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**ABSTRACT**

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Annual Report
on
Research Accomplished Under
Grant AFOSR 84-0159
July 1, 1985 - June 30, 1986
F.J. Samaniego, Principal Investigator

During this period, work was completed on six research projects, each reported upon in a separate technical report and submitted for publication. These six papers are abstracted below:


Abstract
Consider an experiment in which only record-breaking values (e.g., values smaller than all previous ones) are observed. The data available may be represented as $X_1, K_1, X_2, K_2, \ldots$, where $X_1, X_2, \ldots$ are successive minima and $K_1, K_2, \ldots$ are the numbers of trials needed to obtain new records. We treat the problem of estimating the mean of an underlying exponential distribution, and we consider both fixed sample size problems and inverse sampling schemes. Under inverse sampling, we demonstrate certain global optimality properties of an estimator based on the 'total time on test' statistics. Under random sampling, it is shown that an analogous estimator is consistent; however, an improved estimator is derived with smaller mean squared error for any fixed sample size. Applications to reliability and stress testing are indicated.

Abstract

Nomination sampling is a sampling process in which every observation is the maximum of a random sample from some population. Assuming that all samples are taken from a single underlying distribution $F$, data may be viewed as consisting of pairs $(X_i, K_i)$, where $K_i$ is the size of the $i$th sample and, given $K_i = k_i$, $X_i$ is distributed according to $F$. Willemain (1980) discusses nomination sampling in the context of health care delivery. It also arises naturally in certain reliability experiments, and in situations where the available physical measurements (e.g., peak flows, maximum floods) are extreme values. The problem of estimating $F$ is considered. The nonparametric maximum likelihood estimator of $F$ is derived, its consistency is demonstrated, and its asymptotic behavior as a stochastic process is identified. Conditions are given under which these asymptotic results hold with $K$ nonrandom.


Abstract

Redundancy is a well understood and widely used design factor which can contribute significantly to improve the reliability of a system or network. Reliability is improved, for example, when any fixed component is replaced by
a parallel system of \( K \) identically distributed components. In this study, we discuss and treat problems in which the level of redundancy \( K \) is a random variable governed by a (known or unknown) discrete probability model. Given repeated observations on \( K \) and on the lifetime \( X \) of a parallel system with \( K \) independent, identically distributed components, an estimator is derived for the reliability of the individual components. The consistency of the estimator is established, and its asymptotic distribution theory is discussed.


**Abstract**

This paper investigates some families of joint lifetime distributions for components of a system; these are motivated in part by physical models for failure and in part by considerations of mathematical convenience. A large family motivated solely by the latter is characterized by the property of lack of memory (L.U.M.) or age-independent residual lifetimes. Its mathematical convenience stems from the fact that it is the multivariate analogue of the one-dimensional exponential distribution. Unfortunately, it is quite unrealistic for most applications, because it implies (and is implied by) the stochastic independence of the first-failure time and vector of residual lifetimes of the surviving components. The distributions studied here comprise extensions of the L.O.M. class, obtained by considering variations which might be more realistic for lifetimes. Three variations are studied: i) replacing the canonical form of L.O.M. distributions by a form which is less restrictive; ii) extending the functional equation which characterizes the L.O.M. class by
one which incorporates an aging factor; and iii) incorporating an aging mechanism by means of shock models.

V. L.R. Whitaker and F.J. Samaniego (1985), Estimating the reliability of Systems Subject to Imperfect Repair; submitted for publication.

Abstract

This study of statistical inference for repairable systems focuses on the development of estimation procedures for the life distribution \( F \) of a new system based on data on system lifetimes between consecutive repairs. The Brown-Proshan 'Imperfect Repair' model postulates that, at failure, the system is repaired to a condition as good as new with probability \( p \) and is otherwise repaired to the condition just prior to failure. In treating issues of statistical inference for this model, we begin by pointing out the lack of identifiability of the pair \( (p,F) \) as an index of the distribution of interfailure times \( T_1, T_2, \ldots \). We show that data pairs \( (T_i, Z_i) \), \( i = 1, 2, \ldots \) are necessary to render the parameter pair \( (p,F) \) identifiable, where \( Z_i \) is a Bernoulli variable which records the mode of repair (perfect or imperfect) following the \( i \)th failure. We demonstrate that the nonparametric maximum likelihood estimator of \( F \) exists only in special cases, but that a neighborhood maximum likelihood estimator \( \hat{F} \) (using the language of Kiefer and Wolfowitz, 1956) always exists and may be derived in closed form. We demonstrate the strong uniform consistency of \( \hat{F} \) under mild assumptions, and prove weak convergence of an appropriately scaled version of \( \hat{F} \) to a Gaussian process. These results are shown to apply to various experimental designs (including renewal testing and inverse sampling) and to extend to the age-dependent imperfect repair model of Block, Borges and Savits (1956).

Abstract

A distribution function is said to be "New Better than Used in Expectation" (NBUE) if for all $t>0$, the expected residual life length of a used item of age $t$ is no larger than the expected life length of a new item. We address the problem of constructing a consistent estimator of $F$ which also belongs to the NBUE class. A minimum distance estimator is obtained as a solution to a nonlinear programming problem. We develop a numerical algorithm for finding this solution, and establish the strong uniform consistency of the estimator.

Substantial progress has been made on several other research problems. Worthy of special mention are the following: (1) Nonparametric estimation based on record-breaking observations. In situations where the process of recording successive record-breaking observations can be replicated, the nonparametric maximum likelihood estimator of the underlying population distribution has been derived, and its asymptotic behavior has been identified. In particular, it is shown that this estimator is consistent, and that a properly normalized version of the estimator converges to a Gaussian process. In an appropriate context, the nonparametric approach to this problem is compared with the parametric approach taken in Paper I above. This work will soon appear in a technical report by Samaniego and Whitaker. (2) Estimating survival when new is better than used at age $t_0$. Reneau and Samaniego have derived an estimator of the survival curve when this curve is
known to belong to the NUB(t₀) class. The estimator belongs to the NBU(t₀) class, is a consistent estimator, and converges to the true survival curve at an optimal rate. A technical report on this problem is under preparation.

Work continues in a number of research areas, including nonparametric estimation problems in reliability, and modeling, inference and optimization problems involving repairable systems.
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