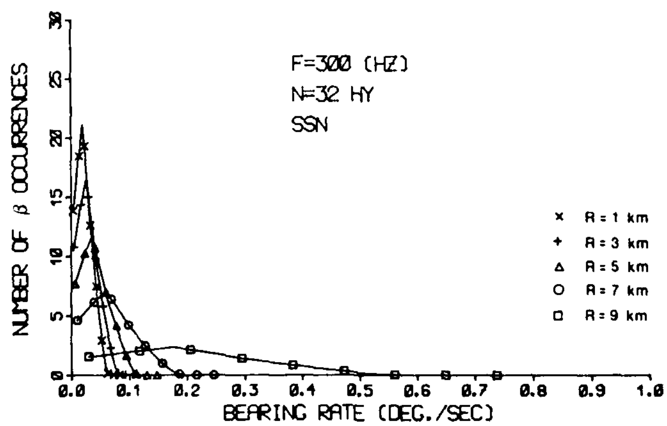


Fig. 5. Time domain analysis: probability density function of bearing rate (β) for a nuclear submarine target.

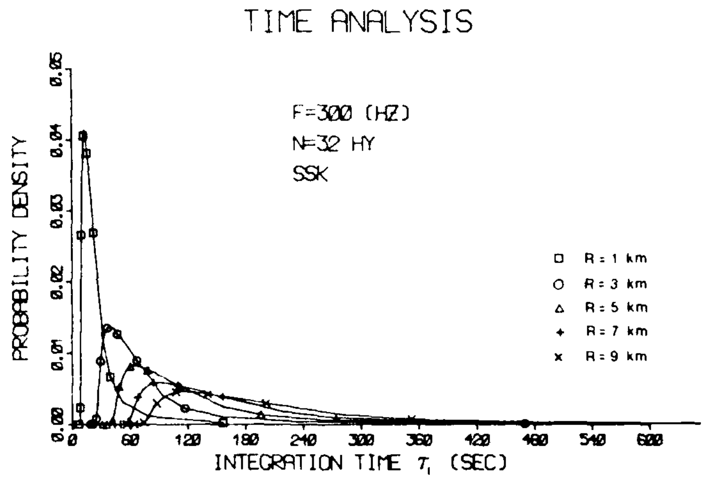


Fig. 6. Time domain analysis: probability density function of integration time (τ_i) for a conventional submarine target.

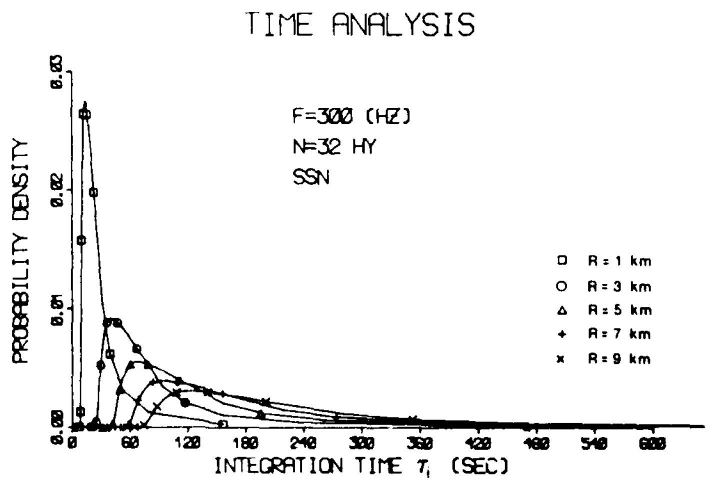


Fig. 7. Time domain analysis: probability density function of integration time (τ_i) for a nuclear submarine target.

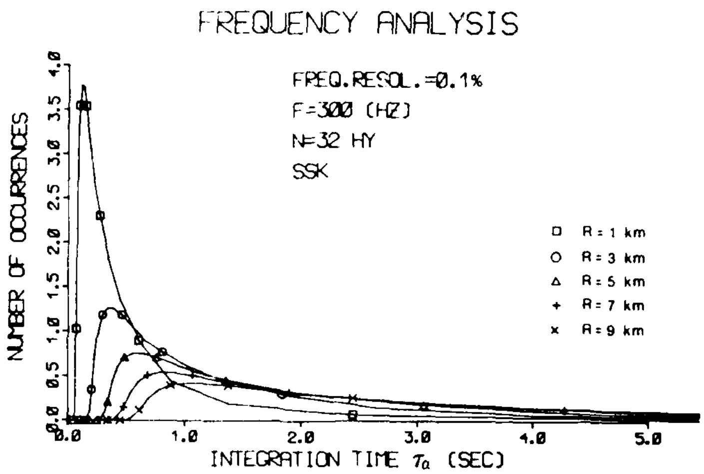


Fig. 8. Frequency domain analysis: probability density function of available obsevation time (τ_a) for a conventional submarine target.

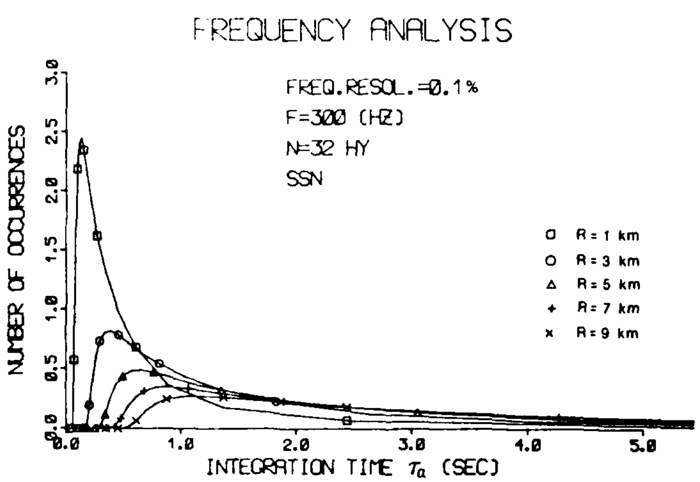


Fig. 9. Frequency domain analysis: probability density function of available observation time (τ_a) for a nuclear submarine target.

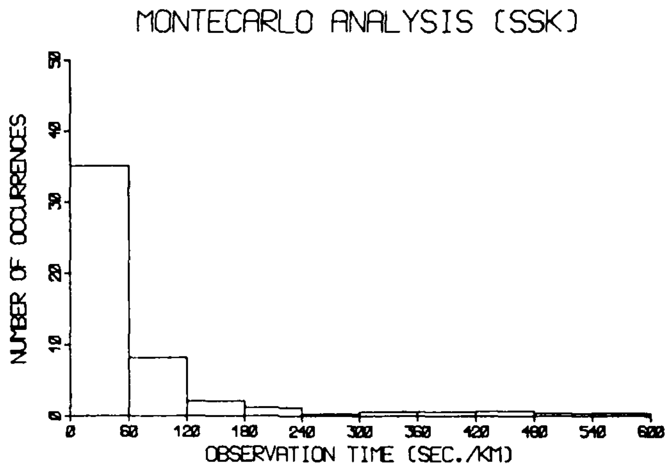


Fig. 10. Montecarlo analysis: probability density function of the integration time (τ) for a conventional submarine target.

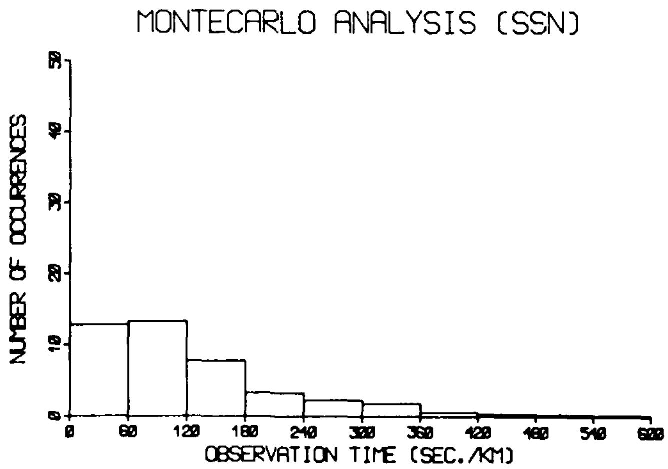


Fig. 11. Montecarlo analysis: probability density function of the integration time (τ) for a nuclear submarine target.

3. Conclusions

The results as presented in Figs. 4 through 11 are valid for the case of a 32-hydrophone array with half-wavelength spacing and a frequency of 300 Hz. They are considered to be representative of a realistic situation, although it would be of interest to extend the study by incorporating a broader range of parameters.

From the PDFs of the speeds considered in this study the results all show a strong dependence on target-observer range, as one would expect, since the change in bearing rate of the target plays a major role. At short ranges the PDF of the integration time is narrow with the peak occurring at a small value of time. At longer ranges the maximum PDF occurs at a higher value of time and the spread increases. This is why at long ranges one has more time available for integration, while at short ranges the time for integration is limited.

Two methods have been used to calculate the probability distribution of the integration time: the analytical method described in Appendix A and the Montecarlo method described in Appendix B. The shape of the curves in Figs. 10 and 11 of the Montecarlo results show a good relation to these obtained with the more complicated analytical method (Figs. 6 to 9). However, the analytical method gives, in addition, mathematical expressions for the mean and the standard deviation of the available integration time, and these can be used for predictions of processing performance.

APPENDICES

A - Calculation of the probability density of the available observation time

B - Montecarlo technique

Appendix A

CALCULATION OF THE PROBABILITY DENSITY OF THE AVAILABLE OBSERVATION TIME

The general calculation procedure is to derive the probability density function (PDF) of the integration time for the ASW observer from the knowledge of the PDFs of the observer and target velocities. In the random encounter case the directions of movement of observer and target are uniformly distributed in $(0, 2\pi)$.

A.1 TIME DOMAIN ANALYSIS

In the time domain analysis (bearing versus time display), the relative crossing speed for the random encounter case is (see Sect. 2 of the main text)

$$w_c = v \sin\alpha - u \sin\beta \ . \qquad \text{(Eq. A1)}$$

The bearing rate is

$$\dot{\beta} = \frac{w_c}{r} \ , \qquad \text{(Eq. A2)}$$

and the available integration time is related to the bearing rate by the relation

$$\tau = \frac{\theta}{\dot{\beta}} \ , \qquad \text{(Eq. A3)}$$

where θ is the 3-dB beamwidth of the towed-array pattern.

A.1.1 Calculation of PDF of crossing speed

From Eq. A1, with $v \sin\alpha = z_1$ and $u \sin\beta = z_2$ we have

$$w_c = z_1 - z_2 . \qquad \text{(Eq. A4)}$$

Under the hypothesis that z_1 and z_2 are random independent variables, the PDF of w_c from the theorem of Papoulis* (p. 189), is

$$p(w_c) = \int_{-\infty}^{+\infty} p_{z_1}(w_c + y)\, p_{z_2}(y)\, dy \ . \qquad \text{(Eq. A5)}$$

In order to determine the PDFs of z_1 and z_2, we define that

$$x_1 = v \quad \text{and} \quad x_2 = \sin\alpha \longrightarrow z_i = v \sin\alpha \ ,$$

and from Papoulis* (p. 205)

$$p(z_1) = \int_{-\infty}^{+\infty} \frac{1}{x_1} p_1(x_1)\, p_2(x_2)\, dx_1 = \int_{-\infty}^{+\infty} \frac{1}{x_1} p_1(x_1)\, p_2(z_1/x_1)\, dx_1 ,$$

where $p(x_2) = p(\sin\alpha)$, and since $p(\sin\alpha)\, d(\sin\alpha) = p(\alpha)\, d\alpha$,

$$p(\sin\alpha) = \frac{p(\alpha)}{d(\sin\alpha)/d\alpha} = \frac{p(\alpha)}{\cos\alpha} \ .$$

With α uniformly distributed between $(0, 2\pi)$,

$$p(\alpha) = \frac{1}{2\pi} \ ,$$

and therefore

$$p(\sin\alpha) = \frac{2}{2\pi} \times \frac{1}{\cos\alpha} \quad ; \qquad \text{[see Papoulis* (p. 133)].}$$

From $z_1 = v \sin\alpha$,

$$\sin\alpha = \frac{z_1}{v} \ ,$$

and therefore $\cos\alpha = (1 - \sin^2\alpha)^{1/2} = 1 - [1 - (z_1/v)^2]^{1/2}$.

*Papoulis, A. "Probability, random variables, and stochastic processes", McGraw Hill, New York, 1962.

Hence

$$p(z_1) = \int_{-\infty}^{+\infty} \frac{1}{v}\, p(v) \times \frac{2}{2\pi} \times \frac{1}{\sqrt{1 - (z_1/v)^2}}\, dv = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{1}{v}\, p(v) \times \frac{v}{\sqrt{v^2 - z_1^2}}\, dv$$

$$= \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{p(v)}{\sqrt{v^2 - z_1^2}}\, dv = p(v \sin\alpha) \; ; \qquad \text{[target]}.$$

At this point we need to specify $p(v)$, the analytical form of the PDF of the target velocity.

The same procedure is followed for the observer term $u \sin\beta$:

$$x_2 = u \; , \qquad x_3 = \sin\beta \qquad z_2 = u \sin\beta,$$

and

$$p(z_2) = \int_{-\infty}^{+\infty} \frac{1}{x_2}\, p_2(x_2)\, p_3(x_3)\, dx_2 = \int_{-\infty}^{+\infty} \frac{1}{x_2}\, p_2(x_2)\, p_3(z_2/x_2)\, dx_2 .$$

With β uniformly distributed between $(0, 2\pi)$,

$$p(\beta) = \frac{1}{2\pi}$$

and therefore $p(\sin\beta) = \dfrac{1}{2\pi} \times \dfrac{2}{\cos\beta}$.

From $z_2 = u \sin\beta$,

$$\sin\beta = \frac{z_2}{u}$$

and therefore $\cos\beta = (1 - \sin^2\beta)^{1/2} = \sqrt{1 - (z_2/u)^2}$.

Hence

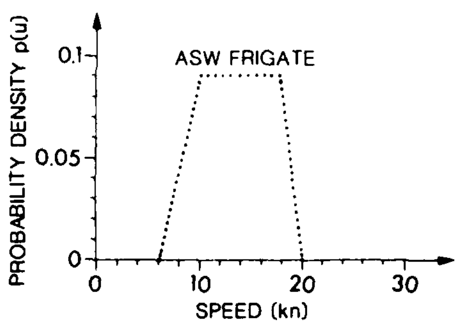
$$p(z_2) = \int_{-\infty}^{+\infty} \frac{1}{u}\, p(u) \times \frac{1}{\pi} \times \frac{u}{\sqrt{u^2 - z_2^2}}\, du = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{p(u)}{\sqrt{u^2 - z_2^2}}\, du = p(u \sin\beta);$$

[observer].

We assume a specific analytical form for $p(u)$, the PDF of the observer (ship) velocity.

The calculation of the integrals $p(z_1)$ and $p(z_2)$ for the cases in which the ASW frigate is the observer and the conventional or nuclear submarine is the target are developed in this report from given distributions of the velocities u, v.

A.1.2 Integrals calculation: observer (ASW frigate)



$u_{m_1} = 10$

$u_{m_2} = 18$

$u_{m_3} = 20$

$p(u)$ is the PDF of the observer.

Interval: $6 < u < 10$

$$p(u) = m_1(u-6),$$

$$m_1 = \frac{0.09 - 0}{4} = 0.0225 \quad \text{(slope)},$$

$$p(u) = -0.1350 + 0.225u, \qquad \begin{cases} b_1 = -0.1350 \\ B_1 = 0.0225. \end{cases}$$

Interval: $10 < u < 18$

$$p(u) = 0.09, \qquad \begin{cases} b_2 = 0.09 \\ B_2 = 0. \end{cases}$$

Interval: $18 < u < 20$

$$p(u) = 0.09 + m_3(u-18),$$

$$m_3 = \frac{0 - 0.09}{20 - 18} = \frac{-0.09}{2} = -0.0450,$$

$$p(u) = 0.9 - 0.0450u, \qquad \begin{cases} b_3 = 0.9 \\ B_3 = -0.0450. \end{cases}$$

The probability term is given by

$$p(z_2) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{p(u)}{\sqrt{u^2 - z_2^2}} \, du.$$

Since $z_2 = u \sin\beta$,

$$z_2/u \leq 1 \quad ; \quad u \geq z_2.$$

The limits of the integral are as follows:

$$I_2 = p_2(z_2) = \frac{1}{\pi} \int_{z_2}^{u_{max}} \frac{p(u)}{\sqrt{u^2 - z_2^2}} \, du.$$

Thus the total probability is given by

$$p(z_2) = F_1 + F_2 + F_3,$$

where

$$F_1 = \frac{1}{\pi} \int_{z_2}^{u_{m1}} \frac{(b_1 + B_1 u)}{\sqrt{u^2 - z_2^2}} \, du,$$

$$F_2 = \frac{1}{\pi} \int_{z_2}^{u_{m2}} \frac{(b_2 + B_2 u)}{\sqrt{u^2 - z_2^2}} \, du,$$

$$F_3 = \frac{1}{\pi} \int_{z_3}^{u_{m2}} \frac{(b_3 + B_3 u)}{\sqrt{u^2 - z_2^2}} \, du.$$

Evaluating these integrals, we have

$$F_1 = \frac{b_1}{\pi} \times \ln \left[\frac{u_{m1} + \sqrt{u_{m1}^2 - z_2^2}}{z_2} \right] + \frac{B_1}{\pi} \times \sqrt{u_{m1}^2 - z_2^2} ,$$

$$F_2 = \frac{b_2}{\pi} \times \ln \left[\frac{u_{m2} + \sqrt{u_{m2}^2 - z_2^2}}{z_2} \right] + \frac{B_2}{\pi} \times \sqrt{u_{m2}^2 - z_2^2} ,$$

$$F_3 = \frac{b_3}{\pi} \times \ln \left[\frac{u_{m3} + \sqrt{u_{m3}^2 - z_2^2}}{z_2} \right] + \frac{B_3}{\pi} \times \sqrt{u_{m3}^2 - z_2^2} .$$

Finally, we have

<u>for $z_2 < u_{m1}$</u>

$$p(z_2) = F_1(z_2) + F_2(z_2 = u_{m1}) + F_3(z_2 = u_{m2}),$$

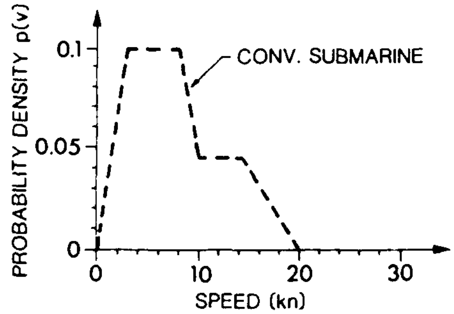
<u>for $u_{m1} < z_2 < u_{m2}$</u>

$$p(z_2) = F_2(z_2) + F_3(z_2 = u_{m2}),$$

<u>for $u_{m2} < z_2 < u_{m3}$</u>

$$p(z_2) = F_3(z_2).$$

A.1.3 Integrals calculation: target (conventional SSK)



$v_{m_1} = 3$

$v_{m_2} = 8$

$v_{m_3} = 10$

$v_{m_4} = 16$

$v_{m_5} = 20$

Interval: $0 < v < 3$

$$p(v) = y_0 + m_1(v - x_0),$$

$$p(v) = y_0 + m_1 x_0 + m_1 v = a_1 + m_1 v \quad ; \qquad a_1 = y_0 + m_1 x_0,$$

$$m_1 = \frac{0.1 - 0}{3 - 0} = 0.0333,$$

$$p(v) = 0.0333v, \qquad \begin{cases} a_1 = 0 \\ A_1 = m_1 = 0.0333. \end{cases}$$

Interval: $3 < v < 8$

$$p(v) = a_2 + m_2 v,$$

$$p(v) = 0.1, \qquad \begin{cases} a_2 = 0.1 \\ A_2 = m_2 = 0. \end{cases}$$

Interval: $8 < v < 10$

$$p(v) = 0.1 + m_3(v-8),$$

$$m_3 = \frac{0.05 - 0.1}{10 - 8} = \frac{-0.05}{2} = 0.0250,$$

$$p(v) = 0.1 + 0.0250\,v - 8 \times 0.0250,$$

$$p(v) = -0.1 + 0.0250\,v, \qquad \begin{cases} a_3 = -0.1 \\ A_3 = 0.0250. \end{cases}$$

Interval: $10 < v < 16$

$$p(v) = 0.045, \qquad \begin{cases} a_4 = 0.045 \\ A_4 = 0. \end{cases}$$

Interval: $16 < v < 20$

$$p(v) = 0.045 + m_5(v-16),$$

$$m_5 = \frac{0 - 0.045}{20 - 16} = \frac{-0.045}{4} = -0.0113,$$

$$p(v) = 0.045 - 0.0113(v-16),$$

$$p(v) = 0.2250 - 0.0113v \qquad \begin{cases} a_5 = 0.2250 \\ A_5 = -0.0113. \end{cases}$$

The total $p(z_1)$ is given by

$$p(z_1) = \frac{a_1}{\pi} \times \ln\left[\frac{v_{m1} + \sqrt{v_{m1}^2 - z_1^2}}{z_1}\right] + \frac{A_1}{\pi} \times \sqrt{v_{m1}^2 - z_1^2}$$

$$+ \frac{a_2}{\pi} \times \ln\left[\frac{v_{m2} + \sqrt{v_{m2}^2 - z_1^2}}{z_1}\right] + \frac{A_2}{\pi} \times \sqrt{v_{m2}^2 - z_1^2}$$

$$+ \frac{a_3}{\pi} \times \ln\left[\frac{v_{m3} + \sqrt{v_{m3}^2 - z_1^2}}{z_1}\right] + \frac{A_3}{\pi} \times \sqrt{v_{m3}^2 - z_1^2}$$

$$+ \frac{a_4}{\pi} \times \ln\left[\frac{v_{m4} + \sqrt{v_{m4}^2 - z_1^2}}{z_1}\right] + \frac{A_4}{\pi} \times \sqrt{v_{m4}^2 - z_1^2}$$

$$+ \frac{a_5}{\pi} \times \ln\left[\frac{v_{m5} + \sqrt{v_{m5}^2 - z_1^2}}{z_1}\right] + \frac{A_5}{\pi} \times \sqrt{v_{m5}^2 - z_1^2} .$$

Since

$a_1 = 0,$	$A_1 = 0.0333,$	$v_{m1} = 3,$
$a_2 = 0.1,$	$A_2 = 0,$	$v_{m2} = 8,$
$a_3 = -0.1,$	$A_3 = 0.0250,$	$v_{m3} = 10,$
$a_4 = 0.045,$	$A_4 = 0,$	$v_{m4} = 16,$
$a_5 = 0.2250,$	$A_5 = -0.0113,$	$v_{m5} = 20,$

and

$$p(z_1) = F_1 + F_2 + F_3 + F_4 + F_5,$$

we have

<u>for $v_{m1} < z_1 < v_{m2}$</u>

$$p(z_1) = F_1(z_1) + F_2(z_1 = v_{m1}) + F_3(z_1 = v_{m2}) + F_4(z_1 = v_{m3}) + F_5(z_1 = v_{m4})$$

for $v_{m_1} < z_1 < v_{m_2}$

$$p(z_1) = F_2(z_1) + F_3(z_1 = v_{m_2}) + F_4(z_1 = v_{m_3}) + F_5(z_1 = v_{m_4})$$

for $v_{m_2} < z_1 < v_{m_3}$

$$p(z_1) = F_3(z_1) + F_4(z_1 = v_{m_3}) + F_5(z_1 = v_{m_4})$$

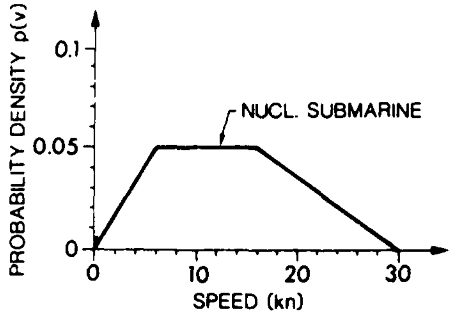
for $v_{m_3} < z_1 < v_{m_4}$

$$p(z_1) = F_4(z_1) + F_5(z_1 = v_{m_4})$$

for $v_{m_4} < z_1 < v_{m_5}$

$$p(z_1) = F_5(z_1)$$

A.1.4 Integrals calculation: target (nuclear SSN)



$v_{m_1} = 6$

$v_{m_2} = 16$

$v_{m_3} = 30$

Interval: $0 < v < 6$

$$p(v) = mv, \qquad m = \frac{0.05}{6} = 0.8333 \times 10^{-2}.$$

Interval: $6 < v < 16$

$$p(v) = 0.05.$$

Interval: $16 < v < 30$

$$p(v) = 0.05 + m(v-16), \qquad m = \frac{-\,0.05}{14} = -\,0.3571 \times 10^{-2}.$$

Thus

$$0 < v < 6, \qquad p(v) = 0.8333 \times 10^{-2}\, v,$$

$$6 < v < 16, \qquad p(v) = 0.05,$$

$$16 < v < 30, \qquad p(v) = +\,0.1071 - 0.3571 \times 10^{-2} \times v = a + A\, v,$$

with

$$a = y_0 + mx_0 \quad ; \qquad y_0 = 0.05 \quad , \quad m = -\,0.3571 \times 10^{-2},$$

$$A = m \quad ; \qquad x_0 = 16 \quad , \quad a = 0.1071.$$

The total $p(z_1)$ is given by

$$p(z_1) = \frac{a_1}{\pi} \times \ln\left[\frac{v_{m1} + \sqrt{v_{m1}^2 - z_1^2}}{z_1}\right] + \frac{A_1}{\pi} \times \sqrt{v_{m1}^2 - z_1^2}$$

$$+ \frac{a_2}{\pi} \times \ln\left[\frac{v_{m2} + \sqrt{v_{m2}^2 - z_1^2}}{z_1}\right] + \frac{A_2}{\pi} \times \sqrt{v_{m2}^2 - z_1^2}$$

$$+ \frac{a_3}{\pi} \times \ln\left[\frac{v_{m3} + \sqrt{v_{m3}^2 - z_1^2}}{z_1}\right] + \frac{A_3}{\pi} \times \sqrt{v_{m3}^2 - z_1^2} \; .$$

Since

$$a_1 = 0, \qquad A_1 = 0.8333 \times 10^{-2}, \qquad v_{m_1} = 6,$$
$$a_2 = 0.05, \qquad A_2 = 0, \qquad v_{m_2} = 16,$$
$$a_3 = 0.1071, \qquad A_3 = -\,0.3571 \times 10^{-2}, \qquad v_{m_3} = 30,$$

and

$$p(z_1) = F_1 + F_2 + F_3,$$

we have

<u>for $z_1 < v_{m_1}$</u>

$$p(z_1) = F_1(z_1) + F_2(z_1 = v_{m_1}) + F_3(z_1 = v_{m_2}),$$

<u>for $v_{m_1} < z_1 < v_{m_2}$</u>

$$p(z_1) = F_2(z_1) + F_3(z_1 = v_{m_2}),$$

<u>for $v_{m_2} < z_1 < v_{m_3}$</u>

$$p(z_1) = F_3(z_1).$$

Consequently,

<u>for $z_1 < v_{m_1}$</u>

$$p(z_1) = \frac{a_1}{\pi} \times \ln\left[\frac{v_{m_1} + \sqrt{v_{m_1}^2 - z_1^2}}{z_1}\right] + \frac{A_1}{\pi} \times \sqrt{v_{m_1}^2 - z_1^2}$$
$$+ \frac{a_2}{\pi} \times \ln\left[\frac{v_{m_2} + \sqrt{v_{m_2}^2 - v_{m_1}^2}}{v_{m_1}}\right] + \frac{A_2}{\pi} \times \sqrt{v_{m_2}^2 - v_{m_1}^2}$$
$$+ \frac{a_3}{\pi} \times \ln\left[\frac{v_{m_3} + \sqrt{v_{m_3}^2 - v_{m_2}^2}}{v_{m_2}}\right] + \frac{A_3}{\pi} \times \sqrt{v_{m_3}^2 - v_{m_2}^2},$$

for $v_{m_1} < z_1 < v_{m_2}$

$$p(z_1) = \frac{a_2}{\pi} \times \ln\left[\frac{v_{m_2} + \sqrt{v_{m_2}^2 - z_1^2}}{v_{m_2}}\right] + \frac{A_2}{\pi} \times \sqrt{v_{m_2}^2 - z_1^2}$$

$$+ \frac{a_3}{\pi} \times \ln\left[\frac{v_{m_3} + \sqrt{v_{m_3}^2 - v_{m_2}^2}}{v_{m_2}}\right] + \frac{A_3}{\pi} \times \sqrt{v_{m_3}^2 - v_{m_2}^2},$$

for $v_{m_2} < z_1 < v_{m_3}$

$$p(z_1) = \frac{a_3}{\pi} \times \ln\left[\frac{v_{m_3} + \sqrt{v_{m_3}^2 - z_1^2}}{v_{m_2}}\right] + \frac{A_3}{\pi} \times \sqrt{v_{m_3}^2 - z_1^2}\ .$$

A.1.5 Calculation of PDF of bearing rate

From the equation

$$\dot{\beta} = \frac{w_c}{r},$$

where w_c = crossing speed and r = range, we obtain

$$p_\beta(\dot{\beta})\, d\dot{\beta} = p_{w_c}(w_c)\, dw_c$$

or

$$p_\beta(\dot{\beta}) = p_{w_c}(w_c) \times \left|\frac{d\dot{\beta}}{dw_c}\right|^{-1},$$

where

$$\frac{d\dot{\beta}}{dw_c} = \frac{1}{r}\ .$$

The PDF of $\dot{\beta}$ is then

$$p_\beta(\dot{\beta}) = r\, p_{w_c}(w_c).$$

A.1.6 ...lation of PDF of integration time

From the equation

$$\tau = \frac{\theta}{\dot{\beta}}$$

we obtain

$$p_\tau(\tau)\, d\tau = p_{\dot{\beta}}(\dot{\beta})\, d\dot{\beta}$$

or

$$p_\tau(\tau) = p_{\dot{\beta}}(\dot{\beta}) \times \left(\frac{d\tau}{d\dot{\beta}}\right)^{-1}$$

where

$$\frac{d\tau}{d\dot{\beta}} = -\frac{\theta}{(\dot{\beta})^2} = -\frac{\tau^2}{\theta} .$$

The PDF of τ is then

$$p_\tau(\tau) = \frac{\theta}{\tau^2}\, p_{\dot{\beta}}\left(\frac{\theta}{\tau}\right)$$

with the 3-dB beamwidth given by

$$\theta = \lambda/(N-1)\, d,$$

in which λ = wavelength,
N = number of hydrophones,
d = hydrophone spacing.

A.2 FREQUENCY DOMAIN ANALYSIS

In the frequency domain analysis (frequency versus time display), the available observation time is given by

$$\text{Time in a frequency bin} = \frac{\text{bin width}}{\text{frequency rate}} ,$$

i.e. $$\tau_a = \frac{\Delta f_0}{\dot{f}} . \qquad \text{(Eq. A6)}$$

Frequency rate is given by

$$\dot{f} = \frac{\partial}{\partial t}(\Delta f) \;, \qquad \text{(Eq. A7)}$$

where Δf is the *frequency* Doppler shift

$$\Delta f = \frac{w_r}{c} \times f \;, \qquad \text{(Eq. A8)}$$

in which w_r = radial velocity component,
c = speed of sound.

From Eqs. A7 and A8 we have

$$\dot{f} = \frac{1}{c} \times f \times \dot{w}_r \;. \qquad \text{(Eq. A9)}$$

Based on the geometry of Sect. 2.1 of the main text

$$\dot{f} = \frac{f}{c} \left[\frac{w_c^2}{r} + \dot{\theta}_0 \, u \sin\beta - \dot{u} \cos\beta - \dot{\theta}_T \, v \sin(\beta + \theta_T - \theta_0) + \dot{v} \cos(\beta + \theta_T - \theta_0) \right]. \qquad \text{(Eq. A10)}$$

Therefore, from Eqs. A9 and A10 in the steady-state case we have

$$\dot{f} = \frac{f}{c}\left(\frac{w_c^2}{r}\right) = \frac{1}{\lambda} \times \frac{w_c^2}{r} \;, \qquad \text{(Eq. A11)}$$

in which w_c = crossing speed component (m/s),
λ = wavelength (m),
r = range (m).

From Eqs. A6 and A11 we have

$$\tau_a = \lambda \times \Delta f_0 \times r \times \frac{1}{w_c^2} \quad , \qquad \text{(Eq. A12)}$$

where Δf_0 is frequency resolution.

Alternatively,

$$\tau_a = \frac{k_a}{w_c^2} \; , \qquad \text{(Eq. A13)}$$

with $k_a = \lambda \times \Delta f_0 \times r$.

To calculate the PDF of τ_a, let $\tau_a = y$ and $w_c = z$ in Eq. A13. Hence

$$y = \frac{k_a}{z^2} \; .$$

And from

$$p_y(y)\ dy = p_z(z)\ dz \qquad \text{(Eq. A14)}$$

we obtain $p_y(y) = p_z(z) \times \left| \frac{dy}{dz} \right|^{-1}$.

But from Eq. A14, $\frac{dy}{dz} = \frac{-2k_a}{z^3}$,

and thus $p_y(y) = \frac{1}{2k_a} \times z^3 \times p_z(z)$.

Now since

$$z^2 = \frac{k_a}{y} \quad ; \qquad z = \left(\frac{k_a}{y}\right)^{1/2} \quad ; \qquad z^3 = \left(\frac{k_a}{y}\right)^{3/2},$$

we have $p_y(y) = \frac{1}{2k_a} \times \left(\frac{k_a}{y}\right)^{3/2} \times p_z(z)$.

Finally, reverting to our original notation of Eq. A13, we obtain

$$p_{\tau_a}(\tau_a) = \frac{1}{2k_a} \times \left(\frac{k_a}{\tau_a}\right)^{3/2} \times p_{w_c}\left[\left(\frac{k_a}{\tau_a}\right)^{1/2} + p_{w_c} - \left(\frac{k_a}{\tau_a}\right)^{1/2}\right]. \quad \text{(Eq. A15)}$$

And, if p_{w_c} is an even function of w_c,

$$p_{\tau_a}(\tau_a) = \frac{1}{k_a} \times \left(\frac{k_a}{\tau_a}\right)^{3/2} \times p_{w_c}\left(\frac{k_a}{\tau_a}\right)^{1/2} . \qquad \text{(Eq. A16)}$$

Appendix B

MONTECARLO TECHNIQUE

The integration time τ is a function of the following random variables (see Sect. 2 of the main text):

u = speed of observer,
v = speed of target,
α = bow angle of target,
β = bearing relative of target to observer,

and is derived from three equations:

$$w_C = v \sin\alpha - u \sin\beta, \qquad \text{(Eq. B1)}$$

$$\dot{\beta} = \frac{w_C}{r} \qquad (r = \text{range}), \qquad \text{(Eq. B2)}$$

$$\tau = \frac{\theta}{\dot{\beta}} \qquad (\theta = \text{3-dB beamwidth of array}). \qquad \text{(Eq. B3)}$$

Therefore the integration time is a function of four random variables as follows:

$$\tau \equiv F(u, v, \alpha, \beta) \qquad [r \text{ and } \theta \text{ are constants}].$$

To calculate the PDF of τ in terms of the PDFs of u, v, α, β, the Montecarlo method is applied to the case of a random encounter in which α and β are uniformly distributed between $(0, 2\pi)$ and the distributions of u and v are given (see Appendix A for the assumed distribution of u, v).

The Montecarlo calculation is based on the subroutine RANDO 1(X), which generates random numbers in the interval (0, 1), and is available on the Univac Computer at SACLANTCEN.

The operations on which the Montecarlo computer simulation is based are as follows:

1. Extract a random number from
   CALL RANDO 1(X)
   The output is a number between 0 to 1 and indicates the interval of Σ (see Table B1).

2. Again, extract a random number from
   CALL RANDO 1(X)
   The output is again a number between 0 and 1 and will determine the value of the random variable in the selected interval (see Fig. A1 in Appendix A ($0 < x < 1$) and Table B1).

3. Calculate τ corresponding to the random values obtained from Operation 2.

TABLE B1

Cumulative interval probability (Σ) for observer and target

(a) ASW frigate (Observer)

PROBABILITY	Σ	CURVE ($0 < x < 1$)
$p(6 < u < 10) = 0.18$	0.18	$u_1 = 6 + 4x$
$p(10 < u < 18) = 0.72$	0.90	$u_2 = 10 + 8x$
$p(18 < u < 20) = 0.10$	1	$u_3 = 18 + 2x$

(b) Conventional submarine (Target)

PROBABILITY	Σ	CURVE ($0 < x < 1$)
$p(0 < v < 3) = 0.14$	0.14	$v_1 = 3x$
$p(3 < v < 8) = 0.45$	0.59	$v_2 = 3x$
$p(8 < v < 10) = 0.12$	0.71	$v_3 = 8 + 2x$
$p(10 < v < 14) = 0.17$	0.88	$v_4 = 10 + 4x$
$p(14 < v < 20) = 0.12$	1	$v_5 = 14 + 6x$

(c) Target bearing (β) relative to observer

PROBABILITY	Σ	CURVE ($0 < x < 1$)
$p(0 < \beta < 2\pi) = 1$	1	$\beta_1 = 2\pi x$

(d) Target bow angle (α)

PROBABILITY	Σ	CURVE ($0 < x < 1$)
$p(0 < \alpha < 2\pi) = 1$	1	$\alpha_1 = 2\pi x$

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