A procedure is developed for the allocation of an operations and support (O&S) cost target. The procedure is specifically tailored to an aircraft system. It involves the use of the Program Evaluation and Review Technique (PERT), regression analysis and other standard statistical methods to derive a predicted subsystem operating cost per flight hour. In addition, it provides a method of adjusting cost distributions among the subassemblies when a defined risk value is exceeded. The procedure can be used as a cost allocating tool in applying an O&S cost guarantee. The study contains examples of the procedure's use.
ALLOCATION OF OPERATIONS AND SUPPORT COST TARGETS STUDY

J.O. Kolson, G. W. Tyner, and E. O. Wehlander
Vought Aero Products Division
LTV Aerospace and Defense Company
P.O. Box 225907
Dallas, Texas 75265

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The Air Force wants to hold contractors responsible for product performance during operational use. The use of an Operations and Support (O&S) cost guarantee is one such method. However, for industry there is a general reluctance to offer these guarantees, because of the contractor's inability to determine the risk involved, and the operating cost of the delivered product. Although the government's position on the use of O&S cost guarantees is not known, there is increasing emphasis on warranties that are similar to an O&S cost guarantee. Examples are Reliability Improvement Warranties, the KC-10 Warranty Program, the service life policy of the C-17, and the product-warranty provisions in the 1984 Defense Department Appropriations Bill.

This study develops a procedure for allocating government aircraft system cost targets into manageable system component sub-targets. This procedure uses standard statistical techniques, the central limit theorem, regression analysis, and the Program Evaluation and Review Technique (PERT). Statistical cost equation values are derived using Air Force 66-1 data, the Air Force Visibility and Management of Operations and Support Cost (VAMOSC) data, and engineering judgement.

The derivation of the allocated cost is based on the performance of six major operations:

- Establish an acceptable contractor risk. This is an assumed value in the study, and risk determination methodology is not addressed.
- Define an approximate distribution of O&S cost for each subsystem.
- Derive an approximate cost distribution for the combined subsystems.
- Compute the probability of exceeding the cost guarantee.
Accept or reject the probability of exceeding the cost guarantee as determined by the acceptable risk value.

Adjust the subsystem cost distribution to obtain an acceptable risk value.

Each of these operations is illustrated by examples which are accompanied by evaluations of the strength and scope of each operational step.

The procedure, which is both flexible and iterative, offers a systematic framework for determining an O&S cost target allocation. Its primary decision problem is the determination of initial cost values. However, techniques that reduce the effect of this problem have recently been developed.
<table>
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<td>37</td>
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<tr>
<td>Appendix B</td>
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</tr>
</tbody>
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1.0 INTRODUCTION

Background and Objectives

Program decisions are required in the course of a weapon system life-cycle to determine the best balance among three parameters: Cost, Schedule, and Performance. Cost is probably the most dynamic factor in the life cycle. For example, support costs account for 80% [1] of the total life-cycle cost (LCC) of some systems, and if equipment performance is less than expected or specified, long-term support cost is directly affected.

To counter undesirable support cost trends, the government is considering various types of long-term guarantees. These guarantees will hold a contractor responsible for a designated O&S cost envelope over a stated period. However, two factors that must be considered in the implementation of such guarantees are: the willingness of the industry to accept them, and the manner in which contract requirements will be monitored.

Contractors have been reluctant to accept the implementation of a long-term guarantee because of the risk involved, and their inability to monitor the cost performance of a delivered product. [2] Historically, there has been little emphasis on O&S cost control because contractors consider such control to be the government's responsibility. O&S cost consciousness in industry must be enhanced.


In implementing an O&S cost guarantee, the government initially establishes a firm O&S cost target for the system. Next, the contractor allocates the system O&S cost target to specified system components as cost subtargets. The Allocation of O&S Cost Targets study presented here provides a procedure for performing and evaluating the firm cost target down to specific components. The procedure fulfills the requirement to increase industry's O&S cost consciousness by providing cost visibility at the two to three digit work unit code (WUC) levels. It also provides a basis for contractor risk reduction simply by breaking a high risk venture into smaller more manageable items. This makes it possible for each item to have a separate guaranteed O&S cost with separate profit and loss probabilities. The contractor, as a result, is provided a method of spreading his risk and increasing his risk adjustment capability. Finally, the VAMOSC system provides a basis for monitoring O&S cost performance.

In a situation in which an O&S cost guarantee is implemented, the top-level O&S cost target is established by the government. Only the allocation of the top-level O&S cost target into manageable subtargets is addressed here. The selection of the system's allocation cost guarantee candidates, the determination of the customer and contractor responsibilities for O&S cost elements, and risk analysis are outside the scope of this study.

Although the selection of a system's O&S cost guarantee candidates is beyond the study's scope, some selection methodologies that might merit consideration are as follows:

- The total system could be guaranteed.
- System items could be selected based on O&S cost histories of similar systems.
- System design could be done using probabilistic techniques as opposed to deterministic techniques as discussed by E. B. Haugen and P. H. Wirsching, Associate Professors, Dept. of Aerospace and Mechanical Engineering, University of Arizona, Tucson, Arizona in 1975. Each designed
assembly could have an assigned uncertainty value that would cover the following uncertainties:

- in reliability predictions and evaluations
- in the maintainability predictions and evaluations
- in the maintenance philosophy
- in discounting and inflation rates

The assemblies with the highest uncertainty values would be possible candidates.

- System could be subject to a stress screening program. System items would be eligible for guarantee candidacy when their ratio of assembly to parts in the assembly stress screening is low, for example, where 1 out of 4 assemblies are stress screened compared to 6 out of 6 parts in each assembly being stress screened.

The application of the allocation procedure is the focus of the study, which begins with a description of the procedure and the statistical approach. A hypothetical scenario that sets the stage for the application is developed. The six operations involved in the procedure are then performed and discussed, and an acceptable risk level is determined. The approximate cost distributions are defined for each subsystem and for the combined subsystems. The probability of overstepping the cost guarantee is determined, along with risk acceptability. Finally, the method of adjusting the subsystem O&S cost allocation is developed for situations in which the probability of exceeding the cost guarantee is greater than the acceptable risk. Results, discussion, conclusions and recommendations follow.
2.0 METHOD

2.1 Procedure Development

The O&S cost target allocation procedure employs various analytical techniques that make it both flexible and iterative. This procedure provides management with a tool to systematically and objectively allocate cost targets to subsystem levels. Figure 1 shows the six major operations of the procedure.

To assess with what probability a cost guarantee can be met, it is necessary to approximate the distribution of the O&S cost of each relevant subsystem. If a relational method can be determined for the selected subsystems, then a combined probability distribution can be found. In considering the O&S cost associated with a given subsystem, we find that the cost is subject to random variation; consequently, it is considered a random variable. Moreover, there is a statistical distribution associated with a random variable that describes its behavior. Once this statistical basis for the O&S subsystem cost is established, a technique for defining the cost distribution needs to be determined.

The particular technique chosen to define an approximate O&S cost distribution for a given subsystem depends upon available information. Since the initial allocation of the O&S cost will most likely occur during the conceptual design phase, when detailed engineering information is often limited, the approach to defining distributions should not depend heavily upon historical data or detailed design and performance inputs. The statistical approach developed for the PERT system [3] was found to be an appropriate technique for deriving approximate O&S cost distributions during conceptual design. The approach requires that three O&S cost values be estimated for each subsystem. These cost values are as follows:

a = "Optimistic" O&S subsystem cost: an O&S subsystem cost that would be undercut only 5 percent of the time (a 5-percentile value of the distribution)

m = "Most Likely" or modal value: The most frequently occurring O&S subsystem cost

b = "Pessimistic" O&S subsystem cost: an O&S subsystem cost that would be exceeded only 5 percent of the time (a 95-percentile value of the distribution)

Based on the cost values selected for a, m, and b, the O&S subsystem cost distribution may appear skewed or symmetric, as shown in Figure 2.

The original development of PERT [4, 5] defined a and b as the 0 and 100 percentiles, respectively. A later study by Moder and Rodgers [6] which gave preference to the 5 and 95 percentile definitions, is more applicable to the O&S cost allocation procedure requirements.


Figure 1  Overview of O&S Cost Allocation Procedure

1. Establish acceptable level of risk, \( r \)
2. Define approximate subsystem O&S cost distribution
3. Define approximate combined subsystem O&S cost distribution
4. Compute cost guarantee overstep probability (K)
5. Adjust subsystem O&S cost distribution
6. Acceptable if \( K > r \) or unacceptable if \( K < r \)
7. Stop
Figure 2 O&S System Cost Distributions
The manner in which the values for \( a, m \) and \( b \) are estimated depends upon the information that is available and how the subsystem compares with similar subsystems that are currently operational. In some cases, O&S cost values may be based on vendor and development test data. In others, the estimates may be based on subjective information such as the opinions of engineers and logisticians. Reducing the uncertainty in cost value estimating appears to be a feasible objective.

A more objective estimating approach is the development of prediction equations that are based on historical VAMOSC and 66-1 maintenance data. Recent work in this area [7, 8] indicates the general feasibility of developing O&S cost estimating relationships through regression analysis that employs VAMOSC and corresponding operational data. For the near future, the limited availability of VAMOSC data may inhibit the use of this approach.


Assuming that data is available and that the subsystem to be estimated is a logical extension or combination of existing subsystem techniques, an O&S cost estimating relationship can be developed. The form is as follows:

\[ y = b_0 + b_1x_1 + b_2x_2 + \ldots + b_nx_n \]

WHERE: \( y \) is some functional form of O&S cost, such as dollars per flight hour, dollars per available aircraft, etc. \( x_1, x_2, \ldots, x_n \) are operational factors selected as predictor variables such as failure rates and mean time to repair, and \( b_0, b_1, \ldots, b_n \) are coefficients derived through regression analysis.

In using the equation to estimate \( a, b, \) and \( m \) for the subsystem, appropriate values are required for the operational factors (x's) included in the equation. The choice of operational factors for the equation would be limited to information readily available at the time the equation is applied; otherwise, considerable estimation will be required to derive the appropriate x values.

2.2 Scenario

The following hypothetical situation is used in subsequent sections to illustrate the cost target allocation procedure's application. A contractor's aircraft system is in the conceptual design phase. The contract calls for an O&S cost guarantee on four systems: landing gear, flight controls, engine core module and radar. The guarantee is expressed as a total not-to-exceed cost of $45.00 per flight hour for the systems. There is a penalty of one-tenth of 1 percent (0.1%) of the fixed-price fee for each dollar of O&S costs exceeding the contract limits. The cost guarantee covers a period of 5 years or 300,000 accumulated flight hours, whichever occurs first. The O&S cost status will
be reviewed quarterly. The scenario rationale is partially based on AFR 173-13 data for a typical F-16 Squadron repair support costs.

2.3 Establishing the Risk Level

The contractor normally desires to optimize his ability to make a profit. Determining the probability of avoiding any penalty adds to the assurance of maximum profits under a fixed-fee contract; therefore, it is assumed that management has set an acceptable risk level \( r \) at 0.05, meaning that management is willing to take a chance that the average quarterly O&S cost per flight hour for the four systems will exceed the $45.00-per-flight hour limit 5 percent of the time. Table 1 lists characteristics that might be used in determining the acceptable risk level. As previously stated, risk analysis lies outside the scope of this study.

2.4 Defining the Approximate Cost Distribution

Using the techniques of comparisons, opinions, and prediction equations outlined in Section 2.1, the three PERT cost values, optimistic \( a \), pessimistic \( b \), and most likely \( m \), were derived for each of the four selected systems.

**Landing Gear** - The values for \( a \), \( b \), and \( m \) are estimated directly from VAMOSC cost data for the landing gear systems of the F-15A and F-16A. The landing gear system on the aircraft in the hypothetical situation is assumed to be similar to those of the F-15A and F-16A. The aircraft is also assumed to be somewhat larger and heavier than the F-16A, but smaller and lighter than the F-15A. The average F-15A O&S cost ($24.00/flight hour) is used as the pessimistic value \( b \), the average F-16A O&S cost ($9.00/flight hour) as the optimistic value \( a \), and the weighted average of F-15A and F-16A O&S costs as the most likely value \( m \). While this approach is subjective, an examination of the failure characteristics of a landing gear system
<table>
<thead>
<tr>
<th>SYSTEM CHARACTERISTICS</th>
<th>LOW</th>
<th>MEDIUM</th>
<th>HIGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Technology</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Present State of Technology</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Majority of System's Subassemblies Bought</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Majority of System's Subassemblies Made</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three-level Maintenance Concept - Org-Int-Depot</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Two-level Maintenance Concept - Org-Depot</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Majority of System Is Modular</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Majority of System Is Nonmodular</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Operational System Semifixed - Permanent Basing</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operational System Mobile - Tactical</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Newly Activated Units Will Use System</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>System will Transition into Operational Units</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>System's Logistics Element is Directly Controllable</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>System's Logistics Element Possesses Interfacings</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Number of Events in Maintenance Analysis Is High and Serialised</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Number of Events in Maintenance Analysis Is Low and Nonserialised</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Most of the System's Subassemblies Have Component Interconnectivity</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>System's Subassemblies Have Limited Component Interconnectivity</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>System Design Has Redundancy</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>System Maintenance Analysis Depends on Embedded Diagnostics</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>System Maintenance Analysis Depends on Peculiar Support Equipment</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Operational Environment is Well Defined</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Dual Role System</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Single Role System</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Number of Systems Bought &gt; 650</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Number of Systems Bought &lt; 650</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
reveals that it is reasonable. The correctness of this approach is supported by the fact the primary contributor to the system failure rate is tires and the primary contributors to tire failures are aircraft weight and take-off/landing speed. This example illustrates how subjectivity and logistics/engineering expertise may be used in combination to determine initial cost values when "hard" data is not available.

**Flight Controls** - The technique applied to obtain the landing gear system PERT cost values is also used to estimate the initial a, b, and m values for the flight controls system. The VAMOSC-determined O&S cost for the F-16A with its fly-by-wire system is assumed to be the optimistic value (a). The pessimistic value (b) is assumed to be equal to the F-15A with a "conventional" system, and the weighted average of the F-15A and F-16A is assumed to be equal to the most likely value (m).

**Engine Core Module** - The engine core module cost equation (1) was developed using F-15A and F-16A VAMOSC and 66-1 maintenance data, and a Vought-developed cost equation that relates operations and maintenance factors to O&S costs using regression methods.[7]

\[
\text{O&S Cost/Flight Hour} = 15.6473 - (0.0164410)(\text{MFHBF}) - (0.0490074)(\text{MFHBMA})
\] (1)

It is assumed that the manufacturer has predicted that mean flight hours between failures (MFHBF) will be 450 and mean flight hours between maintenance actions (MFHBMA) will be 150 for the engine core module. Historically, it is assumed that the actual MFHBF and MFHBMA of this manufacturer's engine parts have been from 70 to 90 percent and from 50 to 80 percent, respectively, of the values predicted.

[7] ibid p 11
Considering the engine core module as an existing design, an MFHBF of 405 (90 percent of 450) and an MFHBMA of 120 (80 percent of 150) are used in the equation to estimate the most likely value \((m)\). An MFHBF of 315 (70 percent of 450) and an MFHBMA of 75 (50 percent of 150) are used to estimate pessimistic value \((b)\). The manufacturer's values of an MFHBF of 450 and an MFHBMA of 150 are used to estimate the optimistic value \((a)\).

**Radar** - The radar cost equation (2) is developed in a manner similar to that used for developing the engine core module equation. The equation is as follows:

\[
\text{O&S Cost/Flight Hour} = 0.204657 + (1545.58)(\text{Failure Rate})
\]

The reliability of the subsystem is expected to be similar to that for the F-16A's radar. Thus, low, high, and most likely failure rates of 0.0048, 0.0025, and 0.0035, respectively, are determined based on the historical failure rate trends of the F-16A. This determination is made based on the review of four quarters of failure data. The quarters with the lowest and highest numbers of failures are used to compute the low and high failure rates; the average of these two constitutes the most likely rate. These values, in turn, are used in the equation to estimate, \(a\), \(b\), and \(m\) for the radar system.

The data used to develop the equation for the engine core module and radar system are discussed in Appendix A. The sole purpose of developing these equations is to demonstrate the cost estimating relationship approach. Due to the limited data from which they are derived, the sample equations themselves may not be useful predictors in actual practice. The values of \(a\), \(b\), and \(m\) for the four subsystems are summarized in Table 2.
TABLE 2  INITIAL ESTIMATES FOR PERT REQUIRED VALUES FOR HYPOTHETICAL O&S COST GUARANTEE

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>MOST LIKELY O&amp;S COST PER FLT HR</th>
<th>OPTIMISTIC O&amp;S COST PER FLT HR</th>
<th>PESSIMISTIC O&amp;S COST PER FLT HR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landing Gear</td>
<td>12.00</td>
<td>9.00</td>
<td>24.00</td>
</tr>
<tr>
<td>Flight Controls</td>
<td>15.00</td>
<td>11.00</td>
<td>24.00</td>
</tr>
<tr>
<td>Engine Core Module</td>
<td>3.10</td>
<td>0.90</td>
<td>6.80</td>
</tr>
<tr>
<td>Radar</td>
<td>5.60</td>
<td>4.10</td>
<td>7.60</td>
</tr>
</tbody>
</table>
2.5 Defining an Approximate Combined Subsystem O&S Cost Distribution

Since the O&S cost of each subsystem included in the cost guarantee is a random variable, the O&S cost of the combined subsystem is also a random variable. In fact, the combined, or total, O&S cost random variable, \( T \), is the sum of the individual subsystem random variables, and the associated statistical distribution may be mathematically derived based on the statistical distributions for the subsystems.

Using the PERT statistical approach, two parameters - mean and variance - are estimated for each subsystem's O&S cost. These parameters are estimated using the values for \( a \), \( b \), and \( m \) derived in Section 2.4. For each subsystem, the mean \( (D) \) and variance \( (V) \) are:

\[
D = \frac{(a + b + 4m)}{6} \tag{3}
\]

\[
V = \left[ \frac{(b - a)}{3.2} \right]^2 \tag{4}
\]

The rationale behind the choice of these formulas is discussed by Moder and Phillips. [3]

For combined subsystem random variable, \( T \), the mean \( (D_T) \) and variance \( (V_T) \) are derived as follows:

\[
D_T = \sum D = \text{Sum of the subsystem O&S cost means} \tag{5}
\]

\[
V_T = \sum V = \text{Sum of the subsystem O&S cost variances} \tag{6}
\]

[3] ibid p 7
The parameter estimates for the cost allocation example are shown in Table 3. Variance, $V_T$, is estimated under the assumption that the subsystem costs, as random variables, are statistically independent, that is, the O&S cost associated with a given subsystem is independent (unrelated) to the O&S cost of all other subsystems subject to the cost guarantee.

The central limit theorem states that the distribution of the sum of $n$ independent random variables tends to be normally distributed, with a mean and variance equal to the sum of the means and variances of the $n$ independent random variables as $n$ becomes infinitely large. Thus, the distribution of the total system, $T$, is said to be approximately normal. In practice, increasing the number of subsystems that are summed together increases the quality of the approximation. In addition, the more normal the individual subsystem O&S cost distributions are, the better the normal distribution approximates the sum of the subsystem O&S cost distribution.

2.6 Computing the Cost Guarantee Overstep Probability

Using the central limit theorem, $T$, the O&S cost for the total of the subsystems under the cost guarantee is distributed in a near-normal manner with mean $D_T$ and variance $V_T$. Thus, the probability of exceeding the O&S cost guarantee value, $C$, may be determined by straightforward statistical means. Namely.

$$P(T > C) = P \left( \frac{T - D_T}{\sqrt{V_T}} > \frac{C - D_T}{\sqrt{V_T}} \right)$$

(7)

$$\approx P \left( Z > \frac{C - D_T}{\sqrt{V_T}} \right) = K$$

(8)
<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>m</th>
<th>a</th>
<th>b</th>
<th>D</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landing Gear</td>
<td>12.00</td>
<td>9.00</td>
<td>24.00</td>
<td>13.50</td>
<td>21.97</td>
</tr>
<tr>
<td>Flight Controls</td>
<td>15.00</td>
<td>11.00</td>
<td>24.00</td>
<td>15.83</td>
<td>16.50</td>
</tr>
<tr>
<td>Engine Core Module</td>
<td>3.10</td>
<td>0.90</td>
<td>6.80</td>
<td>3.35</td>
<td>3.40</td>
</tr>
<tr>
<td>Radar Set</td>
<td>5.60</td>
<td>4.10</td>
<td>7.60</td>
<td>5.68</td>
<td>1.20</td>
</tr>
</tbody>
</table>

TOTAL SYSTEM UNDER GUARANTEE \[ D_T = 38.36 \]
and \[ V_T = 43.07 \]
Where:

\[ T = \text{O&S Cost of combined subsystems} \]
\[ D_T = \text{Mean of } T \]
\[ V_T = \text{Variance of } T \]
\[ C = \text{Cost guarantee value} \]
\[ Z = \text{Standard normal random variable (i.e., a normal random variable with mean 0 and variance 1)} \]

and \( K \) = Value derived from a table of probabilities for the standard normal distribution

This calculation is important because it allows us to compare the value of \( K \) with \( r \), the acceptable level of risk previously set in Section 2.3. If \( K \) is less than or equal to \( r \), there is nothing more to be done. As long as the subsystem costs occur as defined by the values of \( a \), \( b \), and \( m \), the probability of exceeding the cost guarantee is less than or equal to the accepted risk. In short, the allocation can be based on the available estimates of the subsystem O&S cost. If \( K \) is greater than \( r \), the subsystems that are subject to the cost guarantee require further study and alteration.

For the hypothetical situation, the probability of exceeding the cost guarantee is computed as follows from equations (7) and (8):

\[ C = \text{O&S cost guarantee} = \$45.00 \text{ per flight hour} \]
\[ D_T = \text{Mean of } T = \$38.36 \text{ per flight hour} \]
\[ V_T = \text{Variance of } T = \$43.07 \text{ per flight hour} \]

\[
P (T > 45.00) = P \left[ \frac{T - 38.36}{\sqrt{43.07}} > \frac{45.00 - 38.36}{\sqrt{43.07}} \right]
\]

\[
P (Z > 1.01) = 0.15625 *
\]

* From tables of standard normal distribution probabilities
The assumed management risk level \( (r) \) is 0.05, as indicated in Section 2.3. The probability of exceeding the cost guarantee is about 0.16; therefore, adjustments in the subsystems O&S cost distributions are required to lower the probability of exceeding $45.00 per flight hour.

2.7 Establishing the Priority for Subsystem O&S Cost Alteration

When the probability of exceeding the cost guarantee is greater than the acceptable risk, further investigation of the subsystems and their associated O&S cost is required. The goal of this additional investigation is to determine areas in which subsystem improvements must be made in order to lower the risk of penalty. One way to lessen the risk is to try to improve all subsystems under the cost guarantee. However, this approach seems potentially inefficient and largely ineffective, because subsystems are likely to differ with respect to their O&S cost distributions and their attainable improvement. A more efficient and effective approach is to systematically set priorities on subsystems as candidates for O&S cost improvement, giving highest priority to those subsystems with the greatest potential for lowering risk.

Various quantitative and qualitative measures may be used to help establish a priority on the subsystems. Many of the factors, such as the mean, variance, and skewness, may relate to the distribution. Others may relate to the potential for further changes in the O&S cost, such as technology improvement, confidence in the estimates for \( a, b, \) and \( m \), and modularity of design. To reduce subjectivity, the measures should be quantitative. For qualitative factors, techniques such as ranking the subsystems with respect to the characteristic of interest may be used to quantify the information.

Subsystems which merit further review are determined by combining the various measures considered to be of significance. In assimilating the information, those factors considered to be of
greater importance are given greater relative weight. Steps are also taken to ensure that all factors have comparable units of measurement.

In the cost allocation example, six quantitative measures are used to determine which subsystem or subsystems should be investigated for O&S cost improvements. For each subsystem, the following measures or factors are considered:

- **Mean = D**: Represents the central or average value for O&S cost per flight hour.
- **Variance = V**: Measures dispersion or variability in the O&S cost per flight hour.
- **Asymmetry Quotient = AQ = (b - m)/(m - a)**: With AQ = 1, the distribution is symmetric. If AQ < 1 or AQ > 1, the distribution is skewed to the left or right, respectively. The further the value is from 1, the greater the skewness.
- **Coefficient of Variation = CV = \sqrt{\text{V}/D}**: A unitless measure of the relative degrees of uncertainty or variation in the O&S cost per flight hour when comparing subsystems.

- **Percent Reliability Improvement**: An attainable percentage of improvement in subsystem reliability due to improvements in design, technology, vendor parts, redundancy, etc.
- **Percent Maintainability Improvement**: An attainable percentage of improvement in subsystem maintainability due to improvements in equipment accessibility, testability, modularity, etc.

The six quantitative measures for each subsystem are shown in Table 4. For each measure, the larger the value, the greater the potential impact for improving or reducing the subsystems O&S cost per flight hour distribution. To avoid the distortion caused by using different units of measurements, the subsystem values for each measure in Table 4 are normalized by summing each column and
### TABLE 4 COMPARATIVE MEASURES USED FOR HYPOTHETICAL O&S COST IMPROVEMENT ASSESSMENT

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>MEAN</th>
<th>VAR</th>
<th>ASYMM QUOT</th>
<th>COEFFT OF VAR</th>
<th>% REL IMPROVE</th>
<th>% MAINT IMPROVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landing Gear</td>
<td>13.50</td>
<td>21.97</td>
<td>4.00</td>
<td>0.347</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>Fit Controls</td>
<td>15.83</td>
<td>16.50</td>
<td>2.25</td>
<td>0.257</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>Engine Core Module</td>
<td>3.35</td>
<td>3.40</td>
<td>1.68</td>
<td>0.550</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Radar</td>
<td>5.68</td>
<td>1.20</td>
<td>1.33</td>
<td>0.193</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>
dividing the column sum into each subsystem's column value. Using
the normalized values, a composite value for all six measures is
derived, and each normalized factor is given equal contribution
to a composite index. Consequently, the six normalized values
are summed for each subsystem. The normalized subsystem values
and composite index are shown in Table 5.

The composite index indicates that improvements in the land-
ing gear and flight controls have the best potential for lowering
the risk of exceeding the cost guarantee. The high priority placed
on these two subsystems resulted primarily from the magnitude of
the means and variances. Since the allocation procedure is oriented
toward adjusting one subsystem O&S cost distribution at a time,
the landing gear was chosen for further investigation.

2.8 Adjusting Subsystem O&S Cost Distributions

Making various types of adjustments to a subsystem's O&S cost
distribution will reduce the combined subsystem probability of
exceeding the cost guarantee. For example, any alteration in
the values of a, b, and m that reduces the mean, variance, or
asymmetry of the subsystem, and, in turn, the combined subsystems,
reduces the risk of penalty. Figure 3 shows the impact of reduc-
ing the mean, variance or asymmetry.

From equation B-4 in Appendix B, the relationship that must
be satisfied to achieve the acceptable level of risk is

\[
K^2_r \left[ \frac{V^1}{T} + \left( \frac{b^1 - a^1}{3.2} \right)^2 \right] \leq \left[ (C - D^1) - \left( \frac{a^1 + b^1 + 4m}{b} \right) \right]^2
\]  (9)
Where:

\[ V^1_T = \text{Variance for the combined subsystems excluding the subsystem under adjustment.} \]

\[ D^1_T = \text{Mean for the combined subsystems excluding the subsystem under adjustment.} \]

\[ r = \text{Previously defined acceptable level of risk.} \]

\[ K_r = \text{The upper } r \text{ (100 percent) point of a standard normal distribution.} \]

\[ C = \text{O&S cost guarantee value,} \]

\[ a^1, b^1, \text{ and } m^1 = \text{revised values of } a, b, \text{ and } m \text{ for the subsystem O&S cost distribution under adjustment.} \]

Any revision or adjustment to the estimated O&S cost distribution for a selected subsystem should be justified. For example, more detailed information on the subsystem's capabilities, design, or anticipated changes may reduce the estimated values of \( a, b, \) and \( m \). In short, the revised estimates must be attainable values.

If the selected subsystem O&S cost adjustments do not provide the acceptable level of risk, another subsystem cost distribution may require revision. This will be apparent from the direction of the inequality in equation (9). The suggested approach is to incorporate revised information on the first subsystem examined before assessing the potential for improving the O&S cost distribution of the second subsystem.

In the hypothetical situation, the landing gear subsystem of the conceptual aircraft is reexamined. Further discussions with design engineers indicated that the reliability of the landing gear is better than initially expected. All factors considered indicate
<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>D</th>
<th>V</th>
<th>AQ</th>
<th>CV</th>
<th>% RI</th>
<th>% MI</th>
<th>INDEX TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landing Gear</td>
<td>0.352</td>
<td>0.510</td>
<td>0.432</td>
<td>0.258</td>
<td>0.10</td>
<td>0.40</td>
<td>2.052</td>
</tr>
<tr>
<td>Flt Controls</td>
<td>0.413</td>
<td>0.383</td>
<td>0.243</td>
<td>0.191</td>
<td>0.50</td>
<td>0.20</td>
<td>1.930</td>
</tr>
<tr>
<td>Engine Core Module</td>
<td>0.087</td>
<td>0.079</td>
<td>0.181</td>
<td>0.408</td>
<td>0.20</td>
<td>0.40</td>
<td>1.355</td>
</tr>
<tr>
<td>Fire Control</td>
<td>0.148</td>
<td>0.028</td>
<td>0.144</td>
<td>0.143</td>
<td>0.20</td>
<td>0.00</td>
<td>0.663</td>
</tr>
</tbody>
</table>

1.0 1.0 1.0 1.0 1.0 1.0
Figure 3  Examples of O&S Cost Distribution Changes Due to Parameter Reduction
that the O&S cost for the landing gear is consistently better than that of the F-15A which served as the basis for estimating b. Thus, revised estimates for a, b, and m for the landing gear subsystem were derived as follows:

\begin{align*}
    a^1 &= 8.00 \text{ versus } 9.00 \\
    b^1 &= 18.00 \text{ versus } 24.00 \\
    m^1 &= 9.50 \text{ versus } 12.00
\end{align*}

From information previously established:

\begin{align*}
    V_T^1 &= \text{Variance for the combined subsystems excluding the landing gear} = 21.10 \\
    D_T &= \text{Mean for the combined subsystems excluding the landing gear} = 24.86 \\
    r &= \text{Acceptable level of risk} = 0.05 \\
    K_r &= K_{0.05} = \text{upper 5 percent point of a standard normal distribution} = 1.645 \\
    C &= \text{O&S cost guarantee value} = 45.00
\end{align*}

Substituting these values into equation (9), we obtain:

\begin{align*}
    K_r^2 
    \left[ V_T^1 + \left( \frac{b^1 - a^1}{3.2} \right)^2 \right] 
    &= (1.645)^2 \left[ 21.10 + \left( \frac{18.00 - 8.00}{3.2} \right)^2 \right] \\
    &= 83.52 \\
    \left[ (C - D_T^1) - \left( \frac{a^1 + b^1 + 4m^1}{6} \right) \right]^2 
    &= \left[ (45.00 - 24.86) - \left( \frac{8.00 + 18.00 + 38}{6} \right) \right]^2 \\
    &= 89.74
\end{align*}
As 83.52 < 89.74, the adjustments made in the landing gear O&S cost
distribution are sufficient to achieve the 0.05 acceptable risk
level.

2.9 Revising the Combined Subsystem O&S Cost Distribution and Cost
Guarantee Overstep Probability

Once the subsystem O&S cost distribution for a specific sub-
system has been adjusted, the procedure outlined in Sections 2.4 and
2.5 is repeated using the revised information. If the inequality
shown in equation (9) was satisfied using new estimates for a, b,
and m, the new probability of exceeding the cost guarantee will be
less than the acceptable risk. Thus, the allocation of the O&S
cost guarantee can be based on the revised estimates of the sub-
system O&S costs. If the inequality does not hold, a second sub-
system will require further investigation. The new values for D_T
and V_T will replace the initial values derived.

As further subsystem changes are proposed or incorporated, the
impact of these changes on the O&S cost distribution and the proba-
bility of exceeding the cost guarantee should be reviewed.

For the cost allocation example, new estimates for landing
gear a, b, and m necessitated a revision of the estimates for D, V,
D_T, and V_T. The new parameter estimates for the landing gear and
subsystems total are shown in Table 6. With D_T = 35.53 and
V_T = 30.87, the probability of exceeding the O&S cost guarantee
was recalculated from equations (7) and (8) as follows:

\[ P(T > 45.00) = P \left[ \left( \frac{T - 35.53}{30.87} \right) > \left( \frac{45.00 - 35.53}{\sqrt{30.87}} \right) \right] \]

\[ \Rightarrow P(Z > 1.70) = 0.04457 < 0.05 \quad [\text{The acceptable risk level}] \]
### TABLE 6 REVISED PARAMETER ESTIMATES
FOR SYSTEMS UNDER HYPOTHETICAL O&S COST GUARANTEE

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>m</th>
<th>a</th>
<th>b</th>
<th>D</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landing Gear</td>
<td>9.50</td>
<td>8.00</td>
<td>18.00</td>
<td>10.67</td>
<td>9.77</td>
</tr>
<tr>
<td>Flight Controls</td>
<td>15.00</td>
<td>11.00</td>
<td>24.00</td>
<td>15.83</td>
<td>16.50</td>
</tr>
<tr>
<td>Engine Core Module</td>
<td>3.10</td>
<td>0.90</td>
<td>6.80</td>
<td>3.35</td>
<td>3.40</td>
</tr>
<tr>
<td>Radar</td>
<td>5.60</td>
<td>4.10</td>
<td>7.60</td>
<td>5.68</td>
<td>1.20</td>
</tr>
</tbody>
</table>

**TOTAL SYSTEM UNDER GUARANTEE**

- $D_T = 35.53$
- $V_T = 30.87$
If the estimated values for a, b, and m for the four systems are realized, the risk of penalty for exceeding $45.00 per flight hour as a quarterly average combined O&S cost is less than 0.05.
3.0 RESULTS AND DISCUSSION

The allocation procedure results are shown by the examples, discussions of operational characteristics, and tables in Section 2. The results show that the procedure provides a method of distributing a system level O&S cost goal to the individual components of an aircraft system. Furthermore, it is an aid in achieving the objective of designing low-cost-to-operate systems and it tailors the size of a contractor's commitment to a set risk level. The simple, easily applied procedure has fewer parameters than some other techniques, and the model can be easily written as a computer routine.

The use of the procedure could have several ancillary benefits. For example, it could be used as a design trade-off tool. It could also serve as a method for identifying the most critical O&S cost components of a system, as well as spreading out the costs of a high-risk venture by breaking down these costs into smaller items with separate prices and separate profit and loss probabilities; thereby, providing a means of distributing and analyzing the costs. Finally, the procedure can be used iteratively to update and refine the component costs of the system as the design progresses.

There are several facts that must be considered before implementing the procedure. The first is that O&S cost measurement is primarily dependent on VAMOSC system data. Presently, these data are limited and somewhat unstable; however, improvements are being made, and VAMOSC may soon become a reliable data source. Secondly, the O&S cost characteristics of a new aircraft system may not be similar to those of currently operational aircraft. In such a situation, the available cost and maintenance data must be initially biased in some way. The allocation procedure is quite flexible and can be adjusted to offset
such a technology change. Work in this area [7, 8] indicates the feasibility of developing O&S cost estimating relationships through regression analysis using VAMOSC and corresponding operational data. Finally, the time-frame for which comparable aircraft data are available must be considered. For example, if F-15 and F-16 aircraft cost and maintenance data are used in predicting or allocating O&S costs for a new system, it must be recognized that O&S costs for these mature weapon systems will not reflect the impact of early learning curve experience and infant mortality.[9]

These facts do not weaken the allocation procedure, but they must be considered when it is applied. Various statistical methods to minimize negative impacts are available. The O&S cost target allocation procedure is a tool that can provide visibility in making contract management decisions.

[7] ibid p 11

[8] ibid p 11

4.0 CONCLUSIONS

This study has developed and presented a hypothetical example of a procedure that allocates an O&S cost target into manageable subtargets for an aircraft system. The major conclusions of the study are as follows:

- The procedure can be used as an O&S cost allocating tool.
- The simplicity of the procedure and the few parameters used make it suitable for computer modeling.
- The use of the procedure could reduce industry's reluctance to implement an O&S cost guarantee.
- The procedure's effectiveness needs to be evaluated in the real world, where all the dynamics of the weapon system acquisition process occur.
5.0 RECOMMENDATIONS

It is recommended that:

A. A study be performed to verify the procedure's use. The study would involve:

1. Selecting an operational aircraft system that is approaching maturity and has both a maintenance and cost data history.

2. Using the cost and maintenance criteria on which the system procurement was based to determine the initial procedure parameters and perform a cost allocation.

3. Using the actual costs and maintenance data to determine the procedure parameters and perform a cost allocation.

4. Comparing the results of items 2 and 3.

B. The study be presented to the Logistics community, both government and contractor, for critique.

C. Consideration be given to developing a computer model of the procedure.
APPENDIX A
DATA SOURCES

Two Air Force data systems were accessed to obtain cost and usage data for the O&S Cost Target Allocation Procedure. They were the VAMOSC and the D056E Product Performance System.

VAMOSC was designed to fulfill the DoD's requirements for the operating and support portion of life cycle-costing. The system provides the capability to gather, portray, and retain for historical reference the operating and support cost of resources (labor, material, services, and overhead) directly and indirectly associated with the logistics support of aircraft and ground communication-electronic systems.[1] VAMOSC has three subsystems: Weapon System Support Cost (WSSC) System, Ground Communication-Electronic (C-E) System, and the Component Support Cost System (CSCS). CSCS was the only system applicable to the requirements of the allocation procedure. The first CSCS reports were distributed for the fourth quarter of FY 1982, followed by the first quarter of FY 1983. Subsequent reports will be delayed until a Data Automation Request making extensive changes to the system has been completed.

It was determined that a minimum of four quarters of cost data would be required for the allocation procedure to produce acceptable results. Consequently, O&S costs for the third quarter of FY 1982 and the second quarter of FY 1983 were estimated using statistical techniques and D056E data for those time periods. These four quarters of data were used to produce a reasonable example for illustrating the procedure. They will not be required when the modifications to CSCS have been completed.

The D056E Product Performance System provides numerous analytical maintenance reports and magnetic tapes of unprocessed Maintenance Data Collection System (MDCS) records (commonly referred to as 66-1 data) to contractors and various government agencies. Contractors commonly use the D056E products to perform maintenance analyses, logistics support analyses, reliability studies and life-cycle-cost studies. The well-documented D056E data products have been used by prime contractors for a number of years.

The MDCS is designed for the collection and dissemination of data produced by maintenance technicians to document hours and resources required to maintain Air Force weapon systems and equipment. It is a people-dependent system designed to interface with a computer system. As is common with such systems, there are numerous errors of both commission and omission with MDCS products. Coding, keypunch, and transposition errors are not uncommon, as is the failure to document maintenance actions. However, the huge volume of data produced over long periods of time tend to statistically moderate or even negate the reporting system's shortcomings. This is also the case with data used in this study. One year or more of data at the system or even the subsystem level is sufficiently accurate to be useful as an analytical and predictive tool.
APPENDIX B

RISK INEQUALITY DEVELOPMENT

Let $D_i = $ Mean of the ith subsystem under a cost guarantee
$V_i = $ Variance of the ith subsystem under a cost guarantee

Then $D_T = \sum_{i=1}^{n} D_i = $ Mean of the total of the subsystems under guarantee

And, $V_T = \sum_{i=1}^{n} V_i = $ Variance of the total of the subsystems under guarantee (assuming statistically independent subsystems)

Given an acceptable level of risk and using the PERT statistical approach, the objective is to satisfy the relationship

$$P(T > C) = P\left[\frac{T - D_T}{\sqrt{V_T}} > \frac{C - D_T}{\sqrt{V_T}}\right]$$

$$\leq r$$

Where, $T = $ The random variable representing the cost of the combined subsystems
$C = $ Cost guarantee value
$Z = $ Standard normal random variable (i.e., a normal random variable with mean 0 and variance 1)
$r = $ Previously defined acceptable level

To satisfy the inequality,

$$\frac{C - D_T}{\sqrt{V_T}} \geq K_r$$

(B-1)
Where, \( K_r \) = Upper r (100\%) point of a standard normal distribution

Without loss of generality, assume the cost distribution of the first subsystem with mean \( D_1 \) and variance \( V_1 \) has changed. All other subsystem distributions remain constant.

Substituting into equation (B-1),

\[
\frac{C - D}{\sqrt{V_T}} = \frac{C - \left( \sum_{i=1}^{n} D_i \right)}{\sqrt{\sum_{i=1}^{n} V_i}} = \frac{C - \left( \sum_{i=2}^{n} D_i \right) - D_1}{\sqrt{\sum_{i=2}^{n} V_i + V_1}} \geq K_r
\]

This implies

\[
K_r \left( \sqrt{\sum_{i=2}^{n} V_i + V_1} \right) \leq (C - \sum_{i=2}^{n} D_i) - D_1
\]

or,

\[
K_r^2 \left( \sum_{i=2}^{n} V_i + V_1 \right) \leq \left[ (C - \sum_{i=2}^{n} D_i) - D_1 \right]^2 \tag{B-2}
\]

Let \( a_1, b_1, \) and \( m_1 \) be the optimistic, pessimistic, and most likely values, respectively, for the first subsystem. Then under the PERT statistical approach,

\[
D_1 = \frac{(a_1 + b_1 + 4m_1)}{6} \quad \text{and} \quad V_1 = \left( \frac{b_1 - a_1}{3.2} \right)^2
\]

Implying,

\[
K_r^2 \left[ \sum_{i=2}^{n} V_i + \left( \frac{b_1 - a_1}{3.2} \right)^2 \right] \leq \left[ (C - \sum_{i=2}^{n} D_i) - \left( \frac{a_1 + b_1 + 4m_1}{6} \right) \right]^2 \tag{B-3}
\]
Since the choice of which subsystem has changed is totally arbitrary, let,

\[ V^*_T = \sum_{i=2}^{n} V_i \quad \text{Variance of the total of the subsystems except the adjusted subsystem,} \]

\[ D^*_T = \sum_{i=2}^{n} D_i \quad \text{Mean of the total of the subsystems except the adjusted subsystem,} \]

and let \(a_1, b_1,\) and \(m_1\) be \(a^1, b^1,\) and \(m^1,\) respectively. Substituting into equation (B-3),

\[ \kappa_r^2 \left[ V^*_T + \left( \frac{b^1 - a^1}{3.2} \right)^2 \right] \leq \left[ (C - D^*_T) - \left( \frac{a^1 + b^1 + 4m^1}{6} \right) \right]^2 \quad (B-4) \]

is the relationship which must be satisfied in order that

\[ P(T > C) \leq r. \]