DYNAMIC RESPONSE OF FLUID TRANSMISSION LINES

THESIS

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Major, Canadian Armed Forces

AFIT/GAE/AA/85S-1

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
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Wright-Patterson Air Force Base, Ohio
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Approved for public release; distribution unlimited
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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Aeronautical Engineering

Major, Canadian Armed Forces

September 1985

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Preface

The analysis of the dynamic response of pneumatic transmission lines has been the subject of an ongoing series of studies at the Air Force Institute of Technology, School of Engineering. This thesis is the latest in this series. The purpose of the study has been to examine specific areas in an attempt to answer questions raised in the earlier studies. The emphasis has been on the continuation of work conducted by Captain P. G. Wilkins in 1984.

I would like to take this opportunity to thank several individuals without whose assistance this study would not have been possible. My thesis advisor, Dr. M. E. Franke, provided invaluable guidance and advice. Dr. W. Elrod and Captain W. Cox also contributed welcome assistance in specific areas. Captain Phil Wilkins, whose earlier thesis provided a basis and direction for this study, gladly answered my questions and offered insights into the subject. I would also like to take this opportunity to express a special thank you to my wife Suzanne for her encouragement and confidence which has made my studies at AFIT so much easier.

— Stephen J. McKenna
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<td>A</td>
<td>line cross-sectional area</td>
<td>$in^2$</td>
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<tr>
<td>a</td>
<td>adiabatic speed of sound</td>
<td>in/sec</td>
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<tr>
<td>B</td>
<td>phase angle</td>
<td>radians</td>
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<td>C</td>
<td>capacitance/unit length</td>
<td>$in^4/lbf$</td>
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<td>$C_a$</td>
<td>adiabatic capacitance/unit length</td>
<td>$in^4/lbf$</td>
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<td>$C_i$</td>
<td>isothermal capacitance/unit length</td>
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<td>$C_T$</td>
<td>turbulent capacitance/unit length</td>
<td>$in^4/lbf$</td>
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<td>inside line diameter</td>
<td>in</td>
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<td>f</td>
<td>friction factor</td>
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<td>conductance/unit length</td>
<td>$in^4/lbf-sec$</td>
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<td>$(-1)^i$</td>
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<td>k</td>
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<td>$K_{LT}$</td>
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<td>L</td>
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<td>mass flow rate</td>
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<td>$P_{in}$</td>
<td>inlet pressure</td>
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<td>$R_{DC}$</td>
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<td>$R_g$</td>
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<td>ambient temperature</td>
<td>R</td>
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<tr>
<td>$u$</td>
<td>velocity</td>
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<td>$V$</td>
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<td>x</td>
<td>distance in downstream direction</td>
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<td>Y</td>
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<td>in^4/lbf-sec</td>
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<td>impedance/unit length</td>
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<td>rad/in</td>
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<td>lbf-sec/in^2</td>
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<td>kinematic viscosity</td>
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<td>nondimensional frequency</td>
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<td>\Omega_B</td>
<td>nondimensional break frequency</td>
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<td>density</td>
<td>lbf-sec^2/in^4</td>
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<td>\sigma</td>
<td>(Prandtl number)^{\frac{1}{2}}</td>
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<td>\chi_0</td>
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Theoretical predictions of the dynamic response of circular cross-section pneumatic transmission lines were compared with experimental results. Tests were conducted in blocked line situations as well as in both laminar and turbulent mean flow conditions. The lines used in the study were 24 in. in length with inside diameters of 0.041, 0.119, and 0.195 in. Tests were conducted over the frequency range 20-1000 Hz and over a range of mean flow Reynolds numbers from 0-16000.

Theoretical solutions were obtained from computer simulations using a laminar flow model and two turbulent flow models. These models were developed from the Navier-Stokes equation and include components to account for viscosity and radial heat transfer in the fluid.

An examination of the turbulent models was conducted based on a nondimensional frequency and attenuation. This led to modification of one of the turbulent models to include a frequency dependent term in the conductance.

The effect of flow conditions upstream of the test line and the effect of orifice design on the dynamic response were studied.
Experimental results showed good agreement with results of earlier studies. The use of a bypass device upstream of the test line was found to have no effect on line response.

The reconfiguration of the orifice devices to more closely match theoretical models produced no noticeable change in the results.
I. Introduction

Background

The behavior of periodic pressure signals in fluid lines has been the topic of many studies in the past. The results of these studies are important in applications such as pneumatic and liquid control systems in aircraft, jet engine test facilities and measuring systems. Both analytical and experimental studies have been conducted in an effort to develop accurate models for the prediction of the dynamic response of fluid filled lines.

As a signal travels along a fluid line, the magnitude and phase angle of the signal are modified, depending on such parameters as the line length, diameter, signal frequency and fluid characteristics. The ratio of the downstream magnitude to the upstream magnitude is referred to as the gain and the change in the phase angle is referred to as the phase shift. The ability to accurately predict the gain and phase shift is of major importance in many system designs.

Studies have shown that the model developed by Nichols (Ref 1), and modified by Krishnaiyer and Lechner
(Ref 2) accurately predicts the response of signals in lines with blocked and laminar mean flow conditions. Attempts to extend the use of this model to turbulent mean flow conditions have shown good agreement at high frequency but result in lower attenuation than observed experimentally at low frequencies. Studies by Brown, Margolis and Shaw (Ref 3), Moore (Ref 4), Briski (Ref 5) and Wilkins (Ref 6) have concentrated on the development of a model which can accurately predict the response under turbulent flow conditions. Briski (Ref 5) and Wilkins (Ref 6) worked with the model developed by Moore (Ref 4) to produce modified models which provide accurate and relatively simple means of predicting line response under turbulent mean flow conditions.

Objectives

The following objectives were established to study the dynamic response of pneumatic transmission lines.

1. Review current models to ensure proper application of theory to experimental conditions.

2. Verify experimental procedures and extend results obtained in previous studies.

3. Investigate the effect of conditions upstream of the test line on line response.
4. Revise existing computer program to incorporate revised models and improve documentation.

5. Investigate effect of improved orifice termination devices on correlation of theoretical and experimental results.
II. Theory

Electrical Transmission Line Analogy

The theory of electrical transmission lines is well developed and may be found in many references. This theory may be used in the analysis of fluid transmission lines by considering pressure and volumetric flow rate in fluid systems to be analogous to potential and current respectively in electrical systems. With this analogy Ohms law becomes

\[ \Delta P = ZQ \]  

or

\[ Q = Y\Delta P \]

where

\[ \Delta P \] = change in pressure
\[ Q \] = volumetric flow rate
\[ Z \] = complex series impedance
\[ Y \] = complex shunt admittance = \( \frac{1}{Z} \)

The complex impedance is composed of the resistance and inertance, while the admittance is composed of the conductance and capacitance. Using these components, the
above equations may be expressed in terms of the distance along the line as

$$\frac{dp}{dx} = ZQ = (R + j\omega L)Q \quad (3)$$

and

$$\frac{dQ}{dx} = YP = (G + j\omega C)P \quad (4)$$

where

- $R$ = resistance
- $L$ = inertance
- $G$ = conductance
- $C$ = capacitance

all taken per unit length.

Based on these equations, the analogous electrical circuit to a fluid transmission line is shown in Figure 1.

**Blocked and Laminar Flow**

The equations developed by Nichols (Ref 1) provide a means of calculating values of $Z$ and $Y$ for circular lines with either laminar mean flow or blocked line conditions. These equations, however, involve the use of Bessel functions, and the calculations can become complicated. Krishnaiyer and Lechner (Ref 2) have presented algebraically simplified equations which provide accurate results over the frequency range $0.1 \omega_y < \omega < \infty$. The characteristic frequency $\omega_y$, is defined as
Fig. 1. Electrical Analogy to Fluid Transmission Line
\[ \omega_{\nu} = \frac{8\pi \nu}{A} \]  \hspace{1cm} (5)

where A is the line cross-sectional area.

The modified equations for Z and Y are

\[ Z = \frac{8\pi \nu}{A^2} (DR) + j \left[\frac{\omega \phi}{A} + \frac{8\pi \nu}{A^2} (DL)\right] \]  \hspace{1cm} (6)

and

\[ Y = \frac{- [\omega (k-1) A/kP (DG)]}{(DC)^2 + (DG)^2} + j \omega \left[\frac{A}{kP} + \frac{(k-1) A/kP (DC)}{(DC)^2 + (DG)^2}\right] \]  \hspace{1cm} (7)

with the expressions DR, DL, DG and DC described as

\[ DR = \frac{3}{8} + \frac{h_{\nu}}{4} + \frac{3}{8h_{\nu}} \]  \hspace{1cm} (8)

\[ DL = \frac{h_{\nu}}{4} - \frac{15}{64h_D} \]  \hspace{1cm} (9)

\[ DG = \frac{h_t}{2} - \frac{1}{4h_t} \]  \hspace{1cm} (10)

\[ DC = \frac{1}{4} + \frac{h_t}{2} + \frac{1}{4h_t} \]  \hspace{1cm} (11)

where

\[ h_{\nu} = 2 (\omega/\omega_{\nu})^{\frac{1}{4}} \]  \hspace{1cm} (12)

\[ h_t = \sigma h_{\nu} \]  \hspace{1cm} (13)

are velocity and temperature distribution parameters respectively.
The transmission line may now be described by the propagation operator $\Gamma$ and the characteristic impedance $Z_c$.

$$\Gamma = (ZY)^k = \alpha + j\beta \quad (14)$$

$$Z_c = \left(\frac{Z}{k}\right)^k \quad (15)$$

where

$\alpha = \text{attenuation per unit length}$

$\beta = \text{phase shift per unit length}$

For the general case of a line with terminal impedance $Z_L$, as shown in Figure 2, Kirshner and Katz (Ref 7) express the input impedance as

$$Z_{in} = \frac{P_{in}}{Q_{in}} = Z_c \left[\frac{Z_L \cosh(\Gamma x) + Z_C \sinh(\Gamma x)}{Z_L \sinh(\Gamma x) + Z_C \cosh(\Gamma x)}\right] \quad (16)$$

The pressure transfer function relating the pressure at a distance $x$ along the line to the input pressure is

$$\frac{P}{P_{in}} = \frac{\sinh(\Gamma x)(1-x/Z_l) + (Z_L/Z_C)\cosh(\Gamma x)(1-x/Z_l)}{\sinh(\Gamma x) + (Z_L/Z_C)\cosh(\Gamma x)} \quad (17)$$

and the pressure gain is defined as the magnitude of the transfer function

$$g = \left|\frac{P}{P_{in}}\right| \quad (18)$$
Fig. 2. Generalized Transmission Line with Terminal Impedance
or, in decibels,

\[ g_{dB} = 20 \log g \]  (19)

The phase shift is defined as the angle formed by the real part and the complex part of the transfer function in the complex plane

\[ B = \tan^{-1} \left( \frac{\text{Im}(P/P_{in})}{\text{Re}(P/P_{in})} \right) \]  (20)

**Turbulent Flow**

While Equations (6) and (7) have proven accurate for use in blocked line and laminar flow cases, it has been shown that their use in the turbulent flow case results in attenuation values lower than those observed experimentally (Refs 3-5).

The velocity profile for fully developed laminar steady state flow is parabolic with the boundary layer thickness equal to the tube radius. When a periodic signal is applied to the steady state flow a dynamic boundary layer is created. As the frequency of the signal is increased this dynamic boundary layer becomes thinner and the velocity gradients near the wall increase. As a result, the dynamic layer is thinner than the steady state boundary layer at frequencies of practical interest and dictates the line response (Ref 13).
In turbulent flow the steady state velocity profile exhibits a dependency on the Reynolds number. The boundary layer thickness decreases with increasing Reynolds number and at the limit of an infinite Reynolds number a uniform profile, or slug flow, is obtained. With this variation in steady state boundary layer thickness, the dynamic boundary layer created by the application of a periodic signal will no longer necessarily be thinner than the steady state boundary layer. With the response determined by the thinner of the two layers, it is clear that variations in Reynolds number and signal frequency would affect line response.

In order to classify the response based on Reynolds number and frequency, Brown et al. (Ref 3) presented a plot of nondimensional attenuation versus nondimensional frequency. An example of this type of plot is shown in Figure 3. The two nondimensional parameters are defined as

\[
\text{nondimensional attenuation} = \frac{a_\alpha}{\nu} \quad (21)
\]

\[
\text{nondimensional frequency} \ \Omega = \frac{R \omega}{\nu} = 8 \frac{\omega}{\omega_v} \quad (22)
\]

This plot has been divided into three frequency regions by Moore (Ref 4) as suggested by Brown et al. (Ref 3). In the low frequency region, the dynamic boundary layer is thicker than the steady state boundary layer. In the mid frequency region, the dynamic layer and the steady
Fig. 3. Nondimensional Plot of Attenuation vs Frequency
layer are of comparable thickness. In the high frequency region the dynamic layer has become much thinner than the steady state layer and as a result the behavior in this region is the same as in laminar flow. The frequency at which the steady state layer becomes the thicker of the two and the response transitions to laminar behavior is termed the break frequency, \( \Omega_B \).

Several turbulent models have been suggested to predict dynamic response. Moore applied a constant LRC model to the low and mid frequency regions, the components of which were selected based on analysis of the low frequency laminar flow case (Ref 4). The adiabatic inertance was defined as

\[ L_a = \rho/A \]  
(23)

A steady state (DC) resistance was defined as

\[ R_{DC} = \frac{f \text{Reu}}{2Ad^2} \]  
(24)

and using the Blasius resistance formula for turbulent flow in smooth lines

\[ f = 0.3164 \text{Re}^{-0.25} \]  
(25)

This was rewritten as

\[ R_{DC} = 0.1582 \frac{\text{Re}^{0.75}u}{Ad^2} \]  
(26)
The capacitance was defined as

\[ C = \frac{k}{n} C_a \quad (27) \]

where \( n \) is the polytropic exponent and the adiabatic capacitance. \( C_a \) is defined as

\[ C_a = A/kP \quad (28) \]

The value of \( n \) is seen to vary from \( k \) at high frequencies to 1.0 at low frequencies.

Wilkins presented modifications to the constant LRC model in an attempt to improve the attenuation predictions of his model. The attenuation curve in the mid frequency region of Figure 3 is seen to be dependent only on the Reynolds number. The modifications to the resistance, inertance and capacitance proposed by Wilkins were based on this variation in attenuation with Reynolds number.

A linearized (AC) resistance suggested in Ref 8

\[ R_{AC} = R_{DC} + Re \left[ \frac{3R_{DC}}{3Re} \right] \quad (29) \]

was used which using the \( R_{DC} \) in Equation (26) reduces to

\[ R_{AC} = 1.75 R_{DC} \quad (30) \]

Moore and Franke have proposed a turbulent inertance to reflect the effect of the velocity profile on
the inertance (Ref 9). This turbulent inertance is defined as

\[ L_T = K_{LT} L_a \]  \hspace{1cm} (31)

where \( K_{LT} \) is the inertance constant defined as

\[ K_{LT} = \frac{(f^{0.5} + 1)(f^{0.5} + 2)^2}{4(2f^{0.5} + 1)} \] \hspace{1cm} (32)

\( L_T \) is now seen to be a function of the Reynolds number through the friction factor \( f \).

The turbulent capacitance was formed by combining the imaginary part of the conductance with the isothermal capacitance. The conductance accounts for the effects of radial heat transfer in a gas. In laminar flow the increased attenuation due to this effect is well understood and has been included in the equations by Krishnaiyer and Lechner (Equations (6) and (7)).

The conductance for turbulent flow as presented by Brown et al. (Ref 3) as

\[ G = \frac{(k-1)C_a L_a \omega^2}{R_{AC} + j\omega L_T} \] \hspace{1cm} (33)

If the quantity \( R_{AC}/\omega \) is assumed to be much less than \( L_T \), Wilkins (Ref 6) showed that after some algebraic manipulation, this reduces to
\[
G = (k-1) \frac{R_{AC} a L_a}{L_T^2} - j\omega (k-1) \frac{C_a K_L T L_a^2}{L_T^2}
\]  

(34)

From this expression, the turbulent capacitance is written as

\[
C_T = C_i - (k-1) \frac{C_a K_L T L_a^2}{L_T^2}
\]

\[
= kC_a - (k-1) \frac{C_a K_L T L_a^2}{L_T^2}
\]

\[
= C_a \left[ k - \frac{k-1}{K_{LT}} \right]
\]  

(35)

which is again seen to be dependent on the Reynolds number through the inertance constant.

The second turbulent model presented by Wilkins (Ref 6) was the constant LRCG model. This model uses the same resistance, inertance and capacitance components as the modified constant LRC but also includes the real part of the conductance from Equation (34). Each component of these models is seen to be independent of the signal frequency and a function of only the Reynolds number.

If the quantity $\frac{R_{AC}}{\omega}$ cannot be assumed to be much less than LT, the complete equation for G must be utilized (Equation (33)). Expansion of this expression gives
It should be noted that use of the full expression results in a component that is dependent on frequency as well as the Reynolds number.

If, as before, a portion of the imaginary part of $G$ is combined with $C_i$ to form the turbulent capacitance, the following result is obtained.

$$C_T = C_i - \frac{\omega^2 (k-1) L_T L_a C_a}{R_{AC}^2 + \omega^2 L_T^2}$$

$$= C_a \left[ k - \frac{\omega^2 (k-1) L_T L_a}{R_{AC}^2 + \omega^2 L_T^2} \right]$$

The turbulent capacitance thus formed is also seen to be frequency dependent in contrast to the component used by Wilkins in the Constant LRCG model.

A new model can be formed by using the frequency dependent expression for $G$ and $C_T$ in place of the respective frequency independent terms in the Constant LRCG model. The resulting expressions will be referred to as the Modified LRCG model.

It must be noted that although the expression for the conductance used in both LRCG models is based upon that presented by Brown et al. (Ref 3), there is an
important difference between the components as defined by Brown and those used in this study. As an example, Brown defines the effective inertia, I (equivalent to the adiabatic inductance) as

$$ I = \rho $$

(38)
as compared to the definition of $L_a$ in Equation (23). The two components are seen to differ by the area factor such that

$$ I = L_a A $$

(39)
A similar multiplicative factor must be included in the resistance components used in this study to obtain Brown's results.

Similarly, Brown's admittance components are related to those used in this study by

$$ Y_{\text{Brown}} = \frac{Y}{A} $$

(40)

Though the admittance and impedance terms each differ by a factor of the area, these factors cancel out when the two components are combined to form the propagation operator $\Gamma$ producing the same result regardless of the method that is used. Care must be taken, however, to ensure that the two types of components are not combined without regard to the multiplicative factor by which they differ.
**Terminal Impedance**

Several methods have been proposed to accurately model the terminal impedance of a dynamic system. Using a linearized DC expression such as that proposed by Franke, Malanowski and Martin (Ref 10)

\[ Z_L = \frac{\Delta P_L}{\Delta Q_L} \]  \hspace{1cm} (41)

shows that in the limiting cases, for an open line, \( \Delta P_L = 0 \) and \( Z_L = 0 \); while for the blocked line case, \( \Delta Q_L = 0 \) and \( Z_L = \infty \).

In this study two primary methods have been used to model terminal impedance. In the open line case, the radiation impedance as suggested by acoustic theory has been employed.

The air at the end of the transmission line is described as a massless piston which transmits some of the energy into free space while reflecting some energy back into the line. This finite impedance is seen to differ from the zero impedance resulting from the linearized impedance model mentioned above. Morse and Ingard (Ref 11) define the radiation impedance as

\[ Z_R = \left( \frac{\varrho a}{A} \right) \left( \theta_0 - j\lambda_0 \right) \]  \hspace{1cm} (42)
where $\theta_o$ and $\chi_o$ are radiation functions. For the case of a flanged exit (a flush opening in an infinite wall) these functions may be approximated as

$$\theta_o = 0$$  \hspace{1cm} (43)

$$\chi_o = 0.4 \, \omega / a$$  \hspace{1cm} (44)

Using these approximate values results in an expression for the terminal impedance of

$$Z_R = -0.4 j \omega D / a$$  \hspace{1cm} (45)

The second method used to model the terminal impedance involves the use of reflection coefficients. The orifice of the transmission line was modeled as illustrated at Figure 4. Kensler and Frey (Ref 12) give the terminal impedance for this situation as

$$Z_L = (\frac{\Delta a}{A}) \left( \frac{1 + W}{1 - W} \right)$$  \hspace{1cm} (46)

where $W$ is the reflection coefficient for the sudden change in line area between 1 and 2. The reflection coefficient for the situation where line 2 is terminated by its characteristic impedance is given as

$$W = (A_1 - A_2) / (A_1 + A_2)$$  \hspace{1cm} (47)
Compressible flow theory states that waves which reflect from a boundary and travel at the speed of sound will not be able to propagate upstream against a sonic flow. The reflected waves will be swept downstream by the flow and the presence of the orifice will not be sensed at the input. The line will appear to be of infinite length and Equation (47) may be applied.

In those situations where the flow was found to be choked by the orifice, the above analysis was assumed to be applicable and Equation (47) was used to determine the reflection coefficient.

In those situations where choked flow conditions are not present, the orifice was represented as an infinitesimally short open line and the remainder of area 1 (Figure 4) was treated as a blocked line. The reflection coefficients for the two areas were taken respectively to be $-1$ and $1$. These two values were averaged by area to obtain a composite reflection coefficient

$$ W = \frac{(A_1 - 2A_2)}{(A_1 + A_2)} $$

(48)

**Effect of Upstream Conditions**

A review of the various equations describing the dynamic response of a pneumatic transmission line reveals dependence only on the conditions at the line inlet.
Schlichting (Ref 13) gives the pressure drop along a line as

$$\Delta P = - f(\Delta x) \rho \left( \frac{u^2}{2D} \right)$$  \hspace{1cm} (49)

and by substituting the applicable expression for the friction factor, the following results may be obtained.

**Laminar Flow**

$$\Delta P = -0.106 (\Delta x) \frac{M}{D^4 \rho}$$  \hspace{1cm} (50)

**Turbulent Flow**

$$\Delta P = -(7.18 \times 10^{-6}) (\Delta x) \frac{M^{0.25} \mu^{1.75}}{D^{4.75} \rho}$$  \hspace{1cm} (51)

From these expressions it can be seen that, for a given fluid at ambient conditions and a given line configuration, the mass flow rate is dependent only on the pressure drop along the line. In test situations where an upstream bypass device is used to simulate a branch line in a more complex system, the pressure drop along the test line may be used to establish a required mass flow rate.

**Error Analysis**

In order to properly assess the correlation of theoretical predictions with experimental results, a determination of the error due to the readability of measurement instruments is necessary. In this study the method of absolute error was used to calculate this error (Ref 15).

For some quantity $F$, defined by

$$F = F(e_1, e_2, \ldots, e_n)$$  \hspace{1cm} (52)
where $e_1, e_2, \ldots, e_n$ are independent measured quantities, the absolute error in $F$ due to instrument readability is

$$\Delta F = \sum_{i=1}^{n} \Delta e_i \left| \frac{\partial F}{\partial e_i} \right|$$  \hspace{1cm} (53)

where $\Delta e_i$ is the readability of the instrument used to measure $e_i$.

The absolute error may be used to determine the accuracy of experimental results. If the error analysis shows an error of 5 percent in a calculation, for example, the effort spent on resolving a difference of 2 percent between experimental and theoretical results might be better spent improving instrumentation or experimental techniques.

**Computer Simulation**

The computer program developed by Malanowski (Ref 14) and modified by Briski (Ref 5) and Wilkins (Ref 6) was used to provide theoretical simulations for comparison with experimental results.

The test line is divided into several short segments over which the properties are assumed to remain constant. Using Schlichting's equation for the pressure drop along a line (Equation (49)) and the respective expressions for the friction factor for laminar and turbulent flow, Wilkins developed expressions to determine the
pressure drop along a line segment of known length given the mass flow rate and the inlet pressure. The final expressions are, for laminar and turbulent flow respectively.

\[
\Delta P = P_{in} - \left[ P_{in}^2 + 64\mu \frac{Re_T T_T}{D^3} \right]^{\frac{1}{2}}
\]  

(54)

\[
\Delta P = P_{in} - \left[ P_{in}^2 + 0.3164\mu \frac{Re^{1.75} T_T T_T}{D^3} \right]^{\frac{1}{2}}
\]  

(55)

Using the appropriate expression the program calculates the pressure drop for each segment in succession and thereby the inlet pressure for each of the line segments. The program then uses the segment pressures and other properties to calculate the propagation operator and from this, the theoretical gain and phase shift.

The program is interactive and offers the choice between the laminar model (Equations (6) and (7)) and either the Modified Constant LRC or Modified LRCG models. For the turbulent models, the attenuation is calculated by both the selected turbulent model and the laminar model. The program compares the two values and selects the larger for the calculation of gain and phase shift. In this manner the program is able to identify the break frequency between the mid frequency and high frequency ranges and then use the laminar model for calculations beyond the break frequency.
The program also contains an interactive module to allow the selection of the appropriate reflection coefficient expression. The radiation impedance is used for open line cases.

The expressions for the ratio of turbulent/adiabatic capacitances and the inertance constant were added to the program to reduce the amount of input data required.

The documentation and layout of the program were improved to make it more easily understood and to provide a more useful analytical tool for future studies in this area. A complete listing of the program is included at Appendix B.
III. Experiment

Apparatus

The test sections used in this study consisted of 24-inch lengths of stainless steel line which were permanently mounted in plexiglass blocks at each end. Three inside line diameters were used: 0.041, 0.119 and 0.195 in. The mounting blocks were machined to provide fittings for air input line, static pressure transducer, and dynamic pressure transducer. The bypass device and several features of the upstream mounting block as listed below are shown in Figure 5.

A - Bypass device
B - Static pressure transducer
C - Dynamic pressure transducer
D - Test line

A typical test line is shown in Figure 6 and the actual line dimensions are listed in Table I. The downstream mounting block was also machined to allow the fitting of different termination devices. There were four termination devices for each test line; a blocked line fitting, an open line fitting and two orifice fittings with diameters of 0.0135 in. and 0.016 in. (Figure 7). Dimensions of the termination devices are also shown in Table I.

A plexiglass calibration block shown in Figure 8 was used to calibrate the dynamic pressure transducer/
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Note: All dimensions in inches.
charge amplifier combinations. The calibration block positioned the transducers at the same point in the line to allow comparison of the measured signals. The block was machined similar to the test line mounting blocks for an air supply line connection.

The experimental instrumentation used in the study is shown in Figures 9 and 10. The equipment is also illustrated in schematic form in Figure 11.

The sending wave analyzer provided a sinusoidal input signal of variable frequency, which was measured by a frequency counter, to the amplifier/power supply and the pneumatic driver. This input signal was suitably conditioned and passed to the pneumatic driver itself which imposed a sinusoidal pressure signal on the mean air flow passing through the driver. The pressure signal was measured at upstream and downstream locations by piezoelectric dynamic pressure transducers. The charge output of each dynamic transducer was fed to a charge amplifier where it was converted to a voltage and input to a wave analyzer where it was displayed as a RMS voltage. The charge amplifier output was also input to a digital recording oscilloscope where the waveforms could be compared and recorded if desired. A static pressure transducer mounted in the upstream mounting block provided a measure of the static line pressure at that point. The pneumatic driver, test line and charge amplifiers are shown in Figure 12.
Fig. 10. Nicolet Digital Oscilloscope
Fig. 12. Pneumatic Driver (A), Test Line (B) and Charge Amplifiers (C)
For those runs involving bypass flow upstream of the test line, a complete bypass device (Figure 5) was used. This device consisted of a tee fitting located between the pneumatic driver and the upstream mounting block. The bypass outlet of the tee was fitted with an inside threaded cap with a floating disc. Adjustment of the threaded cap allowed varying amounts of the flow to escape providing increased flow through the pneumatic driver for a given flow through the test line. This device was also used for the blocked line cases where the entire flow was diverted from the test line.

A calibrated rotometer was used to measure the mass flow rate through the test line. As the presence of the rotometer downstream of the test line altered the line's response, flow measurements were made and the rotometer removed before data was taken. When the bypass device was being used, the rotometer again could not be used as its presence changed the respective flows through the test line and the bypass device. To determine the mass flow through the test line with the bypass device open, it was assumed that the difference between a given upstream static pressure and atmospheric pressure resulted in a given flow through the line.

With the bypass device closed, the air supply was adjusted to give the desired static pressure reading. The rotometer was then used to measure the flow and
removed. The bypass device was then opened and the air supply increased until the previous static pressure reading was re-established.

A complete listing of the instrumentation and ancillary equipment used in this study is provided in Appendix C.

Procedures

Dynamic Transducer Calibration. Throughout the study each dynamic pressure transducer was paired with a specific charge amplifier and the combination treated as a single unit. To calibrate the transducer/amplifier units the transducers were mounted in the calibration block which was then connected to the air supply line in place of the test line. The remainder of the experimental equipment was set up as for a normal test run. At a selected frequency, the signals from the transducers were displayed on the oscilloscope and positioned such that the signals overlayed each other. With the magnitude switches for each channel set to the same setting, the sensitivity dial on the downstream charge amplifier was adjusted until the signal traces on the oscilloscope exactly matched. The amplitude of the signals was verified by setting the downstream wave analyzer frequency to obtain maximum voltage.
It was found that using the procedure of matching the signals visually on the oscilloscope produced voltages on the wave analyzers which were equal to within the accuracy of the analyzer meters.

This sensitivity setting was then verified over a range of frequencies and air supply levels. In all cases the signals displayed negligible differences over the range of frequency and air supply levels used in this study. After calibration the transducers were transferred to the applicable test line and the line connected in place of the calibration block.

This calibration procedure was repeated each time the transducers were placed in a new test line. Initial tests using the calibration block showed that the same results were obtained regardless of the position in which the transducers were placed. Calibration tests were repeated with the transducer positions reversed without measurable differences.

**Experimental Procedure.** The experimental data collected during this study consisted of the sending (upstream) pressure, the receiving (downstream) pressure and the phase shift between the two pressure signals. The data was measured as a function of the frequency over the frequency range 20 to 1000 Hz.
Given a selected Reynolds number, the required static pressure setting was determined from Equations (50) or (51). Following the warmup period of the pneumatic driver amplifier/power supply and the zeroing of the wave analyzers, the air pressure was adjusted to obtain the correct reading on the voltmeter connected to the static pressure transducer. The voltage reading was converted to psig using the calibration equation determined for the transducer. The ambient-atmospheric pressure and temperature were recorded with the atmospheric pressure being corrected for temperature and gravity effects.

The frequency was selected on the sending wave analyzer using the frequency counter as the measuring device. The frequency on the receiving wave analyzer was adjusted until a maximum voltage was obtained. This procedure was used rather than simply matching the frequencies using the frequency counters because it was discovered early in the study that there were sometimes differences of up to 7 Hz between the sending frequency and the receiving frequency at which the maximum voltage was measured. In many instances if the frequencies were simply matched, the receiving pressure signal would decrease from a maximum of anywhere from 50 to 300 mV to almost zero mV. This problem is attributed to the wave analyzers used in this study. The problem should be eliminated by the scheduled acquisition of new wave analyzers.
When the maximum voltage was obtained, both charge amplifiers were grounded using the ground button on the front panel to ensure accurate readings and the two pressure signal voltages recorded.

The storage feature of the Nicolet oscilloscope was utilized to record the traces of the pressure signals. The time difference between identifiable common points on the two signals was determined using the movable cursor and the digital display of the oscilloscope. Any traces which showed distinctive or unusual features were stored on the oscilloscope's magnetic disc for later review and analysis. This procedure was repeated over the usable frequency range of the pneumatic driver of 20 to 1000 Hz at varying intervals.

The flow rate through the test line was measured using the rotometer before and after the test run. The rotometer measurement was converted to a mass flow rate using calibration curves previously prepared for the rotometer. The calibration of the most commonly used rotometer tube was verified during this study.

The air supply was monitored using the static pressure transducer output. Fluctuations in the pressure signal voltage readings could be related to sharp fluctuations in the supply line pressure. Data measurements were not taken during periods of line pressure fluctuation.
The precaution of grounding the charge amplifiers before taking the readings was implemented after it was noted that the output of the amplifiers would decay significantly over a period of time. Although the length of time required for this effect to occur was longer than that usually taken to obtain the readings, this precaution was taken to ensure accurate readings in the event that there was some delay.

In the cases where the bypass device was used, the required static pressure reading was obtained with the bypass device closed by adjusting the air supply and the flow rate was measured. The bypass device was then opened the optimum amount to provide maximum pressure signal voltages and clear oscilloscope traces and the air supply increased to re-establish the static pressure reading.

Test Designation

A five-digit numeric code was assigned to each test run. The code was the same as that used by Wilkins (Ref 6) to allow direct comparison of experimental results and to maintain consistency within the data base.

The first digit identifies the line used; 1 for the 0.041, 2 for the 0.119 and 3 for the 0.195 in. diameter line. The second digit identifies the line termination condition; 0 for blocked, 1 and 2 respectively for the
0.0135 and 0.016 in. diameter orifices, and 3 for open.

The last three digits indicate the nominal Reynolds number in hundreds.

An alpha character suffix was used in some instances to identify significant test characteristics. The characters and their meaning are listed below.

- b - bypass device open
- n - modified orifice device
- r - transducer positions reversed
IV. Results and Discussions

Test Conditions

A summary of line configurations and ambient conditions for which experimental tests were conducted is presented in Table II. The shaded area of Figure 13 shows the portion of the nondimensional frequency vs. attenuation region covered by these results ($3.52 < \Omega < 9201.8$).

Analysis of Instrument Error

Table III presents the absolute error in the experimental gain for typical pressure measurements. This error is based solely on the readability of the wave analyzers and was determined by analysis of Equation (19) using the method of Equation (53).

A similar analysis was conducted for the phase angle calculation based on the readability of the Nicolet oscilloscope and the frequency counters. The experimental phase shift is calculated from the equation

$$B = -0.36 \text{freq} \times \text{phd} \quad (56)$$

where

- freq = signal frequency in Hz
- phd = phase delay in msec.

The resultant absolute error for typical phase delay measurements is presented in Table IV.
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<th>$P_T$</th>
<th>$T_a$</th>
<th>$M$</th>
<th>Re</th>
<th>$\Omega_{\text{min}}$</th>
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### TABLE III

**ABSOLUTE ERROR IN EXPERIMENTAL GAIN DUE TO INSTRUMENT READABILITY**

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<tr>
<th>$P_S$ (mV)</th>
<th>$P_R$ (mV)</th>
<th>Gain (dB)</th>
<th>Error (dB)</th>
<th>%</th>
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<td>69</td>
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<td>230</td>
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### TABLE IV

**ABSOLUTE ERROR IN EXPERIMENTAL PHASE SHIFT DUE TO INSTRUMENT READABILITY**

<table>
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<tr>
<th>Freq (Hz)</th>
<th>(msec)</th>
<th>Phase Shift (deg)</th>
<th>Error (deg)</th>
<th>(%)</th>
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</thead>
<tbody>
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<td>60</td>
<td>1.2</td>
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</table>
Though the Nicolet oscilloscope features sample rates up to $50 \times 10^{-9}$ secs., the sample rates used in this study were determined largely by the signal frequency. It was necessary to select a sample rate which provided a display of several cycles to allow a significant number of phase delay measurements to be made at each frequency.

The major source of error in the phase shift calculation is the determination of the actual peak location on the upstream signal in certain situations. In these instances the signal either degenerated into a multipeak signal as illustrated in Figure 14 or displayed random noise which obscured the location of the peak. The range of possible peak locations in these situations was several sample steps wide and presents significant error in addition to that resulting from instrument readability. Almost all cases where the experimental phase shift data differs significantly from theoretical predictions can be traced to situations where this problem was evident.

General Gain and Phase Shift Results

All cases presented in Table II were evaluated using the computer simulation to compare experimental results with theoretical predictions. Figures 15-32 present the correlation between experimental and theoretical results for some representative cases from this study. The results of the remaining cases will be used to
Fig. 14. Atypical Sending Pressure Signal (Upper Signal)
illustrate points discussed in the following sections. In most cases the experimental results show good agreement with the results of Briski (Ref 5) and Wilkins (Ref 6). Blocked line and laminar mean flow experimental results show close correlation with laminar theory predictions (Figures 15-18 and 27-30), while in turbulent mean flow cases the laminar theory predicts significantly lower attenuation than observed experimentally (Figures 19-26 and 31-32). The Modified Constant LRC model (MCLRC) developed by Wilkins (Ref 6) predicts increased attenuation in turbulent mean flow cases but the attenuation is still below experimental results. The use of the Constant LRCG model (Ref 6) results in a further increase in attenuation over that predicted by the MCLRC model, but in most cases the theoretical attenuation still results in peak gain magnitudes less than those observed experimentally. The phase shift results are generally in close agreement between experimental and theoretical models with some deviation at high Reynolds numbers. The deviations are consistent with results obtained by Wilkins (Ref 6) but are of unknown cause at this time.

Analysis of Radiation Impedance

For the open line cases in this study the radiation impedance was used in the computer simulation to determine the terminal impedance. A comparison of the
radiation impedance and the assumption that \( Z_L = 0 \) for an open line (Ref 10) was made and the effect of each method on the line response was studied. For the line configurations and conditions of this study the value of the radiation impedance was between \(-1 \times 10^{-6}\) and \(-3 \times 10^{-3}\) with the largest value occurring at high frequency in the 0.119 in. diameter line. Figures 33-39 show the relative effects of the two impedances on the results of the MCLRC model and the Constant LRCG model. In all cases setting \( Z_L = 0 \) caused a slight increase in the gain over the entire frequency range. The increase was so small, however, that it is considered insignificant in relation to the differences between experimental and theoretical results. The effect of the radiation impedance is seen to be negligible from these results.

**Effect of Upstream Conditions**

Several test runs were conducted to confirm that variations in the conditions upstream of the test section would not affect line response. The variation in upstream conditions was obtained by diverting a significant amount of the mass flow out of the line upstream of the test section. The input flow rate was increased to maintain the same flow rate through the test section in both situations. Operation of the system in the bypass configuration would simulate a branch line upstream of the test section.
Figures 40-48 show the comparison of experimental gain between the bypass and no-bypass conditions.

The experimental results match very closely in most cases confirming that flow variations upstream of the test section have no effect on line response.

The results of Case 21004 (Figure 45), however, show an unexpected positive offset of the gain over the entire frequency range in the bypass condition. This offset resulted from a switch of the transducer locations and indicated that there was a difference in sensitivity between the two transducer/charge amplifier pairs despite the same settings being used on both amplifiers. This difference was confirmed by the results of two other sets of runs where the transducer positions were reversed. These results led to the implementation of the calibration procedure discussed in Chapter III. Results obtained after calibration of the transducers show no such offset. The theoretical gain has not been plotted for those cases conducted before transducer calibration as the difference in sensitivity caused an offset of both the bypass and no-bypass results from the actual values.

Effect of Modified Orifice Devices

Due to the machining methods used in their production, the original orifice devices did not match the characteristics of the orifice model shown in Figure 4. The
orifice devices contained a significant length of area reduction before the actual orifice as shown by the dotted lines of Figure 7. The devices were modified to more closely match the model by machining out the inlet passage to a constant diameter. The dimensions noted on Figure 7 and listed in Table I apply to the modified orifice devices.

The gain and phase shift results in Figures 49-60 show close correlation with results of similar cases presented by Wilkins with the exception of Case 12034 (Figures 66-67). The experimental gain data from this case shows significant deviation from Wilkins' results in similar cases (12020 and 12050). The results show higher peak gains over the frequency range 20-500 Hz. Particularly noticeable is the peak gain of 6.89 dB at 350 Hz compared with values of approximately zero in Wilkins' results. The gain results over the remainder of the frequency range and the phase shift results show good agreement with Wilkins' results. The deviation between the experimental results and the theoretical predictions is generally the same as displayed in Wilkins' results. The reason for this anomaly is not known but the almost exact agreement between Cases 12034 and 12034b reduces the likelihood of experimental error. It should be noted that there is a significant difference in the Reynolds numbers between this case and comparable cases of Wilkins (3425 vs. 2044 and 4992).
Modified LRCG Model

Analysis of theoretical results obtained using the Constant LRCG model developed by Wilkins (Ref 6) revealed that this model did not display the characteristic decrease in nondimensional attenuation at low nondimensional frequency as described by Brown et al. (Ref 5). (All further references to attenuation and frequency will refer to the nondimensional parameters unless otherwise specified.) A similar analysis of the MCLRC model showed that the MCLRC model did exhibit the expected decrease in attenuation. This led to a review of the development of the Constant LRCG model to determine the cause of the deviation.

The assumption that $\frac{RAC}{\omega}$ is much less than $L_T$ was used by Wilkins to simplify the model and remove any dependency on the frequency. In his results Wilkins (Ref 6, p. 41) discusses the limitations imposed on the model by the assumption. A review of Wilkins' cases reveals that for a wide range of line configurations and Reynolds numbers this assumption is not valid over a significant portion of the frequency range under consideration. Table V presents a review of cases from both this study and Wilkins and illustrates the frequency limitation of this assumption for various line configurations and flow conditions.
TABLE V

COMPARISON OF $R_{AC}$ VS. $L_T$

<table>
<thead>
<tr>
<th>Case</th>
<th>Re</th>
<th>$\frac{lb \cdot s}{(\frac{f}{6}) \times 10^4}$</th>
<th>$\frac{lb \cdot s^2}{(\frac{f}{6}) \times 10^4}$</th>
<th>$\frac{rad}{s}$</th>
<th>(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11030*</td>
<td>2968</td>
<td>1434.8</td>
<td>2.1388</td>
<td>6283</td>
<td>1000</td>
</tr>
<tr>
<td>12018n</td>
<td>1780</td>
<td>906.96</td>
<td>1.8190</td>
<td>4986</td>
<td>794</td>
</tr>
<tr>
<td>23070*</td>
<td>7326</td>
<td>40.542</td>
<td>.10723</td>
<td>3781</td>
<td>602</td>
</tr>
<tr>
<td>23090</td>
<td>9094</td>
<td>43.857</td>
<td>.10892</td>
<td>4027</td>
<td>641</td>
</tr>
<tr>
<td>23100*</td>
<td>10450</td>
<td>48.683</td>
<td>.11197</td>
<td>4348</td>
<td>692</td>
</tr>
<tr>
<td>23105</td>
<td>10490</td>
<td>48.812</td>
<td>.11037</td>
<td>4423</td>
<td>704</td>
</tr>
<tr>
<td>23165</td>
<td>16513</td>
<td>68.598</td>
<td>.12297</td>
<td>5578</td>
<td>888</td>
</tr>
<tr>
<td>31003b</td>
<td>300</td>
<td>.46812</td>
<td>.086477</td>
<td>54.1</td>
<td>8.6</td>
</tr>
<tr>
<td>31009</td>
<td>927</td>
<td>1.0746</td>
<td>.11011</td>
<td>97.6</td>
<td>15.5</td>
</tr>
<tr>
<td>31052</td>
<td>5156</td>
<td>3.9743</td>
<td>.037176</td>
<td>1069</td>
<td>170</td>
</tr>
<tr>
<td>32010n</td>
<td>979</td>
<td>1.1190</td>
<td>.12665</td>
<td>88.4</td>
<td>14.1</td>
</tr>
<tr>
<td>33020*</td>
<td>1818</td>
<td>1.8192</td>
<td>.037723</td>
<td>482</td>
<td>76.8</td>
</tr>
<tr>
<td>33070*</td>
<td>7177</td>
<td>5.5296</td>
<td>.038131</td>
<td>1450</td>
<td>231</td>
</tr>
<tr>
<td>33079</td>
<td>7862</td>
<td>5.3430</td>
<td>.037187</td>
<td>1437</td>
<td>229</td>
</tr>
</tbody>
</table>

*denotes cases from Wilkins (Ref 6).

$\omega_b$ is defined as the frequency at which $R_{AC}/\omega = 0.1(L_T)$ and above which the assumption $R_{AC}/\omega$ much less than $L_T$ is taken to be valid.
The results in Table V show that the assumption is invalid for the 0.041 in. diameter line over most of the frequency range except at very low Reynolds numbers. In the case of the 0.195 in. diameter line the assumption is generally valid except at low frequencies (<200 Hz) and high Reynolds numbers (>7000).

Results obtained using the Modified LRCG model which incorporates the full expression for G (Equation 36) display the expected decrease in attenuation with negligible changes in attenuation at higher frequencies. Figures 61-63 show comparisons of the attenuation predicted by the Constant LRCG model with that predicted by the Modified LRCG model. The difference in attenuation at low frequencies is immediately obvious.

Though the inclusion of the frequency dependent term drastically alters the nondimensional behavior of the model, the effect on the gain and phase shift predictions is less significant. Figures 64-73 compare the effect of the Modified LRCG model versus the Constant LRCG model for typical cases from this study. These figures indicate an increase in the gain over that predicted by the Constant LRCG model, particularly at low frequency. The increase in the gain depends on the line configuration and signal frequency and generally reflects the validity of the limiting assumption discussed earlier. Review of the theoretical data from Case 11033 (Figure 64), for example,
shows an increase of 3.5 dB at 1 Hz and of 0.9 dB at 100 Hz. The differences decreases with increasing frequency and in this case is negligible above about 700 Hz. This increase in the gain produces better agreement with experimental results yet a significant difference still remains between theoretical and experimental results.

Examination of the expression for the modified $C_T$ (Equation (37)) shows that as $\omega$ varies from 0 to infinity, $C_T$ varies from $kC_a$ to $(k - \frac{k-1}{kLT})C_a$. The latter value is the same as the original expression for $C_T$ (Equation (35)). As illustrated in Table VI the use of Equation (37) results in an increase in $C_T$ of from approximately 38 percent at 1 Hz to less than 1 percent at 1000 Hz over the value of $C_T$ determined by Equation (35). It is also noted that the value of the original $C_T$ is less than 2 percent greater than $C_a$ for the cases examined in this study.

Table VII shows the effect of the modification of $C_T$ on the attenuation predicted by the Modified LR and G model. To obtain this data the Modified LRC model was used with values of $C_T$ calculated from Equations (35) and (37). The use of Equation (37) results in an increase in the attenuation over that predicted using Equation (35). Though the percentage increase is large at low frequencies, the absolute increase is on the order of 2 dB and is much less significant when compared with the 25-55 dB (300-600 percent) decrease in attenuation resulting from the use of
### TABLE VI

**VARIATION OF $C_T$ DUE TO FREQUENCY**

<table>
<thead>
<tr>
<th>Case</th>
<th>Re</th>
<th>$C_T$ (Eq 35) $\frac{\text{in}^4}{\text{lb}_F} \times 10^4$</th>
<th>Range of $C_T$ (Eq 37) $\frac{\text{in}^4}{\text{lb}_F} \times 10^4$ **</th>
</tr>
</thead>
<tbody>
<tr>
<td>11012</td>
<td>1180</td>
<td>.334</td>
<td>.459 - .337</td>
</tr>
<tr>
<td>12018n</td>
<td>1780</td>
<td>.315</td>
<td>.435 - .318</td>
</tr>
<tr>
<td>23070*</td>
<td>7326</td>
<td>5.285</td>
<td>7.308 - 5.292</td>
</tr>
<tr>
<td>23090</td>
<td>9094</td>
<td>5.151</td>
<td>7.126 - 5.159</td>
</tr>
<tr>
<td>23105</td>
<td>10490</td>
<td>5.038</td>
<td>6.972 - 5.048</td>
</tr>
<tr>
<td>23110*</td>
<td>10450</td>
<td>5.048</td>
<td>6.986 - 5.058</td>
</tr>
<tr>
<td>23165</td>
<td>16513</td>
<td>4.537</td>
<td>6.286 - 4.551</td>
</tr>
</tbody>
</table>

*denotes case from Wilkins (Ref 6).

**(1 < $\omega$ < 1000).**

### TABLE VII

**PERCENT INCREASE IN ATTENUATION (MODIFIED LRCG MODEL) DUE TO MODIFIED $C_T$ (EQ 37) OVER $C_T$ (EQ 35)**

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Case</th>
<th>11033</th>
<th>23090</th>
<th>23105</th>
<th>23165</th>
<th>33079</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>17.6</td>
<td>17.5</td>
<td>17.5</td>
<td>17.6</td>
<td>17.3</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>9.9</td>
<td>7.8</td>
<td>8.5</td>
<td>10.4</td>
<td>1.65</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>4.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.9</td>
<td>0.44</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>1.4</td>
<td>0.84</td>
<td>1.0</td>
<td>1.6</td>
<td>0.11</td>
</tr>
<tr>
<td>300</td>
<td></td>
<td>0.64</td>
<td>0.38</td>
<td>0.46</td>
<td>0.73</td>
<td>0.05</td>
</tr>
<tr>
<td>500</td>
<td></td>
<td>0.24</td>
<td>0.14</td>
<td>0.17</td>
<td>0.27</td>
<td>0.02</td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td>-</td>
<td>0.04</td>
<td>0.04</td>
<td>0.07</td>
<td>-</td>
</tr>
</tbody>
</table>
the Modified LRCG model vice the Constant LRCG model. This indicates that the largest part of the difference in attenuation between the two LRCG models is due to the use of the full expression for Re (G).

Correlation of Modified LRCG Model with Brown's Modified IRC Model

In Reference 3, Brown et al. present the results of their Modified IRC model for turbulent flow accounting for heat transfer. The laminar reference curve (Figure 3) produced using the simplified equations developed by Krishnaiyer and Lechner (Ref 3) matches the laminar references shown in Figure 9 of Reference 3 and in Wilkins (Ref 6) except for the difference in the slope of the reference compared to the Modified LRCG curves in the low frequency region. Brown shows a change in the slope of the laminar reference at \( \Omega = 2.5 \) which does not appear in the laminar reference produced using the equations of Krishnaiyer and Lechner (Equations (6) and (7)).

The mid frequency range attenuation levels predicted by the Modified LRCG model agree with those shown on Brown's figure to within the resolution allowed by the figure.

Brown (Ref 3) gives an expression for the break frequency between the mid and high frequency regions as

\[
\Omega_b = 2.5 \times 10^{-3} \, \text{Re}^{1.6} \quad (57)
\]
This expression identifies a point of inflection on Brown's curves rather than the intersection of the turbulent model curves and the laminar reference curve. The results of this study give the expression for the intersection point for the Modified LRCG model as

$$\Omega_b = 9.68 \times 10^{-4} \ Re^{-1.58}$$

(58)

assuming a power form solution as presented by Brown. This break frequency agrees closely with the actual intersection point of the curves in Brown's figure.

**Correlation of Results with Nichols (Ref 1)**

Figure 74 presents a comparison of the attenuation predicted by the modified equations of Krishnaiyer and Lechner (Equations (6) and (7)) with that given by Nichols (Ref 1, Figure 6). The large deviation shown is attributed to the use of an approximation to the Bessel functions used to derive Equations (6) and (7). The simplified equations are stated to be limited to the region \( \omega/\omega_v > 0.1 \). This is confirmed by Figure 74.

Figures 75 and 76 compare the attenuation predictions of the various models discussed in this study. The deviation of the Constant LRCG at low frequencies is again seen to be significant. The MCLRC model and the Modified LRCG model show comparable behavior with the Modified LRCG model displaying the higher attenuation of the two models.
V. Conclusions

1. Changes in conditions upstream of the test section as represented by a bypass device with variable flow rates did not affect the line response as measured experimentally. This was determined to be true in both laminar and turbulent mean flow conditions.

2. The elimination of small differences between the orifice devices and the orifice model used to determine the reflection coefficient caused no change in the experimental results.

3. The absolute error in the experimental calculations due to instrument readability was determined to be approximately 0.5 dB in the gain calculations and approximately 5 degrees in the phase shift calculations. With the current experimental apparatus these values represent realistic limits on the resolution of differences between theoretical predictions and experimental results.

4. The effect of the radiation impedance on the theoretical predictions in open line cases is negligible. The use of $Z_L = 0$ rather than the radiation impedance resulted in an insignificant increase in the gain.

5. The Modified LRCG model, which is formed by adding the full frequency dependent expression for the
conductance to the MCLRC model, was seen to predict lower attenuation than the Constant LRCG model at low frequency. The resultant increase in the gain varied from as much as 6 dB for the small diameter line to less than 1 dB for the large diameter line. The change was negligible at high frequency.
VI. **Recommendations**

1. Automated data processing equipment currently on order should be utilized to increase accuracy, facilitate low frequency measurements and significantly reduce the time required to collect and correlate experimental data. With automated experimental equipment a large database could be easily built up and more time would be available for analytical work.

2. Efforts should be made to extend the range of experimental results, particularly into the low frequency region. Digital wave analyzers teamed with automated data processing equipment would hopefully allow low frequency measurements with current line configurations and at least would greatly improve data collection capability.
Appendix A: Comparison of Experimental and Theoretical Results

Case Number Explanation

Case ABxxx

A - line diameter
1 = 0.041 in.
2 = 0.119 in.
3 = 0.196 in.

B - line termination
0 = blocked
1 = 0.0135 in. diameter orifice
2 = 0.016 in. diameter orifice
3 = open

xxx = Nominal Reynolds number in hundreds
Fig. 16. Theoretical and Experimental Phase Shift—Case 10000—Laminar Theory
Fig. 18. Theoretical and Experimental Phase Shift--Case 11006--Laminar Theory
Fig. 19. Theoretical and Experimental Gain—Case 23090
Fig. 20. Theoretical and Experimental Phase Shift--Case 23090
Fig. 21. Theoretical and Experimental Gain--Case 23129
Fig. 22. Theoretical and Experimental Phase Shift—Case 23129
Fig. 23. Theoretical and Experimental Gain--Case 23165
Fig. 25. Theoretical and Experimental Gain--Case 23165
Fig. 26. Theoretical and Experimental Phase Shift—Case 23165
Fig. 27. Theoretical and Experimental Gain--Case 30000--Laminar Theory
Fig. 28. Theoretical and Experimental Phase Shift--Case 30000--Laminar Theory
Fig. 29. Theoretical and Experimental Gain—Gase 31093B—Laminar Theory
Fig. 31. Theoretical and Experimental Gain—Case 33079
Fig. 34. Effect of Radiation Impedance on Theoretical Gain--Case 23107--Mod. Const. LRC
Fig. 35. Effect of Radiation Impedance on Theoretical Gain—Case 23107—Constant LRCG
Fig. 36. Effect of Radiation Impedance on Theoretical Gain—Case 23165—Constant LRCG
Fig. 37. Effect of Radiation Impedance on Theoretical Gain--Case 33052--Constant LRCG

- $Z_L = 0$
- $Z_L = Z_R$ (Eq. 45)
Fig. 39. Effect of Radiation Impedance on Theoretical Gain—Case 33079—Constant LRCG
Fig. 40. Effect of Upstream Conditions on Experimental Gain--Case 11012
Fig. 41. Effect of Upstream Conditions on Experimental Gain---Case 11014n
Fig. 43. Effect of Upstream Conditions on Experimental Gain--Case 11033
Fig. 44. Effect of Upstream Conditions on Experimental Gain—Case 12034n
Fig. 45. Effect of Upstream Conditions on Experimental Gain--Case 21004
Fig. 47. Effect of Upstream Conditions on Experimental Gain--Case 23107
Fig. 49. Theoretical and Experimental Gain—Case 11030n—Mod. Const. LRC
Fig. 50. Theoretical and Experimental Phase Shift--Case 11030n--Mod. Const. LRC
Fig. 54. Theoretical and Experimental Phase Shift--Case 21015n--Laminar Theory
Fig. 56. Theoretical and Experimental Phase Shift--Case 22020n--Laminar Theory
Fig. 59. Theoretical and Experimental Gain—Case 32010n—Laminar Theory
Fig. 60. Theoretical and Experimental Phase Shift--Case 32010n--Laminar Theory
Fig. 61. Nondimensional Behavior—Constant LRCG vs. Modified LRCG—Case 11033
Fig. 63. Nondimensional Behavior--Constant LRCG vs. Modified LRCG--Case 33079
Fig. 65. Effect of Modified LRCG on Theoretical Phase Shift--Case 11033
Fig. 66. Effect of Modified LRCG on Theoretical Gain—Case 12034
Fig. 68. Effect of Modified LRCG on Theoretical Gain—Case 23165
Fig. 70. Effect of Modified LRCG on Theoretical Gain--Case 33060
Fig. 71. Effect of Modified LRCG on Theoretical Phase Shift--Case 33060
Fig. 72. Effect of Modified LRCG on Theoretical Gain—Case 33079
Fig. 73. Effect of Modified LRCG on Theoretical Phase Shift--Case 33079
Fig. 75. Comparison of Attenuation for Theoretical Models--Case 11033
Fig. 76. Comparison of Attenuation for Theoretical Models—Case 33079
Appendix B: Simulation of the Dynamic Response of the Pneumatic Transmission Line

PROGRAM MSB

C

C TRANSMISSION LINE STUDY/Z-IN METHOD WITH BRANCH
D

C DIMENSION DBT(300),RVT(i00),DI(i00),AD(i00),AR(100),CV(100),
& OMT(i00),CNA(100),GMA(100),AGM(100),FN(100),RN(100),GN(100),
& ALN(100),CN(100),AN(100),BTN(100),AMC(100),AZRN(100),BZRN(100),
& AZIN(100),BZIN(100),DC(100),DG(100),DD(I00),P(I00),RHO(I00),
& ANU(I00),CA(I00),QTCA(I00),OMG(300),RTP(100),GP(100),BETA(100),
& AZOT(100),BZOT(100),REY(I00),Q(I00),BET(100),GR(300),GI(300),
& DB(300),OMGX(300),OMGP(300),PHASE(300),PH(300)

INTEGER TURB,CASE

C

C Character TITLE*50, THEOR*50
D DATA PI,TP1/3.1415926,6.2831853/
D DATA AMU,RE,GAM,SIG/0.2679E-08,247104.1.4,0.8414/
D DATA N/10/

C

C TO NEGATE THE PLOT ROUTINE SET ICAS TO ZERO.

C

C******************************************************************************

C


C

C******************************************************************************

C

C ACROUT IS THE TABULAR OUTPUT FILE OF THE FOLLOWING PRESENTED BY LINE SEGMENT

129
OPEN(11,FILE='D20000')
REWIND 11
OPEN(12,FILE='ACROUT')
REWIND 12
OPEN(13,FILE='ATTEN')
REWIND 13
OPEN(14,FILE='GAIN')
REWIND 14
CALL PLOTS(0.,0.,9)

OPEN(11,FILE='D20000')
REWIND 11
OPEN(12,FILE='ACROUT')
REWIND 12
OPEN(13,FILE='ATTEN')
REWIND 13
OPEN(14,FILE='GAIN')
REWIND 14
CALL PLOTS(0.,0.,9)

TAPE 11 IS THE INPUT FILE CONTAINING THE FOLLOWING:
ICAS IS THE CASE NUMBER
OP IS THE LINE CONDITION; 1.0 FOR OPEN LINE, ELSE 0.0
TR IS THE AMBIENT TEMPERATURE IN DEGREES RANKINE
PG IS THE BAROMETRIC PRESSURE (PSIA)
PT IS THE STATIC LINE PRESSURE (PSIG)
FL IS THE MASS FLOW RATE (LBM/SEC)
D1 IS THE OUTSIDE LINE DIAMETER (IN)
DORF2 IS THE SQUARE OF THE ORIFICE DIAMETER (IN**2)
DI IS THE LINE SEGMENT DIAMETER (IN)
AD IS THE LINE SEGMENT LENGTH (IN)
LINE SEGMENTS 1 & 2 ARE THE DOWNSTREAM DYNAMIC PRESSURE TRANSDUCER BRANCH.
THE UPSTREAM END OF THE TEST SECTION IS DENOTED AS SEGMENT NUMBER 10.

THE FOLLOWING VARIABLES ARE USED IN THIS PROGRAM
CT IS THE RATIO OF TURBULENT/ADIABATIC CAPACITANCE
TL IS THE INERTANCE CONSTANT
ZEND IS THE REAL PART OF THE TERMINAL IMPEDANCE (PSI/CIS)
ZENDI IS THE IMAGINARY PART OF THE TERMINAL IMPEDANCE (PSI/CIS)
AMU IS THE DYNAMIC VISCOSITY (PSI-SEC)
RE IS THE GAS CONSTANT FOR AIR (IN/SEC-R)
GAM IS THE RATIO OF SPECIFIC HEATS
SIG IS THE SQUARE ROOT OF THE PRANDTL NUMBER
N IS THE NUMBER OF LINE SEGMENTS
REYT IS THE REYNOLDS NUMBER

READ(11,*)ICAS,OP
READ(11,*)TR,PG,PT,FL,D1
IF(OP.EQ.1.)GOTO 110
READ(11,*)DORF2
110 CONTINUE
DO 898 I=1,N
READ(11,*)DI(I),AD(I)
898 CONTINUE
IF(OP.EQ.1.)GOTO 100

IF(DORF2 = 0.) THEN
CASE=3
N1=3
GOTO 96
ENDIF
100 CONTINUE

131
THIS INTERACTIVE SECTION ALLOWS THE SELECTION OF A LAMINAR OR TURBULENT MODEL. IT ALSO ALLOWS THE SELECTION OF ONE OF TWO LOAD IMPEDANCE MODELS. CASE IS THE MODEL INDICATOR FOR THE LAMINAR OR TURBULENT MODEL. IF A COMBINATION OF LAMINAR AND TURBULENT MODELS IS SELECTED, THE PROGRAM WILL SWITCH TO THE TURBULENT MODEL IF THE REYNOLDS NUMBER IS GREATER THAN 2300 AND THE TURBULENT ATTENUATION IS GREATER THAN THE LAMINAR VALUE. OTHERWISE THE LAMINAR MODEL IS USED. NI IS THE LOAD IMPEDANCE INDICATOR. YOU HAVE A CHOICE OF EITHER CHOKED OR NON-CHOKED FLOW THROUGH THE ORIFICE.

PLEASE ENTER VALUE OF CASE:

1  LAMINAR/ MOD. CONST. LRC
2  LAMINAR/ MODIFIED LRCG
3  LAMINAR THEORY ONLY

PLEASE ENTER VALUE OF NI:

READ CASE
IF (OP.EQ.1.0) GOTO 96

PLEASE ENTER VALUE OF N1:
PRINT*, 'N1'
PRINT*, '
PRINT*, 1 CHOKEO FLOW IN THE ORIFICE'
PRINT*, ' 2 NON-CHOKED FLOW IN THE ORIFICE'
PRINT*, '
PRINT*, '? N1'
PRINT*, '
READ*, N1
96 WRITE(12,612) ICAS
612 FORMAT(1X,T10,'RESULTS OF RUN NUMBER',T32,I7)
   IF(CASE.EQ.3) THEN
      WRITE (12,804)
804 FORMAT(1HO,T10,'CALCULATED USING LAMINAR THEORY THROUGHOUT')
   ELSE
      IF(CASE.EQ.1) THEN
         WRITE (12,807)
807 FORMAT(1HO,T10,'CALCULATED USING MOD. CONST. LRC THEORY IN THE TUR
   &BULENT REGION')
      ELSE
         WRITE (12,808)
808 FORMAT(1HO,T10,'CALCULATED USING THE MODIFIED LRCG THEORY IN THE
   TURBULENT REGION')
   END IF
   IF(OP.EQ.1.0) GOTO 500
   IF(N1.EQ.1) THEN
      WRITE(12,820)
820 FORMAT(1X,T10,'REFLECTION COEFFICIENT CALCULATED ASSUMING NON-
   CHOKED FLOW IN THE ORIFICE')
   ELSE
      WRITE(12,820)
500 CONTINUE
ENDIF

THE FOLLOWING VARIABLES ARE USED AS PROGRAM COUNTERS.
ICT=1
IND=1
NNN=1

OP=0 SIGNIFIES BLOCKED OR RESTRICTED FLOW.
CALCULATES INITIAL REYNOLDS NUMBER BASED ON THE LINE DIAMETER
UPSTREAM OF THE TEST SECTION.
(4/(AMU*PI*32.2*12)) = 1.22999E06

IF(OP.EQ.0.) THEN
   REYT=1.22999E+06*FL/D1
ELSE
C TOTL IS THE LINE LENGTH (INCHES) FROM THE STATIC
C PRESSURE TRANSUDER TO THE TERMINATION OF THE
C LINE. THE DISTANCE FROM THE STATIC TRANSUDER
C TO THE BEGINNING OF THE TEST SECTION IS 2.0 INCHES.
C
TOTL=2.0
DO 901 I=3,N
    TOTL=TOTL+AD(I)
901    CONTINUE

REYT=PT*(PT/2.+PG)*(DI(N)**3.)/(32.*AMU**2)*TOTL*RE*TR
IF(REYT.LT.2300.) GOTO 902
REYT=(REYT*32./.1582)**(4./7.)
902    CONTINUE
ENDIF

PT=PT+PG

C
C THE PRESSURE DROP BETWEEN THE STATIC PRESSURE
C TRANSUDER AND THE BEGINNING OF THE TEST LINE IS
C CALCULATED. P (N) IS THE LINE PRESSURE AT THE UPSTREAM
C END OF THE TEST LINE. PF IS THEN SET EQUAL TO P (N).
C
C
C******************************************************************************
IF(REYT.LT.2300.) THEN
    DP=PT-SQRT(PT**2.+2.27E-10*REYT*TR/(DI**3.))
ELSE
    DP=PT-SQRT(PT**2.+1.12226E-12*REYT**1.75*TR/(DI**3.))
ENDIF
P(N)=PT+DP-PF
PF=P(N)
IF(REYT.EQ.0.)GOTO 999
IF(REYT.LT.0.)GOTO 999
F=64./REYT
ELSE
    F=0.3164/(REYT**0.25)
ENDIF
F2=SQRT(F)
TL=((F2+1.)*((F2+2.)**2))/(4.*((2*F2)+1.0))
CT=GAM-((AM-I.)/TL)

C******************************************************************************
C THIS PROGRAM USES THE TERM (CT*AGM(I))
C THIS IS THE TURBULENT CAPACITANCE. IF THE
C ADIABATIC CAPACITANCE IS TO BE USED, SET CT=1.0
C AFTER THIS POINT.
C THE TERM (TL*RHO(I)/AR(I)) IS THE TURBULENT
C INERTANCE. FOR ADIABATIC INERTANCE SET TL=1.0
C
C******************************************************************************
999    CONTINUE
DO 899 I=1,N-2
IF(OP.EQ.1.) GOTO 895
899    CONTINUE
C CALCULATES THE REYNOLDS NUMBER IN THE TEST SECTION.
C
REYT=1.22999E+06*FL/DI(10)
895 CONTINUE
C
*** THE PRESSURE DROP ACROSS EACH SEGMENT OF THE LINE IS ***
C NOW CALCULATED. THE INDEX I IS SET FROM 1 TO N-2 TO ***
C SKIP THE DOWNSTREAM TRANSDUCER BRANCH. THE INDEX K ***
C IS USED TO REVERSE THE ORDER OF THE SEGMENTS SINCE #10 ***
C IS THE UPSTREAM END OF THE LINE.
C
PF=PF+PG
K=N+1-I
IF(REYT.LT.2300.) THEN
DO=PF-SQRT(PF**2.+1.135E-10*AD(K)*REYT*TR/(DI(K)**3.))
ELSE
DP=PF-SQRT(PF**2.+5.61128E-13*REYT**1.75*AD(K)*TR/(DI(K)**3.))
ENDIF
P(N-I)=PF+DP-PG
L=(N-I)
WRITE(*,66) L,P(L)
66 FORMAT(T5,'P(',II,')= ',F10.7)
PF=P(N-I)
C PEND IS THE LINE PRESSURE AT THE DOWNSTREAM END OF THE
C TEST SECTION.
C
PEND=P(2)
IF(OF.EQ.1.) PEND=0.0
899 CONTINUE
C THE PRESSURE IN THE DOWNSTREAM DYNAMIC TRANSDUCER BRANCH
C IS SET EQUAL TO THE PRESSURE IN THE LINE AT THE BRANCH POINT.
P(2)=P(5)
P(1)=P(2)
C DETERMINE REYNOLDS NUMBER FOR A LINE
TOTL=0.0
DO 2 I=3,N
TOTL=TOTL+AD(I)
2 CONTINUE
C RHO(N) IS THE DENSITY AT THE BEGINNING OF THE TEST LINE.
RHO(N)=(PG+P(N))/(RE*TR)
REYT=((P(N)-PEND)*((P(N)+PEND)/2.+PG)*(DI(N)**3.))/32./AMU/AMU
REYT=REYT/TOTL/RE/TR
IF (REYT.LT.2300.) GOTO 3
REYT=(REYT*32./1582)**(4./7.)

QM IS THE MASS FLOW RATE (LBF-S/IN)

QM*386.04=QM(LBM/S)

QM=REYT*PI*DI(N)*AMU/4.
QM=386.04*QM
WRITE(*,67)REYT

67 FORMAT(T5,'THE REYNOLDS NUMBER IS ','F8.2)
WRITE(*,68)QM1

68 FORMAT(T5,'THE MASS FLOW RATE IS ',1PE11.4,' LBM/SEC')

C
C THE VARIABLES IN THE FOLLOWING SECTION ARE DEFINED AS:
C * RHO=DENSITY CALCULATED BY THE IDEAL GAS LAW (LBF-S**2/IN**4)
C * ANU=KINEMATIC VISCOSITY (IN**2/S)
C * CA=ADIABATIC SPEED OF SOUND (IN/S)
C * AR= CROSS-SECTIONAL AREA OF LINE (IN**2)
C * CV=VISCOS CHARACTERISTIC FREQUENCY (OMEGA-NU) (RAD/S)
C * OMT= THERMAL CHARACTERISTIC FREQUENCY (RAD/S)
C * CNA=VISCOS RESISTANCE (LBF-S/IN**6)
C * Q=VOLUMETRIC FLOW RATE (IN**3/S)
C * AGM=ADIABATIC CAPACITANCE
C * RVT=LINE RESISTANCE
C * FN=FUNDAMENTAL FREQUENCY (RAD/S)
C * PBR=LINE SEGMENT PRESSURE (PSIA)

FICI

DO 23 I=1,N
PBR=P(I)+PG
RHO(I)=PBR/(RE*TR)
ANU(I)=AMU/RHO(I)
CA(I)=SQRT(PBR*GAM/RHO(I))
AR(I)=PI*DI(I)**2./4.
CV(I)=(8.*PI*ANU(I))/AR(I)
OMT(I)=CV(I)/(SIG*SIG)
CNA(I)=(8.*PI*AMU)/(AR(I)*AR(I))
Q(I)=QM/RHO(I)

23 CONTINUE

C THE FLOW IN THE DYNAMIC TRANSDUCER BRANCH IS SET
C EQUAL TO ZERO.
Q(1)=0.0
Q(2)=0.0

GMNI=.5*(GAM-1.)

DO 26 I=1,N
REY(I)=(4.*RHO(I)*Q(I))/(PI*DI(I)*AMU)
PBR=P(I)+PG
AGM(I)=AR(I)/(GAM*PBR)
QTCA(I)=0.25*CA(I)

RVT(I) IS THE DC LINE RESISTANCE

136
C 1.75*RVT(I) IS THE AC (INCREMENTAL) LINE RESISTANCE
C
RVT(I)=.3164*AMU*(REY(I)**.75)/(2.*AR(I)*(DI(I)**2.))
RVT(I)=RVT(I)*1.75
FN(I)=QTCA(I)/AD(I)
26 CONTINUE
NST=1
DW=0.
Y=1.
M=0
C
C*****************************************************************************
C LOOP 80 CALCULATES THE THEORETICAL VALUES FOR THE
C PLOT ROUTINE. THE MODIFIED EQUATIONS OF KRISHNAIYER
C AND LECHNER ARE USED FOR BLOCKED AND LAMINAR FLOW.
C THE PROGRAM ALSO CALCULATES THE PROPAGATION CONSTANT
C BASED ON THE SELECTED TURBULENT MODEL AND COMPARES
C THE LAMINAR AND TURBULENT VALUES. IT SELECTS THE
C GREATER VALUE FOR USE IN THE OUTPUT ROUTINE.
C
C THE VARIABLES USED ARE:
C
C RN=REAL PART OF Z (EQN 6 WILKIN)
C DC=COEFFICIENT (EQN 11 WILKIN)
C DG=COEFFICIENT (EQN 10 WILKIN)
C DD=DENOMINATOR OF EQN 7 (WILKIN)
C GN=REAL PART OF Y (EQN 7 WILKIN)
C ALN=IMAGINARY PART OF Z (EQN 6 WILKIN)
C CN=IMAGINARY PART OF Y (EQN 7 WILKIN)
C TEM1=REAL PART OF ZY
C TEM2=IMAGINARY PART OF ZY
C AN=REAL PART OF SQRT(ZY)
C BTN=IMAGINARY PART OF SQRT(ZY)
C OX=NONDIMENSIONAL FREQUENCY
C OY=NONDIMENSIONAL ATTENUATION
C TURB=TURBULENT MODEL INDICATOR
C
C*****************************************************************************
C 40 DO 80 J=1,NST
M=M+1
C
C INCREMENT THE FREQUENCY BY 5 HZ AND CONVERT TO RAD/SEC.
C
Y=Y+DW
W=TPI*Y
DO 27 I=1,N
ARG=.5*SQRT(W/CV(I))
RN(I)=CNA(I)*(.375+ARG+(.375/(4.*ARG))))
\[ DC(I) = 0.25 + \sqrt{W/OMT(I)} + 0.125*\sqrt{OMT(I)/W} \]
\[ DG(I) = \sqrt{W/OMT(I)} - 0.125*\sqrt{OMT(I)/W} \]
\[ DD(I) = DC(I) + DG(I) \]
\[ GN(I) = W*(GAM-1.)*AGM(I)*DG(I)/DD(I) \]
\[ ARG = 5**\sqrt{CV(I)/(W*64.))}/AR(I) \]
\[ ARG = RHO(I)*(1. + ARG - (ARG*(15.*CV(I)/(W*64.))))/AR(I) \]
\[ TURB = 0 \]
\[ CT = GAM - (W**2)*(GAM-1.)*LT*RHO(I)/(AR(I)*DENOM) \]
\[ ALN(I) = RHO(I)*(1. + (GAM-1.)*DC(I)/DD(I)) \]
\[ TEMP = W*W \]
\[ LT = TL*RHO(I)/AR(I) \]
\[ CT = GAM - (W**2)*(LT**2) \]

C  **THIS SECTION CALCULATES THE PROPAGATION**
C  OPERATOR SQRT(ZY). THE THEORY SELECTED AT
C  THE BEGINNING OF THE PROGRAM IS USED IN
C  THIS SECTION.
C  **410 MODIFIED CONSTANT LRC**
C  **420 MODIFIED LRCG**
C  **430 LAMINAR**

C  GOTO(410,420,430) CASE

410  TEM1 = RN(I)*GN(I) + TEMP*ALN(I)*CN(I)
     TEM2 = W*(RN(I)*CN(I) + GN(I)*ALN(I))
     CALL RTCMP(ARG1,ARG2,TEM1,TEM2)
     AN(I) = ARG1
     BTN(I) = ARG2
     TEM1 = TEMP*TL*RHO(I)/AR(I)*CT*AGM(I)
     TEM2 = W*RVT(I)*CT*AGM(I)
     CALL RTCMP(ARG1,ARG2,TEM1,TEM2)
     TAN = ARG1
     TBTN = ARG2
     IF(REY(I) .LE. 2300) GOTO 27
     IF(AN(I) .GE. TAN) GOTO 27
     AN(I) = TAN
     BTN(I) = TBTN
     TURB = 1
     GOTO 27

420  TEM1 = RN(I)*GN(I) + TEMP*ALN(I)*CN(I)
     TEM2 = W*(RN(I)*CN(I) + GN(I)*ALN(I))
     CALL RTCMP(ARG1,ARG2,TEM1,TEM2)
     AN(I) = ARG1
     BTN(I) = ARG2
     CT = GAM - (W**2)*(GAM-1.)*LT*RHO(I)/(AR(I)*DENOM)
     TEM1 = (((RVT(I)*(GAM-1.)*AGM(I)*RHO(I)*(W**2))/(AR(I)*DENOM))
     & + (TEMP*TL*RHO(I)*CT*AGM(I)/AR(I)))
     TEM2 = (((RVT(I)*(GAM-1.)*AGM(I)*RHO(I)*LT*(W**3))/(AR(I)*DENOM))
     & + (W*RVT(I)*CT*AGM(I))
     CALL RTCMP(ARG1,ARG2,TEM1,TEM2)
     TAN = ARG1
     TBTN = ARG2
     IF(REY(I) .LE. 2300) GOTO 27
     IF(AN(I) .GE. TAN) GOTO 27
AN(I)=TAN
BTN(I)=TBTN
TURB=1
GOTO 27

430 TEM1=RN(I)*GN(I)+TEMP*ALN(I)*CN(I)
TEM2=W*(RN(I)*CN(I)+GN(I)*ALN(I))
CALL RTCMP(ARG1,ARG2,TEM1,TEM2)
AN(I)=ARG1
BTN(I)=ARG2

27 CONTINUE

OX=AR(10)*W/ANU(10)/PI
OY=AR(10)*CA(10)*AN(10)/PI/ANU(10)
WRITE(13,75)OX,OY

75 FORMAT(T5,F10.5,T20,F10.5)

C CALCULATE ZO

DO 32 I=1,N
TEM1=W*ALN(I)
TEM2=W*CN(I)
TEM3=RN(I)
TEM4=GN(I)
CALL CMFDV(ARG1,ARG2,TEM3,TEM1,TEM4,TEM2)
CALL RTCMP(AZRNI,BZRN,ARG1,ARG2)
AZRN(I)=AZRNI
BZRN(I)=BZRN

IF(TURB.EQ.0) GOTO 32
TEM1=RHO(I)*(P(I)+PG)/AR(I)
TEM2=-RVT(T)*(P(I)+PG)/W/AR(I)
CALL RTCMP(ARG1,ARG2,TEM1,TEM2)
AZRN(I)=ARG1
BZRN(I)=ARG2

32 CONTINUE

C CALCULATE Z IN I

I=1
TEMP=AN(I)*AD(I)
IF(TEMP.GT.88.) GO TO 80
ARG1=COSH(TEMP)
ARG2=SINH(TEMP)
TEM5=BTN(I)*AD(I)
TEMP=COS(TEM5)
TEM1=ARG1*TEMP
TEM3=ARG2*TEMP
TEMP=SIN(TEM5)
TEM2=ARG2*TEMP
TEM4=ARG1*TEMP
CALL CMFDV(ARG1,ARG2,TEM1,TEM2,TEM3,TEM4)
TEM1=AZRN(I)
TEM2=BZRN(I)
CALL CMPPM(TEM3,TEM4,TEM1,TEM2,ARG1,ARG2)
AZIN(I)=TEM3
BZIN(I)=TEM4

139
C CALCULATE Z IN 3
C SEGMENT 3 IS THE DOWNSTREAM END OF THE TEST SECTION
I=3
  TEM1=AZRN(I)
  TEM2=BZRN(I)
C
C******************************************************************************
C CALCULATES THE TERMINAL IMPEDANCE: IF OP=1 (UNRESTRICTED FLOW), SETS REAL
C PART EQUAL TO ZERO
C CALCULATES IMAGINARY PART
C IF OP=0 (RESTRICTED FLOW), SETS IMAGINARY PART TO ZERO
C CALCULATES REAL PART
C FOR BLOKED LINE RE= INFINITY
C FOR ORIFICE FLOW RE CALCULATED USING REFLECTION COEFFICIENTS
C******************************************************************************
C
C IF (OP.EQ.1.) THEN
ZEND=0.0
ZENDI=-W*.4*RHO(I)*DI(I)/AR(I)
ELSE
ZENDI=-0.0
  IF (DORF2.EQ.0.) THEN
    ZEND=1.0E99
  ELSE
    SQD3=DI(I)**2
  ENDIF
ENDIF
C N1 IS AN INDICATOR TO SELECT THE METHOD OF CALCULATING THE REFLECTION
C COEFFICIENT.
C N1=1  CHOKED FLOW IN THE ORIFICE
C N1=2  NON-CHOKED FLOW IN THE ORIFICE
C IF(N1.EQ.1.)THEN
RC=(SQD3-DORF2)/(SQD3+DORF2)
ELSE
RC=(SQD3-2.*DORF2)/(SQD3+DORF2)
ENDIF
  RCC=(1.+RC)/(1.-RC)
  ZEND=4.*RHO(I)*CA(I)*RCC/(PI*SQD3)
ENDIF
ENDIF
TEM3=ZEND
TEM4=ZENDI
TEM5=AN(I)
TEM6=AD(I)
TEM7=BTN(I)
CALL CALZIN(AARG,BARG,TEM1,TEM2,TEM3,TEM4,TEM5,TEM6,TEM7)
AZIN(I)=AARG
BZIN(I)=BARG
C CALCULATE Z IN 4 AND 5
DO 34 I=4,5
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AARG, BARG, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AARG
BZIN(I)=BARG
C CALCULATE Z IN 2
I=2
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AARG, BARG, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AARG
BZIN(I)=BARG
C CALCULATE RECEIVING Z FOR LINE 6
I=6
TEM1=AZIN(I-1)
TEM2=BZIN(I-1)
TEM3=AZIN(I-4)
TEM4=BZIN(I-4)
CALL ZEBRA(AZOTI, BZOTI, TEM1, TEM2, TEM3, TEM4)
AZOT(I)=AZOTI
BZOT(I)=BZOTI
C CALCULATE Z IN 6 INCLUDING R-TRANSDUCER
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AN(I)
TEM4=AD(I)
TEM5=BIN(I)
CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, AZOTI, BZOTI, TEM4, TEM5)
AZIN(I)=AZINI
BZIN(I)=BZINI
C CALCULATE Z IN 7,8,9 AND 10 INCLUDING R-TRANSDUCERS
DO 39 I=7,10
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AZINI
BZIN(I)=BZINI
34 BZIN(I)=BARG
C CALCULATE Z IN 2
I=2
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AARG, BARG, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AARG
BZIN(I)=BARG
C CALCULATE RECEIVING Z FOR LINE 6
I=6
TEM1=AZIN(I-1)
TEM2=BZIN(I-1)
TEM3=AZIN(I-4)
TEM4=BZIN(I-4)
CALL ZEBRA(AZOTI, BZOTI, TEM1, TEM2, TEM3, TEM4)
AZOT(I)=AZOTI
BZOT(I)=BZOTI
C CALCULATE Z IN 6 INCLUDING R-TRANSDUCER
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AN(I)
TEM4=AD(I)
TEM5=BIN(I)
CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, AZOTI, BZOTI, TEM4, TEM5)
AZIN(I)=AZINI
BZIN(I)=BZINI
C CALCULATE Z IN 7,8,9 AND 10 INCLUDING R-TRANSDUCERS
DO 39 I=7,10
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AZINI
BZIN(I)=BZINI
C CALCULATE Z IN 7,8,9 AND 10 INCLUDING R-TRANSDUCERS
DO 39 I=7,10
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AZINI
BZIN(I)=BZINI
C CALCULATE Z IN 7,8,9 AND 10 INCLUDING R-TRANSDUCERS
DO 39 I=7,10
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AZINI
BZIN(I)=BZINI
C CALCULATE Z IN 7,8,9 AND 10 INCLUDING R-TRANSDUCERS
DO 39 I=7,10
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=AZIN(I-1)
TEM4=BZIN(I-1)
TEM5=AN(I)
TEM6=AD(I)
TEM7=BIN(I)
CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AZINI
BZIN(I)=BZINI
39  BZIN(I)=BZINI
C CALCULATE P5/P6 INCLUDING R-TRANSDUCER

I=6
TEMP=BTN(I)*AD(I)
CSBI1=COS(TEMP)
SNBI1=SIN(TEMP)
TEMP=AN(I)*AD(I)
ARG1=COSH(TEMP)
ARG2=SINH(TEMP)
TEM1=ARG1*CSBI1
AZOTI=AZOT(I)
BZOTI=BZOT(I)
AZINI=AZIN(I)
BZINI=BZIN(I)
CALL CMPDV(TEM7,TEM8,AZOTI,BZOTI,AZINI,BZINI)
CALL CMPMP(TEM3,TEM4,TEM7,TEM8,TEM1,TEM2)
TEM5=ARG2*CSBI1
TEM6=ARG1*SNBI1
TEM9=AZRN(I)
TEM10=AZRN(I)
CALL CMPDV(TEM7,TEM8,AZOTI,BZOTI,TEM9,TEM10)
CALL CMPMP(TEM1,TEM2,TEM7,TEM8,TEM5,TEM6)
TEM1=TEM3-TEM1
TEM2=TEM4-TEM2
TEMP=TEM1*TEM1+TEM2*TEM2
BET(I)=ATAN2(TEM2,TEM1)
BET(I)=(180./PI)*BET(I)
RTP(I-1)=SQRT(TEMP)
GP(I-1)=20.*ALOG10(RTP(I-1))

C CALCULATE P6/P7, P7/P8, P8/P9, AND P9/P10 INCLUDING R-TRANSDUCERS
DO 43 I=7,10
TEMP=BTN(I)*AD(I)
CSBI1=COS(TEMP)
SNBI1=SIN(TEMP)
TEMP=AN(I)*AD(I)
ARG1=COSH(TEMP)
ARG2=SINH(TEMP)
TEM1=ARG1*CSBI1
TEM2=ARG2*SNBI1
AZIN1=AZIN(I-1)
BZIN1=BZIN(I-1)
AZINI=AZIN(I)
BZINI=BZIN(I)
CALL CMPDV(TEM7,TEM8,AZIN1,BZIN1,AZINI,BZINI)
CALL CMPMP(TEM3,TEM4,TEM7,TEM8,TEM1,TEM2)
TEM5=ARG2*CSBI1
TEM6=ARG1*SNBI1
TEM9=AZRN(I)
TEM10=AZRN(I)
CALL CMPDV(TEM7,TEM8,AZIN1,BZIN1,TEM9,TEM10)
CALL CMPMP(TEM1,TEM2,TEM7,TEM8,TEM5,TEM6)
TEM1 = TEM3 - TEM1
TEM2 = TEM4 - TEM2
TEMP = TEM1 * TEM1 + TEM2 * TEM2
BETA(I) = ATAN2(TEM2, TEM1)
BET(I) = (180./PI) * BETA(I)
RT = (I - 1) SQRT(TEMP)

43 GP(I-1) = 20.* ALOG10(RTP[I-1])
RTPT = RTP(6) * RTP(7) * RTP(8) * RTP(9) * RTP(5)
GPT = 20.* ALOG10(RTPT)
BETAR = BETA(6) + BETA(7) + BETA(8) + BETA(9) + BETA(10)

35 IF (BETAR LE 0.) GOTO 36
BETAR = BETAR - TPI
GOTO 35
36 IF (BETAR GT TPI) GOTO 37
BETAR = BETAR + TPI
GOTO 36
C CONVERTS BETAR(RADS) TO BETAD(DEGS)
37 BETAD = (180./PI) * BETAR
C SKIPS WRITE STATEMENT FOR OUTPUT TABLE HEADINGS
IF (NNN GT 1) GOTO 600
WRITE(12,810)
810 FORMAT(1H0, T3, 'SEGMENT', T16, 'LENGTH', T29, 'DIAMETER', T43,
& 'PRESSURE', T59, 'C ADB', T73, 'FN', T86, 'DENSITY', T98, 'REYNOLDS NO')
DO 803 I = 1, N
IF (OP EQ 1.0) GOTO 803
REY(I) = 0.
ENDIF
803 WRITE(12,802) I, AD(I), DI(I), P(I), CA(I), FN(I), RHO(I), REY(I)
802 FORMAT(1H0, T6, I2, T11, 7(1PE12.4, 2X))
WRITE(12,706)
706 FORMAT(1H//)
WRITE(12,890)
890 FORMAT(T25, '***************')
WRITE(12,706)
WRITE(12,700)
700 FORMAT(T40, 'THEORETICAL RESULTS')
WRITE(12,706)
WRITE(12,830)
830 FORMAT(1X, T10, 'FREQUENCY', T35, 'ATTENUATION', T52, 'PHASE SHIFT', T79
& 'GAIN', T101, 'TURBULENCE')
WRITE(12,850)
850 FORMAT(1X, T10, 'INDICATOR')
WRITE(12,840)
840 FORMAT(1X, T5, '(HZ)', T13, '(NON-DIMENSIONAL)', T32, '(NON-DIMENSIONAL)
& T55, '(DEG)', T70, '(DB)', T82, '(NON-DIMENSIONAL)')
WRITE(12,706)
C NNN IS THE COUNTER TO SKIP THE HEADINGS WRITE STATEMENT.
NNN = 2
600 WRITE(12,702) Y, OX, OY, BETAD, GPT, RTPT, TURB
702 FORMAT (1X,T4,F5.0,T14,F10.5,T34,F10.5,T54,F7.2,T66,1PE12.4,T83,1PE
&12.4,T105,T2)
WRITE (14,97) Y,GPT
97 FORMAT (5,F5.0,T20,1PE12.4)
DBT(M)=GPT
OME(M)=Y
PHASE(M)=BETAD
80 CONTINUE
   GO TO (85,611),IND
85 Y=0.
C DW IS THE FREQUENCY INCREMENT (HZ), NST IS THE NUMBER
C OF THEORETICAL VALUES CALCULATED (0-1000 HZ)
   DW=5.
   NST=200
   IND=2
   GO TO 40
C
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C THIS SECTION IS FOR THE CALCOMP PLOTTER. ONE SET OF
C EXPERIMENTAL DATA MUST BE INCLUDED WITH EACH RUN.
C
C NPTS IS NUMBER OF EXPERIMENTAL POINTS TO BE INPUT
C LSMB IS THE PLOTTER SYMBOL TO BE USED FOR GPX AND PHD.
C SEE A CALCOMP PLOTTER USERS MANUAL FOR DESCRIPTION.
C FREQ IS THE EXPERIMENTAL FREQUENCY (HZ)
C PS IS THE SENDING PRESSURE, EXPERIMENTAL (MV)
C PR IS THE RECEIVING PRESSURE, EXPERIMENTAL (MV)
C PHT IS THE EXPERIMENTAL PHASE DELAY (MSEC)
C
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C NEGADES THE PLOT ROUTINE IF ICAS .LE. ZERO
   611 IF (ICAS)502,502,610
   610 CALL FACTOR(0.625)
        CALL RECT(0.,0.,9.6,12.0,0.,3)
        CALL PLOT(.6,.6,-3)
        OMG(M+1)=0.
        OMG(M+2)=100.
        DBT(M+1)=-20.
        DBT(M+2)=5.0
C DRAWS AXES FOR GAIN PLOT
        CALL AXIS(0.,0.,'FREQUENCY (HERTZ)',-17,10.0,0.,OMG(M+1),OMG
&     (M+2))
CALL AXIS(0.,0.,'GAIN (DECIBELS)',15,8.,90.,DBT(M+1),DBT & (M+2))

C PLOTS CURVE FOR THEORETICAL GAIN
CALL LINE(OMG,DTB,M,1,0,4)
READ(11,705)TITLE
705 FORMAT(A50)
READ(11,**NPTS
WRITE(12,706)
WRITE(12,890)
WRITE(12,706)
WRITE(12,860)
WRITE(12,706)
WRITE(12,870)
860 FORMAT(T45,'EXPERIMENTAL DATA')
WRITE(12,207)
207 FORMAT(1H//,T14,'N' ,T20,'FREQUENCY (HZ)' ,T42,'PRESSURE (MV)' ,T66, &'GAIN (DB)' ,T82,'PHASE SHIFT (DEG)')
WRITE(12,870)
870 FORMAT(1H//,T39,'SENDING' ,T49, 'RECEIVING')
WRITE(12,706)
J=0
DO 69 I=1,NPTS
READ(11,**FREQ,PS,PR,PHT
GPX=20.*ALOG10(PR/PS)
PHD=-PHT*FREQ*.36
IF(PHD.LT.-360.)THEN
PHD=PHD+360.
ENDIF
WRITE(12,209) I,FREQ,PS,PR,GPX,PHD
209 FORMAT(X,T12,3,T23,F5.0,T39,F6.1,T50,F6.1,T64,F10.5,T84,F10.5)
DB (I)=GPX
OMGX (I)=FREQ
IF (PHD.GT.0) GOTO 69
J=J+1
PH(J)=PHD
OMGP(J)=FREQ
JMAX=J
69 CONTINUE
OMGX(NPTS+1)=OMG(M+1)
DB(NPTS+1)=DBT(M+1)
OMGX(NPTS+2)=OMG(M+2)
DB(NPTS+2)=DBT(M+2)

C PLOTS EXPERIMENTAL GAIN DATA
LSMB=0
CALL LINE(OMGX,DTB,NPTS,1,-1,LSMB)
CALL SYMBOL(3,13,...,165,TITLE,0.,50)
GOTO(111,112,113)CASE
111 THEOR='MOD CONST LRC'
GOTO 114
112 THEOR='CONST LRCG'
GOTO 114
113 THEOR='LAMINAR'

145
CONTINUE
CALL SYMBOL(3.,12.5.,165.,THEOR,0.,50)
CALL PLOT(15.0.,-6.,-3)
CALL FACTOR(0.625)
CALL RECT(0.,0.,9.6.,12.0.,0.,3)
CALL PLOT(.6.,.6.,-3)
OMG(M+1)=0.
OMG(M+2)=100.
PHASE(M+1)=360.
PHASE(M+2)=45.
C
CALL AXIS(0.,0.,'FREQUENCY (HERTZ)',-17.,10.,0.,OMG(M+1), &OMG(M+2))
CALL AXIS(0.,0.,'PHASE ANGLE (DEGREES)',21.,8.,90.,PHASE(M+1), &PHASE(M+2))
C
OMG(JMAX+1)=OMG(M+1)
OMG(JMAX+2)=OMG(M+2)
PH(JMAX+1)=PHASE(M+1)
PH(JMAX+2)=PHASE(M+2)
C
CALL LINE(OMG,PHASE,M,1,0,4)
CALL LINE(OMG,PH,JMAX,1,-1,LSMB)
CALL SYMBOL(3.,13.,165.,TITLE,0.,50)
CALL SYMBOL(3.,12.5.,165.,THEOR,0.,50)
1003 CALL PLOT(10.0.,0.,-3)
C
CALL PLOTE
502 STOP
END
C
SUBROUTINE RTCMP(X,Y,A,B)
CALL ANGL(TEMP,A,B)
TEMP=.5*TEMP
Y=A*A+B*B
X=SQRT(Y)
X=SQRT(X)
Y=X*SIN(TEMP)
X=X*COS(TEMP)
RETURN
END
C***********************************************
C
SUBROUTINE CMPMP(X,Y,A1,A2,B1,B2)
C MULTIPLIES TWO COMPLEX NUMBERS
C (A1+A2J)*(B1+B2J)=X+YJ
C
X=A1*B1-A2*B2
Y=A1*B2+A2*B1
RETURN
END
C
C***********************************************
C
C FINDS THE QUOTIENT OF TWO COMPLEX NUMBERS
SUBROUTINE CMPDV(C1,C2,A1,A2,B1,B2)
C CALCULATES THE QUOTIENT OF TWO COMPLEX NUMBERS
C
A1+A2J/B1+B2J
C1=A1*B1+A2*B2
C1=C1/TEMP
C2=C2/TEMP
RETURN
END
C
C***********************************************
C
SUBROUTINE HSINX(ARG,X)
C CALCULATES THE HYPERBOLIC SINE OF X
A=EXP (X)
B=EXP (-X)
A=A-B
ARG=.5*A
RETURN
END
C
C***********************************************
C
SUBROUTINE HCOSX(ARG,X)
C CALCULATES THE HYPERBOLIC COSINE OF X
A=EXP (X)
B=EXP (-X)
A=A+B
ARG=.5*A
RETURN
END
SUBROUTINE ANGL(C,A,B)
C CALCULATES THE MAGNITUDE OF ANGLE X (IN RADS) AND ASSIGNS C
THE PROPER SIGN BASED ON THE CARTESIAN COORDINATES (A,B)
DATA PI/3.1415926/
C=ABS(B/A)
C=ATAN(C)
IF (A.GT.0.) GO TO 5
IA=1
GO TO 7
5 IA=0
7 IF (B.GT.0.) GO TO 10
IB=2
GO TO 15
10 IB=0
15 IA=IA+IB+1
GO TO (35,30,25,20),IA
20 C=C-PI
GO TO 35
25 C=-C
GO TO 35
30 C=PI-C
35 RETURN
END

SUBROUTINE CALZIN(AZIN1,BZIN1,AZRN1,BZRN1,AZIN2,BZIN2,AN1,DII,
&BTN1)
TEMP=AN1*DII
CALL Hcosx(ARG1,TEMP)
CALL HSinx(ARG2,TEMP)
TEMP=BTN1*DII
CSBI1=COS(TEMP)
SNBI1=SIN(TEMP)
ZR=0.
CALL CMPMP(TEM1,TEM2,AZIN2,BZIN2,ARG1,ZR)
CALL CMPMP(TEM3,TEM4,AZRN1,BZRN1,ARG2,ZR)
CALL CMPMP(TEM5,TEM6,AZIN2,BZIN2,ARG2,ZR)
CALL CMPMP(TEM7,TEM8,AZRN1,BZRN1,ARG1,ZR)
A1=TEM1+TEM3
B1=TEM2+TEM4
A2=TEM5+TEM7
B2=TEM6+TEM8
CALL CMPMP(TEM1,TEM2,A1,B1,CSBI1,ZR)
CALL CMPMP(TEM5,TEM6,A2,B2,CSBI1,ZR)
CALL CMPMP(TEM7,TEM8,A1,B1,ZR,SNBI1)
CALL CMPMP(TEM3,TEM4,A2,B2,ZR,SNBI1)
TEM3 = TEM3 + TEM1
TEM4 = TEM4 + TEM2
TEM7 = TEM7 + TEM5
TEM8 = TEM8 + TEM6
CALL CMPDV (TEM1, TEM2, TEM3, TEM4, TEM7, TEM8)
CALL CMPMP (AZIN1, BZIN1, TEM1, TEM2, AZRN1, BZRN1)
RETURN
END

C
C****************************************************************************
C
C SUBROUTINE ZEBRA(C1, C2, A1, A2, B1, B2)
C Computes the inverse of the sum of the inverses of two
C complex numbers.
C
C C1+C2J = 1/((1/A1+A2J)+ (1/B1+B2J))
C D1 = 1.
C D2 = 0.
CALL CMPDV( ARG1, ARG2, D1, D2, A1, A2)
CALL CMPDV( ARG3, ARG4, D1, D2, B1, B2)
ARG1 = ARG1 + ARG3
ARG2 = ARG2 + ARG4
CALL CMPDV( C1, C2, D1, D2, ARG1, ARG2)
RETURN
END
Appendix C: Equipment

<table>
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<th>Item</th>
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<th>Model</th>
<th>Serial No.</th>
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<td>-297</td>
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<td>Measurement</td>
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<td>ShoRate</td>
<td>USAF9175-83</td>
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<td>Gaertner</td>
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Bibliography


VITA

Major Stephen J. McKenna was born in 1950. He graduated from the Royal Military College of Canada in Kingston, Ontario in 1972 and was selected to attend the Royal Military College of Canada in Kingston, Ontario under the Regular Officer Training Plan. He graduated in 1976 with a Bachelor of Chemical Engineering (Nuclear Science) and received his commission in the Canadian Armed Forces as an Aerospace Engineering Officer. He has served as a maintenance officer on C-130 and CC-138 aircraft, and following specialized training in the United Kingdom, as a training officer at the Canadian Forces School of Aerospace and Ordnance Engineering. Major McKenna entered the School of Engineering, Air Force Institute of Technology, in August 1983.

Permanent address:
Title: DYNAMIC RESPONSE OF FLUID TRANSMISSION LINES

Thesis Chairman: Dr. Milton E. Franke
The theoretical predictions of the dynamic response of circular cross-section pneumatic transmission lines were compared with experimental results. Tests were conducted in blocked line situations as well as in both laminar and turbulent mean flow conditions. The lines used in the study were 24 in. in length with inside diameters of 0.041, 0.119, and 0.195 in. Tests were conducted over the frequency range 20 to 1000 Hz and over a range of mean flow Reynolds numbers from 0 to 16000.

Theoretical solutions were obtained from computer simulations using a laminar flow model and two turbulent flow models. These models were developed from the Navier-Stokes equation and include components to account for viscosity and radial heat transfer in the fluid.

An examination of the turbulent models was conducted based on a nondimensional frequency and attenuation. This led to modification of one of the turbulent models to include a frequency dependent term in the conductance.

The effect of flow conditions upstream of the test line and the effect of orifice design on the dynamic response were studied.

Experimental results showed good agreement with results of earlier studies. The use of a bypass device upstream of the test line was found to have no effect on line response.

The reconfiguration of the orifice devices to more closely match theoretical models produced no noticeable change in the results.