USE OF STATE ESTIMATION TO CALCULATE ANGLE-OF-ATTACK
POSITION ERROR FROM (U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI. T H THACKER
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USE OF STATE ESTIMATION TO CALCULATE
ANGLE-OF-ATTACK POSITION ERROR
FROM FLIGHT TEST DATA

THESIS
Thomas H. Thacker
Captain, USAF
AFIT/GAE/AA/85J-3

DEPARTMENT OF THE AIR FORCE
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AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology Air University
In Partial Fulfillment of the Requirements for the Degree of Master of Science in Aeronautical Engineering

Thomas H. Thacker, B.S.
Captain, USAF

October 1985

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Preface

The purpose of this project was to determine the position errors of the angle-of-attack (AOA) sensors on aircraft using state estimation with flight test data. Aircraft from the USAF Test Pilot School (TPS) were used to obtain flight test data, and Kalman filtering was used to process the data. The results of this project are significant to future flight test projects where an accurate AOA measurement is required.

Aircraft AOA position errors are caused by aerodynamic factors such as local flow and upwash. The first step in finding those errors was to determine the equations for calculating the true AOA from other available flight test parameters. Since the inputs to those equations were from instrumentation on flight test aircraft, they were noise corrupted and had to be filtered. I used state estimation in a Kalman filter program to calculate an "optimal" true AOA. The data were obtained from flights in a T-38A Talon, a two-seat supersonic trainer modified with an instrumented Vought yaw and pitch system noseboom. The position errors calculated in this report are only good for that aircraft and nose boom configuration. However, the methods used are applicable to all properly instrumented aircraft.

I would like to thank my thesis advisors, Major (Dr.) James T. Silverthorn of the USAF TPS and Dr. Robert A. Calico of AFIT, for their help in this project. I would also like to
thank the test pilots I flew with on the data flights, Major Philip B. Arnold and Captain David J. Eichhorn of the USAF TPS. But most of all I would like to thank my wife, Diana, for her help and understanding over the last two years of AFIT and TPS.
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<td>cg</td>
<td>center of gravity</td>
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<td>degrees</td>
<td>degrees</td>
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<tr>
<td>F</td>
<td>applied force</td>
<td>pounds</td>
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<td>component of force in z direction</td>
<td>pounds</td>
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<td>g</td>
<td>acceleration due to gravity</td>
<td>feet/sec&lt;sup&gt;2&lt;/sup&gt;</td>
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<td>h</td>
<td>vertical velocity</td>
<td>feet/sec</td>
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<td>hertz</td>
<td>cycles/sec</td>
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<td>mass</td>
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<td>MAC</td>
<td>mean aerodynamic chord</td>
<td>feet</td>
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<td>NI&lt;sub&gt;j&lt;/sub&gt;</td>
<td>noseboom instrumentation unit</td>
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<td>n&lt;sub&gt;z&lt;/sub&gt;</td>
<td>normal load factor</td>
<td>g</td>
</tr>
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<td>radians/sec</td>
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<tr>
<td>Q, Q&lt;sub&gt;o&lt;/sub&gt;, q</td>
<td>pitch rate</td>
<td>radians/sec</td>
</tr>
<tr>
<td>R</td>
<td>yaw rate</td>
<td>radians/sec</td>
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<tr>
<td>rms</td>
<td>root mean squared</td>
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<td>$U, U_o$</td>
<td>true airspeed</td>
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<td>component of vehicle velocity along x-axis</td>
<td>feet/sec</td>
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<td>$u(t), u(t_j)$</td>
<td>control input vector</td>
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<tr>
<td>$V, V_o, v$</td>
<td>component of vehicle velocity along y-axis</td>
<td>feet/sec</td>
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<td>$\mathbf{V}_T$</td>
<td>vehicle velocity vector</td>
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</tr>
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<td>$W, W_o, w$</td>
<td>component of vehicle velocity along z-axis</td>
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<td>input noise vector</td>
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<td>feet</td>
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<td>$\mathbf{x}(t)$</td>
<td>state vector</td>
<td>---</td>
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<tr>
<td>$\hat{\mathbf{x}}(t), \hat{\mathbf{x}}(t_j)$</td>
<td>estimated state vector</td>
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<td>$\mathbf{x}(t_0)$</td>
<td>initial condition of state vector</td>
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<td>XYZ</td>
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<td>vehicle body axis</td>
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<td>$y_a$</td>
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<td>yaw and pitch system</td>
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</tr>
<tr>
<td>$z(t), z(t_j)$</td>
<td>measurement history vector</td>
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<tr>
<td>$a$</td>
<td>angle-of-attack</td>
<td>degrees</td>
</tr>
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<td>$a_c$</td>
<td>true AOA</td>
<td>degrees</td>
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<td>$a_m$</td>
<td>measured AOA at aircraft sensor</td>
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<td>$a_o$</td>
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<td>$a_{o_T}$</td>
<td>bias between true and measured AOA</td>
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<td>Δcg</td>
<td>longitudinal distance from cg to fixed point</td>
<td>feet</td>
</tr>
<tr>
<td>Δt</td>
<td>time interval</td>
<td>seconds</td>
</tr>
<tr>
<td>γ</td>
<td>flight path angle</td>
<td>degrees</td>
</tr>
<tr>
<td>ω̇</td>
<td>vehicle rotation vector</td>
<td>radians/sec</td>
</tr>
<tr>
<td>σ₂aₖ</td>
<td>variance of normal acceleration</td>
<td>feet²/sec⁴</td>
</tr>
<tr>
<td>σ₂q</td>
<td>variance of pitch rate</td>
<td>deg²/sec²</td>
</tr>
<tr>
<td>σ₂θ</td>
<td>variance of pitch angle</td>
<td>degrees²</td>
</tr>
<tr>
<td>θ</td>
<td>pitch angle</td>
<td>degrees</td>
</tr>
<tr>
<td>θₘ</td>
<td>measured pitch angle</td>
<td>degrees</td>
</tr>
<tr>
<td>(</td>
<td>time rate of change</td>
<td>---</td>
</tr>
<tr>
<td>-q</td>
<td>strength of system noise</td>
<td>---</td>
</tr>
<tr>
<td>r</td>
<td>strength of measurement noise</td>
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Abstract

This project determined the position errors of an aircraft's angle-of-attack (AOA) sensor using state estimation with flight test data. The position errors were caused by local flow and upwash and were found to be a function of AOA and Mach number. The test aircraft used in this project was a T-38A Talon supersonic trainer from the USAF Test Pilot School configured with a Vought yaw and pitch system noseboom and an internal Aydin-Vector data acquisition system (DAS).

The position errors were found by calculating the true AOA using equations of motion and DAS parameters. The data from the DAS were noise corrupted and had to be filtered. This was accomplished using state estimation in a Kalman filter. The estimated AOA was compared to the measured AOA from the noseboom sensor to obtain the position error. Accurate position errors were obtained, even in dynamic maneuvers. The method was accurate enough to identify a hysteresis error in the T-38A's AOA sensor of +/- 0.5 degrees, which was confirmed by ground calibration. This method should be considered in future AOA error testing.
USE OF STATE ESTIMATION TO CALCULATE
ANGLE-OF-ATTACK POSITION ERROR
FROM FLIGHT TEST DATA

I. Introduction

Problem

Angle-of-attack (AOA) is a primary parameter of performance and stability-and-control in flight test. Unfortunately, the AOA measured by the aircraft sensors has a position error caused by the aerodynamic influence of the aircraft body. The first source of this position error is local flow about the AOA sensor caused by aerodynamic interference, boundary layer effects, and shock interaction. The second source of the position error is upwash from aircraft components such as the fuselage and wing. The accurate determination of AOA position error is a significant problem in flight test (1:7).

Background

The magnitude of the AOA position error is evident with the USAF/CAL variable stability NT-33 airplane, a jet trainer used in the USAF Test Pilot School (TPS) curriculum. The NT-33 has a fuselage-mounted AOA vane which is subject to large flow and upwash effects. Figure 1 shows the NT-33 AOA (and sideslip, which has similar errors) position error
correction factors (2:163). At Mach 0.6, the NT-33 has an AOA position error correction factor of 1.75, which means the measured AOA is 1.75 times the true AOA. The AOA position errors were determined by combining wind tunnel and flight test data. One data point was determined, and a line was extrapolated over a range of Mach numbers. AOA position error was assumed to be a function of Mach number alone (2:162-163).

Figure 1. USAF/CAL NT-33 Angle-of-Attack and Sideslip Position Error Correction Factors (2:163)
Wind tunnel calibration is a commonly used method of determining AOA position error at the Air Force Flight Test Center (AFFTC). An entire noseboom instrumentation unit (NBIU) can be installed in a wind tunnel and tested over a range of conditions. One of the AFFTC standard NBIUs, a Conrac adapter with a Rosemount Model 852G pitot-static probe, was tested in the NASA/Ames Research Center wind tunnels in 1973 (3). The AOA position error was found to be small, less than 7.5% (3:38). The wind tunnel test showed AOA position error to be a function of Mach number and sideslip angle. Reynolds number effects were not discovered.

Very little flight testing has been accomplished to determine AOA position error. One technique that has been used is to mount flight path accelerometers on the test aircraft and fly 1 g, wings level stable points over a range of Mach numbers, sideslips, and aircraft weights. True AOA ($\alpha$) is determined from the equation:

$$\alpha = \theta - \gamma$$  \hspace{1cm} (1)

where $\theta$ is pitch angle in wings level flight and $\gamma$ is the flight path angle. The T-46 jet trainer Combined Test Force is planning to use this technique to calibrate their AOA sensors when flight testing begins in October 1985. They plan to eliminate noise in the data by using a 2 hz low bypass Butterworth filter. Unfortunately, not all aircraft can be
equipped with flight path accelerometers due to the size and required cost. Furthermore, the range of AOA that is attainable at a particular Mach number is very limited for straight and level flight, since altitude is the only variable that can be adjusted. As an example, the T-38A, at Mach 0.83, flies at +2.5 degrees AOA at 25,000 feet and at +1 degrees AOA at 15,000 feet. A technique to obtain AOA position error during dynamic maneuvers is required.

A new flight test method of determining AOA position error is through the use of MMLE3, a modified maximum likelihood estimation program (4). MMLE3 uses the aircraft mathematical model with estimated stability and control (S&C) derivatives. Flight test maneuvers such as elevator doublets are flown, and MMLE3 tries to match the time history of the maneuver with the time history of the math model by changing the estimated S&C derivatives. MMLE3 also calculates an AOA position error factor for the maneuver (4:3). MMLE3 is not extremely accurate and requires numerous flight test maneuvers to increase its accuracy. An easier and more accurate technique is needed to calibrate AOA sensors.

Scope

The purpose of this project was to determine the position errors on the AOA sensors of aircraft using flight test data available from standard data acquisition systems (DAS). Initially, AOA position error was determined using deterministic equations from straight and level flight.
Problems with this technique suggested a more general approach. State estimation in the form of Kalman filtering was used to filter out noise on the flight test data and calculate an "optimal" true AOA during dynamic maneuvers. This true AOA was compared to the measured AOA to determine the position errors. USAF TPS T-38A aircraft were used to collect data and the AOA position errors are valid for those aircraft. However, the technique will work for any properly instrumented aircraft.

Objectives

The objectives of this project were to:

(1) Determine the equations necessary to calculate the true AOA from flight test data.

(2) Use state estimation (Kalman filtering) to filter noise from the flight test data and calculate an "optimal" true AOA.

(3) Collect the flight test data needed to compute the "optimal" true AOA.

(4) Calculate the AOA position error correction factors for the test aircraft.
II. Angle-of-Attack Equations

AOA Correction Factor

The AOA position error correction factor, $K_a$, is calculated from the equation (4.3):

$$a_c = \frac{a_m}{K_a} + \frac{q (x_a + \Delta cg)}{U} - \frac{p (Y_a)}{U} \tag{2}$$

where $a_c$ is the true AOA of the aircraft and $a_m$ is the AOA measured by the aircraft's sensor.

The term:

$$\frac{q (x_a + \Delta cg)}{U} \tag{3}$$

corrects the measured AOA for pitch rate ($q$) effects. The terms $x_a$ and $\Delta cg$ account for the longitudinal distance from the cg to the AOA sensor. $U$ is aircraft true airspeed.

The term:

$$\frac{p (Y_a)}{U} \tag{4}$$

corrects the AOA for roll rate ($p$) effects. The term $Y_a$ is the lateral distance from the aircraft centerline to the AOA sensor.

All flight testing for this project was accomplished wings level. Since there was no roll rate, equation (4) drops out of equation (2). Pitch rate, true airspeed, and measured AOA are parameters measured by the aircraft DAS. Longitudinal
distance from the cg to the AOA sensor is a function of aircraft fuel weight and is easily calculated. The only remaining unknown is true AOA.

Equations of Motion

In order to calculate the true AOA, an equation was needed that used parameters available from the aircraft DAS. Equation (1) showed the angular relationship between AOA, pitch angle, and flight path angle in wings level flight. Since many test aircraft, including the TPS aircraft used in this project, do not have flight path accelerometers, flight path angle \( \gamma \) must be calculated by:

\[
\sin \gamma = \frac{h}{U}
\]

where \( h \) is the vertical velocity of the aircraft. Vertical velocity can be calculated as the time rate of change of the altitude from the DAS.

Another equation to calculate true AOA comes from the aircraft's equations of motion (5.3.21-3.51). The vector equation for applied force (F) is:

\[
\bar{F} = m \frac{d\bar{V}}{dt} \bigg|_{XYZ}
\]

which applies to inertial space. Assuming the forces resulting from the earth's rotation and coriolis effects to be negligible, a fixed earth axis system can be used instead of
inertial space. The movement of a vehicle with respect to a fixed earth axis is shown in Figure 2.

Figure 2. Relationship of Fixed Earth Axis (XYZ) to Vehicle Body Axis (xyz) (5:3.22)

The vector equation for the time rate of change of velocity from one axis system to another is:

$$\frac{d\vec{V}_T}{dt} \bigg|_{XYZ} = \frac{d\vec{V}_T}{dt} \bigg|_{xyz} + \vec{\omega} \times \vec{V}_T$$

(7)

where XYZ is the fixed earth axis and xyz is the aircraft body axis.
Equation (6) now becomes:

\[ \mathbf{F} = m \left[ \frac{d\mathbf{V}_T}{dt} \right]_{xyz} + \mathbf{\dot{\omega}} \times \mathbf{V}_T \]  

(8)

where aircraft velocity (\(\mathbf{V}_T\)) can be written as:

\[ \mathbf{V}_T = U \mathbf{i} + V \mathbf{j} + W \mathbf{k} \]

and aircraft rotation (\(\mathbf{\dot{\omega}}\)) can be written:

\[ \mathbf{\dot{\omega}} = P \mathbf{i} + Q \mathbf{j} + R \mathbf{k} \]

Equation (8) now becomes:

\[ \mathbf{F} = m \begin{bmatrix} U \mathbf{i} + V \mathbf{j} + W \mathbf{k} \end{bmatrix} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ P & Q & R \\ U & V & W \end{bmatrix} \]

(9)

Taking the cross product of the inner term and expanding:

\[ \mathbf{F} = m \left[ U \mathbf{i} + V \mathbf{j} + W \mathbf{k} + (QW - RV) \mathbf{i} - (PW - RU) \mathbf{j} + (PV - QU) \mathbf{k} \right] \]

(10)

Looking at only the z component of force gives:

\[ F_z = m \left( \dot{W} + PV - QU \right) = m \left( a_z \right) \]

(11)

where (\(\dot{W} + PV - QU\)) equals the normal acceleration, \(a_z\).

Assuming that the aircraft motion consists of small deviations from an initial reference condition, the above values can be written as:
\[ \dot{W} = \dot{W}_o + \dot{w} \]
\[ P = P_o + p \]
\[ V = V_o + v \]
\[ Q = Q_o + q \]
\[ U = U_o + u \]

where the small case values are the small perturbations from the initial values. Assuming the aircraft starts from wings level, steady straight symmetrical flight:

\[ \dot{W}_o = P_o = V_o = Q_o = 0 \]

Equation (11) now becomes:

\[ m \left( \dot{w} + pv - qU \right) = m \left( a_z \right) \quad (12) \]

All testing during this project was done wings level, so roll rate (p) is zero. The change in velocity (u) is assumed to be small, so U = U_o. Dividing each side by m(U_o) gives:

\[ \frac{\dot{w}}{U_o} - q = \frac{a_z}{U_o} \quad (13) \]

Assuming small AOA gives the relationship:

\[ \dot{a} = \frac{\dot{w}}{U_o} \quad (14) \]

which can be substituted into equation (13) and rearranged to give:

\[ \dot{a} = q + \frac{a_z}{U_o} \quad (15) \]
where both pitch rate and normal acceleration are measured by the aircraft DAS. Assuming a finite time interval \( \Delta t \), \( \dot{a} = \Delta a \). An iterative equation can be formed where:

\[
a_{i+1} = a_i + \Delta a (\Delta t)
\]

(16)

**Computer Program AOAOP**

A FORTRAN computer program was designed to calculate true AOA using both the angle relationship (equations (1) and (5)) and the iterative relationship (equations (15) and (16)). This program is called AOAOP and is shown in Appendix B. The program reads the required flight test parameters from the DAS, makes necessary pitot-static corrections, calculates true AOA using both methods, and calculates the AOA position error correction factor (from equation (2)). The program works with flight test data from either USAF TPS T-38As or RF-4Cs. No data filtering is accomplished.

**Results**

Sample RF-4C flight test data from 1 g, wings level flight was processed by the program AOAOP. Test data was sampled at the highest rate possible for the aircraft DAS, 8 times per second (a complete DAS description is in Chapter IV). Figure 3 is a plot of the AOA calculated using the angle relationship. The calculated AOA was very sensitive to noise in the altitude channel and was only accurate by averaging over a time span of 3 to 4 seconds in 1 g, wings level flight.
Figure 3. True AOA Calculated Using the Angle Method With Unfiltered Data Compared to Measured AOA From the Aircraft AOA Sensor
Figure 4 is a plot of the AOA calculated using the iterative method. The resulting AOA is less subject to noise, as the pitch rate and normal acceleration channels were fairly noise free. The data pitch rate and normal acceleration values were also corrected for bias measured while on the ground (bias was measured from 0 deg/sec pitch rate and 1 g normal acceleration). The major problem with the iterative method is its initial value, $\alpha_0$, which must be calculated beforehand. The program AOAOP uses as $\alpha_0$ the AOA calculated from the angle relationship averaged over a time interval of 3 to 4 seconds of 1 g flight. However, the AOA values are still corrupted by noise and are only as good as the resolution of the DAS. Some type of filtering is needed to optimize the true AOA.
Figure 4. True AOA Calculated Using the Iterative Method With Unfiltered Data Compared to Measured AOA From the Aircraft AOA Sensor
III. State Estimation

State Equations

In order to use a digital computer to filter the flight test data and compute an "optimal" true AOA, the system dynamics need to be modelled (6:174). One way to model the system is with linear differential equations of the form:

\[
\dot{x}(t) = F(t) x(t) + B(t) u(t) \quad (17)
\]

\[
\dot{z}(t) = H(t) x(t) \quad (18)
\]

where \( x(t) \) is the state vector, \( u(t) \) is the control input, and \( z(t) \) is the measurement history. One differential equation for angle-of-attack comes from equation (15):

\[
\dot{\alpha} = q + \frac{a_z}{U_o} \quad (15)
\]

Another equation to use in the state equations is the pitch rate equation valid for wings level flight:

\[
\dot{\theta} = q \quad (19)
\]

Since \( \dot{\alpha} \) and \( \dot{\theta} \) are not directly related, they become functions of the inputs \( q \) and \( a_z \). The only useful parameter to measure is pitch angle, \( \theta \), since the measured AOA has an undetermined position error. Combining equations (15) and (19) together gives the state equations:

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{1}{U_o} \\
a_z
\end{bmatrix}
\begin{bmatrix}
q
\end{bmatrix} \quad (20)
\]
The measurement equation is:

\[
\begin{bmatrix}
\theta_m
\end{bmatrix} = \begin{bmatrix}
0 & 1
\end{bmatrix} \begin{bmatrix}
a \\
\theta
\end{bmatrix}
\]  \hspace{1cm} (21)

The above equations define the matrices \( F(t) \), \( B(t) \), and \( H(t) \). This is the math model to compute a true AOA, but nothing in the model filters the noise in the data.

Kalman Filtering

A Kalman filter provides the best method to "optimize" the true AOA from available flight test data. A Kalman filter will combine the pitch angle measurements, plus prior knowledge about the system and measuring devices, to produce an estimate of true AOA in such a manner that the error in true AOA is minimized statistically (7:5). The filter uses the state equations plus a statistical description of the system noises, measurement noises, and uncertainty in the dynamics model (7:4). The Kalman filter assumes that the system can be described by a linear model, and that system and measurement noises are white and Gaussian (7:7).

The original system model, equations (17) and (18), is augmented by (7:146):

\[
\begin{align*}
\dot{x}(t) &= F(t) x(t) + B(t) u(t) + G(t) w(t) \\
\dot{z}(t) &= H(t) x(t) + v(t)
\end{align*}
\]  \hspace{1cm} (22) (23)

where the system is now driven by the input vector \( u(t) \) and noise vectors \( w(t) \) and \( v(t) \).
The vector $w(t)$ models the system noise as white and Gaussian with mean of zero and strength $\bar{q}$, described as:

$\mathbb{E}[w(t)] = 0$ \hspace{1cm} (24)

$\mathbb{E}[w(t) w^T(t + \tau)] = \bar{q} \delta(\tau)$ \hspace{1cm} (25)

where $\bar{q}$ is a measure of the uncertainty in the input vector $u(t)$. The noise in the values from the aircraft DAS is assumed to be white since it is random and uncorrelated.

The vector $v(t)$ models the measurement noise as white and Gaussian with mean zero and strength $\bar{r}$, described as:

$\mathbb{E}[v(t)] = 0$ \hspace{1cm} (26)

$\mathbb{E}[v(t) v^T(t)] = R$ \hspace{1cm} (27)

where $R$ is a measure of the uncertainty in the measurement vector $z(t)$.

In order for the Kalman filter to propagate the system, the estimated state vector (denoted by $\hat{x}$) must be given an initial condition, $x(t_0)$, where $E$:

$\mathbb{E}[x(t_o)] = \hat{x}(t_o)$ \hspace{1cm} (28)

$\mathbb{E}[x(t_o) - \hat{x}(t_o)][x(t_o) - \hat{x}(t_o)]^T = P(t_o)$ \hspace{1cm} (29)

The equations to propagate and update the optimal estimate using Kalman filtering are fully derived in *Stochastic Estimation and Control Systems* (6:210-233). Since
the matrix $F(t)$ is a zero matrix (equation (20)), the propagation equations from a measurement at time $t_{j-1}$ to time $t_j$ become:

$$\hat{X}(t_j^-) = \hat{X}(t_{j-1}^+) + \Delta t \, B(t_j) \, \hat{X}(t_j^-)$$  \hspace{1cm} (30)

$$P(t_j^-) = P(t_{j-1}^+) + \Delta t \, G(t_j) \, Q \, G^T(t_j)$$  \hspace{1cm} (31)

where $^-$ denotes prior to the update and $^+$ denotes after the update. The matrix $B(t_j)$ is defined in equation (20). Matrix $G(t_j)$ is set equal to $B(t_j)$ so the noise in the input vector, $u(t_j)$, is modeled by the values in matrix $Q$. $Q$ becomes a $2 \times 2$ matrix which contains the uncertainties in the inputs $q$ and $a_z$. These two inputs are assumed independent, therefore $Q$ becomes a diagonal matrix of the form:

$$Q = \begin{bmatrix} \sigma_q^2 & 0 \\ 0 & \sigma_{a_z}^2 \end{bmatrix}$$  \hspace{1cm} (32)

where the diagonal values are constant with time.

The update equations at measurement time $t_j$ are:

$$K(t_j) = P(t_j^-) \, H^T(t_j) \, [ \, H(t_j) \, P(t_j^-) \, H^T(t_j) + R \, ]^{-1}$$  \hspace{1cm} (33)

$$\hat{X}(t_j^+) = \hat{X}(t_j^-) + K(t_j) \, [ \, Z(t_j) - H(t_j) \, \hat{X}(t_j^-) \, ]$$  \hspace{1cm} (34)

$$P(t_j^+) = P(t_j^-) - K(t_j) \, H(t_j) \, P(t_j^-)$$  \hspace{1cm} (35)

The matrix $H(t_j)$ is defined in equation (21). $K(t_j)$ is the gain matrix which specifies how much the measurement, $Z(t_j)$, is weighted in the update. $R$ is a $1 \times 1$ value modelling the
noise in the measurement, \( Z(t_j) \), which is pitch angle. \( R \) is in the form:

\[
R = \begin{bmatrix}
\sigma^2 & 0 \\
0 & \theta^2
\end{bmatrix}
\]

(36)

and is also constant in time.

The Kalman filter is ready to be put into a digital computer routine. Equations (30), (31), (33), (34), and (35) will propagate and update the system over time. The matrix \( x(t_j^+) \) contains the "optimal" values for \( \alpha \) and \( \theta \), of which \( \alpha \) is the "optimal" true AOA desired. Before the routine can be implemented, the Kalman filter must be "tuned" to determine the values for \( P(t_0) \), \( Q \), and \( R \).

**Filter Tuning**

The objective of filter tuning is to achieve the best possible estimation performance from a filter that is totally specified except for \( P(t_0) \), \( Q \), and \( R \). The covariance values in those matrices account for the actual noises and disturbances in the system and determine how adequately the model represents the real world system. The \( P(t_0) \) matrix determines the initial performance of the filter, and the \( Q \) and \( R \) matrices determine the long term performance (7:337).

The method of filter tuning used here is "covariance analysis" (7:337-339). The filter program is run with some assumed covariance values in the three matrices, \( P(t_0) \), \( Q \), and \( R \). The "true" root mean squared (rms) error, which is the error at each update between the filter's estimate and the
actual measurement, is plotted over time. The time history of the computed rms error, or what the filter calculates as its error, is plotted with the true error.

Figure 5 is an example of what these plots show. Plot 5(a) shows a filter that has a low computed error and weighs
the measurement too little. Plot 5(b) shows a filter that has too high a computed error and weighs the measurements too much. Plot 5(c) shows a filter that is just right - its computed error and true error are equal (7:339). For the Kalman filter used in this project, the values of $P(t_0)$, $Q$, and $R$ were varied until the computed rms errors and true rms errors were about equal. The Kalman filter was complete and ready to filter test data.

**Computer Program KALOPT**

A FORTRAN computer program was designed to calculate an "optimal" true AOA using the Kalman filter equations. This program is called KALOPT and is shown in Appendix C. The program reads the required flight test parameters from the DAS, makes necessary pitot-static corrections, and calculates an "optimal" true AOA each iteration. The program works with flight test data from either USAF TPS T-38As or RF-4Cs.

The Aydin-Vector DAS does not read normal acceleration, $a_z$, but instead reads normal load factor, $n_z$. The sign of $n_z$ is opposite from the standard body axis system: positive $n_z$ is through the top of the canopy. Also, $n_z$ includes acceleration due to gravity. The following equation, which assumes small pitch and roll angles, corrects $n_z$ to $a_z$:

$$a_z = -(n_z - 1) \times 32.2 \quad (37)$$

The Kalman filter needs to know the initial values for AOA and pitch angle to use as $x(t_0)$. KALOPT uses the first
DAS value for pitch angle as $\theta_0$. However, an initial AOA needs to be calculated since the DAS AOA values have the yet-to-be-determined position error. The value of $\theta_0$ is calculated using the angle method (equations (1) and (5)) used in the program AOA OPT. It averages the unfiltered true AOAs over a 3 to 4 second period in 1 g flight to calculate $\theta_0$.

KALOPT calculated the computed and true rms errors after each iteration. The values were varied from 0.001 to 1.0 during the covariance analysis. Changing the values of $P(t_0)$ changed the initial rms values, but had little effect on the overall results. When the $Q$ values were increased, the measurement was weighted more; the true rms error was less than the computed rms error. Increasing the $R$ value caused the measurement to be weighted less; the true rms error was greater than the computed rms error. These results agreed with the theory behind filter tuning. Based on the covariance analysis conducted using T-38A data, the following values caused the true and computed errors to be equal:

$$P(t_0) = \begin{bmatrix} 0.100 & 0 \\ 0 & 0.030 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.025 & 0 \\ 0 & 0.002 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.300 \end{bmatrix}$$

KALOPT reads in the covariance values from a separate file, so they can be easily changed without changing the program.
Results

The first attempt at Kalman filtering included another state equation formed by combining equations (1) and (5):

\[ h = (\theta - a) U_0 \]

The DAS altitude readout was used as a measurement along with pitch angle. Unfortunately, the noise of the altitude transducer in the T-38A DAS was too erratic and could not be modelled as Gaussian. Altitude was not used in KALOPT.

The program KALOPT processed the same RF-4C flight test data that was used in Chapter II. The test data was sampled 8 times per second. Figure 6 is a plot of the "optimal" true AOA calculated by KALOPT from that data. The measured AOA and the unfiltered true AOA calculated by AOAOPT are also shown. The "optimal" true AOA is quicker to return to a steady state value than the unfiltered true AOA. It is impossible to tell which AOA is more accurate as the actual true AOA is unknown.

A better way to see how the Kalman filter is working is to compare the measured pitch angle to the "optimal" pitch angle to see how well it filters over noise and resolution increments. Figure 7 is a plot of measured pitch angle and "optimal" pitch angle. The measured pitch angle only had a resolution of 0.7 degrees, and after two samples it immediately increased by that amount. The "optimal" pitch angle is a fairly smooth curve over the time span, which shows that the Kalman filter is working. The next step is to use flight test data to calculate the AOA position error.
Figure 6. Optimal True AOA Calculated by the Kalman Filter Compared to Measured AOA and Unfiltered True AOA
Figure 7. Optimal Pitch Angle Calculated by the Kalman Filter Compared to Measured Pitch Angle
IV. Flight Test

Test Item Description

The test aircraft was a USAF TPS T-38A Talon. The T-38A is a two place (tandem) jet trainer which is used extensively in the TPS curriculum. The aircraft is powered by two J85-GE-5 turbojet engines which give it a maximum capability of Mach 1.2 in level flight (8:6-6). Figure 8 is a photograph of a T-38A used at the TPS. A single T-38A, serial number (S/N) 68-8205, was used for all data flights in this project. Statistics on that airplane are shown in Table I. An important measurement is the distance from the cg of the aircraft to the AOA measuring vane, 25 feet. This length ($x_a$)

Figure 8. USAF T-38A Talon
is used to correct the true AOA for pitch rate (see equation (3)). The cg of the T-38A only shifts 0.3% mean aerodynamic chord (MAC) while consuming fuel, which is only 0.25 inches. Therefore, the Acg term from equation (3) can be neglected.

TABLE I
USAF T-38A Talon Statistics (S/N 68-8205)

<table>
<thead>
<tr>
<th>Engines:</th>
<th>Two J85-GE-5 Turbojets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions:</td>
<td></td>
</tr>
<tr>
<td>Length:</td>
<td>46 ft</td>
</tr>
<tr>
<td>CG to AOA Vane:</td>
<td>25 ft</td>
</tr>
<tr>
<td>Wingspan:</td>
<td>25 ft</td>
</tr>
<tr>
<td>MAC:</td>
<td>7 ft</td>
</tr>
<tr>
<td>Height:</td>
<td>12 ft</td>
</tr>
<tr>
<td>Weights:</td>
<td></td>
</tr>
<tr>
<td>Operating Weight:</td>
<td>8,533 lbs</td>
</tr>
<tr>
<td>Fuel (JP-4) Weight:</td>
<td>3,790 lbs</td>
</tr>
<tr>
<td>Takeoff Gross Weight:</td>
<td>12,323 lbs</td>
</tr>
<tr>
<td>Center of Gravity Movement:</td>
<td></td>
</tr>
<tr>
<td>CG w/ 3,790 lbs fuel:</td>
<td>18.4% MAC</td>
</tr>
<tr>
<td>CG w/ 400 lbs fuel:</td>
<td>18.1% MAC</td>
</tr>
</tbody>
</table>

The test T-38A, S/N 68-8205, was modified for flight testing. The most important modification concerning this project is a fully instrumented yaw and pitch system (YAPS) noseboom (9:A.1), shown in Figures 9(a) and 9(b). A complete diagram of the YAPS noseboom is in Appendix E. The YAPS noseboom has two vane-type sensors, one for AOA and one for sideslip angle. These vanes are in front of the fuselage,
Figure 9. T-38A Yaw and Pitch System Noseboom
away from the aerodynamic influence of the aircraft, so the AOA (and sideslip) position errors should be less than for fuselage mounted sensors.

The YAPS noseboom on the T-38A is made out of aluminum alloy. It has been structurally tested up to 8.3 g and only minimal bending resulted (9:D.43-D.53). As a result, bending was ignored during this evaluation. The YAPS noseboom is canted 4 degrees down from the aircraft centerline.

Instrumentation

An internal Aydin-Vector SAU-537 DAS was installed in the aircraft to measure flight test parameters. The following components of the DAS were used in this project: a vertical gyro installed in the nose section to measure pitch and roll angles; a three axis rate gyro installed in the nose section to measure pitch, roll, and yaw rates; a three axis accelerometer installed in the center fuselage (at the nominal cg location) to measure acceleration in the x, y, and z axes (10:1.1-1.8). Other instruments were installed in the aircraft for flight test, but they were not used in this project.

The test aircraft was equipped with an internal Conrac ATR-580T70 magnetic tape recorder in the aft cockpit to record the data parameters (9:A.1). Forty-eight data channels were recorded. Indicated airspeed and altitude were recorded with 16 bit precision, the other parameters for this project had 8 bit precision. Table II is a summary of the parameters used.
in this project with their maximum/minimum values, precision, and accuracy. The parameters were recorded 8 times per second.

**TABLE II**
Summary of Flight Test Parameters

**USAF T-38A S/N 68-8205**
Aydin-Vector SAU-537 DAS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Min Value</th>
<th>Max Value</th>
<th>Resolution</th>
<th>Accuracy</th>
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</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>feet</td>
<td>0</td>
<td>65000</td>
<td>1.030</td>
<td>0.103</td>
</tr>
<tr>
<td>Airspeed</td>
<td>knots</td>
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<td>1250</td>
<td>0.019</td>
<td>0.0019</td>
</tr>
<tr>
<td>AOA</td>
<td>degrees</td>
<td>-22</td>
<td>28</td>
<td>0.202</td>
<td>*0.101</td>
</tr>
<tr>
<td>Sideslip</td>
<td>degrees</td>
<td>-20</td>
<td>20</td>
<td>0.164</td>
<td>0.082</td>
</tr>
<tr>
<td>Pitch</td>
<td>degrees</td>
<td>-80</td>
<td>80</td>
<td>0.704</td>
<td>0.704</td>
</tr>
<tr>
<td>Roll</td>
<td>degrees</td>
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<td>180</td>
<td>1.408</td>
<td>2.816</td>
</tr>
<tr>
<td>Pitch Rate</td>
<td>deg/sec</td>
<td>-20</td>
<td>20</td>
<td>0.163</td>
<td>0.163</td>
</tr>
<tr>
<td>N</td>
<td>g</td>
<td>-3</td>
<td>6</td>
<td>0.037</td>
<td>0.0037</td>
</tr>
</tbody>
</table>

* Actual accuracy +/- 0.5 degrees due to hysteresis

Before any flight test was performed, all of the DAS instruments were ground calibrated and their calibration files updated. All important instruments were found to be working correctly except for the AOA transducer. It had a large hysteresis problem due to wear on its internal gearing. This hysteresis is shown in Figure 10. There is a +/- 0.5 degree error in true AOA depending on whether the vane is moving up or down. The AOA transducer was designed in the 1960s, and no replacement parts are available. The worn gears could not be fixed or replaced. This hysteresis will have a large effect on the flight test data.
Figure 10. Hysteresis Error in T-38A Angle-of-Attack Transducer
Flight Test Method

The purpose of the flight testing was to gather data to calculate true AOA and to see what factors affected the AOA position error. AOA position error is primarily a function of AOA, therefore the testing covered large AOA changes. Other possible factors that were considered in designing the maneuvers were Mach number, Reynold's number, and sideslip angle. All testing was conducted wings level due to the assumptions used in the AOA equations (see Chapter II).

Since Reynold's number was a possible factor, testing was performed at different altitudes. Due to the altitude restrictions of available supersonic airspace, 25,000 feet and 15,000 feet were chosen for the testing. In order to see the effects of sideslip on AOA, the first maneuver to be performed was a wings level, slowly varying sideslip using maximum rudder deflection in both directions. Thrust was varied to maintain Mach number constant. This maneuver was performed at different Mach numbers. Actual data points are shown in Appendix D (11:3).

After the sideslip maneuver, a roller coaster maneuver was performed to vary AOA as much as possible. From a 1 g trim condition, the aircraft nose was pulled up slightly, then pushed forward to the minimum load factor specified for that data point. An onset rate of 3 seconds per g minimum was desired throughout the maneuver. At the minimum load factor, the aircraft nose was pulled back to the maximum load factor.
specified for that data point. The aircraft nose was then pushed forward to regain 1 g level flight. All data points and load factor limits are in Appendix D (11:3). Thrust was varied to maintain constant Mach number during the maneuver.

All testing was performed in the cruise configuration (gear and flaps up) with no external stores. All T-38A Flight Manual (8) limitations were complied with. Additional restrictions in the T-38A AOA Position Error Test Plan (11) were followed.

Test Results

Three T-38A test flights were flown at the USAF Flight Test Center, Edwards AFB, California. A summary of these flights is shown in Appendix D. No data were gathered on one flight due to bad weather. The same aircraft, 68-8205, was used on all three flights due to scheduling availability. Future test programs using this method should fly different tail numbers to prevent bias from one aircraft's own peculiarities.

The wings level sideslip maneuver was performed at all data points. The maximum sideslip angle generated was +/- 3.8 degrees at 25,000 feet pressure altitude (Hc), Mach 0.45. No change in AOA was found at this point or any of the others. In wind tunnel testing performed on a Conrac NBIU, a noseboom similar to the T-38A YAPS noseboom, no AOA position error change was discovered until five degrees of sideslip (3:38-39).
The roller coaster maneuver was performed at all data points, and repeated at the 25,000 feet H, points. The data was reduced using the FORTRAN program KALOPT (see Chapter III). Measured AOA was plotted against the "optimal" true AOA (henceforth referred to as true AOA) at each point tested. These plots are Figures 12 - 29, Appendix A. The data points plot out fairly linear, which shows that the equations and Kalman filtering worked. Most of the test points flown up to 6 g were terminated at that point due to the Mach number decreasing outside tolerances (+/- 0.02 Mach desired). Also, at many of the high g points the data trace becomes erratic. This was due to aerodynamic buffet. Future test maneuvers for this method do not have to go to such high g limits, as the data collected at lower g limits is satisfactory.

As expected, the hysteresis error due to mechanical lag in the AOA gears was evident in the results. All eighteen plots show two lines of data, depending on whether the AOA vane was moving down or up. The error between the two lines ranges from +/- 0.5 to +/- 0.8 degrees, similar to the hysteresis error in Figure 10. In order to average the error, a straight line was drawn down the middle of the two lines. The slope of this line is the AOA position error correction factor, K, (from equation (2)). The x-axis intercept, aoT, was also determined from these plots. These values are summarized in Table III. The Reynold's numbers were calculated using MAC (7 feet) as the constant length.
### TABLE III
Summary of Flight Test Results

<table>
<thead>
<tr>
<th>Date of Flight</th>
<th>Altitude (feet)</th>
<th>Mach</th>
<th>Reynolds # (x 10^7)</th>
<th>$K_a$</th>
<th>$\alpha_{OT}$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23 Jul 85:</td>
<td>25,000</td>
<td>0.84</td>
<td>1.99</td>
<td>1.26</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.94</td>
<td>2.23</td>
<td>1.38</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.98</td>
<td>2.30</td>
<td>1.32</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>1.07</td>
<td>2.51</td>
<td>1.48</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.62</td>
<td>1.46</td>
<td>1.16</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.44</td>
<td>1.03</td>
<td>1.16</td>
<td>1.20</td>
</tr>
<tr>
<td>23 Jul 85:</td>
<td>15,000</td>
<td>0.83</td>
<td>2.69</td>
<td>1.10</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>15,000</td>
<td>0.94</td>
<td>3.03</td>
<td>1.14</td>
<td>3.20</td>
</tr>
<tr>
<td></td>
<td>15,000</td>
<td>0.96</td>
<td>3.10</td>
<td>1.33</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>15,000</td>
<td>1.07</td>
<td>3.44</td>
<td>1.45</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>15,000</td>
<td>0.64</td>
<td>2.08</td>
<td>1.30</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>15,000</td>
<td>0.42</td>
<td>1.35</td>
<td>1.20</td>
<td>1.90</td>
</tr>
<tr>
<td>26 Jul 85:</td>
<td>25,000</td>
<td>0.81</td>
<td>1.90</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.92</td>
<td>2.16</td>
<td>1.20</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.96</td>
<td>2.24</td>
<td>1.30</td>
<td>*-1.9</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>1.06</td>
<td>2.47</td>
<td>1.36</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.65</td>
<td>1.53</td>
<td>1.16</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>0.44</td>
<td>1.03</td>
<td>1.20</td>
<td>0.50</td>
</tr>
</tbody>
</table>

* Exceeds 2 standard deviations from mean

The values for $K_a$ are plotted versus Mach number in Figure 11. A curve was drawn through the points and shows a large increase in $K_a$ as Mach number increases above 0.8. No apparent Reynold's number effects in $K_a$ are evident in comparing the 15,000 feet points to the 25,000 feet points. The $K_a$ values from the Conrac NBIU wind tunnel testing are also plotted in Figure 11 (3:38). Although the shapes of the curves are similar, the values for $K_a$ are different.
both nosebooms are so similar in size and shape, the
difference is mainly due to the lack of fuselage and wing
effects on the wind tunnel results.

Equation (2), the equation to calculate true AOA, is:

\[ \alpha_c = \frac{a_m}{K_a} + \frac{q (x_a + \Delta cg) - P (V_a)}{U} \]  (2)

For T-38A 68-8205, \((x_a + \Delta cg)\) is assumed a constant 25 feet.
\(y_a\) is 6 inches (see Appendix E). The values for \(K_a\) are shown
in Figure 11 as a function of Mach number. However, equation
(2) assumes that no AOA position error exists at zero degrees
AOA. According to Figures 12 to 29, this is not true for the
T-38A. Some bias exists, which is the x-axis intercept, \(\alpha_o_T\).
Adding this bias to equation (2), and neglecting pitch and
roll rate, gives:

\[ \alpha_c = \alpha_o_T + \frac{a_m}{K_a} \]  (38)

The values for \(\alpha_o_T\) from Figures 12 to 29 are shown in Table
III. The values vary randomly, and do not seem to be
functions of Mach number. The average of all 18 values is
1.26 degrees, with a standard deviation of 1.06 degrees. One
value, for 25,000 feet \(H\) and Mach 0.96, is -1.9 degrees,
which exceeds two standard deviations from the mean.
Neglecting that point as erroneous, the average of the
remaining values is 1.45 degrees, with a standard deviation of 0.73 degrees. The equation to solve for the true AOA of the T-38A with a Vought YAPS noseboom is:

\[ a_c = 1.45 + \frac{a_m}{K_a} \]  \hspace{1cm} (39)

The pitch and roll rate terms from equation (2) should be included when applicable.
V. Conclusions and Recommendations

All project objectives were met. Conclusions and recommendations follow in order of importance:

The Kalman filter program KALOPT calculated "optimal" true angle-of-attack (AOA) values for a T-38A Talon using equations of motion for wings level flight and pitch angle measurements. From these true AOA values, the AOA position error correction factors were determined and were found to be functions of Mach number. Only standard T-38A flight test instrumentation was used - no flight path accelerometers were needed. This method proved to be accurate in gathering data with minimal instrumentation over a large range of AOAs.

1. THIS STATE ESTIMATION/KALMAN FILTERING METHOD OF CALCULATING AOA POSITION ERROR SHOULD BE CONSIDERED IN FUTURE AOA ERROR TESTING.

The Kalman filter needed an initial AOA to start propagating an "optimal" true AOA. This initial AOA was calculated from 1 g wings level flight prior to the roller coaster flight test maneuver. Also, aerodynamic buffet at high load factors caused some data scatter. The data gathered prior to the buffet were enough to calculate the AOA position error.

2. FUTURE MANEUVERS TO GATHER DATA FOR THIS METHOD SHOULD START FROM A 1 G TRIM SHOT FOR 3 TO 4 SECONDS. THE MANEUVERS SHOULD TERMINATE PRIOR TO AERODYNAMIC BUFFET.
The T-38A flight test data were not completely accurate due to hysteresis errors in the AOA transducer. The accuracy of the method would be better determined using an aircraft with no AOA hysteresis error. Also, testing with an aircraft with a fuselage-mounted AOA sensor would show larger AOA position errors and would further validate the method.

3. FURTHER TESTING SHOULD BE CONDUCTED USING AN RF-4C OR OTHER SUITABLE AIRCRAFT THAT HAS NO AOA HYSTERESIS ERROR AND HAS A FUSELAGE-MOUNTED AOA SENSOR.

4. THE USAF TEST PILOT SCHOOL NEEDS TO INSTALL NEW AOA TRANSDUCERS IN THEIR T-38A AIRCRAFT TO ELIMINATE THE AOA HYSTERESIS ERRORS.
Appendix A

Flight Test Results
Figure 13. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet $H_e$, Mach 0.94
Figure 14. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet $H_C$, Mach 0.98
Figure 15. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet $H_c$, Mach 1.07
Figure 16. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet \( H_c \), Mach 0.62
Figure 17. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom AT 25,000 feet H_c, Mach 0.44
Figure 18. T-38A Angle-of-Attack Position Error for a Vought YAP5S Noseboom at 15,000 feet $H_C$, Mach 0.83
USAFT-38A TALON  
S/N 68-8205
GROSS WEIGHT: 10,183 LBS  
CG: 18.2% MAC
15,000 FEET PRESSURE P:T  
MACH 0.94
FLIGHT TEST DATA  
23 JUL 85

Figure 19. T-38A Angle-of-Attack Position Error for a Vought
YAPS Noseboom at 15,000 feet $H_c$, Mach 0.94
Figure 20. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 15,000 feet H_c, Mach 0.96

\[ K_a = 1.33 \]

\[ \alpha_0 = 1.20 \]

MEASURED ANGLE-OF-ATTACK (DEG)

TRUE ANGLE-OF-ATTACK (DEG)
Figure 22. T-38A Angle-of-Attack Position Error for a Vought YAP-5 Noseboom at 15,000 feet H.C., Mach 0.64

USAF T-38A TALON 9,583 LBS GROSS WEIGHT 15,000 FEET PRESSURE ALT FLIGHT TEST DATA

CG: S/N 68-8205 MACH 0.64
23 JUL 85

K_a = 1.30

a_T = 0.80

MEASURED ANGLE-OF-ATTACK (DEG)
USAF T-38A TALON
GROSS WEIGHT: 11,433 LBS
CG: 18.3% MAC
25,000 FEET PRESSURE FLT
MACH 0.81
FLIGHT TEST DATA
26 JUL 85

Figure 24. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet $H_c$, Mach 0.81
Figure 25. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet $H_c$, Mach 0.92
Figure 26. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet \( H_c \), Mach 0.96
Figure 27. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet $H_c$, Mach 1.06
Figure 28. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet $H_c$, Mach 0.65
Figure 29. T-38A Angle-of-Attack Position Error for a Vought YAPS Noseboom at 25,000 feet H_c, Mach 0.44
Appendix B

Computer Program AOAOPT
PROGRAM AROPT

PROGRAM AROPT IS A FORTRAN PROGRAM DESIGNED TO CALCULATE
ANGLE-OF-ATTACK FROM PARAMETERS RECORDED INFLIGHT ON AN
AVDIN-VECTOR DATA ACQUISITION SYSTEM. AROPT USES TIME (T1),
driven velocity (IAS, AS), indicated altitude (ALT), measured angle of attack (AOA), pitch
angle (TH), pitch rate (Q), normal acceleration (ZN), sideslip angle (SS), and sink angle (BA) obtained from dynamics
files. AROPT CORRECTS FOR PITOT-STATIC ERRORS TO
DETERMINE PRESSURE ALTITUDE (HC), MACH NUMBER (AMC), AND
TRUE AIRSPEED (UTAS). THE PROGRAM CALCULATES TRUE AOA BY TWO
METHODS: ANGLE RELATIONSHIP WHERE AOA = PITCH - FLIGHT PATH
AND ITERATIVE METHOD WHERE AOA-DOT = PITCH RATE - NORMAL G.

PROGRAM CAN BE USED WITH T-38A OR RF-4C DATA.

BYTE NAME(12), ENAME(12), DOF(7),ATYPE(5),TAIL(3)
DIMENSION TI(300), AS(300), ALT(300), AOA(300), TH(300), Q(300),
SS(300), SS(300), BA(300), HC(300), AMC(300), UTAS(300), AOA2(300),
AOA3(300), HDOT(300), FPANG(300), AOR1(300), ERROR(300), ADOT(300),
ERROR(300), ERROR(300), ERROR(300), ERROR(300), ERROR(300),
BYTE FILE1(15), FILE1(17)
BYTE FILE2(15), FILE2(17)
BYTE FILE3(15), FILE3(17)
BYTE FILE4(15), FILE4(17)
BYTE FILE5(15), FILE5(17)
BYTE FILE6(15), FILE6(17)
DATA FILE1 //E, S**, **, E, U', P'
DATA FILE2 //E, S**, **, E, U', S'
DATA FILE3 //E, S**, **, D, A', T'
DATA FILE4 //E, S**, **, D, A', T'
DATA FILE5 //E, S**, **, D, A', T'
DATA FILE6 //E, S**, **, D, A', T'

WRITE (S, 100)
100 FORMAT ('ENTER NAME OF DYNAMICS FILE: NNNN.EJP',/)
ACCEPT 110, (FILE1(I), I=1, 4, 11)
FORMAT (99)
OPEN (UNIT-1, NAME=FILE1, TYPE='UNKNOWN')
WRITE (S, 120)
120 FORMAT ('ENTER NAME OF DYNAMICS FILE: NNNN.EUS',/)
ACCEPT 110, (FILE2(I), I=1, 4, 11)
OPEN (UNIT-2, NAME=FILE2, TYPE='UNKNOWN')
WRITE (S, 130)
130 FORMAT ('ENTER NAME FOR DATA FILE 1 : NNNN.DAT',/)
ACCEPT 110, (FILE3(I), I=1, 4, 11)
OPEN (UNIT-3, NAME=FILE3, TYPE='UNKNOWN')
WRITE (S, 140)
140 FORMAT ('ENTER NAME FOR DATA FILE 2: NNNN.DAT',/)
ACCEPT 110, (FILE4(I), I=1, 4, 11)
OPEN (UNIT-4, NAME=FILE4, TYPE='UNKNOWN')
WRITE (S, 150)
150 FORMAT ('ENTER NAME FOR DATA FILE 3: NNNN.DAT',/)
ACCEPT 110, (FILE5(I), I=1, 4, 11)
OPEN (UNIT-5, NAME=FILE5, TYPE='UNKNOWN')
WRITE (S, 160)
160 FORMAT ('ENTER NUMBER OF DATA POINTS DESIRED ',/)
READ (S, 170) N
170 FORMAT (I5)
WRITE(5,180)
FORMAT (' ENTER FIRST LINE OF DATA DESIRED ' ,'/')
READ(5,180) M
180 FORMAT (15)
WRITE(5,190)
190 FORMAT (' ENTER NUMBER OF INITIAL CONSTANT AOA LINES ' ,'/')
READ(5,195) NA
195 FORMAT (15)
WRITE(5,200)
200 FORMAT (' ENTER CORRECTIONS FOR PITCH RATE AND NZ: XXX,YYY'/
1' CORR ARE FROM GROUND BLOCK, ABOVE/BELLOW 0 FOR Q'/,
2' ABOVE/BELLOW 1 FOR NZ'/)
READ(5,210) QCORR, ZNCORR
210 FORMAT (2F7.3)

THIS PORTION READS DATA FROM THE DYNAMICS EU? AND EU? FILES
AND FORMATS THREE DATA FILES (XXX.DAT) WHERE THE PROGRAM
RESULTS WILL BE SENT. DATA FILE #1 RECORDS THE RAW DATA FROM
THE EU? AND EU? FILES. DATA FILE #2 CONTAINS AOA COMPUTED
BY THE ANGLE METHOD. DATA FILE #3 CONTAINS AOA COMPUTED BY
THE ITERATIVE METHOD.

READ (1,220) PNAME, ENAME, DOF
220 FORMAT (8X, 12A1, 15X, 12A1, 21X, 7A1)
WRITE (3, 230) PNAME, ENAME, DOF
WRITE (4, 230) PNAME, ENAME, DOF
WRITE (6, 230) PNAME, ENAME, DOF
WRITE (3, 260) PNAME, ENAME, DOF
230 FORMAT (' PILOT ', 1X, 12A1, SX, ' ENGINEER ', 1X, 12A1, SX,
1'DATE OF FLIGHT ', 1X, 7A1)
READ (1, 240) ATYPE, ATAIL
240 FORMAT (11X, 5A1, 13X, 3A1)
WRITE (3, 260) ATYPE, ATAIL
WRITE (4, 260) ATYPE, ATAIL
WRITE (6, 260) ATYPE, ATAIL
WRITE (3, 260) ATYPE, ATAIL
250 FORMAT (' A/C TYPE ', 1X, 5A1, SX, ' TAIL # ', 1X, 3A1)
WRITE (3, 260)
260 FORMAT ('//, 7X, 'LINE', 6X, 'TIME', 2X, 'AIRSPEED', 2X, 'ALTITUDE', 7X,
1'AOA', 5X, 'PITCH', 2X, 'PITCH RT', 8X, 'NZ', 6X, 'AOSS', 2X, 'ROLL ANG')
WRITE (4, 270)
1 AS', 7X, 'ROG', 2X, 'FLT PATH', 5X, 'AOA-1', 5X, 'AOA-1', 5X, 'ERROR')
WRITE (6, 280)
15X, 'AOA-M', 5X, 'ERROR', 5X, 'AOA-3', 5X, 'AOA-T', 5X, 'ERROR', 3X,
2'K-ALPHA')
WRITE (3, 290)
290 FORMAT (16X, '(SEC)', 3X, '(KNOTS)', 4X, '(FEET)', 5X, '(DEG)',
15X, '(DEG SEC)', 7X, '(G)', 5X, '(DEG)', 5X, '(DEG/SEC)')
WRITE (4, 300)
300 FORMAT (16X, '(SEC)', 4X, '(FEET)', 12X, '(FT/SEC)', 2X,
1'(FT/SEC)', 5X, '(DEG)', 5X, '(DEG)', 5X, '(DEG)')
WRITE (6, 310)
310 FORMAT (16X, '(SEC)', 1X, '(DEG SEC)', 1X, '(DEG SEC)', 5X, '(DEG)',
15X, '(DEG)', 5X, '(DEG)', 5X, '(DEG)', 5X, '(DEG)', 5X, '(DEG)')
READ (1, 320)
320 FORMAT (//, //)
READ (2,330)
  
330  FORMAT (/,,/)
  
DO 350 J=1,M
  
READ(1,340)
READ(2,340)
  
340  FORMAT ( )
  
CONTINUE
DO 390 I=1,N
  
READ(1,360)T(I),AS(I),ALT(I),AOA(I),TH(I),Q(I),ZN(I)
  
  
READ(2,370)ADSS(I),BA(I)
  
370  FORMAT (31X,10X,F10.3)
  
THE NEXT TWO STEPS CORRECT PITCH RATE AND NORMAL G FOR BIAS
FOUND ON THE GROUND (BIAS COMPUTED FROM 0 DEG/SEC FOR PITCH RATE AND 1 G FOR NORMAL G)
  
Q(I)=Q(I)-QC0RR
ZN(I)=ZN(I)-ZN0RR
WRITE(3,38)IT(I),AS(I),ALT(I),AOA(I),TH(I),Q(I),ZN(I),ADSS(I),
  
1BA(I)
  
380  FORMAT(I10,10F10.3)
  
CONTINUE
  
THE FOLLOWING PORTION PERFORMS THE PITOT-STATIC CORRECTIONS TO COMPUTE
PRESSURE ALT, MACH NUMBER, AND TRUE AIRSPEED. IT USES THE
STANDARD PITOT-STATIC EQUATIONS AND ERROR COEFFICIENTS FROM THE
USAF TEST PILOT SCHOOL FILES.
  
DO 710 I=1,N
  
IF(ALT(I) .GT. 36089)60 TO 500
  
DELTA=(1-(6.87559E-6*ALT(I))**5.2561
THETA=(6.87559E-6*ALT(I))
GO TO 510
  
500  DELTA=(4.80635E-5*(ALT(I)-36089))
THETA=.751874
  
510  IF(AS(I) .GT. 661.48)60 TO 520
  
QCICPA=((1+(.2*(AS(I)/661.48)**2))**3.5)-1
GO TO 530
  
520  QCICPA=((166.922*(AS(I)/661.48)**7)/(((7*(AS(I)/661.48)**2)
1-1)**2.5))**1
  
530  QCICPS=QCICPA/DELTA
  
AMIC=SQRT(((QCICPS+1)**(.2857143)-1))
  
IF(TYPE(I) .NE. 'T')60 TO 600
  
C=
  
(997+(10270*AMIC)+(-44495*AMIC**2)+(93931*AMIC**3))
  
1+(-5540*AMIC**4)+(37208*AMIC**5))*THETA
  
IF(AMIC .GT. 1)DHPC=0
GO TO 700
  
600  IF(AMIC .LT. .955)GO TO 620
IF(AMIC .LT. .957)GO TO 640
IF(AMIC .LT. 1.025)GO TO 660
C0=-46325
C1=8253
C2=-36567
GO TO 680
  
63
620 C0=118
   C1=-478
   C2=912
   Go to 680
640 C0=2676675
   C1=-5636739
   C2=2360026
   Go to 680
660 C0=-141471
   C1=308123
   C2=-166187
   Go to 680

680 DHPC=(C0+(C1*AMIC)+(C2*(AMIC**2)))*THETA
700 HC(I)=ALT(I)+DHPC
   DPPPS=(3.61332E-5*DHPC)/THETA
   DMPC=((1+(.2*(AMIC**2)))*DPPPS)/(1.4*AMIC)
   AMC(I)=AMIC+DMPC
   VTAS(I)=1.6878*AMC(I)*369.96763*SQRT(THETA*288.15)

710 CONTINUE

THIS PORTION COMPUTES TRUE AOA BY THE ANGLE METHOD. FLIGHT PATH ANGLE IS CALCULATED BY:

FLIGHT PATH ANGLE (FPANG) = INV SIN (VERTICAL VEL (HDOT) / TRUE AIRSPEED (VTAS))

TRUE AOA IS CALCULATED BY: AOA = PITCH ANGLE - FLT PATH
AOA IS THEN AVERAGED OVER A TIME INTERVAL FOR USE AS THE INITIAL AOA FOR THE ITERATIVE METHOD

DO 750 I=1,NA
   IF(I.EQ.1)GO TO 720
   IF(I.EQ.NA)GO TO 720
   DTIME=TI(I)-TI(I-1)
   DALT=HC(I)-HC(I-1)
   HDOT(I)=DALT/DTIME
   FPANG(I)=ASIN(HDOT(I)/VTAS(I))
   FPANG(I)=FPANG(I)*7.2957B
   AOAI(I)=(TH(I)-FPANG(I))/COS(BR(I)/7.29578)
   ERROR(I)=AOAI(I)-AOAI(I-1)
720 WRITE(4,740),TI(I),HC(I),AMC(I),VTAS(I),HDOT(I),FPANG(I),
   AOAI(I),AOAI(I-1),ERROR(I)
740 FORMAT(1X,10,9F10.3)
750 CONTINUE

THIS PORTION AVERAGES THE ALTITUDE, MACH, PITCH, PITCH RATE, NZ, AOA-MEAS, AOA-CALC, AND ERROR

DO 752 I=2,NA
   DTIME=TI(I)-TI(I-1)
   TALT=TALT+(HC(I)*DTIME)
   THCH=THCH+(AMC(I)*DTIME)
   TTH=TTH+(TH(I)*DTIME)
   TQ=TQ+(Q(I)*DTIME)
   TNZ=TNZ+(ZN(I)*DTIME)
   TAOA=TAOA+(AOAI(I)*DTIME)
   TAOAI1=TAOA1+(AOAI(I)*DTIME)
   TERROR=TERROR+(ERROR(I)*DTIME)
TTIME+TIME+DTIME
CONTINUE
ALT=ALT+TIME
AMACH+TMACH+TIME
ATH+TH+TIME
AQ+TQ+TIME
NZ=TNZ+TIME
AOA=TAOA+TIME
AERROR+ERROR+TIME
DO 755 I=2,NA-1
XALT=ALT-HE(I)
XMACH=AMACH-AMC(I)
XTH=ATH-TH(I)
XQ=AOA-AT(I)
XNZ=ANZ-ZN(I)
XAOA=AOA-AD(I)
XAOA1=AOA1-AD(I)
XERROR+ERROR-ERROR(I)
XALT=ABS(XALT)
XMACH=ABS(XMACH)
XTH=ABS(XTH)
XQ=ABS(XQ)
XNZ=ABS(XNZ)
XAOA=ABS(XAOA)
XAOA1=ABS(XAOA1)
XERROR=ABS(XERROR)
IF(XALT.GT.YALT)YALT=XALT
IF(XMACH.GT.YMACH)YMACH=XMACH
IF(XTH.GT.YTH)YTH=XTH
IF(XQ.GT.YQ)YQ=XQ
IF(XNZ.GT.YNZ)YNZ=XNZ
IF(XAOA.GT.YAOA)YAOA=XAOA
IF(XAOA1.GT.YAOA1)YAOA1=XAOA1
IF(XERROR.GT.YERROR)YERROR=XERROR
755 CONTINUE
WRITE (4,758)ALT,YALT,AMACH,YMACH,ATH,YTH,AQ,YQ,ANZ,YNZ,
1XAOA,YAOA,XAOA1,AERROR,YERROR
758 FORMAT(' AVG ALT = ',F10.3, '+/- ',F6.3, '
  AVG MACH = ',F10.3, '+/- ',F6.3, '
  AVG PITCH = ',F10.3, '+/- ',F6.3, '
  AVG PITCH RATE = ',F10.3, '+/- ',F6.3, '
  AVG AOA = ',F10.3, '+/- ',F6.3, '
  AVG ERROR = ',F10.3, '+/- ',F6.3,')
CC
THIS PORTION COMPUTES THE AOA OF THE REMAINING
CC POINTS BY COMPUTING FLIGHT PATH ANGLE, THEN
CC USING: AOA = PITCH ANGLE - FLIGHT PATH ANGLE
CC
IF(N.EQ.NA)GO TO 772
DO 770 I=NA+1,N
770 DTIME=TI(I+1)-TI(I-1)
DALT=HC(I+1)-HC(I-1)
HDOT(I)=DALT/DTIME
FPANG(I)=ASIN(HDOT(I)/Vtas(I))
FPANG(I) = FPANG(I) * 57.29578
AOA1(I) = (TH(I) - FPANG(I)) / COS(TH(I) / 57.29578)
ERROR(I) = AOA1(I) - AOA(I)
WRITE(4,765)I,TI(I),HC(I),AMC(I),UTAS(I),HDOT(I),FPANG(I),
AOA1(I),AOA(I),ERROR(I)
760 FORMAT(1X,110,9F10.3)
770 CONTINUE

C THIS PORTION COMPUTES ANGLE-OF-ATTACK BY
C COMPUTING ALPHA-DOT, THEN ADDING IT TO
C THE PREVIOUS AOA TO COMPUTE A NEW AOA.

772 DO 800 I=1,N
AOA2(I) = AOA(I)
AOA3(I) = AOA1
IF(1,1.EQ.1) GO TO 775
AVGZ = (Z(I) + Z(I-1)) / 2
AUGBA = (BA(I) + BA(I-1)) / 2
AVGTH = (TH(I) + TH(I-1)) / 2
AVGQ = (Q(I) + Q(I-1)) / 2
AVGTS = (UTAS(I) + UTAS(I-1)) / 2
AVGZ = AVGZ * COS(AUGBA / 57.29578) * COS(AVGTH / 57.29578)
ADOT(I) = AVGZ / (AVGTH / 57.29578 / AVGTS)
DTIME = TI(I) - TI(I-1)
AOA2(I) = AOA(I-1) + (ADOT(I) * DTIME)
AOA3(I) = AOA3(I-1) + (ADOT(I) * DTIME)

C THIS PORTION CORRECTS AOA CALCULATED AT CG FOR
C PITCH RATE: AOA CG = AOA VANE + (Q X X / UTAS)
C WHERE X IS THE DISTANCE BETWEEN THE VANE AND THE
C ACCELEROMETER LOCATION ON THE AIRCRAFT.

X = 17 IF(AType(I),EQ.'T',X = 25
ADOT = AOA3(I) - (Q(I) * X / UTAS(I))
ALPHA = AOA(I) + ADOT
ERROR(I) = AOA2(I) - AOA(I)
ERROR2(I) = AOA3(I) - AOA(I)
ADOTM(I) = (AOA3(I) - AOA(I-1)) / DTIME
WRITE(6,780)I,TI(I),ADOT(I),ADOTM(I),AOA2(I),AOA(I),ERROR1(I),
1AOA3(I),ADOT,ERROR2(I),ALPHA
780 FORMAT(1X,110,9F10.3)
800 CONTINUE

C STOP
END
Appendix C

Computer Program KALOPT
PROGRAM KALOPT

PROGRAM KALOPT IS A FORTRAN PROGRAM DESIGNED TO OPTIMIZE ANGLE-OF-ATTACK USING KALMAN FILTER EQUATIONS. THE PROGRAM USES DATA PARAMETERS RECORDED INFLIGHT ON AN AIRDIV-VECTOR DATA ACQUISITION SYSTEM. KALOPT USES TIME (TI), IAS (AS), INDICATED ALT (ALT), MEASURED AOA (AOA), PITCH ANGLE (TH), PITCH RATE (Q), NORMAL ACCELERATION (ZN), SIDESLIP ANGLE (AOSS), AND BANK ANGLE (BA) OBTAINED FROM DYNAMICS EUP AND EUS FILES. KALOPT CORRECTS FOR PITOT-STATIC ERRORS TO DETERMINE PRESSURE ALTITUDE (HC), MACH NUMBER (AMC), AND TRUE AIRSPEED (VTAS). THE PROGRAM CALCULATES AN "OPTIMAL" TRUE AOA BY COMBINING STATE ESTIMATION OF AOA AND PITCH ANGLE WITH ACTUAL PITCH ANGLE MEASUREMENTS. THE PROGRAM WEIGHS THE PITCH ANGLE MEASUREMENT AND ADDS THE WEIGHTED VALUE TO THE ESTIMATED AOA AND PITCH ANGLE. THE UPDATED AOA IS THE "OPTIMAL" TRUE AOA.

THESE PROGRAM CAN BE USED WITH T-38A OR RF-4C DATA.

BYTE PNAME(12), ENAE(12), DOF(7), ATYPE(S), ATAIL(3)
DIMENSION TI(300), AS(300), ALT(300), AOA(300), TH(300), Q(300),
12N(300), AOSS(300), BA(300), HC(300), AMC(300), VTAS(300)
DIMENSION B(2,2), G(2,2), QA(2,2), P(2,2), AK(2,1), XA(2,1), BT(2,2),
1U(2,1), XP(2,1), GT(2,2), QG(2,2), PP(2,2), H(1,2),
2HT(2,1), PHT(2,1), AK2PP(2,1), AKHPP(2,2)
BYTE FILE1(1S), FILE2(1S), FILE3(1S), FILE4(1S),
FILE6(1S), FILE7(1S), FILE8(1S)
DATA FILE1/'E', 'S', 'B', 'X', '0', ' ', 'E', 'U', 'P',
DATA FILE2/'E', 'S', 'B', 'X', '0', 'D', 'A', 'T',
DATA FILE3/'E', 'S', 'B', 'X', '0', 'D', 'A', 'T',
DATA FILE4/'E', 'S', 'B', 'X', '0', 'D', 'A', 'T',
DATA FILE6/'E', 'S', 'B', 'X', '0', 'D', 'A', 'T',
DATA FILE7/'E', 'S', 'B', 'X', '0', 'T', 'N', 'P',
DATA FILE8/'E', 'S', 'B', 'X', '0', 'T', 'N', 'P',

WRITE (S, 100)
100 FORMAT(' ENTER NAME OF DYNAMICS FILE: NNNNN.EUP',/)
ACCEPT 110, (FILE1(I), I=4, 11)
110 FORMAT (BA1)
OPEN(UNIT=1, NAME=FILE1, TYPE='UNKNOWN')
WRITE (S, 120)
120 FORMAT(' ENTER NAME OF DYNAMICS FILE: NNNNN.EUS',/)
ACCEPT 110, (FILE2(I), I=4, 11)
OPEN(UNIT=2, NAME=FILE2, TYPE='UNKNOWN')
WRITE (S, 125)
125 FORMAT(' ENTER NAME OF INPUT FILE: NNNNN.INP',/)
ACCEPT 110, (FILEB(I), I=4, 11)
OPEN(UNIT=6, NAME=FILEB, TYPE='UNKNOWN')
WRITE (S, 130)
130 FORMAT(' ENTER NAME FOR DATA FILE 1: NNNNN.DAT',/)
ACCEPT 110, (FILE3(I), I=4, 11)
OPEN(UNIT=3, NAME=FILE3, TYPE='UNKNOWN')

68
WRITE(S,140)
FORMAT(' ENTER NAME FOR DATA FILE 2: NNNNN.DAT',/)
ACCEPT 110,(FILE4(I),I=4,11)
OPEN(UNIT=4,NAMES=FILE4,TYPE='UNKNOWN')
WRITE(S,150)
FORMAT(' ENTER NAME FOR DATA FILE 3: NNNNN.DAT',/)
ACCEPT 110,(FILE6(I),I=4,11)
OPEN(UNIT=6,NAMES=FILE6,TYPE='UNKNOWN')
WRITE(S,155)
FORMAT(' ENTER NAME FOR DATA FILE 4: NNNNN.DAT',/)
ACCEPT 110,(FILE7(I),I=4,11)
OPEN(UNIT=7,NAMES=FILE7,TYPE='UNKNOWN')
WRITE(S,160)
FORMAT(' ENTER NUMBER OF DATA POINTS DESIRED ',/)
READ(S,170)N
WRITE(S,180)
FORMAT(' ENTER FIRST LINE OF DATA DESIRED ',/)
READ(S,190)M
WRITE(S,200)
FORMAT(' ENTER NUMBER OF INITIAL CONSTANT AOA LINES ',/)
READ(S,210)NA
READ(S,220)PNAME,ENAME,DOF
WRITE(3,230)PNAME,ENAME,DOF
WRITE(4,230)PNAME,ENAME,DOF
WRITE(6,230)PNAME,ENAME,DOF
WRITE(7,230)PNAME,ENAME,DOF
READ(S,240)ATYPE,ATAIL
WRITE(3,250)ATYPE,ATAIL
WRITE(4,250)ATYPE,ATAIL
WRITE(6,250)ATYPE,ATAIL
WRITE(7,250)ATYPE,ATAIL
READ(S,260)AC_TYPE,TAIL#,ATAIL
WRITE(3,270)
FORMAT(' PILOT:',1X,12A1,SX,'ENGINEER',1X,12A1,SX,'DATE OF FLIGHT':1X,7A1)
READ(1,220)NAME,NAME,NAME
WRITE(3,220)NAME,NAME,NAME
WRITE(4,220)NAME,NAME,NAME
WRITE(6,220)NAME,NAME,NAME
WRITE(7,220)NAME,NAME,NAME
FORMAT(' CORR ARE FROM GROUND BLOCK, ABOVE/Below 0 FOR Q',/,
1' ABOVE/Below 1 FOR NZ',/
READ(S,220)QCORR,ZNCORR
WRITE(S,220)
FORMAT(' T/LINES',6X,'TIME',2X,'AIRSPEED',2X,'ALTITUDE',7X)
1 'AOA', 'Sx', 'PITCH', '2X', 'PITCH RT', '6X', 'K2', '6X', 'AOSS', '2X', 'ROLL ANG')
WRITE (4, 270)
270 FORMAT ('/7X', 'LINE', '6X', 'TIME', '1X', 'PRESS ALT', '6X', 'MACH', '3X', 'TRUE
1 AS', '5X', 'PITCH', '3X', 'PITCH +', '5X', 'AOA +', '5X', 'Q4X-V', '2X', 'AOA-TRUE',
22X', 'AOA-VANE', '5X', 'ERROR', '3X', 'K-ALPHA')
WRITE (6, 280)
280 FORMAT ('/7X', 'LINE', '6X', 'TIME', '5X', 'P-1)', '5X', 'P-2)', '5X', 'P-3)',
15X', 'P-4)', '6X', 'K(1)', '6X', 'K(2)')
WRITE (7, 285)
285 FORMAT ('/7X', 'LINE', '6X', 'TIME', '5X', 'P+1)', '5X', 'P+2)', '5X', 'P+3)',
2'DIFF', '7X', 'SUM', '5X', 'PSQRT')
WRITE (3, 290)
290 FORMAT (16X, '(SEC)', '3X', '(KNOTS)', '4X', '(FEET)', '5X', '(DEG)',
15X', '(DEG)', '1X', '(DEG/SEC)', '7X', '(G)', '6X', '(DEG)', '5X', '(DEG)', '/)
WRITE (4, 300)
300 FORMAT (16X, '(SEC)', '4X', '(FEET)', '12X', '(FT/SEC)', '5X',
1'(DEG)', '5X', '(DEG)', '5X', '(DEG)', '5X', '(DEG)', '5X', '(DEG)', '/)
WRITE (6, 310)
310 FORMAT (16X, '(SEC)', ')/
WRITE (7, 320)
320 FORMAT (1/, ')
READ (1, 330)
330 FORMAT (1/, ')
DO 360 J=1, H
READ (1, 340) TI(I), AS(I), ALT(I), AOA(I), TH(I), PN(I), ZN(I)
READ (2, 370) AOSS(I), BA(I)
370 FORMAT (31X, F10.3, 10X, F10.3)

THE NEXT TWO STEPS CORRECT PITCH RATE AND NORMAL G FOR BIAS
FOUND ON THE GROUND (BIAS COMPUTED FROM 0 DEG/SEC FOR PITCH
RATE AND 1 G FOR NORMAL G)
Q(I)=Q(I)-Gcorr
ZN(I)=ZN(I)-Zncorr
WRITE (3, 380) TI(I), AS(I), ALT(I), AOA(I), TH(I), Q(I), ZN(I), AOSS(I),
BA(I)
380 FORMAT (1X, 110.9F10.3)
390 CONTINUE

THIS PORTION PERFORMS THE PITOT-STATIC CORRECTIONS TO COMPUTE
PRESSURE ALT, MACH NUMBER, AND TRUE AIRSPEED. IT USES THE
STANDARD PITOT-STATIC EQUATIONS AND ERROR COEFFICIENTS FROM THE
USAF TEST PILOT SCHOOL FILES.

DO 710 I=1, H
IF (ALT(I) .GT. 3600) GO TO 500
DELTA*S=(1-(6.87559E-6*ALT(I)))*S.2561

70
THETA = 1 - (6.87569E-6 * ALT(1))
GO TO 510
500 DELTA = 0.23375 * EXP(-4.88635E-5 * (ALT(1) - 36089))
THETA = .751874
510 IF (AS(1) .GT. 661.48) GO TO 520
QCIPA = (((1 + (.2 * ((AS(1) / 661.48) ** 2))) ** 3.5) - 1)
GO TO 530
520 QCIPA = (((166.922 * (AS(1) / 661.48) ** 7) / (((7 * ((AS(1) / 661.48) ** 2)) ** 2.5) - 1)))
GO TO 530
530 QCIPA = QCIPA / DELTA
AMIC = SQRT(5 * ((QCIPA + 1) ** (2.857143) - 1))
IF (ATYPE(1) .EQ. 'T') GO TO 600

POSITION ERROR COEFFICIENTS COME FROM THE
USAF TEST PILOT SCHOOL PITOT-STATIC FILES

DHPC = (-907 + (10270 * AMIC) + (-44495 * (AMIC ** 2)) + (93931 * (AMIC ** 3))
+ (-95540 * (AMIC ** 4)) + (37208 * (AMIC ** 5)) * THETA
IF (AMIC .GT. 1) DHPC = 0
GO TO 700
600 IF (AMIC .LT. .955) GO TO 620
IF (AMIC .LT. .967) GO TO 640
IF (AMIC .LT. 1.025) GO TO 660
CO = 46325
C1 = 82985
C2 = 36667
GO TO 680
620 CO = 118
C1 = 476
C2 = 912
GO TO 680
640 CO = 2676675
C1 = 5636789
C2 = 29688036
GO TO 680
660 CO = -141471
C1 = 308123
C2 = -165107
GO TO 680
680 DHPC = (CO + (C1 * AMIC) + (C2 * (AMIC ** 2))) * THETA
700 HC(I) = ALT(I) + DHPC
DPPPS = (3.61382E-5 * DHPC) / THETA
DMPC = (((1 + (.2 * (AMIC ** 2))) * DPPPS) / (1.4 * AMIC))
AMC(I) = AMIC + DMPC
VTAS(1) = 1.6878 * AMIC(I) ** 38, 96763 * SQRT(THETA ** 288.15)
CONTINUE

THIS PORTION COMPUTES TRUE AOA BY THE RELATIONSHIP: TRUE AOA =
PITCH ANGLE - FLIGHT PATH ANGLE. FLIGHT PATH ANGLE IS
COMPUTED BY: FLIGHT PATH ANGLE (FPANG) = INV SIN (VERTICAL
VEL (HDOT) / TRUE AIRSPEED (VTAS)). THE TRUE AOA IS THEN
AVERAGED OVER A TIME INTERVAL TO USE AS AOA(1) IN THE STATE
ESTIMATION. THE DATA USED HERE IS UNFILTERED.

IF (NA .EQ. 0) AOA = AOA(1)
IF (NA .EQ. 0) GO TO 790
DO 750 I=2,NA-1
DTIME=TI(I+1)-TI(I-1)
DALT=HC(I+1)-HC(I-1)
HDOT=DALT/DTIME
FPANG=ASIN(HDOT/VTAS(I))
AOA1=TH(I)-FPANG

THIS PORTION AVERAGES THE CALCULATED AOA TO USE AS AOA(0) IN THE STATE ESTIMATION

TTIME=TTIME+DTIME
TAOA=TAOA+AOA1*DTIME
750 CONTINUE

THIS SECTION PRINTS OUT THE INITIAL LINE OF DATA

WRITE(4,800)TI(CI),HC(I),AMC(I),VTAS(I),TH(I),AOA1,TH(I)
WRITE(6,810)TI(I)
810 FORMAT(10X,'1',7F10.3,20X,F10.3)
WRITE(7,610)TI(I)

C THIS PORTION READS IN THE VALUES FOR THE MATRICES P(T0), Q(T), AND R(T) (CALLED P, QA, AND R) FROM AN INPUT FILE CALLED XXXX.INP

DO 910 I=1,2
READ(8,900)P(I,1),P(I,2)
900 FORMAT(11X,F10.3,10X,F10.3)
910 CONTINUE

DO 930 I=1,2
READ(8,920)QA(I,1),QA(I,2)
920 FORMAT(11X,F10.3,10X,F10.3)
930 CONTINUE

READ(8,940)R
940 FORMAT(11X,F10.3)

C THIS PORTION Initializes THE STATE MATRIX X(T0) AND THE MATRICES BIT), G(T), H(T), AND U(T).

XA(1,1)=AOA1
XA(2,1)=TH(I)
DO 2000 I=2,N
T=TI(I)-TI(I-1)
B(I,1)=1
B(I,2)=(-32.2)*7.29576/VTAS(I)
2000 CONTINUE

G(1,1)=1
G(1,2)=(-32.2)*7.29576/VTAS(I)
G(2,1)=1
H(1,2)=1
U(1,1)=Q(I)
U(2,1)=Z(N(I))-COS(TH(I)/7.29576)

C THIS PORTION PROPAGATES THE STATE MATRIX FROM THE LAST UPDATE
Lx(T+) TO THE POINT PRIOR TO THIS UPDATE X(T-)

DO 1020 J=1,2
DO 1010 K=1,2
BT(J,K)=B(J,K)*T
1010 CONTINUE
DO 1020 J=1,2
XP(J,1)*X(A(J,1)+(BT(J,1)*U(1,1))+(BT(J,2)*U(2,1))
1020 CONTINUE
C
C THIS PORTION PROPAGATES THE COVARIANCE MATRIX P(T) FROM THE
C LAST UPDATE P(T+) TO THE POINT PRIOR TO THIS UPDATE, P(T-)
C DO 1050 J=1,2
DO 1040 K=1,2
GT(K,J)=G(J,K)
1040 CONTINUE
DO 1070 J=1,2
DO 1060 K=1,2
GGT(K,J)=(G(K,1)*GT(1,J))+(G(K,2)*GT(2,J))
1060 CONTINUE
DO 1090 J=1,2
DO 1080 K=1,2
PP(J,K)=P(J,K)+(T*GGT(J,K))
1080 CONTINUE
1090 CONTINUE
DO 1150 J=1,2
DO 1140 K=1,2
PPH(J,K)-P(J,K)+(T*(P(J,2)*K))
1140 CONTINUE
1150 CONTINUE
C
C THIS PORTION COMPUTES THE GAIN MATRIX, K(T)
DO 1210 J=1,2
HT(J,1)=H(1,J)
1210 CONTINUE
DO 1230 J=1,2
PPHT(J,1)=(PP(J,1)*HT(1,1))+(PP(J,2)*HT(2,1))
1230 CONTINUE
HPHT=(H(1,1)*PPHT(1,1))+(H(1,2)*PPHT(2,1))
HPHTR=HPHT+R
DO 1270 J=1,2
AK(J,1)=PPHT(J,1)/HPHTR
1270 CONTINUE
C
C THIS PORTION PERFORMS THE UPDATE OF THE STATE MATRIX, X(T),
C TO TAKE IT FROM X(T-) TO X(T+) BY WEIGHING THE PITCH ANGLE
C MEASUREMENT (Z) AND ADDING IT TO X(T-)
Z*TH(1)
HXP=(H(1,1)*XP(1,1))+(H(1,2)*XP(2,1))
ZXP=Z-HXP
DO 1320 J=1,2
  AKZMH(J,1)=AK(J,1)*ZH(J,1)
  XA(J,1)=XP(J,1)+AKZMH(J,1)
1320 CONTINUE

THIS PORTION UPDATES P(T) FROM P(T-) TO P(T+)

DO 1400 K=1,2
  HPP(K,1)=H(1,1)*PP(1,K)+H(1,2)*PP(2,K)
1400 CONTINUE

DO 1430 J=1,2
  AKPP(J,K)=AK(J,1)*PP(J,K)
1430 CONTINUE

DO 1420 I=1,2
  P(J,K)=PP(J,1)-AKPP(J,K)
1420 CONTINUE

THIS PORTION CORRECTS AOA CALCULATED AT CG FOR
PITCH RATE: AOA CG = AOA VANE + (Q * X / VTAS)
WHERE X IS THE DISTANCE BETWEEN THE VANE AND THE
ACCELEROMETER LOCATION ON THE AIRCRAFT.

X=17
IF(TYPE(1).EQ.'T') X=25
QXV=Q(1)*X/VTAS(1)

THIS PORTION COMPUTES THE SQUARE OF THE ERROR BETWEEN
PITCH ANGLE-CALC AND PITCH ANGLE-MEAS TO COMPUTE THE
TRUE RMS ERROR.

ZA=XA(2,1)-TH(1)
IF(ZA.LT.0) GO TO 1500
ZA=ZA**2
ZB=0
ZAT=ZAT+ZA
GO TO 1510
1500 ZB=ZA**2
ZA=0
ZBT=ZBT+ZB

1510 DIFF=ZAT-ZBT
SUM=(ZA+ZB)**.5
PSQRT=P(2.2)**.5

THIS PORTION COMPUTES THE ERROR BETWEEN THE "OPTIMAL" TRUE AOA
AND THE MEASURED AOA AND THEN PRINTS OUT THE RESULTS OF
THE KALMAN FILTER PROGRAM.

ERROR=AOAT-AOA(1)
ALPHAK=AOA(1)/AOAT
WRITE(6,1810) ITI(1),HC(1),ARC(1),VTAS(1),TH(1),
1XAO(2,1),XA(1,1),QXV,AOAT,AOA(1),ERROR,ALPHAK
1800 FORMAT(1X,11.0,12F10.3)
WRITE(6,1810) ITI(1),PP(1,1),PP(1,2),PP(2,1),PP(2,2),
1AK(1,1),AK(2,1)
1810 FORMAT(1X,10,7F10.3)
WRITE(7,1820)I, TI(I), P(1,1), P(1,2), P(2,1), P(2,2),
12A, ZB, ZAT, ZBT, DIFF, SUM, P10KT
1820 FORMAT(IX,110,12F10.3)
2000 CONTINUE
STOP
END
Appendix D

Flight Test Summary
### TABLE IV

T-38A Flight Test Points

<table>
<thead>
<tr>
<th>Cruise Configuration</th>
<th>No External Stores</th>
<th>3 seconds per g min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wings Level</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Point</th>
<th>Altitude (feet)</th>
<th>Mach</th>
<th>Min G</th>
<th>Max G</th>
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<td>25,000</td>
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<td>+4</td>
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<td>0.9</td>
<td>-1</td>
<td>+6</td>
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<tr>
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<td>-1</td>
<td>+6</td>
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<td>-1</td>
<td>+6</td>
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<td>-1</td>
<td>+2</td>
</tr>
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<td>-1</td>
<td>+6</td>
</tr>
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<td>15,000</td>
<td>0.9</td>
<td>-1</td>
<td>+6</td>
</tr>
<tr>
<td>9</td>
<td>15,000</td>
<td>0.95</td>
<td>-1</td>
<td>+6</td>
</tr>
<tr>
<td>10</td>
<td>15,000</td>
<td>1.05</td>
<td>-1</td>
<td>+6</td>
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<tr>
<td>11</td>
<td>15,000</td>
<td>0.6</td>
<td>-1</td>
<td>+4</td>
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<tr>
<td>12</td>
<td>15,000</td>
<td>0.45</td>
<td>-1</td>
<td>+2</td>
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\(^{(11:3)}\)
### TABLE V
Summary of T-38A Test Flights

T-38A 68-8205  
Edwards AFB, Cal.

<table>
<thead>
<tr>
<th>Date of Flight</th>
<th>Pilot</th>
<th>Engineer</th>
<th>Flight Time</th>
<th>Test Points Flown</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 Jul 85</td>
<td>Eichhorn</td>
<td>Thacker</td>
<td>0.7 hrs</td>
<td>None - Bad Wx</td>
</tr>
<tr>
<td>23 Jul 85</td>
<td>Eichhorn</td>
<td>Thacker</td>
<td>1.0 hrs</td>
<td>1 through 12</td>
</tr>
<tr>
<td>26 Jul 85</td>
<td>Arnold</td>
<td>Thacker</td>
<td>0.8 hrs</td>
<td>1 through 6</td>
</tr>
</tbody>
</table>

Total Flight Time - 2.5 hrs
Appendix E

T-38A YAPS Noseboom Diagram
Bibliography


Vita

Captain Thomas H. Thacker was born on 24 December 1955 in Casablanca, Morocco. He graduated from Beavercreek High School in Dayton, Ohio in 1973. Captain Thacker was a distinguished graduate of the United States Air Force Academy in 1978. He received the degree of Bachelor of Science in Aeronautical Engineering and a regular commission in the USAF from the Academy. He attended undergraduate navigator training and received his wings in February, 1979. He was a distinguished graduate and received the ATC Commander's Trophy as the top graduate in his class. Captain Thacker served as an F-111 instructor weapons systems officer in the 20th Tactical Fighter Wing, RAF Upper Heyford, United Kingdom, from November 1979 to March 1983. He graduated from the F-111 Fighter Weapons School in March, 1982. Captain Thacker was selected for the combined Air Force Institute of Technology/Test Pilot School program in 1983. He attended the AFIT School of Engineering from June 1983 to June 1984. He graduated from the USAF Test Pilot School in June 1985 as a distinguished graduate and received the R. L. Jones Award as the top flight test engineer/navigator in his class. Captain Thacker is currently assigned with the 3247th Test Squadron, Eglin AFB, Florida.

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USE OF STATE ESTIMATION TO CALCULATE

This project determined the position errors of an aircraft's angle-of-attack (AOA) sensor using state estimation with flight test data. The position errors were caused by local flow and upwash and were found to be a function of AOA and Mach number. The test aircraft used in this project was a T-38A Talon supersonic trainer from the USAF Test Pilot School configured with a Vought yaw and pitch system noseboom and an internal Aydin-Vector data acquisition system (DAS).

The position errors were found by calculating the true AOA using equations of motion and DAS parameters. The data from the DAS were noise corrupted and had to be filtered. This was accomplished using state estimation in a Kalman filter. The estimated AOA was compared to the measured AOA from the noseboom sensor to obtain the position error. Accurate position errors were obtained, even in dynamic maneuvers. This method should be considered in future AOA measurement.
Block 19. (Cont) -error testing. The method was accurate enough to identify a hysteresis error in the T-38A's AOA sensor of +/- 0.5 degrees, which was confirmed by ground calibration.