COMPRESSION OF SPREAD SPECTRUM SIGNALS ON AN EXCESSIVELY DISPERSSIVE CHANNEL

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### Abstract

There are several new techniques for correcting time dispersive distortion of wideband and/or spread spectrum signals using signal processing capabilities which can be embedded into modern communications systems. These include adaptive channel equalization and in the case of spread spectrum signals, synchronous correlation, matched filtering and Rake processing. This paper shows several distinct advantages in the combination of matched filtering and Rake processing for handling dispersion of spread spectrum signals.
Compression of Spread Spectrum Signals on an Excessively Dispersive Channel

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1. INTRODUCTION

The object of this article is to aid in analyzing the trade-offs connected with feasible techniques for compression of wideband spread-spectrum signals after transmission over an excessively dispersive, conventionally narrowband, channel. The term excessively dispersive is used here to describe a condition where the channel's time delay spread is greater than the inverse of the signal bandwidth, a frequent condition on HF channels at data rates above 1 kbps and on troposcatter channels above 1 Mbps. Specifically the RAKE technique will be compared to a pre-compression equalization technique. Both techniques when applied in a spread-spectrum system coherently combine temporally dispersed multipath components and compress (using correlation techniques) the spread-spectrum coding. However, an equalizer does the combining first and the compression last, while the RAKE uses the reverse order.

2. SPREAD SPECTRUM SIGNALING

To evaluate the subtle implications of any spread-spectrum technique we first need to develop the basic concepts involved as well as a standard notation. In general a spread-spectrum signal is generated by "multiplying" a conventional information signal (in this paper only signals of discrete levels such as binary data signals will be considered) by a spreading code which has a bandwidth much greater than the information signal. For instance, a TTL (binary digital) data stream and TTL code stream can be "multiplied" by an exclusive OR gate or they can be AC coupled then multiplied by a true four quadrant multiplier or a double balanced mixer.

Ideally the spreading code, represented here by \( p(t) \), would have a normalized auto-correlation function approaching zero for all lags (delays) except zero where it would equal 1, that is,

\[
R_{pp}(\tau) = \langle p(t) p(t-\tau) \rangle_T = \frac{\int_{-T}^{T} p(t) p(t-\tau) \, dt}{0} = \begin{cases} 0 & \text{for } \tau \neq 0 \\ 1 & \text{for } \tau = 0 \end{cases}
\]
where $T$ is the duration of the averaging period and is usually the period of one code repetition. A transmitted spread spectrum RF signal using this code could be represented as,

$$s(t) = d(t) \ p(t) \ \cos(\omega_c t) \tag{2}$$

where $d(t)$ is the information signal and $\omega_c$ is the frequency of the sinusoidal carrier. The tilde "$\sim$" superscript will be used throughout to designate a complex valued variable. I use it here to allow but not necessitate a multiple phase information baseband signal. If:

a. $d(t)$ and $p(t)$ are AC coupled zero-mean) waveforms then $s(t)$ is a phase shift keyed (PSK) signal.

b. $d(t)$ and $p(t)$ are both high-low (eg. TTL) signals then $s(t)$ is AM or On/Off Keyed (OOK).

c. $d(t)$ and/or $p(t)$ are sinusoidal tones, $s(t)$ could be M-ary FSK.

d. $d(t)$ and/or $p(t)$ are complex and $\cos(\omega_c t)$ is replaced by $\exp(j\omega_c t)$ then any of the above could be single sideband signals.

The effect of the channel on all of the above signals is very similar therefore I will choose a representative signal, binary PSK, and set $d(t)$ to a normalized DC level to illustrate the channel effects on the spread-spectrum signal. Even this is a useful communications signal in that one spreading code $p(t)$ can be chosen from a pre-determined set of codes to represent each information symbol. This is code shift keying (CSK). The representative signal $s(t)$ is,

$$s(t) = \tilde{d}(t) \ \cos(\omega_c t) \tag{3}$$

which is illustrated in Fig. 1 where the carrier function $f_c(t) = \cos(\omega_c t)$.

3. THE CHANNEL

The channel model to be used here is a linear time-invariant (LTI) system transfer function. This model is only valid for as long as there is no significant variation (hence time-invariant) of the channel. Therefore, in an adaptive system with parameters based on the current channel model (eg. a linear equalizer or a RAKE) care must be taken to update the channel model as changes occur and to ensure that no signal is in the linear system when the parameters are changed. This latter restriction indicates a "block processing" mode where a block of signal data can be processed based on the channel model (eg. in a linear equalizer) then the model can be updated and the next block processed. The LTI system model is illustrated in Fig. 2. $s(t)$ will be used throughout to represent the transmitted signal, $r(t)$ the received signal and $h(t)$ the system impulse response. One useful property of linear system theory is that the output signal $r(t)$ is the result of convolving the input $s(t)$ and the impulse response $h(t)$ which is identical to multiplying their Fourier transforms $S(j\omega)$ and $H(j\omega)$ then inverse transforming the product, $(F^{-1}[R(j\omega)] = r(t)) \ [Ref. 1]$. Fig. 2 will also be referred to later during a discussion of equalizers.
The channel model is a tapped delay line or transversal filter with complex weights [Ref. 2],

$$h_{\text{channel}}(t) = \sum_{i=1}^{N} \tilde{a}_i \delta(t - \tau_i). \quad (4)$$

This model correctly represents excessive multipath dispersion, frequency selective fading, and random multipath phase components. Since the model is time invariant it cannot represent Doppler spread or fading rates, these are realized only by updating the model parameters (i.e. Doppler effects cause the phase response to drift over time therefore, the $a_i$'s are functions of time, $a_i(t)$, which cannot be reflected in the LTI model). When implementing this model, especially as a basis for adaptive signal processing, care must be taken to ensure that the time interval between delays satisfies the constraints of the Sampling Theorem [Ref. 2] for the waveform of interest since Eq. (4) really amounts to a discrete time representation of a continuous function. The effect of this channel model on the received waveform is to convolve our transmitted signal (Eq. 3) with the channel impulse response thus producing,

$$r(t) = \int h_{\text{channel}}(u) s(t-u) du \quad \text{which by using Eq. (4)}$$

$$= \int \sum_{i} \tilde{a}_i \delta(u-\tau_i) s(t-u) du$$

$$= \sum_{i} \tilde{a}_i s(t-\tau_i)$$

$$= \sum_{i} a_i p(t-\tau_i) \cos[\omega_c t + \phi_i]$$

$r(t)$ is therefore a superposition of multipath signals all identical to the transmitted signals except that they have unique propagation delays ($\tau_i$), amplitudes ($a_i$) and phases ($\phi_i$). Note that $\phi_i$ now contains the phase information of $\tilde{a}_i$. Because each multipath component may have a unique phase, any system attempting to resolve the multipath must quadrature demodulate if representative amplitudes $|a_i|$ are to be maintained. This process would result in a baseband signal:

$$\tilde{b}(t) = \sum_{i} a_i p(t-\tau_i). \quad (6)$$

After some standard analog processing (amplifying, heterodyning, I and Q splitting and filtering) the complex baseband signal can be compressed (despread) by one of two basic methods, synchronous demodulation or matched filter convolution. These two techniques are illustrated in Fig. 3.

4. SYNCHRONOUS DEMODULATION

The left side of Fig. 3 illustrates synchronous demodulation which performs a cross correlation at one lag value at a time as determined by the time delay offset $\Delta$. To generate an entire cross correlation function the value $\Delta$ must be sequentially increased by increments equal to the inverse of the spreading band-
width and each correlation value $A_i$ is recorded or evaluated in some way. This process of incrementing $\Delta$ can be thought of as a synchronization search mode since it will show which delay corresponds to the channel's propagation delay. Of course on an excessively dispersive multipath channel there are multiple resolvable propagation delays therefore as the delay parameter is scanned $A(\Delta)$ becomes a measurement of the channel's impulse response,

$$A(\Delta) = h_{\text{channel}}(t)$$

After the peak response has been located, the received signal can be continuously demodulated using only that optimum value of $\Delta$ until the channel changes sufficiently to require resynchronization. A simple communications system application for this technique would be to split the received signal and process two parallel channels using the same optimum delay $D_{\text{opt}}$ but different spreading codes $p_0(t)$ and $p_1(t)$. A larger response in channel 0 would represent reception of a binary "0" (a space) where the larger response in channel 1 would represent a binary "1" (a mark).

5. MATCHED FILTER DEMODULATION

The right side of Fig. 3b illustrates a fundamentally different technique but shows that for a linear time invariant channel the output is mathematically equivalent to the technique shown in Fig. 3a. A time domain spread spectrum matched filter is defined [Ref. 3] by an impulse response $h_m(t)$ which is the time inverse of the spreading function $p(t)$. Therefore

$$h_m(t) = p(-t)$$

NOTE: $p(t)$ is assumed to be a zero-mean function such that the matched filter will tend to incoherently average random noise to zero.

With Eq.6 as input, the output of the matched filter has a peak at any time which corresponds to the time delay of a multipath signal component. This technique performs a cross-correlation of $p(t)$ and $b(t)$ for all lag values without (unlike synchronous demodulation) resetting any of the parameters in the demodulation process. Therefore, it is an ideal technique for timing synchronization since it is always in a "search mode". Using Eq. 6 and comparing the result to Eq. 4 we also see that the matched filter output is identically the complex channel impulse response as shown in Eq. 9.

$$\tilde{y}_m(t) = \int_0^T b(u)h_m(t-u)du = \sum_{i=1}^T \tilde{a}_i p(u-\tau_i)h_m(t-u)du$$

$$= \sum_{i=1}^T \tilde{a}_i \int_0^T p(u-\tau_i)p(u-t)$$

$$= \sum_{i=1}^T \tilde{a}_i \delta(t-\tau_i)$$
Again a simple communication system application would be to split the received signal into two parallel channels and process it in two parallel matched filters set up for two different spreading codes $p_0(t)$ and $p_1(t)$ to detect binary data.

One advantage of the matched filter technique over synchronous demodulation is its inherent synchronization property. That is, if the channel response suddenly changes (eg. relative motion of near field metallic structures causes a null in the largest multipath components) it would be immediately apparent in the matched filter output, no resynchronization would be required, and no data need be lost. The major disadvantage is that more signal processing power is required therefore the limiting factor is the state-of-the-art in signal processing devices.

6. **PRE-COMPRESSION EQUALIZATION**

Looking back to Fig. 1 we see a channel equalizer, represented by $1/H(j\omega)$. As illustrated, the effect of the equalizer is to cancel the effect of the channel transfer function thus producing a distortion free replica of the transmitted signal at the receiver. The concept is simple but its realization has been a hot research topic for several years [Ref. 4, 5, 6].

The realization of a linear equalizer almost universally takes the form of Fig. 4 which shows that the equalizer output $y_e(t)$ is a weighted summation of received signal samples including a noise component which is uncorrelated to the received signal or the equalizer response. Therefore

$$ y_e(t) = \sum_{n=-N}^{N} c_n r(t-nT) + \sum_{n=-N}^{N} c_n z(t-nT) \quad (10) $$

where $T$ is one delay interval in the equalizer's delay line and $z(t)$ is the random noise component. By definition $y_e(t)$ is a distortion-free replica of $s(t)$ which means that all intersymbol interference has been removed prior to signal detection, but a penalty, $\sum_{n=-N}^{N} c_n z(t-nT)$, has been paid in signal to noise (SNR) ratio. Furthermore the equalizer due to its spectral characteristics "amplifies" the received noise by a greater factor than the signal. This can be illustrated by Fig. 5 which shows that the equalizer's maximum frequency response is by design precisely where the received SNR is a minimum, resulting in a relative attenuation of the spectral components having the best SNR. This penalty is acceptable in many applications in return for the higher data capacity provided by removal of the intersymbol interference.

Now the corrected signal $y_e(t)$ can be pulse compressed using either the synchronous demodulation or matched filter techniques of Fig. 3. Since the sampled data has already had its superposed multipath components removed there will only be a single correlation peak. Therefore, if synchronization has previously been established (ie. $\tau_1$ is known and the correct $\Delta_1$ has been computed) only one time delay need be processed using either technique. However, if synchronization has not been established or has become unreliable a delay sweep (synchronization search mode) or a matched filter convolution must be performed to determine which correlation lag value corresponds to the signal's propagation
delay (including the delay through the equalizer). The trade-off between the two techniques of Fig. 3 is the same as for an unequalized signal which is basically more computation for the matched filter vs longer and less reliable synchronization procedures for synchronous demodulation. Therefore, matched filtering is the preferred technique providing it is feasible to provide the necessary processing power. The state-of-the-art in correlation signal processing IC's is nicely summarized in Ref. 8 which shows that very fast correlators are available for binary code compression but little is available for tapped delay lines that require complex multiplications at each tap. This will be an important distinction when implementation is considered in Section 8.

7. THE RAKE

There is another technique, the RAKE [Ref. 9, 10] which performs a weighted summation of the multipath components of the received signal after pulse compression rather than before. This approach seems inherently more efficient since the RAKE performs one weighted summation per compressed symbol where the equalizer does a weighted summation after each chip (i.e. uncompressed symbol) is received. Therefore, the equalizer must coherently sum multipath components TB times faster than the RAKE (TB is the time bandwidth product). Note that I do not use the term equalize for what the RAKE does because it does not remove intersymbol interference. Therefore, a simple RAKE is limited to a symbol throughput rate less than the inverse of the total time dispersion on the channel. For an HF channel this has a worst case value of about 200 symbols per second resulting from a 5 ms delay spread. However, if a system is intended to always operate at a processed throughput rate of less than 200 symbols per second either technique is appropriate. Furthermore with multiple pseudo orthogonal (statistically independent) symbol sets, symbols could be allowed to overlap in the received signal and still be resolvable by multiple matched filters. Therefore at the price of providing N parallel matched filters, the 200 symbols per second could be increased by N.

First demonstrated in 1958 [Ref. 9] the RAKE has become feasible for more and more applications due to the increasing signal processing capabilities of modern electronic components. Fig. 6a shows the multiple correlator configuration for the RAKE and upon close examination it can be seen that it is simply several synchronous demodulators working in parallel. Due to the equivalent properties shown earlier, see Fig. 3, another implementation of the RAKE can be realized using a matched filter as shown in Fig. 6b. This configuration is the system on the right side of Fig. 3 with the matched filter outputs at each t summed in phase. That is, each complex sample ym(nT) of the matched filter output is multiplied by a complex phasor cm (a conjugate phase weight) prior to summation such that all multipath components sum in phase while noise sums incoherently, thus some processing gain can be realized.

The conjugate weights used to "line-up" the multipath phasors can be of unit magnitude or optimally can carry the magnitude of the channel multipath component which they operate on. This is an optimal combining diversity technique which can be shown to maximize SNR [Ref. 11]. Therefore, by using cm = hChanne(lmT) both of the RAKE's in Fig. 6 will result in a real (scalar magnitude) value yRAKE(i) for the i-th symbol, plus noise as shown in Eq. 8. Further optimization can be incorporated (readily so if this combining is done digitally) by setting
to zero the conjugate weights corresponding to any delay contributions which "seem to be" only noise (eg. decision criteria could exclude responses of magnitude less than the average, more than 10 dB below the peak, less than 3 dB above the smallest response etc.). This last step helps to reduce the summation of noise components which would otherwise appear in a manner similar to Eq. 10, that is,

\[ y_{RAKE}(i) = \sum_{n=-N}^{N} c(n) y_m(nT) \]

where \( c(n) = h_{channel}(nT) \) for \( h_{channel}(nT) > \text{Threshold} \)

\( = 0 \) for \( h_{channel}(nT) \leq \text{Threshold} \)

The weights \( c_n \) can be estimated to great precision by maintaining a running average of \( y_m(t) \), such as an exponentially decaying average. This running average would coherently integrate the received signal thus improving the s/n ratio proportionately to the number of records averaged. This average can be realized even during information transfer by using decision feedback. For instance if multiple symbol CSK waveforms are used only the output \( y_m(t) \) from the matched filter which received the code would be added to the running average. Another instance would be if phase reversal modulation were used, the \( y_m(t) \) for a "1" symbol would be the negative of that for a "0" symbol, therefore \( y_m(t) \) could be inverted or not (based on decision feedback) before adding it to the running average.

8. COMPARISON OF REFINED RAKE AND PRE-COMPRESSION EQUALIZATION TECHNIQUES FOR WIDEBAND HF CHANNELS

Throughput - The pre-compression equalization technique is not limited mathematically in throughput rate. In fact adaptive equalization has been shown to provide a 4800 bps throughput (without compression) on an HF skywave channel which would otherwise be limited to a few hundred baud [Ref. 12]. However for applications where the compressed symbol rate is below about 400 Hz, either technique is feasible.

SNR - Either technique provides worse SNR performance than an "infinitely coherent" line of sight channel because they both sum delayed signal components coherently and noise incoherently. Both techniques sum noise components twice, once during pulse compression and again during multipath summation, but when summing multipath components either of the RAKE's of Figure 6 could be designed to discriminate against low SNR components and leave them out of the summation. Furthermore, the frequency response of the equalizer is unpredictable and may result in significant degradation of SNR as previously illustrated in Fig. 5. This compares with a known and generally flat response (eg. Fig. 7) through the matched filter which is also the optimum filter to maximize the SNR [Ref. 3] of the coded waveform in the presence of white noise.

Diversity - The name RAKE describes its capability to collect the multipath components scattered in time by the channel impulse response. This multipath combining, which is performed optimally by Eq. 11, provides a powerful diversity which can overcome Faraday (polarization) fading or intramode fading (scintillation) which affect the multipath modes independently. The length of the
equalizer (ie. # of taps times the tap delay interval) must be between 2-5 ms on an HF channel to take advantage of this diversity however, lengths greater than 100 µs for intervals of 1 µs are not currently feasible for very wideband systems due to the computational speed required (see next paragraph). However, the RAKE can provide this filter length easily if a binary convolutional code is used [Ref. 13] as shown in the next paragraph.

Complexity of Implementation - The pre-compression equalizer technique requires \( N \) complex multiplies and \( N \) complex additions after each sample of the received signal is taken. For instance, if 500K chips/sec are being sent and an equalizer length of 128 µs (a 64 tap I equalizer and a 64 tap Q equalizer) is used then 256 million complex multiplies and 256 million adds per second must be performed just in the equalizer. If an orthogonalization (of the Gram-Schmidt type) is performed to speed convergence [Ref. 6] using a lattice filter then another 128 million multiplies and adds per second are required. The equalizer coefficient computation generally requires autocorrelation coefficients as input which necessitates another 128 million multiplies and adds plus some housekeeping. At least a few times per second the equalizer coefficients (and lattice filter parameters if used) must be updated which requires anywhere from about 4,000 multiplies per update for frequency domain conjugation to over 1 million multiplies for the Kalman algorithm. After the equalization, a 1024 chip code compression could be performed 500 times per second using a binary matched filter requiring over 1 million adds/subtracts per second.

For comparison we will consider a RAKE using a 4096 chip (64 x 64) binary convolutional code to BPSK modulate a carrier at 500K chips/sec. The codes could be compressed using two Fairchild FCI CCD 3361Z CCD correlators (40 pin dip packages using 0.4 watts each) as matched filters, one for the I and one for the Q channels. These analog chips would perform the equivalent of 128 million 10-bit digital adds (or subtracts) each second. The sampled matched filter output could be averaged (to estimate \( h_{\text{channel}}(t) \)) with 1 million adds per second, and the conjugate weights, \( c(n) \) could be determined in 100,000 operations per sec (100ms updates). The multipath summation (Eq. 7) only requires 125 multiplies and adds per second (symbol throughput rate of 125 Hz) per multipath processed (eg. a reasonable number might be 25,000 based on processing the largest 200 usec of the channel response). Therefore, the RAKE processor might be as simple as an Intel IAPX 186 with a 187 numeric coprocessor or certainly a Texas Instruments TMS 320 could handle it.

Numeric Computations per Second (approximate):

<table>
<thead>
<tr>
<th>Pre-Compression Equalized</th>
<th>RAKE with CCD Correlators</th>
</tr>
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<tbody>
<tr>
<td>1.1 Billion Operations</td>
<td>1.1 Million Operations</td>
</tr>
</tbody>
</table>

The implementations suggested here are still not equivalent in diversity combining because the 200 usec equalizer length is not sufficient to "rake in" multiple propagation modes, while the RAKE would process over 8ms of delay spread.

Conclusion - The RAKE technique for post-compression multipath combining has the advantages of optimal SNR, optimal diversity combining and ease of implementation with the one drawback of being limited in post-compression throughput speed (data rate). Therefore, for spread-spectrum HF or tropo systems which have throughput rates compatible with the channel's time dispersion the RAKE matched filter compression technique is preferable.
References


2. J. Proakis, Digital Communications, Fig. 7.5.1, McGraw-Hill 1983.


Figure 1

Spread Spectrum Waveform

\[ s(t) = f_c(t) p(t) \]

Ideally:

\[ p(t-a)p(t-b) = \begin{cases} 0 & \text{for } a \neq b \\ 1 & \text{for } a = b \end{cases} \]

For Maximal Length Sequence:

\[ p(t-a)p(t-b) = \begin{cases} -\frac{1}{N} & \text{for } a \neq b + nT \\ 1 & \text{for } a = b + nT \end{cases} \]

If \( N = 1023 \) then Peak/Sidelobe = 60dB
LINEAR SYSTEM MODEL

\[ r(t) = h(u) s(t-u) \, du \]

\[ R(j\omega) = S(j\omega) H(j\omega) \]

\[ R'(j\omega) = S(j\omega) H(j\omega) [1/H(j\omega)] = S(j\omega) \]

Figure 3a

\[ A_0 \sum a_i \xi(t-d_i) p(t-d_i) \]

\[ \bar{A}_1 \sum \tilde{a}_i p(t-d_i) \]

\[ \bar{A}_2 \sum \tilde{a}_i p(t-d_i) \tilde{p}(t-d_i) \]

\[ \bar{A}_2 \{ \tilde{a}_i \text{ for } \Delta = d_i \leq N \} \]

\[ \bar{A}_2 \{ \tilde{a}_i / N \text{ for } \Delta \neq d_i \leq N \} \]

\[ |a_2| \]

\[ d_1 \quad d_2 \quad d_3 \]

Figure 3b

\[ B_0 \sum b_i \xi(t-d_i) p(t-d_i) \]

\[ B_1 \sum \tilde{b}_i p(t-d_i) \]

\[ B_2 \int \sum \tilde{b}_i p(u-d_i) \tilde{p}(u-t) \, du \]

\[ B_2 \{ \tilde{b}_i \text{ for } t = d_i \leq 2N \} \]

\[ B_2 \{ \tilde{b}_i / N \text{ for } t \neq d_i \leq 2N \} \]

\[ |b_2| \]

\[ d_1 \quad d_2 \quad d_3 \]
CHANNEL EQUALIZER FREQUENCY RESPONSE

Figure 5

CHANNEL RESPONSE
EQUALIZER RESPONSE
WHITE NOISE

MAGNITUDE (dB)
TWO RAKE ARCHITECTURES

Figure 6