ECONOMIC APPROACHES TO OVERHEAD COSTS: AN APPLICATION OF MULTIPRODUCT COST THEORY

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An Application of Multiproduct Cost Theory

by

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ABSTRACT

An important element of military production cost is the overhead cost component. At the elemental level, overhead costs represent costs that are indirect - in other words, costs that cannot be unambiguously assigned to more than one product. To the extent that detailed cost accounting can make reasonable allocations of these costs, they are unimportant. However, many of the contractors that produce military hardware also produce similar hardware for commercial applications within the same division. This is good in that it enables them to exploit significant economies of scale. Unfortunately, this also creates ambiguity in the assignment of overhead costs between military and commercial production.

Even if a correct allocation rule for every indirect cost of military production is known, it is impossible to calculate the exact amount of overhead cost assigned to military production. This is because overhead costs for one output legitimately depend on the production level of other outputs. As a result, it is difficult for the military to both evaluate and predict project overhead costs reported by contractors.

Recent developments in multiproduct cost theory have provided a solution to this dilemma. The concept of economies of scope provides the necessary link between the economic concept of indirect costs and the accounting concept of overhead costs in a multiproduct setting. This paper shows how these recent advances can be utilized in order to statistically estimate an econometric cost function which can unambiguously assign overhead costs to different outputs based on casual production relationships. This type of analysis is unique in its ability to reveal the overhead components that are genuinely "caused" by individual outputs.

A number of difficulties have been encountered in the empirical application of this methodology. The paper addresses some of the most important of these problems and examines how they have been dealt with in the literature. These issues include the relationship between accounting data and economic cost information, selection of appropriate functional forms, and trade-offs between detail and statistical validity. Several recent applications of this methodology to the telecommunications industry are discussed, along with current research efforts in the area of naval jet engine production. The paper concludes with a discussion of the general applicability of this approach to other military settings.
1. THE ALLOCATION PROBLEM

For decades accountants and industry managers have wrestled with the problem of allocating and predicting overhead costs. It is extremely important for any company to understand what actual costs it faces in producing its various outputs. The problem becomes a critical issue when a regulated firm, such as a telephone company or an electric utility, is under consideration because of the danger of undesired cross subsidization. Cross subsidization occurs when costs that are related to one output are charged to another. A regulated monopoly may use this procedure to subsidize one regulated output by charging a higher price for another (for example, when the subsidized output has the higher price elasticity of demand). Or, alternatively, a firm with both regulated (monopoly) and unregulated (competitive) outputs may subsidize the competitive output with returns to the regulated output. This type of behavior can be of a predatory nature, but it can also occur unintentionally when a firm doesn't have a complete understanding of its own costs.

In order to deal with this problem the accounting profession has developed a variety of accounting principles and rules for distributing the costs of a firm into functional and financial categories which includes the allocation of overhead costs to individual outputs. These methods generally distribute costs at a very detailed level so that each cost component can be traced to a distinct output. Many of these cost allocations are necessarily arbitrary, such as the allocation of the costs of a building to the different outputs produced within it.

I use the word arbitrary hesitantly, for fear of stirring up a hornets nest. Nevertheless, I would argue that these allocations are indeed arbitrary in two ways. First, the use of any allocation method is the result of a choice between competing goals. Allocation of costs are usually based on some valuation of benefits received or produced by each product, whether units produced, services consumed, revenues produced, or some set of direct costs serves as the basis of an allocation rule. Thus, there is no right allocation rule, and therefore, while there may be a reason for favoring one rule over another, in terms of pure accounting principles, it is arbitrary.

Second, while I cannot agree with Thomas (1974) that the immeasurability of allocations renders them misleading and therefore totally useless, I do admit that at some level the measurement pattern does arise. However, the level of which allocations become arbitrary, depends on the level of detail that the manager or the cost analyst is willing to pursue. For example, let's say one oil can is used to lubricate two machines. In practice, the cost of the oil will probably be allocated using an allocation rule, an allocation rule which is as good as any other but is chosen for some reason above all others. If the manager, however, can measure precisely the oil used for each machine he can eliminate arbitrariness at one level. Nevertheless, he is still left with an oil can, which is not so easily divisible. The cost of the can can be allocated in proportion to the oil dispensed. But then, it could also be divided equally. After all neither machine can run without the can and thus it is in some sense
required equally by each. Examples more obstreperous than this are easy to find.

In this paper, I argue for an allocation technique that is less "arbitrary" in that it makes sense in an economic way, in other words, it assigns cost to outputs based on the true value of services it uses up according to the specific technology of the firm. But I don't deny that this is also an arbitrary allocation technique. A choice is made, among competing options. This method uses statistical estimation to assign joint costs. An experience and knowledgeable manager or cost analyst may be able to do the same thing, ten times better, based on his detailed knowledge of the production process, by eliminating arbitrariness as in the oil example. He is so familiar with his shop that he just knows how output drives both direct and joint costs and can assign them accordingly, using prior information that may be superior to our own biased regression estimaters.

Nevertheless, there are still two problems with this approach. First, his knowledge cannot be transferred or formularized. There is no practical way for an outsider to know how good he really is, or to transport his skill to a different shop. Detailed process models are often used to function in the same way as this knowledgeable manager. But these models are deterministic and unwieldy. To be good they have to be gigantic, and then they are too hard to keep up to date. Ultimately they cannot replace the manager's ability to form prior allocations. The second problem is that this manager or analyst is unable to apply his detailed expertise to the cost reports submitted by contractors. He can verify computations and perform comparisons, but ultimately he must accept most of the allocations adopted by the contractors.
2. A NEW APPROACH

The present paper discusses the application of a new overhead cost allocation technique to the production of military weapon systems and subsystems. The technique is based on the statistical estimation of an economic cost function. Statistical estimation is frequently used to predict project overhead costs, and various techniques and functional relationships have been tried, with varying success. (See Gross, 1984; Wright, 1983; Gross and Dienemann, 1978)

The advantage of an economic cost function is that the full cost structure of the firm is reproduced, which allows for far richer and more robust functions to be utilized. These functions conform to certain properties that we know to exist in firms, such as linear homogeneity in input prices. This means that if the prices of all inputs increase by some equal percentage, the total cost will increase by the same percentage. By working within a structure, rather than searching for a specific cost relationships, we are able to derive results that are theoretically consistent and explain a much broader range of actual cost experience. The conventional overhead cost estimation techniques tend to involve highly restrictive functional forms which imply certain properties of the firm, such as constant returns to scale or limitations on factor substitutions. These implications, however, are often not brought to light.

Another advantage of using an economic cost function in the context of company overhead costs is that this approach essentially enables us to see inside the production process of the firm and evaluate its cost structure without having to examine each individual cost allocation that the accounting process makes. All that is necessary is that we be able to translate the accountants definitions and rules into economic cost concepts in a reasonably consistent way. Once this set of economic cost variables has been obtained, recent advances in multiproduct cost theory allow us to statistically estimate all of the information needed to construct a complete picture of the firm’s production process and cost structure.

The process of estimation reveals how the complete set of cost-relevant variables affect the costs of production over some period of time. By observing these movements in costs the portions of overhead costs that are caused by some distinct output over time can be observed. This type of analysis can be of great importance in military settings by allowing the overhead rates charged by contractors to be evaluated without relying solely on estimates supplied by the contractors themselves and by using data that is much more manageable and readily obtainable than the types of detailed accounting data required to otherwise verify their estimates.

A major problem in the analysis of military overhead costs is their inherent unpredictability. Since overhead charges represent such a large fraction of most military weapon systems production costs, this is a problem of great importance. Accurately predicting project overhead costs, whatever allocation system is employed, is crucial to efficient project costing. Unfortunately, overhead costs are a very
unstable function of direct labor costs and other composites of direct
costs, even for very similar projects. This is because of the shared
components of cost, represented generally by overhead costs, depend on
the levels of production of all outputs of the firm or division, not
just those of the output being considered. For example, a firm that
produces both military and commercial products will allocate some pro-
portion of company fixed overhead to each. If commercial production
decreases, the overhead rate will necessarily increase, since military
production now constitutes a larger share of the business base, which
still shares the same pool of fixed overhead. For accurately predict
these costs an estimation model is preferred to a deterministic
accounting model, since the estimation can reflect the ever-changing
relationship between outputs, direct costs and overhead costs.
Furthermore, such a model should be based on the economic cost con-
cepts discussed above so that the full cost structure of the firm,
and consequently the true causes of costs, can be incorporated into
the simulation of future overhead costs.

In the following sections the procedures for estimation of an economic
cost function will be discussed. Section 3 describes the basics of
economic theory which provide the underpinning of this methodology.
In section 4, a number of empirical issues are discussed. And in sec-
tion 5, several recent applications of the theory are discussed.
3. THEORETICAL FOUNDATIONS

The first analytical problem is to find a function that can accurately depict the cost structure of a firm. In order to do this a function must be able to fully represent the firm's production technology. It can be easily shown that a properly specified cost function using outputs and input prices is the dual of the standard production function using inputs as arguments. It can further be demonstrated that this cost function is able to convey the full set of production relationships as well as all economically important cost relationships. Furthermore, the data required to estimate a cost function are much easier to obtain and to work with than those required for a production function.

Until fairly recently it was impossible to correctly evaluate the cost structure of a firm with more than one output in this manner. The traditional microeconomic analysis restricts evaluation of marginal costs, average costs and economies of scale (AC/MC) to the highly unrealistic single product technology. The difficulty in defining even a simple two product average cost was greatly magnified for more advanced concepts, such as economies of scale. In the last 10 or 15 years an extensive literature has developed on multiproduct cost theory. Although refinements are still being made, the tools are now in place to fully analyze the cost properties of multiproduct firms.

The building blocks of this theory are incremental costs. The incremental cost of a product is defined as the increase in total cost due to producing some output of that product, where none was produced before, or

\[ IC_1 = C(Q_1, Q_2) - C(0, Q_2). \]  

Average incremental cost is the analog to average cost in the one product case, and product specific economies of scale \((S_f = AIC/MC)\) are analogous to single product returns to scale. Since the incremental costs of all the products don't add up to total cost of the firm, there must be some missing element. This element is the shared cost component, referred to as joint, common or indirect cost. These shared costs cannot be unambiguously assigned to the respective outputs. The measurement of these shared costs introduces a new concept: economies of scope. Economies of scope are said to exist when the cost of producing two products together is less than the sum of the cost of producing each separately, or

\[ C(Q_1, Q_2) < C(Q_1, 0) + C(0, Q_2). \]  

A generalizable measure of scope developed by Goetz (1984), is defined as

\[ Sc' = \frac{C(Q) - \sum Q_i AIC_i}{C(Q)}. \]
or the ratio of indirect to total cost.

Finally, the measure of the overall economies of scale of a multiproduct firm can be determined, using both product specific economies of scale and economies of scope as arguments:

\[
S = \frac{W_i S_i}{1 - Sc'},
\]

where \( W_i = \frac{Q_i \frac{dc}{dQ_i}}{\sum Q_i \frac{dc}{dQ_i}} \).

It can be shown that there can be diseconomies of scale in each output but positive overall economies of scale due to the influence of economies of scope. Economies of scope, as the measure of indirect cost, provides the key link in the calculation of indirect costs and the allocation of these costs to individual outputs.

In the next section I will discuss the main empirical issues involved in the estimation of a cost function that can be used to derive the above results.
4. EMPIRICAL ISSUES

Before such a model can be constructed, the relationship between economic and accounting cost must be addressed. In the military setting this requires a general analysis of the contractors' accounting structures and a detailed evaluation of each relevant data item. By establishing the conformity of the data to the concept of economic cost, the duality properties of economic cost theory can be used to generate a full representation of the cost and production relationships.

A second important prerequisite to the study is the establishment of a common usage of cost terminology. Referring to Figure 1, it is clear that in a short run framework, in which economists consider some factors to be fixed, overhead costs would include I, II and possibly III. Direct fixed costs may sometimes be included in overhead costs since they cannot be changed in response to short run changes in any output. However, in a long run economic cost sense, III is absorbed into IV as all factors become variable. But the most important discrepancy between the languages of the economist and the accountant occurs in the assignment of specific functional costs reported by contractors to these four general categories. While block IV, direct labor and direct material costs, is unambiguously defined, blocks I, II, and III are less clear cut. Costs reported as plant and equipment costs cannot be neatly assigned to block III since some of those costs are indirect. Likewise some of the costs reported as administrative and managerial costs could be assigned to either block I or block II.

In order to successfully estimate a cost function there must be a thorough analysis of (1) the accounting methodologies used to distribute these costs and (2) the implicit allocations used to separate real costs into the various reported categories of direct production and factory overhead costs.

Another important issue concerns the depreciation rules used by the contractors. In order to estimate a true economic cost function, the economic depreciation rate should be used. Economic depreciation measures the change in the value of an asset and is independent of physical wearing out. There is an extensive literature on economic depreciation and unfortunately little in the way of useful empirical tools to calculate it. In practice, however, accounting depreciation rates are generally employed. Whenever possible, however, adjustments should be made to take into account the true rates of loss of economic value of the firm's fixed capital.
Figure 1

DECOMPOSITION OF SHORT RUN TOTAL COST

I. Indirect Labor and Materials
II. Administrative, Corporate Headquarters
III. Plant and Equipment
IV. Direct Labor and Material, e.g. Engineering and Manufacturing Labor
Another issue is the specification of an appropriate functional form. A cost function that is properly specified is guaranteed by the duality of cost and production to convey all economically relevant cost information about the producing firm. The precise functional form used depends on the desired balance between generality and computational ease, and also on a number of assumptions concerning the particular characteristics of the industry being studied. Some of the issues to be addressed are:

(a) the choice between a long run and a short run function
(b) modifications in the model due to the unavailability of certain types of data, e.g. variable input prices
(c) the appropriate level of disaggregation
(d) returns to scale
(e) technological change
(f) inter-firm, inter-plant differences, etc.

Also, a statistical model must be chosen that conforms to certain econometric criteria, such as overall fit, coefficient significance, identification, and the diagnosis and correction of all statistical pathologies. When these criteria are satisfied the model results can be considered econometrically valid and defensible.

An appropriate functional form is the generalized transcendental logarithmic (translog) function. It is general enough to deal with multiple outputs, variable elasticities of substitution among factors and variable elasticities of transformation among outputs and it enables one to test for properties of the production technology such as separability, homogeneity and non-jointness. The generalized translog cost function takes the form:

$$\ln C = \alpha_0 + \sum_{i} \beta_{wi} \ln w_i + \sum_{k} \beta_{ql} \ln q_l$$

$$+ \frac{1}{2} \sum_{ij} \gamma_{ij} \ln w_i \ln w_j$$

$$+ \frac{1}{2} \sum_{kl} \rho_{kl} \ln q_k \ln q_l$$

$$+ \sum_{ik} \lambda_{ik} \ln w_i \ln q_k$$

The $\alpha$'s, the $\beta$'s and the remaining parameters are interpreted as the value, the first derivatives and the second derivatives of the log of the underlying function. Therefore the actual value, gradient and
Hessian of the function can be derived through differentiation of the logarithmic function. When a cost function is specified in this form it is possible to derive the full cost properties of the firm's production process, including:

- Marginal, incremental and average costs (long run and short run)
- Product specific economies of scale
- Overall economies of scale in production
- Economies of scope
- Direct and indirect costs
- Technologically consistent allocations of indirect costs
- Own and cross elasticities of factor demand
- Partial elasticities of substitution
- Returns to scale

Using the results derived from this function, we are able to determine direct and indirect costs as in (6).

\[
\text{Total costs} \quad \begin{array}{ccc}
\text{Direct Costs} & \text{Indirect Costs} \\
\end{array} \\
\]

\[
\begin{array}{c}
C(Q) = \sum Q_i AIC_i = Sc' \cdot C(Q) \\
\end{array} \\
\]

where \( C(Q) \) = total costs
- \( Q_i \) = quantity of output \( i \)
- \( AIC_i \) = average incremental cost of output \( i \)
- \( Sc' \) = economies of scope

We can combine estimates of marginal costs with the scope measure to derive allocations of indirect costs to individual outputs, by the following formula:

\[
\text{IDC}_i = Q_i MC_i (S - S_i). \\
\]

In order for a cost function to maintain full duality properties it should include a vector of output quantities, \( Q \), a vector of input prices, \( W \), and a measure of total cost, \( C \). A properly specified model will be likely to also include a measure of capital stock, \( K \) (if it is a short run function), a technical change variable, \( T \), and indicator variables, \( D \), to account for discrete changes over time:

\[
C = f(Q, K, P, WR, PM, T, D). \\
\]
where \( C = \) total company or division costs

These can be constructed from expense accounts.

\( Q = \) total units of each output produced by the company or division

\( K = \frac{BV}{PI} = \) quantified stock for each company

\( BV = \) book value of capital stock by company and capital category, e.g. land and buildings, vehicles, machinery, etc.

If \( K \) includes capitalized labor this component should be purged by using loading ratios provided by the company.

\( PI = \) Price index for each capital category supplied by firm or external proprietary sources;

\[ P = \frac{[(r + \delta)/(1 - t)]}{PI} \] = the price, or user cost, of capital.

\( r = \) rate of return to capital, or the interest rate as proxy

\( \delta = \) depreciation rate

If not reported, this can be derived from depreciation expenses divided by \( K \)

\( t = \) corporate tax rate

This is a composite income tax rates. It includes gross receipt and property taxes, investment tax credits and any other significant taxes. Frequently, the company reports its composite corporate tax rate.

\( WR = \) Wage rate by labor category for each company

If not directly reported, these can be derived from total labor compensation and total hours of employees.

Labor categories usually include professional and administrative, engineering, maintenance, building supply and motor vehicles, construction, etc.
PM = Price of materials and non-wage expenses.

These can be derived from various accounting reports using total labor or some other proxy for units.

T = a discrete time variable

D = indicator (or dummy) variables

When the cost estimation is completed, forecasts of project overheads based on estimates of direct labor or other direct cost components can be simulated by using the cost allocations implicit in the estimated cost function. These forecasts will be completely consistent with the cost structure of the production process and will thus be able, for example, to fully account for changes in non-military output.
5. APPLICATIONS OF THE THEORY

A number of multiproduct cost estimations have been reported in the literature. Studies have been performed on the areas of transportation (Spady and Friedlander, 1976) securities (Mathews, 1981), airlines (Caves, Christianson and Tretheway, 1984) and telecommunications (Goetz, 1984). Most of these studies have involved cross section data or pooled cross section/time series data. The crucial issue is often the availability of sufficient data for the estimation of the generalized functional forms proposed above. For example, when only three input prices and three outputs are used, the translog function contains 22 independent variables, including all of the interaction variables. In a time series estimation procedure, this would require four and a half years of quarterly data just to have enough degrees of freedom to actually perform the estimation. Even if the estimation was possible, the statistical validity of the results would be seriously in question.

A number of techniques have been developed to circumvent this problem, with some success. Spady and Friedlander (1976) use homothetic aggregation and index number techniques. These allow several outputs or inputs to be combined for the purposes of estimation in such a way that their independent effects can be disentangled ex post.

Spady and Friedlander (1978) and Goetz (1984) use hedonic functions to differentiate variations of physical output based on attributes or qualities. Again, these functions must conform to certain rules in order to allow the separate effects of the output attributes to be observed. This technique has broad applicability to industries where technological improvement is rapid or products change in composition from year to year.

Other techniques involve estimating the cost equation in sections, usually by estimating all the associated conditional factor demand equations first and then the remaining terms. Another technique is to apply zero restrictions to the individual terms based, in the first instance, on prior information that some should be zero, and subsequently on the relative importance and impact of that term on the overall equation. Goetz (1984) has estimated a dynamic capital adjustment model for use in the telecommunications industry, using these techniques with great success.

This author has discussed the applicability of this approach to the study of Naval jet engine overhead costs with members of the Cost Analysis Branch, Naval Air Development Center in Warminster, PA. A number of specific methodological issues were identified. In particular, the definition of output must be carefully evaluated. It is clear that the number of completed engines of each type in any period is inadequate since production is lumpy and the period of final assembly may differ from the period in which most of the costs were incurred. The use of engine equivalents is also inadequate due to the fact that individual parts have widely divergent cost properties, and the output measure is determined by the part with the lowest output in terms of engine equivalents. In other words, if one essential part
was not produced at all during a period, output is zero even if all of the parts are being produced at full capacity. The most reasonable approach is to have four or five major engine components as well as finished engines as arguments of the cost function. It must be remembered, however, that the estimated coefficients on the finished engine term reflects only the cost of the value added of putting the components together.

All other relevant cost data needed to estimate jet engine cost appear to be readily available. Table I summarizes the data items required.

In considering applications of this model to other military settings, several limitations become apparent, and also a vast potential is realized. On the limitations side, there are few life-cycle applications of this approach. It is restricted to the production cycle and the larger the product and the longer and steadier the production run, the better. Secondly, we have just grazed the surface of converting accounting terminology to economic terminology and visa versa. A great deal of care must be taken to understand the military and contractor accounting methodologies and to establish conformity with the economic cost model.

On the potential side however there is great promise. First of all, most of the problems that can arise have been resolved satisfactorily in other settings. These are working models used by industry to solve concrete strategic pricing and planning problems. Secondly, this methodology utilizes data that is manageable and is in fact frequently available from outside sources (such as SEC 10k reports). There is no need to replicate and evaluate the minute accounting rules that build up from the plant micro costing level all the way up to the corporate level. This method essentially looks at the cost structure from the outside. And finally, there is no other method that is able to objectively provide this kind of independent check on the overhead cost reported by contractors. In the current environment of military cost hysteria, this method can provide a balanced and defensible means of evaluating overhead costs.
TABLE 1

SUMMARY OF REQUIRED COST INFORMATION

1. Labor
   a) Total labor compensation
   b) Wage rates (actual employee pay)
   c) Number of employees
       Broken down into major labor categories
   d) Cost of fringe benefits, pensions, etc.

2. Capital
   a) Book value of capital stock by capital category
   b) Does the firm have capitalized labor?
       1. What are the loading (capitalized) ratios?
   c) Total depreciation expenses and/or depreciation expense for each
       capital breakdown.
   d) Corporate income tax rate and total taxes paid

3. Material
   a) Total material cost
   b) Material prices
       We must have two of these three variables
   c) Quantity of inputs
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