ELECTROMAGNETIC TRANSMISSION THROUGH A
SLOT IN A PERFECTLY CONDUCTING PLANE

Interim Technical Report

by

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December 1984

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**Title:** Electromagnetic Transmission Through a Slot in a Perfectly Conducting Plane

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- Joseph R. Mautz
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**Abstract:**
When a plane wave is incident on a slot in a perfectly conducting ground plane of infinite extent, three quantities of interest are the tangential electric field in the aperture, the transmission coefficient, and the scattering cross section. It is assumed that the tangential electric field in the aperture is transverse to the slot axis and depends only on the coordinate along the slot axis. A computer program is presented to calculate the tangential electric field in the aperture, the transmission coefficient, and the...
20. ABSTRACT (continued)

scattering cross section. This program is described and listed. Sample input and output data are included for the convenience of the user.
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ELECTROMAGNETIC TRANSMISSION THROUGH A SLOTH IN A PERFECTLY CONDUCTING PLANE

I. INTRODUCTION

Although the computer program in [1] can treat the case where the rectangular aperture is one subsection wide, i.e., \( L_y = 1 \) in Fig. 1 of [1], this computer program is long and complicated. A computer program was written specifically for the \( L_y = 1 \) case. Relatively short, this program is described and listed here. It consists of a main program and the subroutines YMAT, PLANE, DECOMP, and SOLVE.

The formulas that are programmed are presented in Section II. The main program is described and listed in Section III, the subroutine YMAT in Section IV, the subroutine PLANE in Section V, and the subroutines DECOMP and SOLVE in Section VI.

II. FORMULATION

The magnetic current \( M \) on the \( z < 0 \) side of the aperture (see [2, Fig. 2]) is expressed as

\[
M = \sum_{j=1}^{L_x-1} V_j M_j^x
\]

(1)

where \( \{V_j\} \) are unknown coefficients and \( \{M_j^x\} \) are expansion functions defined by [1, Eq. (10)]

\[
M_j^x = u_x T_j(x) P(y), \quad j=1,2,\ldots,L_x-1
\]

(2)

Here, \( u_x \) is the unit vector in the \( x \)-direction, \( T_j(x) \) is the triangle function defined by
\[ T_j(x) = \begin{cases} \frac{x - (j-1)\Delta x}{\Delta x} & (j-1)\Delta x \leq x < j\Delta x \\ \frac{(j+1)\Delta x - x}{\Delta x} & j\Delta x \leq x < (j+1)\Delta x \\ 0 & |x - j\Delta x| \geq \Delta x \end{cases} \] (3)

and \( P(y) \) is the pulse function defined by

\[ P(y) = \begin{cases} 1 & 0 \leq y < \Delta y \\ 0 & \text{all other } y \end{cases} \] (4)

Here, \( \Delta x \) and \( \Delta y \) are, respectively, the aperture subsection lengths in the \( x \) and \( y \) directions.

The magnetic field incident on the aperture-perforated conducting plane of [1, Fig. 1] is either \( H_{\theta y} \) or \( H_{xx} \) where

\[ H_{\theta y} = u_{\theta}(\theta^{\text{inc}}) e^{jk(x \cos \theta^{\text{inc}} + z \sin \theta^{\text{inc}})}, \quad \pi \leq \theta^{\text{inc}} < 2\pi \] (5)

\[ H_{xx} = u_{x} e^{jk(y \cos \phi^{\text{inc}} + z \sin \phi^{\text{inc}})}, \quad \pi \leq \phi^{\text{inc}} < 2\pi \] (6)

Here, \( k \) is the wave number. In (5), the incident wave comes from the direction for which \( y=0 \) and \( \theta=\theta^{\text{inc}} \). In (6), the incident wave comes from the direction for which \( x=0 \) and \( \phi=\phi^{\text{inc}} \). In (5), \( u_{\theta}(\theta^{\text{inc}}) \) is the unit vector in the \( \theta \) direction evaluated at \( \theta=\theta^{\text{inc}} \). In (5) and (6), the first subscript on \( H \) denotes the polarization of the magnetic field, and the second subscript denotes the plane of incidence. In (5), the plane of incidence is the \( y=0 \) plane. In (6), the plane of incidence is the \( x=0 \) plane. Called the incident magnetic field, the magnetic field (5) or (6) is the field that would exist in free-space, i.e., in the absence of the aperture-perforated conducting plane.
When the incident magnetic field is given by (5), the coefficients \( \{v_j\} \) in (1) are the elements of the column vector \( \mathbf{v} \) that satisfies

\[
\mathbf{y}^\dagger \mathbf{v} = - (\mathbf{P}^{\text{inc}})_{\theta y}
\]

where the \( i \)th element of the column vector \( (\mathbf{P}^{\text{inc}})_{\theta y} \) is called \( (\mathbf{P}^{\text{inc}})_{\theta y}^{(i)} \)

and is given by [1, Eq. (59)]

\[
(\mathbf{P}^{\text{inc}})_{\theta y} = 2\Delta x \Delta y \sin \theta^{\text{inc}} \left( \frac{\sin k \Delta x \cos \theta^{\text{inc}}}{2} \right) e^{j k i \Delta x \cos \theta^{\text{inc}}} 
\]

\[i=1, 2, \ldots, \text{L}-1\]

In (7), \( Y \) is the square matrix whose \( ij \) element is called \( Y_{ij} \) and is given by [1, Eq. (23)]

\[
Y_{ij} = \frac{j \Delta x \Delta y}{\eta} \left[ \frac{1}{2} I_c(j-1) - \frac{1}{2} I_x(j-i+1) + \frac{(j-i+3/2)}{2} I_c(j-i+1) \right. \\
+ \frac{1}{2} I_x(j-i-1) - \frac{(j-i-3/2)}{2} I_c(j-i-1) + \frac{1}{(k \Delta x)^2} (I_c(j-i+1) \\
- 2I_c(j-i) + I_c(j-i-1)) \right]
\]

(9)

The \( j \) that appears in the factor \( j \Delta x \Delta y/\eta \) on the right-hand side of (9) is \( \sqrt{-1} \). Each of the rest of the \( j \)'s on the right-hand side of (9) is the subscript \( j \) on \( Y_{ij} \). In (9), \( \eta \) is the impedance of free space.

Moreover, \( I_c \) and \( I_x \) are given by [1, Eqs. (27) and (28)]

\[
I_c(1) = 2 \int_0^y dx \int_0^u dy \frac{e^{-j \sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}
\]

(10)
\[ I_x(i) = \frac{2}{kAx} \int_0^{y_u} \int_{x_L}^{x_u} dy \int dx \frac{x e^{-j \sqrt{x^2 + y^2}}}{\sqrt{x^2 + y^2}} \]  

where

\[ y_u = kAx/2 \]  
\[ x_u = (i + 1/2)kAx \]  
\[ x_L = (i - 1/2)kAx \]  

It is evident that \( I_c(i) \) is even in \( i \) and that \( I_x(i) \) is odd in \( i \).

When the incident magnetic field is given by (6), the coefficients \( \{v_j\} \) in (1) are the elements of the column vector \( \vec{V} \) that satisfies

\[ V^{\dagger} \vec{V} = - (P^{\text{inc}})_{xx} \]  

where the \( i \)th element of the column vector \( (P^{\text{inc}})_{xx} \) is called \( (P^{\text{inc}})_{xx} \) and is given by [1, Eq. (65)]

\[ (P^{\text{inc}})_{xx} = -2\Delta x \Delta y \frac{\sin \frac{kAx \cos \phi}{2}}{\sin \frac{kAx \cos \phi}{2}} e^{j(kAx/2) \cos \phi \Delta y/2} \]  

\[ i = 1, 2, \ldots L_x - 1 \]  

In (9),

\[ 2-L_x < j-i < L_x-2 \]  

but, because (9) is even in \( j-i \),

\[ 0 < j-i < L_x-2 \]  

is sufficient. Therefore,

\[ -1 < i < L_x-1 \]
is sufficient in (10) and (11).

The integrals (10) and (11) are evaluated by using the following four term approximation [1, Eq. (39)]

\[ e^{-jr} \approx e^{-jr_1} \left(1 - j(r-r_1) - \frac{1}{2} (r-r_1)^2 + \frac{j}{6} (r-r_1)^3 \right) \]  \hspace{1cm} (20)

where

\[ r = \sqrt{x^2 + y^2} \]  \hspace{1cm} (21)

\[ r_1 = |ik\Delta x| \]  \hspace{1cm} (22)

Substitution of (20) into (10) gives [1, Eq. (42)]

\[ I_c(i) = (C_1U_1 + C_2U_2 + C_3U_3 + C_4U_4)e^{-jr_1} \]  \hspace{1cm} (23)

where

\[ U_1 = 1 - \frac{r_1^2}{2} + jr_1 \left(1 - \frac{r_1^2}{6}\right) \]  \hspace{1cm} (24)

\[ U_2 = r_1 - j \left(1 - \frac{r_1^2}{2}\right) \]  \hspace{1cm} (25)

\[ U_3 = -\frac{1}{2} (1 + jr_1) \]  \hspace{1cm} (26)

\[ U_4 = \frac{j}{6} \]  \hspace{1cm} (27)

\[ C_n = \int_0^{y_{u}} dy \int_{x_{f}}^{x_{u}} dx r^{n-2} , \quad n = 1,2,3,4 \]  \hspace{1cm} (28)

Substitution of (20) into (11) gives

\[ I_x(i) = (X_1U_1 + X_2U_2 + X_3U_3 + X_4U_4) \frac{2e^{-jr_1}}{k\Delta x} \]  \hspace{1cm} (29)
where $U_1$, $U_2$, $U_3$, and $U_4$ are given by (24)-(27) and

$$X_n = \int dy \int_0^{x_n} dx x^{n-2}$$

(30)

Using the indefinite integrals [1, Eqs. (46)-(54)], we obtain

$$C_1 = A_{xu} - A_{\xi} + A_{yu}$$

(31)

$$C_2 = y u \Delta x$$

(32)

$$C_3 = \frac{y u}{3} (x_u r_4 - x_\xi r_3) + \frac{1}{6} (x_{u,x}^2 - x_{\xi,\xi}^2 + y_{u,yu}^2)$$

(33)

$$C_4 = \frac{1}{3} y u (x_u r_4^2 - x_\xi r_3^2)$$

(34)

$$X_1 = \frac{1}{2} (y u (r_4 - r_3) + x_{u,x} - x_{\xi,\xi})$$

(35)

$$X_2 = r_i C_2 \text{ sign } (i)$$

(36)

$$X_3 = y u \left( \frac{r_3^3 - r_3}{12} + \frac{x_{u,x}^2 - x_{\xi,\xi}^2}{8} + \frac{1}{8} (x_{A,A}^3 - x_{\xi,\xi}^3) \right)$$

(37)

$$X_4 = r_i y u \kappa \Delta x (r_4^2 + \frac{(\kappa \Delta x)^2}{4} + \frac{y_{u,yu}^2}{3}) \text{ sign } (i)$$

(38)

where $\text{sign } (i)$ denotes the algebraic sign of $i$. If $i=0$, then $\text{sign } (i)$ is inconsequential because the term that multiplies it is zero. In (31)-(38),

$$r_3 = \sqrt{\frac{2}{x_\xi^2 + y_{u}^2}}$$

(39)

$$r_4 = \sqrt{\frac{2}{x_u^2 + y_{u}^2}}$$

(40)

$$A_{xu} = x_u \log \left( \frac{y_{u} + r_4}{|x_u|} \right)$$

(41)
\[ A_{xL} = x_L \log \left( \frac{y_u + r_3}{|x_L|} \right) \]  
\[ A_{yu} = y_u \log \left( \frac{x_u + r_4}{x_u + r_3} \right) \]

Here, \( \log \) denotes the natural logarithm.

Substituting \( Y/2 \) for \( Y \) in [2, Eq. (27)], we obtain for the complex power \( P_t \) transmitted through the aperture

\[ P_t = \frac{1}{2} \tilde{V}[Y\tilde{V}]^* \]

where \( \tilde{V} \) is the transpose of \( \tilde{V} \) and \( ^* \) denotes the complex conjugate.

Since the incident magnetic field is given by either (5) or (6), \( Y\tilde{V} \) is given by either (7) or (15) so that (44) reduces to

\[ P_t = - \frac{1}{2} \tilde{\Phi}^* \]

where \( \tilde{\Phi} \) is \( (\tilde{\Phi}^{inc})_{yy} \) if the incident magnetic field is given by (5) and \( \tilde{\Phi} \) is \( (\tilde{\Phi}^{inc})_{xx} \) if the incident magnetic field is given by (6).

The transmission coefficient \( T \) is the ratio of the real power transmitted through the aperture to the real power \( P_{inc} \) incident on the aperture. Since the incident magnetic field is given by either (5) or (6), we obtain

\[ P_{inc} = nL_x \Delta x \Delta y \cos \alpha \]

(46)

where \( \alpha \) is the angle between the propagation vector of the incident wave and \( u_z \). Here, \( u_z \) is the unit vector in the z direction. Since the incident magnetic field is given by (5) or (6), inspection of [1, Fig. 1] reveals that

\[ \cos \alpha = - \sin \beta \]  

(47)
where $\beta$ is either the angle $\theta^{inc}$ in (5) or the angle $\phi^{inc}$ in (6). From (45)-(47), we obtain

$$T = \frac{\text{Real}(\mathbf{V}^*\mathbf{P}^*)}{2\pi L \Delta x \Delta y \sin \beta}$$  \hspace{1cm} (48)

The scattering cross section $T(\theta, \phi)$ is the area that the incident power per unit area must be multiplied by in order to obtain the power which, when radiated omnidirectionally into the $z > 0$ half space, would produce the actual power per unit area at $(\theta, \phi)$. The above definition of $T$ leads to [1, Eq. (72)]

$$\tau / \lambda^2 = \frac{k^4}{32\pi^3 \eta^2} |\mathbf{p}^m| \langle \mathbf{V} \rangle^2$$  \hspace{1cm} (49)

where $\lambda$ is the wavelength and $\mathbf{p}^m$ is the transpose of a measurement vector $\mathbf{P}^m$. For the $\theta$ polarized pattern in the $y=0$ plane, $\tau$ is called $(\tau)_{\theta y}$ and $\mathbf{p}^m$ is $\langle \mathbf{P}^m \rangle_{\theta y}$ where $\langle \mathbf{P}^m \rangle_{\theta y}$ is the column vector whose $i$th element is given by (8) with the angle of incidence $\theta^{inc}$ replaced by the observation or "measurement" angle $\theta^m$.

$$(\tau)_{\theta y} / \lambda^2 = \frac{k^4}{32\pi^3 \eta^2} |\mathbf{p}^m| \langle \mathbf{V} \rangle^2$$  \hspace{1cm} (50)

For the $x$ polarized pattern in the $x=0$ plane, $\tau$ is called $(\tau)_{xx}$ and $\mathbf{p}^m$ is $\langle \mathbf{P}^m \rangle_{xx}$ where $\langle \mathbf{P}^m \rangle_{xx}$ is the column vector whose $i$th element is given by (16) with the angle of incidence $\phi^{inc}$ replaced by the measurement angle $\phi^m$.

$$(\tau)_{xx} / \lambda^2 = \frac{k^4}{32\pi^3 \eta^2} |\mathbf{p}^m| \langle \mathbf{V} \rangle^2$$  \hspace{1cm} (51)
III. THE MAIN PROGRAM

The main program uses the subroutines YMAT, PLANE, DECOMP, and SOLVE to calculate the coefficients \( \{ V_j \} \) appearing in expression (1) for the magnetic current, the transmission coefficient (48), and the scattering cross sections per square wavelength (50) and (51). The main program is described and listed in this section. Sample input and output data are provided so that the user can verify that the program is running properly.

In the listing of the main program, line 4 defines the input data file and line 5 defines the output data file. The input data are read according to lines 10 and 11 which are

```plaintext
READ(20,11) LX, LI, NTH, DX, DY, TH
11 FORMAT(3I3, 3E14.7)
```

Here, \( LX \) is \( L_x \), \( DX \) is \( \Delta x/\lambda \), and \( DY \) is \( \Delta y/\lambda \) where \( \lambda \) is the wavelength, and, as in Section II, \( L \Delta x \) is the length of the aperture in the \( x \) direction and \( \Delta y \) is the width of the aperture in the \( y \) direction. \( LX \) is a positive integer greater than or equal to 2. \( LI \) is either 1 or 2. If \( LI \) is 1, then the incident magnetic field is given by (5) and \( TH \) is \( \theta^\text{inc} \) of (5). If \( LI \) is 2, then the incident magnetic field is given by (6) and \( TH \) is \( \phi^\text{inc} \) of (6). The input variable \( TH \) is in degrees. The normalized cross section (50) is calculated at

\[
\theta^M = (J-1)\pi/(NTH-1), \quad J = 1,2,\ldots NTH
\]

and is written on the output data file under the heading \( \text{TAM} \). The normalized cross section (51) is calculated at
\[ \phi^m = (J-1)\pi/(NTH-1) , \quad J = 1, 2, \ldots, NTH \]  
\hspace{10pt} (53)

and is written on the output data file under the heading TAU2. The right-hand sides of both (52) and (53) are in radians.

Minimum allocations are given by

\[ \text{COMPLEX } Y(N*N), P(2*N), B(N), V(N) \]
\[ \text{DIMENSION IPS(N)} \]

where

\[ N = LX - 1 \]  
\hspace{10pt} (54)

Line 20 stores by columns in Y the elements of the matrix \( \frac{\pi}{j \Delta x \Delta y} Y \)

where \( Y \) appears in (7). Line 24 stores in \( P(1) \) to \( P(N) \) the elements of 

\[ \frac{1}{2 \Delta x \Delta y} (P^{inc})_{\theta y} \]

where \( (P^{inc})_{\theta y} \) appears in (7). Here, \( N \) is given by (54).

Line 24 also stores in \( P(N+1) \) to \( P(2*N) \) the elements of 

\[ \frac{1}{2 \Delta x \Delta y} (P^{inc})_{xx} \]

where \( (P^{inc})_{xx} \) appears in (15). Equation (7) is recast as

\[ \left[ \frac{\pi}{j \Delta x \Delta y} Y \right] \vec{V} = j2\pi \left[ \frac{1}{2 \Delta x \Delta y} (P^{inc})_{\theta y} \right] \]  
\hspace{10pt} (55)

Equation (15) is recast as

\[ \left[ \frac{\pi}{j \Delta x \Delta y} Y \right] \vec{V} = j2\pi \left[ \frac{1}{2 \Delta x \Delta y} (P^{inc})_{xx} \right] \]  
\hspace{10pt} (56)

The square matrix \( \left[ \frac{\pi}{j \Delta x \Delta y} Y \right] \) is common to the left-hand sides of both (55) and (56). This is the matrix that resides in the computer program variable \( Y \). The bracketed quantity on the right-hand side of (55) is the column vector that resides in the computer program variables \( P(1) \) to \( P(N) \) where \( N \) is given by (54). The bracketed quantity on the right-hand side of (56) is the column vector that resides in the computer program variables \( P(N+1) \) to \( P(2*N) \). If LI is 1, DO loop 16 performs
the multiplication by the factor $UV = 2j\pi n$ in (55) in order to store the elements of the right-hand side of (55) in $B(1)$ to $B(N)$. If $LI$ is 2, DO loop 16 stores the elements of the right-hand side of (56) in $B(1)$ to $B(N)$. If $LI$ is 1, lines 33 and 34 put in $V(1)$ to $V(N)$ the elements of the column vector $\hat{V}$ that satisfies (55). If $LI$ is 2, lines 33 and 34 put in $V(1)$ to $V(N)$ the elements of the column vector $\hat{V}$ that satisfies (56). Since the elements of $\hat{V}$ are the $\{V_j\}$ of (1), it is evident that the coefficients $\{V_j\}$ in the expansion (1) for the magnetic current $H$ will reside in $V(1)$ to $V(N)$.

Equation (48) is recast as

$$T = \frac{\text{Real (} \frac{1}{2\Delta x \Delta y} \hat{P}^* \text{)}}{n L_x \sin \beta} \tag{57}$$

In (57), $\beta$ is TH and $\frac{1}{2\Delta x \Delta y} \hat{P}$ is the column vector that stored in either $P(1)$ to $P(N)$ or $P(N+1)$ to $P(2N)$ according as $LI$ is either 1 or 2. With regard to (57), DO loop 17 accumulates $\hat{V} \frac{1}{2\Delta x \Delta y} \hat{P}^*$ in UI. Line 44 puts $T$ of (57) in the computer program variable $T$.

Equation (50) is recast as

$$\frac{(\tau)}{\theta_y} \gamma^2 = \frac{(k^2 \Delta x \Delta y)}{8\pi^3 \eta^2} \frac{1}{2\Delta x \Delta y} (\hat{P}^m)_{\theta_y} \hat{V}^2 \tag{58}$$

Equation (51) is recast as

$$\frac{(\tau)}{\lambda^2} \gamma^2 = \frac{(k^2 \Delta x \Delta y)}{8\pi^3 \eta^2} \frac{1}{2\Delta x \Delta y} (\hat{P}^m)_{\lambda^2} \hat{V}^2 \tag{59}$$

The index $J$ of DO loop 19 obtains

$$\theta^m = (J-1)\pi/(NTH-1) \tag{60}$$

in (58) and

$$\phi^m = (J-1)\pi/(NTH-1) \tag{61}$$
in (59). The value of the right-hand side of (61) is the same as that of (60). Inside DO loop 19, line 53 puts this common value in TH. With regard to (58) and (59), line 54 puts the elements of \( \frac{1}{2\Delta x\Delta y} (\mathbf{P}^m)_{\theta y} \) in P(1) to P(N) and the elements of \( \frac{1}{2\Delta x\Delta y} (\mathbf{P}^m)_{xx} \) in P(N+1) to P(2N). DO loop 21 lies inside DO loop 20 whose index is K. If K = 1, DO loop 21 accumulates \( \frac{1}{2\Delta x\Delta y} (\mathbf{P}^m)_{\theta y} \) in UI. If K = 2, DO loop 21 accumulates \( \frac{1}{2\Delta x\Delta y} (\mathbf{P}^m)_{xx} \) in UI. When K = 1, line 64 puts \( (\mathbf{r})_{\theta y}/\lambda^2 \) of (58) with \( \theta^m \) given by (60) in TAU(1). When K = 2, line 64 puts \( (\mathbf{r})_{xx}/\lambda^2 \) of (59) with \( \phi^m \) given by (61) in TAU(2).
LISTING OF THE MAIN PROGRAM

001 C

002 COMPLEX U, V, T (1600), P (100), B (40), V (40), U1, CONJG

003 DIMENSION IPS (40), TAU (2)

004 OPEN (UNIT=20, FILE='HAUTZ3.DAT')

005 OPEN (UNIT=21, FILE='HAUTZ4.DAT')

006 PI = 3.141593

007 ETA = 376.730

008 U = (0., 1.)

009 UV = 2.*PI*ETA*U

010 READ (20, 11) LX, LI, NTH, DX, DY, TH

011 11 FORMAT (3I3, 3E14.7)

012 WRITE (21, 13) LX, LI, NTH, DX, DY, TH

013 12 FORMAT (' LI LI NTH', 5X, 'DX', 12X, 'DY', 12X, 'TB' / 1X,

014 JI3, 3E14.7)

015 EK = 2.*PI

016 DX = DX * BK

017 LY = DY * BK

018 P8 = 180. / PI

019 TH = TH / P8

020 CALL IBAT (LX, DX, DY, Y)

021 WRITE (21, 15) Y (I), I = 1, 3

022 13 FORMAT (' Y' / (1X, 6E11.4))

023 M = LI - 1

024 CALL PIAE (TH, LX, DX, DY, P)

025 WRITE (21, 16) (P (I), I = 1, 3)

026 14 FORMAT (' P' / (1X, 6E11.4))

027 IA = (LI - 1) * M

028 IB = IA

029 DO 16 J = 1, N

030 IB = IB + 1

031 F (J) = UV * P (IB)

032 16 CONTINUE

033 CALL DECONP (N, IPS, Y)

034 CALL SOLVE (N, IPS, Y, B, V)

035 WRITE (21, 24) (V (I), I = 1, 3)

036 24 FORMAT (' V CF MAGNETIC CURRENT', 1X, EXPANSION FUNCTIONS' / (1X, 6E11.4))

037 1 'EXPANSION COEFFICIENTS V CF MAGNETIC CURRENT', 1X, EXPANSION FUNCTIONS' / (1X, 6E11.4))

038 01 = 0.

039 IB = IA

040 DO 17 J = 1, N

041 IB = IB + 1

042 G1 = UV * V (J) * CONJG (P (IB))

043 17 CONTINUE

044 T = REAL (G1) / (LI * ETA * SIN (TH))

045 WRITE (21, 18) T

046 18 FORMAT (' TRANSMISSION COEFFICIENT T=', E14.7)

047 CT = DI / (PI * ETA)

048 CT = CT * CT / (8. * PI)

049 ETH = PI / (TH - 1)

050 WRITE (21, 23)
051 23 FORMAT ( ' ANGLE', 4X, 'TAU1', 7X, 'TAU2')
052 DO 19 J=1, NTH
053 TH = (J-1) * DTH
054 CALL PLANE (TH, LX, DX, DY, P)
055 TH = TH * P8
056 J1 = 0
057 DO 20 K = 1, 2
058 U1 = 0.
059 DO 21 I = 1, N
060 J1 = J1 + 1
061 U1 = U1 + P (J1) * V (I)
062 21 CONTINUE
063 H = U1 * CONJG (U1)
064 TAU (K) = CT * H
065 20 CONTINUE
066 WRITE (21, 22) TH, (TAU (I), I = 1, 2)
067 22 FORMAT (1X, F7.2, 2E11.4)
068 19 CONTINUE
069 STOP
070 END

INPUT DATA IN THE FILE NAUTZ3.DAT

5 1 19 0.5000000E-01 0.5000000E-01 0.2700000E+03
OUTPUT DATA IN THE FILE "AUTZ4.DAT"

\[\begin{array}{cccc}
\text{LI} & \text{LI} & \text{TH} & \text{DI} \\
5 & 1 & 19 & 0.5000000e-01 \\
& & & 0.2700000e+03 \\
\end{array}\]

\[\begin{array}{cccc}
Y & -0.1531E+02 & -0.6526E-01 & 0.6646E+01 \\
P & 0.1000E+01 & 0.1471E-06 & 0.1000E+01 \\
\end{array}\]

COEFFICIENTS \( V \) OF MAGNETIC CURRENT EXPANSION FUNCTIONS

\[\begin{array}{cccc}
0.4511E+02 & 0.5916E+03 & 0.6238E+02 & 0.8153E+03 \\
\end{array}\]

TRANSMISSION COEFFICIENT \( T = 0.1141263E+00 \)

\[\begin{array}{cccc}
\text{ANGLE} & \text{TAU1} & \text{TAU2} \\
0.00 & 0.0000E+00 & 0.2186E-02 \\
10.00 & 0.5885E-04 & 0.2186E-02 \\
20.00 & 0.2308E-03 & 0.2188E-02 \\
30.00 & 0.5016E-03 & 0.2190E-02 \\
40.00 & 0.8462E-03 & 0.2193E-02 \\
50.00 & 0.1228E-02 & 0.2196E-02 \\
60.00 & 0.1602E-02 & 0.2199E-02 \\
70.00 & 0.1918E-02 & 0.2202E-02 \\
80.00 & 0.2129E-02 & 0.2203E-02 \\
90.00 & 0.2204E-02 & 0.2204E-02 \\
100.00 & 0.2204E-02 & 0.2203E-02 \\
110.00 & 0.2199E-02 & 0.2202E-02 \\
120.00 & 0.1602E-02 & 0.2199E-02 \\
130.00 & 0.1228E-02 & 0.2196E-02 \\
140.00 & 0.8462E-03 & 0.2193E-02 \\
150.00 & 0.5016E-03 & 0.2190E-02 \\
160.00 & 0.2308E-03 & 0.2188E-02 \\
170.00 & 0.5885E-04 & 0.2186E-02 \\
180.00 & 0.2088E-15 & 0.2186E-02 \\
\end{array}\]
IV. THE SUBROUTINE YMAT

The subroutine YMAT(LX, DX, DY, Y) stores by columns in Y the matrix \[ \frac{m}{j\Delta x \Delta y} \] that appears in (55) and (56). The first three arguments of YMAT are input arguments. The aperture is LX subsections long in the x direction and one subsection wide in the y-direction. 

DX is \( k\Delta x \) and DY is \( k\Delta y \) where \( k \) is the wave number, \( \Delta x \) is the subsection length in the x direction, and \( \Delta y \) is the subsection width in the y direction.

Minimum allocations are given by

\[ \text{COMPLEX TC}(LX+1), \text{TX}(LX+1), YXX(N), Y(N*N) \]

where \( N \) is given by (54).

Inside DO loop 16, \( I_c(I-1) \) of (23) is put in the computer program variable TC(I+1). Here, \( I \) is the index of DO loop 16. Also inside DO loop 16, \( I_x(I-1) \) of (29) is put in the computer program variable TX(I+1).

The logic inside DO loop 16 is best understood by building up a table of variables in YMAT versus expressions in terms of variables in Section II.

<table>
<thead>
<tr>
<th>Variables in YMAT</th>
<th>Expressions in Section II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>( i+1 ) where ( i ) appears in (23) and (29)</td>
</tr>
<tr>
<td>( YU )</td>
<td>( y_u ) of (12)</td>
</tr>
<tr>
<td>( XU )</td>
<td>( x_u ) of (13)</td>
</tr>
<tr>
<td>( XL )</td>
<td>( x_L ) of (14)</td>
</tr>
<tr>
<td>( R1 )</td>
<td>( r_1 ) of (22)</td>
</tr>
<tr>
<td>( U1 )</td>
<td>( U_1 ) of (24)</td>
</tr>
<tr>
<td>( U2 )</td>
<td>( U_2C_2 ) of (25) and (32)</td>
</tr>
<tr>
<td>( U3 )</td>
<td>( U_3 ) of (26)</td>
</tr>
<tr>
<td>( U4 )</td>
<td>( \frac{1}{6} ) of (27)</td>
</tr>
</tbody>
</table>
Taking advantage of the fact that $I_c(i)$ is even in $i$, line 46 stores $I_c(-1)$ in $TC(I)$. Taking advantage of the fact that $I_x(i)$ is odd in $i$, line 47 stores $I_x(-1)$ in $TX(I)$.

Inside DO loop 20, lines 51 and 52 put in $YXX(J-1)$ the square bracketed term on the right-hand side of (9) with $j-i = J-2$. Now, we have

$$\frac{\pi n}{\Delta x \Delta y} Y_{ij} = YXX(j-i+1), \quad j-i = 0,1,2,\ldots,L_x-2$$

(62)

Because $Y_{ij}$ is even in $(j-i)$, (62) becomes

$$\frac{\pi n}{\Delta x \Delta y} Y_{ij} = YXX(|j-i|+1), \quad |j-i| = 0,1,2,\ldots, L_x-2$$

(63)

Inside nested DO loops 23 and 21, line 59 puts $\frac{\pi n}{\Delta x \Delta y} Y_{IJ}$ of (63) in the computer program variable $Y(I + (J-1)*N)$ where $N$ is given by (54).
LISTING OF THE SUBROUTINE THAT

SUBROUTINE THAT(LX, DX, DX, Y)

COMPLEX U, U1, U2, U3, U4, EX, TC(100), TX(100)

COMPLEX YXX(100), Y(1600)

DX2 = DX * DX

N = LX - 1

U = (0., 1.)

U4 = 1666667 * U

YU = 5 * DX

YUD = YU * DX

YU2 = YU * YU

YU3 = 3333333 * YU

YU4 = 25 * YU2 + YU3 * YU

DO 16 I = 1, LX

JY = 0

CO 23 J = 1, N

CO 21 I = 1, N

JY = JY + 1

K = IABS(J - I) + 1

CONTINUE

XU = (I - 5) * DX

CONTINUE

YU2 = I * DX

CONTINUE

XL = U - DX

CONTINUE

IL2 = IL * IL

CONTINUE

R1 = (I - 1) * DX

CONTINUE

R2 = R1 * R1

CONTINUE

R01 = 1. - 5 * R2

CONTINUE

U1 = R01 + R1 * (1. - 1666667 * R2) * 0

CONTINUE

U2 = (R1 - R01 * U) * YUD

CONTINUE

U3 = 5. - 5 * R1 * U

CONTINUE

EX = 2. * (COS(R1) - U * SIN(R1))

CONTINUE

R7 = IL2 + YU2

CONTINUE

R8 = UX2 + YU2

CONTINUE

E3 = SQRT(R7)

CONTINUE

R4 = SQRT(R8)

CONTINUE

AUX = YU * ALOG((YU + B4) / XU)

CONTINUE

AXL = XL * ALOG((YU + B3) / ABS(XL))

CONTINUE

AYU = YU * ALOG((XU + B4) / (XL + B3))

CONTINUE

C1 = AUX / AXL

CONTINUE

C3 = YU3 * (XU + B4 - XL * B3) + 1666667 * (XU2 * AUX - XL2 * AXL + YU2 * AYU)

CONTINUE

C4 = YU3 * (XU + B8 - XL * R7)

CONTINUE

TC(JP) = (C1 * U1 + U2 + C3 * U3 + C4 * U4) * EX

CONTINUE

AXU = UX1 / AXU

CONTINUE

AXL = XL * AXL

CONTINUE

X1 = 5. * (YU + (R4 - B3) + AUX - AXL)

CONTINUE

X3 = YU3 * (8333333 - 1 * (B8 * B4 - B7 * B3) + 125 * (XU2 + BU - XL2 * B3))

CONTINUE

1 + 125 * (XU2 * AUX - XL2 * AXL)

CONTINUE

4U = B1 * YUD * (R2 + YU4)

CONTINUE

TX(JP) = (X1 * U1 + B1 * U2 + X3 * U3 + X4 * U4) * EX / EX

CONTINUE

16

CONTINUE

TC(3) = TC(3)

CONTINUE

TX(1) = -TX(3)

CONTINUE

DO 20 J = 2, LX

CONTINUE

JM = J - 1

CONTINUE

JP = J + 1

CONTINUE

YXX(JM) = 5 * (TC(J) - TX(JP) + (J - 5) * TC(JP) + TX(JH) -

CONTINUE

(J - 3.5) * TC(JM) + (TC(JP) - 2 * TC(J) + TC(JM)) / DX2

CONTINUE

20
V. THE SUBROUTINE PLANE

The subroutine PLANE(TH, LX, DX, DY, P) stores in P(1) to P(N) the quantities

\[ \frac{1}{2\Delta x\Delta y} (P_{i}^{\text{inc}})^{x} = \sin \theta_{\text{inc}} \sin \left( \frac{k\Delta x \cos \theta_{\text{inc}}}{2} \right) e^{j k\Delta x \cos \theta_{\text{inc}}} \]

\[ \frac{1}{2\Delta x\Delta y} (P_{i}^{\text{inc}})^{y} = \sin \theta_{\text{inc}} \left( \frac{k\Delta x \cos \theta_{\text{inc}}}{2} \right) e^{j k\Delta x \cos \theta_{\text{inc}}} \]

\[ i=1,2,...,N \quad (64) \]

where (64) comes from (8) and N is \((L_x - 1)\). The aperture is \(L_x\) subsections long in the x direction and one subsection wide in the y direction. In (64), \(k\) is the wave number, \(\Delta x\) is the subsection length in the x direction, and \(\Delta y\) is the width of the aperture in the y direction. Moreover, the subroutine PLANE stores in P(N+1) to P(2N) the quantities

\[ \frac{1}{2\Delta x\Delta y} (P_{i}^{\text{inc}})^{xx} = - \sin \frac{k\Delta y \cos \phi_{\text{inc}}}{2} e^{j(k\Delta y/2) \cos \phi_{\text{inc}}} \]

\[ \frac{1}{2\Delta x\Delta y} (P_{i}^{\text{inc}})^{yy} = \sin \frac{k\Delta x \cos \phi_{\text{inc}}}{2} e^{j(k\Delta x/2) \cos \phi_{\text{inc}}} \]

\[ i=1,2,...,N \quad (65) \]

where (65) comes from (16). The angle \(\phi_{\text{inc}}\) in (65) is assumed to be the same as \(\theta_{\text{inc}}\) in (64). The first four arguments of PLANE are input arguments. In radians, TH is the common value of \(\theta_{\text{inc}}\) and \(\phi_{\text{inc}}\) in (64) and (65). Furthermore, LX is \(L_x\), DX is \(k\Delta x\), and DY is \(k\Delta y\). LX is a positive integer greater than or equal to 2.

The minimum allocation for P is given by

COMPLEX P(2*N)

where N is given by (54).

Lines 5 and 6 set

\[ CX = k\Delta x \cos (TH) \]

\[ CY = \frac{1}{2} k\Delta y \cos (TH) \]
If \( \cos(\text{TH}) \neq 0 \), then lines 11 to 14 set

\[
\begin{align*}
\text{SX} &= \sin(\text{TH}) \left( \frac{kAx \cos(\text{TH})}{2} \right)^2 \\
\text{SY} &= -\frac{\sin \left( \frac{kAy \cos(\text{TH})}{2} \right)}{\frac{kAy \cos(\text{TH})}{2}}
\end{align*}
\]

If \( \cos(\text{TH}) = 0 \), then lines 8 and 9 set SX and SY equal to the limits of the above two expressions as \( \cos(\text{TH}) \) goes to zero, namely

\[
\begin{align*}
\text{SX} &= \sin(\text{TH}) \\
\text{SY} &= -1
\end{align*}
\]

Inside DO loop 13, line 19 puts in \( P(I) \) the right-hand side of (64) with \( i \) replaced by \( I \) where \( I \) is the index of DO loop 13. Note that the right-hand side of (65) does not depend on \( i \). Line 21 puts in \( Ul \) the right-hand side of (65). DO loop 14 puts the right-hand side of (65) in \( P(N+1) \) to \( P(2*N) \) where \( N \) is given by (54).
```
LISTING OF THE SUBROUTINE PLANE

002  SUBROUTINE PLANE (TH, LX, DX, DY, P)

003  COMPLEX U, U1, P(100)

004  CS = CCS (TH)

005  CX = LX * CS

006  CY = 5 * DX * CS

007  IF (CS) 11, 10, 11

008 10  SI = SIN (TH)

009  SY = -1.

010  GO TO 12

011 11  SI = 5 * CX

012  SX = SIN (SI) / SI

013  SX = SIN (TH) * SX * SX

014  SX = -SIN (CY) / CY

015  12  U = (0, 1.)

016  N = LX - 1

017  DO 13 I = 1, N

018  S = I * CX

019  P(I) = SX * (COS (S) + U * SIN (S))

020 13  CONTINUE

021  U1 = SX * (COS (CY) + U * SIN (CY))

022  DO 14 J = 1, N

023  I = J + N

024  P(I) = U1

025 14  CONTINUE

026  RETURN

027  END
```
VI. THE SUBROUTINES DECOMP AND SOLVE

The subroutines DECOMP(N, IPS, UL) and SOLVE(N, IPS, UL, B, X) solve a system of N linear equations in N unknowns. The input to DECOMP consists of N and the N by N matrix of coefficients on the left-hand side of the matrix equation stored by columns in UL. The output from DECOMP is IPS and UL. This output is fed into SOLVE. The rest of the input to SOLVE consists of N and the column of coefficients on the right-hand side of the matrix equation stored in B. SOLVE puts the solution to the matrix equation in X.

Minimum allocations are given by

COMPLEX UL(N*N)
DIMENSION SCL(N), IPS(N)
in DECOMP and by

COMPLEX UL(N*N), B(N), X(N)
DIMENSION IPS(N)
in SOLVE.

More detail concerning DECOMP and SOLVE is on pages 46-49 of [3].
LISTING OF THE SUBROUTINE DECOMP

SUBROUTINE DECOMP(N,IPS,UL)

COMPLEX UL(1600),PIVOT,EM

DIMENSION SCL(40),IPS(40)

DO 5 I=1,N

IPS(I)=I

RM=0.

J1=I

DO 2 J=1,N

ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))

J1=J1+N

IF(RM-ULM) 1,2,2

11

RM=ULM

12

CONTINUE

13 SCL(I)=RM/RM

14 CONTINUE

15 MM=MM-1

16 K2=0

17 DO 17 K=1,MM

18 EIG=0.

19 DO 11 I=K,N

20 IP=IPS(I)

21 IPK=IPS(K)

22 SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)

23 IF(SIZE-BIG) 11,11,10

24 EIG=SIZE

25 IPV=I

26 CONTINUE

27 IP=IPK

28 CONTINUE

29 IF(IPV-K) 14,15,14

30 J=IPS(K)

31 IPS(K)=IPS(IPV)

32 IPS(IPV)=J

33 KPP=IPS(K)+K2

34 PIVOT=UL(KPP)

35 KP=KP+1

36 EO 16 I=KP1,N

37 KP=KPP

38 IP=IPS(I)+K2

39 EM=-UL(IP)/PIVOT

40 UL(IP)=-EM

41 DO 16 J=KP1,N

42 IF=IE+N

43 KP=KE+N

44 UL(IP)=UL(IP)+EM*UL(KP)

45 CONTINUE

46 K2=K2+N

47 CONTINUE

48 RETURN

49 END
LISTING OF THE SUBROUTINE SOLVE

050 C

051 SUBROUTINE SOLVE(N,IPS,UL,B,X)

052 COMPLEX UL(1600),B(40),X(40),SUM

053 DIMENSION IPS(40)

054 IF1=I+1

055 IP=IPS(1)

056 X(I)=B(IP)

057 DO 2 I=2,N

058 IP=IPS(I)

059 IPB=IP

060 IN1=I-1

061 SUM=0.

062 DO 1 J=1,IN1

063 SUM=SUM+UL(IP)*X(J)

064 IP=IP+1

065 2 X(I)=B(IPB)*SUM

066 K2=N*(N-1)

067 IP=IPS(N)+K2

068 X(N)=X(N)/UL(IP)

069 DO 4 IBACK=2,N

070 I=IP1-IBACK

071 K2=K2-N

072 IP1=IPS(I)+K2

073 IP=I+1

074 SUM=0.

075 IP=IP1

076 CO 3 J=IP1,N

077 IP=IP+1

078 3 SUM=SUM+UL(IP)*X(J)

079 4 X(I)=(X(I)-SUM)/UL(IP1)

080 RETURN

081 END
REFERENCES


