X-RAY ELASTIC CONSTANTS AND THEIR MEANING FOR Al AND Fe

by

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INTRODUCTION

In the measurement of residual stresses via diffraction (using x-rays or neutrons) it is strains that are actually determined, by employing the interplanar spacing ($d_{hkl}$) of the {hk} planes as an internal strain gauge. The change in this spacing is measured from the shift of diffraction peaks (and Bragg's law) at several orientations of the sample to the incident beam, and the resultant strains are converted to stresses with the "diffraction elastic constants", $S_1(hk\ell)$ and $S_2(hk\ell)/2$. While these take on the values $(-v/E)$ and $(1+v)/E$ respectively for an isotropic solid, in anisotropic materials their values depend on many factors: preferred orientation, shape and orientation of second phases, interaction between grains. In fact there are reports of variation of these constants with plastic deformation and theory predicts variations with morphology. It is possible to calculate approximate values for these constants from theory and the single crystal elastic constants. $S_1$ and $S_2/2$ are really not elastic constants in the strictest sense because of these other factors, and it is best to measure them. One of us (I. C. Noyan) has recently examined this problem in some detail, and we summarize his results here.

In a measurement of diffraction elastic constants (on a piece of material as identical as possible to the piece whose stress is sought) a series of loads are applied in the elastic range. At each load, the inter-planar spacing is measured vs. $\sin^2\theta$ where $\phi$ is the tilt of the specimen normal from the bisector of the incident and diffracted beams. The slope ($\beta$) is obtained at each applied stress, and plotted vs. this stress. The slope of this second plot is related to one of the desired constants:

$$\beta = \sigma_1 S_1 [S_1 jk\ell, K_j (\phi)] + C(\epsilon_{ij}^r)$$

Here $\sigma_1$ is the applied stress, $S_1 jk\ell$ are elastic constants and the $K_j$ are the stress interaction constants between grains, which depends on their orientation, and hence $\phi$. The term $C$ is a complex function of any residual strain ($\epsilon_{ij}^r$) present in the specimen. It is now well established that

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there can be oscillations in $d$ vs. $\sin^2 \phi$, either due to fluctuations in residual stress\(^7\) or due to preferred orientation and elastic anisotropy.\(^8\) Even if such fluctuations are present, obtained by a least-squares fit is linear vs. the applied load.\(^5\) In fact, Eq. 1 can be exploited to establish the cause of these oscillations. If plastic deformation changes only the residual stress distribution and not the texture or second phase morphology, $S_1$ is unchanged by the process. However, if texture or morphology are altered, $S_1$ changes. In this paper we discuss two examples of these effects.

**EXPERIMENTAL PROCEDURES**

Flat tensile specimens were prepared from 1100 Al plate after annealing at 648°C for 2 hrs., followed by reduction in thickness by rolling on a two-high mill, 0.1 mm per pass, to 65, 82 and 90 pct reduction. Diffraction elastic constants were determined with V filtered CrK\(_\alpha\) radiation in the apparatus described in Ref. 9, by applying loads up to 12,000 psi, in 2000 psi increments. The 311 reflection was employed.

**RESULTS AND DISCUSSION**

Some typical plots of $d$ vs. $\sin^2 \phi$ at various loads are shown in Fig. 1; oscillations are apparent. The slopes vs. applied load are given in Fig. 2, and linearly is reasonable. The evaluated elastic constants are summarized in Table I, where they are compared to calculated values. The agreement is quite good and there is no significant variation with deformation and its resultant texture. As Al is nearly isotropic, Eq. 1, predicts that the elastic constants should be that calculated from theory ignoring the $K_i$ because these are zero, and this is what is observed in Table I. The oscillations in $2\theta$ vs. $\sin^2 \phi$ are due to local fluctuations in residual stress.

On the other hand for Fe it is known that oscillations occur (in $d$ vs. $\sin^2 \phi$), that the $\theta$ vs. applied load is linear, but that the diffraction elastic constant varies with deformation and do not agree with theory which neglects $K_i$.\(^2\) Again, this is expected; the variation in preferred orientation with deformation causes changes in the $K_i$ in this case.

It has been suggested that when oscillations occur in $2\theta$ or "d" vs. $\sin^2 \phi$, that hoo or hhh reflections will not show this, if anisotropic elasticity is the cause.\(^6,10\) This is only the case in the Reuss limit and without grain interaction which is generally not the case in deformed materials. In fact, there are reported cases where the oscillations do not vanish for hoo or hhh peaks.\(^7\) Also Al is nearly isotropic and yet, as we report here, there are oscillations. The source of these oscillations is local fluctuations in residual stress. In such a case (because the depth of penetration varies with $\phi$) an average value of stress is of little use,\(^11\) and attempts to eliminate the oscillations by increasing the depth of penetration\(^12\) (changing the wavelength) simply averages over the (important) fluctuations. The tests described here provide a means to decide on the source of the oscillations.

**ACKNOWLEDGEMENTS**

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**REFERENCES**


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**TABLE I.**

**DIFFRACTION ELASTIC CONSTANTS FOR 1100 Al.**

<table>
<thead>
<tr>
<th>REDUCTION IN THICKNESS</th>
<th>S2/2 x 10⁻⁸</th>
<th>ERROR DUE TO COUNTING STATISTICS x10⁻⁹ ¹</th>
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<tr>
<td>65 pct</td>
<td>12.76</td>
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<tr>
<td>81.7 pct</td>
<td>12.60</td>
<td>1.3</td>
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<tr>
<td>90 pct</td>
<td>13.08</td>
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<tr>
<td>Calculated Value²</td>
<td>13.24</td>
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a. See Ref. 8 for equations to calculate errors due to counting statistics.

b. Average of Reuss and Voight limits.
Fig. 1 Interplanar spacing, $d$ vs. $\sin^2 \phi$ for 1100 Al reduced in thickness 65\% by cold rolling.
Each figure is for the indicated applied load.

Fig. 2 The constant $\beta$ in Eq. 1 vs. applied stress. 1100 Al reduced in thickness 81.7\% by cold rolling.
It is demonstrated that the diffraction elastic constants for residual stress measurements can be determined even in the presence of oscillations in interplanar spacing vs $\sin \theta$. Furthermore, these values can be employed to ascertain whether the oscillations are due to local fluctuations in plastic deformation, or elastic anisotropy.
<table>
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<tr>
<th>KEY WORDS</th>
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<th>LINK C</th>
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