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Injection lasers have potential for replacing many of the high-power conventional gas lasers. The research performed in this project was aimed at the design of high-power semiconductor lasers for communications, printing, and others. Diode lasers have been analyzed for obtaining high output power at single frequencies and for optical pattern stability. We have analyzed the following laser types: 1) Gain guided, 2) Index guided, and 3) W-Guides.

*Injection Laser Structure Design*

**Injection Laser Structure Design**

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INJECTION LASER STRUCTURE DESIGN

FINAL REPORT

by

JEROME K. BUTLER

JANUARY 30, 1985

U. S. ARMY RESEARCH OFFICE

GRANT DAAG29-80-K-004

SOUTHERN METHODIST UNIVERSITY

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1. STATEMENT OF PROBLEM

This research project covered a period of approximately four years. The program was aimed at an understanding of the design and fabrication of semiconductor injection lasers for both communication and high power applications. Lasers fabricated for communication systems are typically of the low power type which emit power in 10 to 30 milliwatt range while high power devices (single element types) have emission powers of over 50 milliwatts. Communication devices, fabricated for long wavelength (approximately 15,000 Angstroms) operation where optical fibers have both low attenuation and dispersion, use compounds of indium, gallium, arsenic, and phosphorus. On the other hand, contemporary high-power devices (with emission wavelengths around 8,000 Angstroms) are fabricated with aluminum, gallium and arsenic. Although the research of this project was concerned with aluminum and gallium arsenide compounds, the theory developed is applicable to all semiconductor lasers.

The electromagnetic resonant modes in a semiconductor laser are obtained from solving Maxwell's equations. The ultimate mode operation is inextricably tied to the geometry of the laser as well as to its interaction or coupling with the external world. The particular mode that operates is determined by its interaction with the active region of the laser. Most system applications require devices that emit power in a single and coherent beam of radiation. However, a majority of lasers radiate in either multiple beams or a single unstable beam (drive sensitive). On the other hand, gas lasers can be fabricated to emit power in a single stable beam of radiation because the modes in a gas laser are
formed by the cavity mirrors; the modal selection process is thus independent of drive. In an injection laser the drive dependent gain region plays a major role in the formation of the waveguide geometry. Consequently, we must carefully design structures that can operate only in a single mode.

Because of design restrictions, many contemporary lasers are constrained to low powers. To increase the output power, several exotic structures have been proposed in the literature. But, these newly conceived devices have pushed the fabrication technology to its limits which adversely affects manufacturing yields.

Design sophistication of single laser elements is one method of achieving high overall emission powers. Another approach is to couple several lasers together to form a linear array. Since the devices are fabricated on a single wafer, locking of the elements to obtain a single coherent source can be accomplished via internal field coupling. This coupling is possible because the waveguides are open types. Nevertheless, proper design of the single element to achieve the "right" coupling is necessary.

Thus, the problem of analyzing the individual laser element, the central theme of this research project, is the single most important aspect required for the ultimate realization of the semiconductor injection laser.
INTRODUCTION

Semiconductor lasers are finding applications in many areas, such as optical recording, high-speed printing, fiber optic systems, communication through free space, optical pumping of various lasers which operate at different wavelength and others. Development of high-power lasers has made it possible to fabricate single element lasers that radiate over 100 milliwatts in continuous operation and much higher powers in pulsed operation. Also, recent work in phased-laser-arrays has shown that it is possible to fabricate individual devices with relatively high packing densities. Thus, the overall power emission from the single coherent source is very high.

Semiconductor lasers will begin to replace many of the very high-power optical sources in the next 5 to 10 years. There are two main reasons that will propel the growth in the high power applications. (1) Semiconductor devices are superior to "vacuum tube" components in terms of reliability. (2) For a given volume, the power emission from a semiconductor laser compared to that obtained from a gas laser is extremely large. High powers are possible because of the packing density of excited electrons or inversion levels is semiconductors.

The operation of injection lasers can be categorized into several areas. These include (1) power emission capacity, (2) operational wavelength, (3) wavelength or mode stability, and (4) spatial radiation stability. Typically, many of the above are interconnected. For example, high-power lasers tend to develop many operational instabilities. These
instabilities occur because of the strong nonlinear relationships between optical waveguide formation in the material and the optical mode intensity.

Much of the work accomplished from this grant concerned the development and understanding of single-component high-power injection lasers. Our models of electromagnetic phenomena in nonplanar lasers have helped researchers in the laser field to push toward optimum designs. At this point in time, many possible structures can produce high powers. These include the channel-substrate-planar (CSP), the constricted-double-heterojunction large-optical-cavity (CDH-LOC), and others. In terms of modeling the various devices, the CDH-LOC laser is the most difficult. We discuss some of its properties below. Extension of these models to other nonplanar lasers such as the CSP is easily accomplished.

MODELS OF NONPLANAR SEMICONDUCTOR LASERS

The CDH-LOC laser provides high single-mode power because of the nonplanarity of the various epitaxially grown layers. Because this device is nonplanar, it is very difficult to model many of its subtle characteristics. However, our model of the CDH-LOC optical modes is confirmed by excellent fits of theoretically generated far-field patterns to experimental ones. A major finding discovered by our model was the combined effect of lateral variations in transverse (optical) confinement factor, \( r \), and gain-induced index depressions on lateral mode control.

Figure 1 depicts schematically the basic geometry of CDH-LOC devices: a "convex-lens-shaped" active layer atop a "concave-lens-shaped" guide layer. The bulk refractive indices are such that \( n_1 > n_2 > n_3 > n_4 \). We have shown that such structures translate into passive W-shaped lateral waveguides. Unlike the waveguiding structures usually analyzed, the W-
guide has no known solutions of the wave equation. Thus it was necessary to employ numerical methods to solve Maxwell's equations for arbitrary dielectric profiles, and thus to solve for the lateral mode content of the W-guides as described below.

![Diagram of a constricted-double-heterojunction laser with a large-optical-cavity](image)

**Fig. 1** Cross section of a constricted-double-heterojunction laser with a large-optical-cavity. Light is confined to the regions 1 and 2 and to the lateral region near $y = 0$.

Solutions to Maxwell's equations were restricted to fields polarized along the junction with $E(x,y) = f(x,y) h(y) \exp(j\omega t - yz)$ where $f(x,y)$ and $h(y)$ describes the transverse and lateral fields respectively. The lateral modes are obtained from a solution of

$$d^2h/dy^2 = \left(\gamma^2 - \gamma_0^2 + k_0^2 \Gamma(y) \delta\kappa_1(y)\right)h = 0$$

where $k_0 = 2\pi/\lambda_0$, with $\lambda_0$ the free-space wavelength and $\delta\kappa_1(y)$ is the current dependent change in the complex dielectric constant of the active layer. The passive structures have $\delta\kappa_1 = 0$. $\gamma_0$ is the one-dimensional propagation constant that determines the effective index of the passive structure, $n_{eo} (n_{eo} = j\gamma_0/k_0)$; and $\gamma$ is the two-dimensional propagation
constant to be determined. The values of $\delta k_1$ are found from the current
distribution flowing into the active layer. An estimate of the current
spreading was obtained by numerically solving an approximate circuit
model.\textsuperscript{5,6} We found the current to be virtually flat under the 10 \textmu m-wide
contact stripe, and tapering to half maximum at approximately 5 \textmu m from the
contact stripe edges. Using the current distribution in the active layer,
we can write the gain\textsuperscript{7}

$$g(y) = 45 J_{\text{eff}}(y)/d(y) - 190 \text{ (cm}^{-1})$$

(2)

where $J_{\text{eff}}(kA/cm^2)$ is the effective current density, and $d(y)$ is the active
layer thickness. The gain affects both the imaginary and real parts of the
dielectric constant\textsuperscript{2}:

$$\delta k_1(y) = 2n_1 \delta n_1(y) + jn_1 g(y)/k_0$$

(3)

where $\delta n_1(y)$ is the gain-induced index change in the active layer. (It is
proportional to the carrier injection.) Since $\delta n_1(y)$ is generally found to
be negative we refer to it as gain-induced index depression. The net
real effective index that characterizes the lateral field is

$$n_{\text{eff}}(y) = n_{e0}(y) + \Gamma(y) \delta n_1(y)$$

(4)

If we assume that the background absorption coefficient $\alpha_0$ in the
unpumped active regions is 190 cm$^{-1}$ we have for $\delta n_1$ the following ad hoc
relationship

$$\delta n_1 = R (g(y) + 190) \text{ cm}^{-1}/k_0$$

(5)

with $R$ being the ratio of index changes to normalized gain changes, also
known as antiguiding factor.\textsuperscript{8,9} Finally, by using (2) and (5) we obtain

$$\delta n_1(y) = 45 R J_{\text{eff}}(y)/k_0 d(y)$$

(6)
The analysis was applied to two different high-power CDH-LOC structures. Active and guide-layer thicknesses were accurately measured in the lateral direction, over the range $10 \mu m < y < 10 \mu m$, by using 50 angle-lapped stained cross-sections. The Al content of the various $Al_{x}Ga_{1-x}As$ layers was determined from photoluminescence data. The layers varied laterally in thickness as shown in Fig. 1 (For one geometry the guide layer was asymmetric due to substrate misorientation effects on growth.\textsuperscript{10}) With the structures fully characterized, the confinement factor $\Gamma(y)$ and the effective index $n_{\text{eo}}(y)$ are determined by solving for the transverse modes at each lateral data point. The $\Gamma(y)$ plots for the two structures are shown in Figs. 2 (a) and (c). In the same figures we plot the respective net effective refractive index profiles, $n_{\text{eff}}(y)$ (i.e., W-guides). Both the passive contribution $n_{\text{eo}}(y)$ and the active contribution, $\Gamma(y) \delta n_{\perp}(y)$, are considered. While $n_{\text{eo}}$ and $\Gamma$ were known, $\delta n_{\perp}(y)$ had to be deduced from fitting of theoretical far-field patterns to experimental ones (see Figs. 2 (b) and (d)). Thus we obtain for the two structures at threshold $\delta n_{\perp}(0)$ values of -0.028 and -0.02, which agree with $\delta n_{\perp}$ values determined by other workers\textsuperscript{11,12} from different experiments. The gain in the active layer at threshold was set at 50 cm\textsuperscript{-1} to offset end and internal cavity losses.

Having generated the proper W-guides, we then solved the wave equation numerically for the lateral modes. Radiation patterns were determined from the Fourier Transform of the near fields.

As far as the discrimination against high-order mode oscillation, two features of Figs. 2 (a) and (c) are relevant: 1) $\Gamma(y)$ peaks at $y = 0$ and drops relatively fast across the lasing region (5-7 $\mu m$ wide); and 2) $n_{\text{eff}}(y)$ profiles are W-guides. Both features favor oscillation in the fundamental mode over the first-order mode. It should be stressed that the
The far-fields were fitted by adjusting only one parameter: $R$ (see Eq. (5)). For run DB-208 (Fig. 2(b)) side lobes at $10^\circ$, indicative of radiative leakage, could only be fit for $R = -4$. Run DB-181 (Fig. 2d) had no leakage for the fundamental mode since the central region of the W-guide was wide. Yet a lateral beamwidth of approximately $6^\circ$ could be fitted only for $R$ values in the -3 to -4 range. Once $R$ was determined, $n_1$ was obtained from (6), which in turn allows the generation of the actual W-guides.

The lateral modal content of the two structures for $R$ values of practical interest and threshold mode gain of 50 cm$^{-1}$ is as follows: only the fundamental mode for run DB-208; and three modes for run DB-181. Figure 2 (e) shows the threshold current density for the first two modes of DB-181 lasers as a function of the antiguiding factor $R$. We also plot the index depression at the center of the waveguide for various $J_{\text{eff}}$ and $R$ values. Note that the fundamental mode (i.e., 0th order mode) has a relatively constant threshold current density for all $R$ values of practical interest. On the other hand, the first order mode has an abrupt
discontinuity at $R = -2.9$. Above that value the first order mode is being "pushed out" of the central index guiding region. Thus, the current density must be increased to large values in order to re-establish the mode. From a practical stand point we can consider the first-order mode to be cut off at $R = -2.9$. It is thus apparent that by varying the gain-induced index depression one can choose between multi-mode and fundamental-mode operation. Changes in $n_1$ can be incurred via changes in device length, facet(s) reflectivity and/or ambient temperature.
Fig. 2 Optical modes of CDH-LOC laser structures are calculated by the effective index method. (a) and (c) are for two different lasers. (b) and (d) show the experimental and theoretical far-field patterns. "Mode cutoff" conditions for DB 181 can be estimated from (e).
REFERENCES


4. SUPPORTED PERSONNEL INVOLVEMENT

1. Dr. Jerome K. Butler, Principal Investigator.

2. Dr. Joseph B. Delaney, Doctoral Student, Received a Ph.D. in August 1980.

3. Dr. Alfredo Linz, Doctoral Student, Received a Ph.D. in May 1984.

4. Mr. George Hagicostas, Master Degree Student, Near completion of MSEE.
5. RESEARCH PROJECT PUBLICATIONS


1. ARO PROPOSAL NUMBER: AMXRO-PRI-16436-EL

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3. TITLE OF PROPOSAL: Injection Laser Structure Design

4. CONTRACT OR GRANT NUMBER: DAAG29-80-K-004

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8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:
   J. K. Butler
   J. B. Delaney

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The research carried out over this past period involved the exploration of the field solutions obtained by a rigorous method developed by the Principal Investigator. A highlight of our work shows that stripe geometry laser modes have characteristics that are similar to leaky waveguide modes. In our previous work we assumed that the dielectric profile in the injection laser had a parabolic functional variation along the lateral direction (parallel to the junction plane). Although the parabolic profile has been extensively used by previous investigators, it has definite realistic limitations. In our present work we are developing a model using a dielectric profile which can more accurately characterize the true one. For example, at large lateral distances (away from the current injecting contact) the dielectric constant must approach a limiting value; in the parabolic profile the dielectric constant continues increasing/decreasing dramatically with lateral distances.
PROGRESS REPORT
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   Jerome K. Butler
   Joseph B. Delaney
   Ph.D. awarded to J. Delaney.
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The work during this past period has been aimed at an understanding of the lateral mode structure in CDH-LOC and stripe geometry lasers. The CDH-LOC devices are basic high power lasers with large optical cavities. Important cavity dimensions are: (1) thickness between heterojunctions that confine the light and (2) the width of the active region which is dictated by the injection current spread. The important results of the study during this period are that structures can be designed with varying cavity thicknesses aimed at the control of lateral modes. Ordinarily, lateral modes are not affected by cavity thicknesses.

We have had several verbal inquiries concerning the results of our recent publication in the Journal of Quantum Electronics. This publication is the first to predict how lateral mode structures are affected by thin optical waveguides. Previous publications have linked radiation pattern sidelobes to leaky modes while we have shown these lobes result from a "mode mixture", a requirement for a proper boundary value solution to the wave equation.
BRIEF OUTLINE OF RESEARCH FINDINGS

The work in this past period has been directed toward the extension of lateral mode analysis in constricted double heterojunction-large-optical (CDH-LOC) devices. These devices are fabricated in such a fashion that the active region as well as the optical mode confining region vary in thickness as a function of lateral position. These lasers are far superior in performance when compared to other injection laser devices. CDH-LOC lasers have excellent power output and spectral purity. (Device powers around 100 mWatts radiate in a single mode.) The Principal Investigator is presently preparing the analysis of CDH-LOC lasers in conjunction with the experimental program at RCA Laboratories.

Concurrent with the above work, we are extending the analysis of the exact mathematical solution of two dimensional modes in dielectric waveguides. It should be mentioned that there are published results for exact mode solutions only for (1) optical fibers, (2) dielectric waveguides of rectangular cross-section, and (3) slab waveguides with localized lateral gain distributions. (The latter work was previously reported by the PI.)
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BRIEF OUTLINE OF RESEARCH FINDINGS

During this past six months much effort has been spent on the development of: (1) an understanding of leaky wave structures found in contemporary high-power injection lasers, and (2) the integrity of the effective index method which is widely used in integrated optics.

The work reported in the first paper listed above contains extensive information on the analysis of leaky mode lasers. This work is the first to report and to properly analyze devices lasing in a true leaky mode. It was previously believed that leaky modes could not reach threshold because of their lossy nature.

The work reported in the second publication compares the modal solutions of a class of dielectric waveguides, obtained by exact and effective index methods. The waveguides studied are those which closely approximate actual guides designed for injection lasers. Our conclusion is that the effective index method is excellent for waveguide analysis.
MODE CHARACTERISTICS OF NON-PLANAR DOUBLE-HETEROJUNCTION AND LARGE-OPTICAL-CAVITY LASER STRUCTURES

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Abstract

Mode behavior of non-planar double-heterojunction (DH) and large-optical-cavity (LOC) lasers is investigated using the effective index method to model the lateral field distribution. The thickness variations of various layers for the devices discussed are correlated with the growth characteristics of liquid-phase epitaxy over topographical features (channels, mesas) etched into the substrate. The effective dielectric profiles of constricted double-heterojunction (CDH)-LOC lasers show a strong influence on transverse mode operation: the fundamental transverse mode (i.e., in the plane perpendicular to the junction) may be laterally index-guided while the first (high)-order mode is laterally index-antiguided. The analytical model developed uses a smoothly varying hyperbolic cosine distribution to characterize lateral index variations. The waveguide model is applied to several lasers to illustrate conditions necessary to convert leaky modes to trapped ones via the active-region gain distribution. Theoretical radiation patterns are calculated using model parameters, and matched to an experimental far-field pattern.

This work has been partially supported by the U. S. Army Research Office under Grant No. DAAG29-80-K-004 and by NASA, Langley Research Center, Hampton, VA under Contract No. NAS1-15440.

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Comparison of Numerical and Effective Index Methods
for a Class of Dielectric Waveguides*

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Abstract

A numerical method and the effective-index method are applied to a three-layer dielectric waveguide with Eckart type dielectric constant variation in the active layer. The results of the 2 methods are compared in terms of the propagation constant $\gamma$.

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BRIEF OUTLINE OF RESEARCH FINDINGS

In the work over the last period we have continued the investigation of approximate methods used for the analysis of mode characteristics of injection laser devices. We have reported that contemporary injection lasers with wide active regions can be accurately modelled with the effective index method. This is a very important result since many new lasers have non-planar layers; the effective index method affords a convenient mode analysis procedure. It should be noted however, that there are no exact methods for analyzing modes of non-planar laser structures. It is assumed that since approximate and exact calculations of mode character on planar structures produce almost identical results, that the approximate solutions are satisfactory for non-planar devices.

In our last publication we have theoretically predicted transverse mode operation of a multimode laser. Mode operation was predicted by calculation of the coupling between the optical field and injected charge density. We experimentally investigated mode switching by heating a diode laser; at low temperatures the device operated in the first high order mode while at high temperatures, fundamental mode operation was observed. The prediction of switching was tied to the fact that the optical field/charge distribution coupling are temperature sensitive.


Modal Solutions of Active Dielectric Waveguides

by Approximate Methods

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Abstract

Approximate methods are used to obtain the modal properties of stripe-contact semiconductor injection lasers using a planar three-layer waveguide model. The central active layer has a dielectric constant that varies smoothly along the direction parallel to the heterojunction boundaries. The complex dielectric constant under the stripe contact is dependent on the gain and approaches a constant value at large lateral distances. The two methods are compared in terms of their modal propagation constants. An application of the effective index method facilitates a physical understanding of dielectric waveguide modes as well as providing an efficient calculation procedure.

*Supported by the U.S. Army Research Office
PREDICTION OF TRANSVERSE MODE SELECTION IN DOUBLE HETEROJUNCTION LASERS BY AN AMBIPOLAR EXCESS CARRIER DIFFUSION SOLUTION*

by

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Dallas, Texas 75275

ABSTRACT

Transverse mode selection is characterized for GaAs/AlGaAs double heterojunction lasers from optical field and electron/hole interaction. The electron/hole distribution determined from a solution of the ambipolar diffusion equation provides the necessary information about gain/mode coupling to predict the current at threshold. Lasing power output versus current solutions provide information about internal differential quantum efficiency. Theory is matched to experiment for a multimode laser with one heterojunction having a very small index step. It is found that the laser's characteristics over a temperature and current range are predicted by adjusting the active layer refractive index as determined from far field measurements.

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   J. K. Butler
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The work over this last period has been concerned with both the theoretical and experimental characterization of high-power semiconductor laser devices. These lasers will have extensive applications in military communication and data processing systems. The particular devices modelled are the constricted-double-heterojunction large-optical-cavity (CDH-LOC) lasers which are fabricated at RCA Laboratories. These lasers have the highest output power (approximately 50 mW in continuous operation) of any contemporary injection laser. Furthermore, their operational range for single mode lasing is excellent. Our laboratory has developed the most comprehensive theoretical model of the modal characteristics of CDH-LOC devices.

Typically, reasonable theoretical/experimental performance comparisons are made via the far field radiation patterns. To augment our theory we have developed computer controlled apparatus for taking far-field data in two dimensions. For example, the attached figure shows the experimental pattern of a typical CDH-LOC laser where $\theta$ and $\phi$ are the two transverse angles in the far field. (The main lobe peaks at $\theta = \phi = 0$.)

We are now developing software to transmit the experimental data to a large computer for the purpose of theoretical/experimental comparisons. The addition of these laboratory experiments to our program will expedite our understanding of high-power injection lasers and thus improve our overall theoretical modelling capabilities.
Modal Solutions of Active Dielectric Waveguides

by Approximate Methods

A. Linz and J.K. Butler
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Abstract

Approximate methods are used to obtain the modal properties of stripe-contact semiconductor injection lasers using a planar three-layer waveguide model. The central active layer has a dielectric constant that varies smoothly along the direction parallel to the heterojunction boundaries. The complex dielectric constant under the stripe contact is dependent on the gain and approaches a constant value at large lateral distances. The two methods are compared in terms of their modal propagation constants. An application of the effective index method facilitates a physical understanding of dielectric waveguide modes as well as providing an efficient calculation procedure.
Abstract

Transverse-mode selection is characterized for GaAs/AlGaAs double heterojunction lasers from optical field and electron/hole interaction. The electron/hole distribution determined from a solution of the ambipolar diffusion equation provides the necessary information about gain/mode coupling to predict the current at threshold. Lasing power out versus current solutions provide information about internal differential quantum efficiency. Theory is matched to experiment for a multimode laser with one heterojunction having a very small index step. It is found that the laser's characteristics over a temperature and current range are predicted by adjusting the active-layer refractive index as determined from far-field measurements.
Abstract—This paper compares the lateral mode patterns of stripe geometry lasers produced by two theories: 1) an approximate effective dielectric method and 2) an exact method. The exact method for calculating modes uses an expansion of Hermite-Gaussian functions. For small normalized frequencies and a depression of the refractive index under the stripe, the fundamental lateral mode shows structure usually associated with leaky waveguide modes.

A common description of the lateral modes of a stripe geometry laser uses the Hermite-Gaussian (HG) function [1]-[8] while the transverse field dependence is dominated by the large index steps of the heterojunctions which sandwich the active layer. So far only two methods have been proposed to tie together the lateral and transverse field dependencies under the stripe. The first is the effective index method [7] and the second is an exact method [8]. The effective index method results in a single term lateral field description with appropriate "effective" parameters derived to account for the influence of the cladding layers on the lateral mode shape. The advantage of this representation is that a single term expression can easily be matched to such experimental observables as near-/far-field shapes and virtual waist positions. The exact description of the lateral mode takes into account the vertical geometry by matching the usual field quantities at the heterojunctions. The result is a lateral mode description by a linear combination of HG functions with appropriate weighting coefficients. This paper reports on a comparison of the two methods. A major difference between the two is the divergence of the modal shape when the lateral index is depressed under the stripe and the normalized frequency for the vertical geometry (see Fig. 1), \( v = (d/2)K_0(n_2^2 - n_1^2)^{1/2} \), is small. In general, the coefficients of the higher order HG functions rise in magnitude as more light escapes into the transverse cladding. For a negative depression in the lateral index, the higher order terms appear as shoulders on the near and far fields of the fundamental mode and the modal shape is deformed from the pure Gaussian. We begin with a brief recapitulation of the wave equation solutions of each method.

The complex dielectric constant can be written as [7] (see Fig. 1)

\[
\kappa(x, y) = \begin{cases} 
K_b(x) = K_0 - k_0^2d^2x^2, & |y| < \frac{d}{2} \\
K_a = K_c, & |y| > \frac{d}{2}
\end{cases}
\]

where \(K_0\) is the complex dielectric constant at \(x = 0\), \(k_0\) is the
free space wavenumber, and $\kappa_e, \kappa_c$ are the dielectric constants of the cladding layers. The parameter "$a$" in (1) is defined as

$$a = \frac{(n_0 \delta g)^{1/2}}{k_0 x_0} \left( 2R + 10^{10}/k_0 \right)^{1/2}$$

(2)

$$R = \frac{\delta n}{\delta g} \text{cm.}$$

(3)

$x_0$ is defined as the point at which gain has fallen to zero, i.e., $\delta g = \delta g_0$. $\delta n$ is the refractive index change at $x = x_0$. In the effective index method, the wave equation

$$\nabla^2 \Psi + [k_e^2 \kappa_e (x, y) + \gamma^2] \Psi = 0$$

(4)

with

$$\Psi = \psi(x) \phi(y) e^{-\gamma z}$$

(5)

$$\frac{d^2 \phi}{dy^2} = \begin{cases} \frac{q^2 \phi}{y < \frac{d}{2}} \\ \frac{p^2 \phi}{y > \frac{d}{2}} \end{cases}$$

(6)

is multiplied by $\Psi^*$ and integrated over $y \in (-\infty, \infty)$ to yield an effective solution

$$k_e^2 \psi \kappa_e \Gamma + k_c^2 \kappa_c (1 - \Gamma) - q^2 \Gamma + p^2 (1 - \Gamma) + \gamma^2 = k_e^2 \psi \kappa_e (21 + 1)$$

(7)

$$\psi(x) = H_l(a_{\text{eff}}^2 k_0 x) e^{-\frac{1}{2} a_{\text{eff}} k_0^2 x^2}$$

(8)

$$a_{\text{eff}} = \Gamma^{1/2} a$$

(9)

where $\Gamma$ is the confinement factor

$$\Gamma = \int_{-d/2}^{d/2} \psi(y)^2 dy$$

(10)

where $\phi^2(y)$ is normalized to unity, and

$$\phi(y) = A \begin{cases} \cos qy & |y| < \frac{d}{2} \\ \cos q \frac{d}{2} e^{p \frac{1}{2} (d/2 - |y|)} & |y| > \frac{d}{2} \end{cases}$$

(11)

The quantities $p$ and $q$ are the eigenvalues of the slab waveguide. In [7], the equation corresponding to our (7) did not include the eigenvalue $p$. The integration over the $y$ direction dictates an effective representation of $p^2 (1 - \Gamma) - q^2 \Gamma$ as included in (7).

The exact method can be summarized [8]

$$\Psi = \sum_{l} A_l \cos (q_{il} y) H_l(a_{\text{eff}} k_0 x) e^{-\frac{1}{2} a_{\text{eff}} k_0^2 x^2}$$

$$|y| < \frac{d}{2}$$

$$= \int_{-d/2}^{d/2} B(x) \cos \chi x \exp \left\{ \left( \frac{d}{2} - y \right) \left( \chi^2 - k_e^2 \kappa_e - \gamma^2 \right)^{1/2} \right\} dx$$

$$|y| > \frac{d}{2}$$

(12)

where $l = 0, 2, 4, \cdots$ for even modes. The vertical eigenvalue $q_{il}$ satisfies

$$\gamma^2 = q_{il}^2 - k_e^2 \kappa_e + k_c^2 \kappa_c (2l + 1).$$

(13)

Parameters used in sample calculations are $n_s = n_{se} = 3.4$, $n_g(0) = 3.6 \lambda = 0.9 \mu m$, $G = 50 \text{ cm}^{-1}$, and $\alpha_g = \alpha_e = 20 \text{ cm}^{-1}$. Comparison of the two models begins with the gain. For the effective index method (7) can be used. The term $k_e^2 \kappa_e (1 - \Gamma) + k_c^2 \kappa_c (1 - \Gamma))$ is the effective dielectric constant, while $p^2 (1 - \Gamma) - q^2 \Gamma$ represents an effective vertical eigenvalue. With $\gamma = -G/2 + i\beta$, the gain in the active region must include the mirror loss, losses in the cladding layers, and losses in the active layer represented by the right-hand side of (7). For instance, with $R < 0$, the gain region requires additional pumping to account for lateral reflection losses. Since these are the same losses accounted for by the exact technique, both methods calculate virtually the same peak gain values for all values of $R$ investigated.

For $R > 0$, the near- and far-field structures coincide for both methods. A peculiarity of the patterns is the behavior as the normalized frequency $\nu$ gets small (d $\rightarrow$ 0). The far-field half-power full width $\delta_\nu$ is plotted in Fig. 2 versus active region width $d$. As the active region shrinks, peak gain values increase to satisfy additional losses. The parameter "$a_e" as computed from (2), requires that the near field is pulled in and the far field spreads as the result of high cavity gain values.

The two methods diverge in modal shape for $R < 0$ and small normalized frequencies. For the refractive indexes of this work this corresponds to $d < 0.2 \mu m$. As power leakage to the cladding increases, the magnitude of the expansion coefficient $A_2$ increases as illustrated in Fig. 3. With $R < 0$, $|a_1| > |a_2|$, where $a = a_s + i a_t$, and shoulders develop on the fundamental mode patterns. An example of a far-field pattern evolution with $d = 0.1 \mu m$ is shown in Fig. 4 where the dashed curve is the fundamental mode pattern for the effective method and the solid curve the fundamental mode pattern for the exact method. The negative value $\delta n_e$ computed from (3), is a measure of index change and antiguiding over the gain region. It can be seen that, for a value of $\delta n = -0.00185$ in Fig. 4(b), the shoulders on the far-field pattern are well developed. Since the Fourier transform of an HG function is an HG function, the near-field modal structure is theoretically similar to the shapes of Fig. 4. From Fig. 4(a) it would appear that there is no difference in the two models for $R = 10^{-6}$.
and $d = 0.1 \mu m$. In fact, since the modal shapes are so similar a curve for this case $R = -10^{-6}$ is included in Fig. 2. A final remark about Fig. 4 is that the position of the shoulders is predominately determined by the gain width $2x_0$ and not $R$. An index depression under the stripe, represented by a negative value of $R$, determines the intensity of the shoulders but not the lateral position.

In conclusion, we have briefly compared two solutions for a parabolic lateral dielectric constant in the active layer of a stripe geometry laser. Both solutions assume that the vertical dependence is due to the well defined index steps at the heterojunctions. The effective and exact methods predict the same peak gain values since both satisfy the same loss mechanisms. The two methods diverge in modal shape for the case of a depression in the refractive index in the gain region below the stripe and small normalized frequencies. Matching the fields at the vertical heterojunction results in a fundamental modal shape that has shoulders on both the near- and far-field patterns in the case of lateral antiindexing guiding.

References


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Of primary importance in many applications is the amount of cw power that can be delivered reliably from a diode laser into a single mode (smaller than in frequency). One approach toward the achievement of high-power mode-stabilized diodes is the use of large-optical-cavity (LOC) structures. It is structures in which the optical mode acquires gain from a thin (0.05-0.2-μm) active layer, while a sizable part of the mode energy propagates in thick (0.5-1.5-μm) guide layer(s) adjacent to the active layer. We have reported previously on high-power leaky-cavity CDH-LOC lasers. Such devices provide single-mode cw operation up to 40 mW from one facet in beams of narrow transverse full width (θ x = 25-30°) and in large lasing spots (1.5 X 6 μm). Here we report on a new type of CDH device: positive-index CDH-LOC, and we present a comprehensive treatment of lasing mode confinement and selection in CDH-LOC structures. Positive-index CDH-LOC devices have a relatively thick (0.20-0.35-μm) convex-lens-shaped active layer grown above a concave-lens-shaped guide layer. By comparison with leaky-cavity devices, (1) no radiation side lobes are observed in the lateral farfield pattern (θ y = 8°); (2) the transverse far field is still relatively narrow θ x = 30°; (3) the threshold currents are lower (80-70 mA vs 90-150 mA) for the same device length; and (4) single-mode operation is achieved to only 12 mW/facet (cw) and 20 mW/facet (pulsed). Also the threshold currents are found to have record high-temperature coefficients for LOC-type structures: T θ = 135°C in both pulsed and cw operation, while the difference in Al concentration between the active and guide layers is only 15%.

By using the effective-index method it is shown that in LOC structures the combination of a convex-lens-shaped active layer and a concave-lens-shaped guide layer provides a W-shaped lateral waveguide. Depending on the CDH-LOC structure geometry, these W-type guides support relatively large (5-7-μm) fundamental optical modes that can be (1) totally leaky (i.e., improper mode); (2) partially guided and partially leaky (i.e., guided-leaky mode); or (3) totally guided. Aside from bringing flexibility in the CDH-LOC single-mode laser design, W-type lateral guides, under certain conditions, strongly suppress oscillation of both high-order lateral and transverse modes, much more so than ridge guides (1-4,8 (local increase in index) or leaky guides). (15 mW)

Mode Characteristics of Nonplanar Double-Heterojunction and Large-Optical-Cavity Laser Structures

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Abstract—Mode behavior of nonplanar double-heterojunction (DH) and large-optical-cavity (LOC) lasers is investigated using the effective index method to model the lateral field distribution. The thickness variations of various layers for the devices discussed are correlated with the growth characteristics of liquid-phase epitaxy over topographical features (channels, mesa) etched into the substrate. The effective dielectric profiles of constructed double-heterojunction (CDH)-LOC lasers show a strong influence on transverse mode operation: the fundamental transverse mode (i.e., in the plane perpendicular to the junction) may be laterally index-guided, while the first (high)-order mode is laterally index-antiguided. The analytical model developed uses a smoothly varying hyperbolic cosine distribution to characterize lateral index variations. The waveguide model is applied to several lasers to illustrate conditions necessary to convert leaky modes to trapped ones via the active-region gain distribution. Theoretical radiation patterns are calculated using model parameters, and matched to an experimental far-field pattern.

I. INTRODUCTION

FABRICATION of semiconductor injection laser diodes over the past several years has progressed to a fairly sophisticated level in the sense that many contemporary devices are grown on substrate material that has been processed by the incorporation of various grooves and undulations at the substrate surfaces [1]-[15]. After the substrate has been properly processed, various layers of GaAs and AlGaAs compounds are grown over the grooves. The effects of grooves are to produce active layers as well as optical confining layers of relatively small lateral thickness variations, which tend to confine the optical radiation in the lateral direction defined by the grown layers. In this paper we discuss the performance of devices with grown layers as illustrated in Figs. 1 and 2. Specifically, we will discuss the character of the effective lateral refractive index and the resulting modes in several types of laser devices. Much of the previous work on laser structures has centered around double-heterojunction devices, where the active region is sandwiched between two optical confining layers, as illustrated in Fig. 1(a). We will discuss mainly the mode character of the fields in structures illustrated in Fig. 1(b) and in high-power LOC (large-optical-cavity) devices [11], where the majority of the optical field intensity is confined to a region adjacent to the active layer supplying optical power to the lasing mode. The LOC devices [10], [11] have been reported to exhibit stable lateral mode operation and high output powers. Basically, there are two types of LOC structures reported recently that exhibit stable lateral mode operation. These devices are derivatives of that illustrated in Fig. 2. In both cases the thickness of the active layer decreases as a function of distance with the maximum value occurring in the region where the optical field peaks. The adjacent layer which forms the large optical cavity, however, differs in the two structures; one [10] has the layer thickness decreasing with lateral distance, while the other [11], [15] has the layer increasing with lateral distance. Thus, the former structure has layers that act as convex lenses confining the optical radiation, while the latter has one layer causing convexity (i.e., guiding) but the other causing concavity (i.e., antiguiding).

After an initial discussion of the waveguide theory that pertains to nonplanar lasers, we will discuss the lateral mode character of lasers as they are influenced by lensing effects. We address how various layer thicknesses of LOC devices...
Early approaches of analyzing two-dimensional waveguides yields field solutions which accurately approximate the actual fields when the transverse dielectric variations are large. We also find that there exists a peculiarity that when the laser operates in the fundamental transverse mode, the mode sees a convex lateral index variation which, of course, tends to confine the mode. On the other hand, when the laser operates in the first-order transverse mode, that mode sees a concave lateral refractive index variation, which tends to defocus the mode.

To correlate effective dielectric variations with practical devices we present typical layer thickness variations in mode-stabilized DH and LOC structures grown on nonplanar substrates. Our particular emphasis will be on constricted-double-heterojunction (CDH) [6], [7], [13] and constricted-double-heterojunction large-optical cavity (CDH-LOC) [11], [15] lasers.

The lateral mode behavior is modeled using a hyperbolic cosine variation of the dielectric constant. This analytical model can be applied and-along to a broad number of nonplanar guides. Another useful aspect is the applicability of the model to leaky modes which might exist in index-antiguided structures. The incorporation of lateral gain in the model allows us to predict when leaky modes become trapped. (True leaky modes have lateral fields that increase without limit, while trapped modes have fields that decay to zero at infinite lateral distances.) After developing fundamental characteristics of the lateral modes found from the analytical field solutions, we apply our model to specific DH and CDH-LOC lasers. The results indicate that most practical devices lase in trapped waveguide modes whether the mode is index-guided or index-antiguided. However, we address the possibility that some CDH lasers [6] might oscillate in a leaky mode. The infinite field strength that characterizes leaky modes never develops because of the scattering of energy into waveguide radiation modes at the facets.

Finally, the radiation patterns are given for the fundamental lateral mode found from analytical field solutions. The pattern of a CDH-LOC device is measured and compared with the theoretical pattern using the device parameters pertinent to the hyperbolic cosine variation of the dielectric constant.

II. Waveguide Theory

The analysis of waveguide modes in contemporary laser structures is typically effected in an approximate manner. Early approaches of analyzing two-dimensional waveguides other than those encountered in optical fibers are to treat the two perpendicular transverse directions independently [16]. More sophisticated approximate methods have been refined to what is generally now recognized as the effective index method [17], [19]. In laser structures we define two directions: x, the direction perpendicular to the grown epitaxial layers; and y, the direction along the various grown layers and in the plane of the mirror facet. The effective index method yields field solutions which accurately approximate the actual fields when the transverse dielectric variations are large compared to the lateral ones. In stripe geometry devices, the lateral dielectric dependencies are due to the lateral gain variations which affect the imaginary part of the dielectric constant, while temperature and carrier injection [20] affect the real part of the dielectric constant (local temperature rises increase the refractive index while carriers reduce it).

The dielectric constant in the grown layers is typically invariant with respect to position. However, in active devices, the dielectric constant of the active layer (region 2 in Fig. 1) must reflect the gain distribution. Then the active-layer dielectric constant \( k_2 \) can be written as

\[
\begin{align*}
k_2(y) &= n_2^2 + ig(y) n_2/k_0 \\
&= k_{02} + k_{2y}(y)
\end{align*}
\]

where \( n_2 \) is bulk refractive index of the active layer, \( g(y) \) is the lateral gain distribution, \( k_0 \) is the free-space propagation constant \( (k_0 = 2\pi/\lambda) \), and \( k_{02} = n_2^2 \). It should be noted that when index changes such as due to temperature occur, this small variation will be included in \( k_{2y}(y) \) as a real part. To make our analysis more general in nature, we write the dielectric constant of the ith layer as

\[
k_i(y) = k_{oi} + k_{yi}(y), \quad i = 1, 2, 3, 4
\]

where \( k_{oi} \) contains only "lateral" variations.

Solutions to Maxwell's equations will be restricted to fields polarized along the junction plane and will be assumed \( \psi(x, y) = \psi(x, y) \exp(\i \omega t - \gamma z) \), where \( \gamma = \alpha/2 + i\beta \) is the complex propagation constant. The wave equation is

\[
\nabla^2 \psi + [k_2^2 \kappa(x, y) + \gamma^2] \psi = 0
\]

where \( \kappa(x, y) \) defines the complex dielectric constant of all space and \( \nabla^2 \) is the two-dimensional Laplacian operator. In the spirit of the effective index method, we write

\[
\psi(x, y) = f(x, y) g(y)
\]

where \( f(x, y) \) describes the transverse fields and its y dependence appears there because of the layer thickness variations. The functional dependence of \( f \) is determined from the \( d_i \)'s and \( k_{oi} \)'s. Substituting (4) into (3) gives

\[
gg + f \frac{\partial^2 g}{\partial y^2} + [k_3^2 \kappa(x, y) + \gamma^2] fg = 0
\]

where we have neglected derivatives of \( f \) with respect to \( y \) (typically thickness variations are relatively slow in the y directions compared to dielectric variations in the x direction). Equation (5) is now multiplied by \( f^* \) and integrated on \( x \) in the range \(-\infty, \infty\). This yields

\[
\frac{\partial^2 g}{\partial y^2} + \gamma^2 g + \left( \sum_i (k_3^2 \Gamma_i(y) k_i - \Gamma_i(y) h_i^2(y)) \right) g = 0
\]

where

\[
h_i^2(y) = -\frac{1}{f} \frac{\partial^2 f}{\partial x^2}
\]

and the overlap parameter \( \Gamma_i(y) \) satisfies

\[
\Gamma_i = \int_{\text{th layer}} |f(x, y)|^2 \, dx.
\]
The transverse functions \( f(x, y) \) are normalized to unity along \( x \) so that \( \Sigma f = 1 \). In the effective index approach a propagation constant \( \gamma_0 \), which appears in a two-dimensional waveguide model, satisfies

\[
\gamma_0^2 = k_0^2 - n^2 k_{0\ell}^2.
\]

Substituting (7) into (6) gives the differential equation describing the lateral fields

\[
\frac{\partial^2 g}{\partial y^2} + \left[ \gamma^2 - \gamma_0^2 + k_0^2 \sum_i \Gamma_i(y) k_{0\ell}(y) \right] g = 0
\]

which is equivalent to the result derived by a more rigorous method [19]. It should be noted that for layered waveguides with no ohmic losses, the effective propagation constant \( \gamma_0 = i\beta_0 \) is imaginary for all proper modes. At this point we should note that there are mainly two types of contemporary lasers considered here: 1) double-heterojunction devices which have \( d_2 = 0 \) and \( d_2 \sim 0.1-0.3 \, \mu m \), and 2) LOC structures with \( d_2 \) as above and with \( d_3 \sim 1-2 \, \mu m \). Ordinarily, the former structure supports only a fundamental transverse "proper" (i.e., trapped) mode, while the latter can support high-order proper modes. In LOC devices the relative values of \( d_2 \) and \( d_3 \) should be chosen such that only the fundamental transverse mode oscillates, and thus the radiation is emitted in a single transverse lobe.

If we limit our discussion to devices that have \( \kappa_{\ell} = 0 \) except for the active layer 2, then (8) assumes a form

\[
\frac{\partial^2 g}{\partial y^2} + \left[ \gamma^2 - \gamma_0^2 + k_0^2 \Gamma_3(y) k_{0\ell}(y) \right] g = 0
\]

where \( \Gamma_3(y) \) is the overlap or confinement factor of the mode power to the active layer. A similar equation was originally introduced by Paoli [21]. If \( n_{\ell 0} = -i\gamma_0/k_0 \) is defined as the effective index in the absence of lateral index variations, then the net lateral effective index is

\[
n_{e0}^2 = \left( \frac{i\gamma_0}{k_0} \right)^2 + \Gamma_3(y) k_{0\ell}(y).
\]

In double-heterojunction devices \( \Gamma_3(y) \) may be as low as 0.3 for narrow active layers, however, for LOC lasers \( \Gamma_3 \) might be less than 0.1.

### III. Effective Index Variations of DH and LOC Structures

In this section the lateral variations of the effective index will be discussed for a variety of nonplanar DH lasers. These calculations have been reported earlier for DH [22], [23] and LOC [24] devices with convex curvature. We will give some approximations for DH lasers and extend the results to LOC devices with concave curvature of the heterojunction boundaries.

For simple DH structures with an active-layer thickness \( d \) (Fig. 1), the effective index in terms of the normalized width \( D = k_0 d(n_2^2 - n_1^2)^{1/2} \) is [25]

\[
n_{e0}^2 = bn_{2e}^2 + (1 - b) n_{1e}^2
\]

where

\[
b = \frac{D^2}{4} \left( \frac{\sqrt{9 + 4D^2} - 1}{2D^2} \right)^2
\]

and by using the relationship between \( b, \Gamma, \) and \( D \) [27]. The approximation formula for \( b \) is accurate within 3 percent over the interval \( 0 < D < 2.5 \), which covers all cases of practical interest in AlGaAs/GaAs and InGaAsP/InP devices. Note that for InGaAsP/InP structures, where \( D \) is virtually independent of \( \lambda \) [28] (i.e., \( D \approx 6.6d \)), one obtains an approximation formula for \( b \), which is a function of active-layer thickness alone.

In Fig. 3 we show the variation of the effective index \( n_{e0} \) as a function of \( y \) for both convex (index guided) and concave (index-antiguided) AlGaAs/GaAs structures. In the convex structure, the waveguide width varies from 0.25 \( \mu m \) to 0.1 \( \mu m \), while the concave structure is of opposite variation; the refractive indexes of the various layers are \( n_1 = 3.4 \) and \( n_2 = 3.6 \). We have assumed that the thickness variation follows \( d = d_e + (d_d - d_e) \exp (-y^2/2\sigma^2) \) with \( \sigma = 10 \, \mu m \).

We now consider the waveguide parameters appropriate to the design of LOC structures. In Fig. 4 the confinement factors corresponding to the fundamental and the first-order transverse modes are shown for planar LOC structures with \( d_3 = 1 \, \mu m \). The confinement factors of these transverse modes (i.e., modes in the plane perpendicular to the junction) play roles in transverse mode selection as well as in the determination of the lateral effective index variations as indicated in (9). At \( y = 0 \), the confinement factor of the fundamental transverse mode must be larger than that of the first-order transverse mode if the device is to operate in the fundamental mode. For example, if \( d_2 = 0.2 \, \mu m \) the Al concentration of layer 3 must be larger than 10 percent, since at the 10 percent point \( \Gamma \sim 0.2 \) for both transverse modes. The refractive indexes of the Al\(_x\)Ga\(_{1-x}\)As layers are assumed to follow \( n_3 = 3.6 - 0.62x \).

A peculiarity of LOC structures is that transverse modes have lateral indexes with different functional dependencies. We will consider specifically a LOC device which is grown as a concave-lens-like structure. Fig. 5 shows the various regions...
with thickness variations as a function of position, as discussed in the next section.

It should be noted that the effects of regions 2 and 3 on the effective index oppose each other, in the sense that one produces focusing while the other defocusing. As mentioned previously, the Al content of layer 3 affects lateral mode operation. In Fig. 6 we show the lateral index variations as a function of y for the fundamental transverse mode. In Fig. 6(a), where \( x = 9.7 \) percent, the lateral wave is index-antiguided when gain and losses are not considered. On the other hand, when \( x = 11.3 \) percent, the wave will be totally index-guided. In going from concavity (\( x < 9.7 \) percent) to convexity (\( x > 11.3 \) percent) the intermediate \( x \) values produce the so-called \( W \)-shaped waveguide [29]. In Fig. 7 we show the lateral index variations as seen by the first-order transverse mode; note that this mode remains index-antiguided for all Al values listed. For \( x = 11.3 \) percent the fundamental transverse mode is laterally index-guided, while the first-order transverse mode in index-antiguided. Similar lateral index variations can be obtained while varying the layers' thicknesses at a fixed Al concentration [15], [29]. For instance, by increasing the active-layer thickness to 0.25-0.3 \( \mu \)m, CDH-LOC devices have been shown [15], [29] to become totally index-guided for the fundamental transverse mode.

IV. LATERAL WAVE CONFINING STRUCTURES GROWN ON NONPLANAR SUBSTRATES

As discussed in the previous section, confinement of the optical mode can be realized by inducing thickness variations of the waveguiding layers along the plane of the junction. Such local variations in thickness can be generally achieved while depositing material by liquid-phase epitaxy (LPE) over nonplanar substrates [1]-[15]. We show in Fig. 8 three types of laser structures that can be obtained by LPE over topographical features etched into the substrate. The grown layers have variations in thickness due to two reasons: 1) a strong dependence of LPE growth on local surface curvature [2], [3], [30], [31], and 2) LPE-growth sensitivity to the degree of substrate misorientation [3], [30], [31].

Fig. 8 displays a DH-type device: the "ridge-guide" CDH laser [13], [31] [Fig. 8(a)], and two LOC-type devices: the nonplanar LOC laser [10] [Fig. 8(b)] and the CDH-LOC laser [11] [Fig. 8(c)]. In the DH device, as in most DH nonplanar-substrate devices [1]-[4], [6]-[8], [12]-[14], [32], the active-layer thickness variation solely determines lateral wave
confinement. By contrast, in LOC-type devices both the thickness variations of the active and guide layer play a role in mode confinement and selection. The nonplanar LOC structure [10] has convex-lens-shaped active and guide layers as a result of growth over a channel. A more complex situation is the CDH-LOC structure for which growth is performed above the mesa separating a pair of channels. Growth above the mesa is slow compared to growth in the planar confinement. By contrast, in LOC-type devices both the thick- (14) index values at the center and edges of the distribution, respectively, and $y_0$ is a lateral displacement characteristic of the index lateral variation. (The values $n_5$ and $n_6$ may be complex.) It is interesting to note when $n_5$ and $n_6$ are real that if $\Delta k = n_5 - n_6 > 0$, proper modes (trapped) exist, but if $\Delta k < 0$, the fundamental mode as well as high-order modes are leaky. Our primary interest here is to understand both trapped and leaky modes in concave structures, as these devices have excellent power output and spectral character. Assuming that $\kappa_{o2} = 0$ and substituting (13) into (9), we have
\[
\frac{\partial^2 g}{\partial x^2} + \left[ k_3^2 \kappa_b + \frac{k_5^2 \Delta k \kappa_b}{\cosh^2(y/y_0)} + \gamma^2 \right] g = 0 \tag{14}
\]
where $\kappa(y)$ is the effective dielectric constant. The solution to (14) is
\[
\xi_s(y) = \cosh^{-b_2} \left( \frac{y}{y_0} \right) C^{b_2 + \epsilon/2} \left( \tanh \frac{y}{y_0} \right),
\]
where $b_2 = b_0 - s$ and $b_0 = (k_2 d^2 \Delta k + \frac{1}{4})^{1/2} - \frac{1}{4}$ and $C^{b_2}(\xi)$ are Gegenbauer polynomials, the first two of which are
\[
C^0_s(\xi) = 1 \tag{16a}
\]
and
\[
C^1_s(\xi) = 2 \lambda \xi. \tag{16b}
\]
The real value of $b_2 = b_0 - s$ determines whether the $s$ mode is proper or improper (leaky). Putting $b_2 = b_2' + i b_2''$, the boundary that separates proper and improper modes occurs when $b_2' = 0$. From (15) it is seen that if $b_2' < 0$ the mode field strength increases without limit at large lateral distances, whereas $b_2' > 0$ gives $|\xi_s(y)| < 0$ as $y \rightarrow \infty$. The boundary defined by $b_2' = 0$ is satisfied by [35]
\[
\Delta' + \Delta'' = \frac{16 \pi (s + 1)(2s + 1)^2 - (2k_2\gamma_0 \Delta''^2)^2}{4(2s + 1)^2 (2k_2\gamma_0 \Delta''^2)^2} \tag{17}
\]
where $\Delta k = \Delta' + i \Delta''$. A similar result has been derived for
Fig. 8. (a) Nonplanar DH laser structure obtained by one-step liquid-phase-epitaxy: "ridge-guide" constricted double-heterojunction (CDH) laser [13], [31], nonplanar LOC laser structures obtained by one-step liquid-phase epitaxy, (b) nonplanar large-optical cavity (NP-LOC) laser [10], (c) constricted-double-heterojunction large-optical-cavity (CDH-LOC) laser [11].

step index variations [36]. Fig. 9 shows regions where the lateral modes are proper or leaky in terms of the complex dielectric step $\Delta \kappa$ and the width $2y_0$. The propagation constant is

$$\gamma^2 = -k_0^2 [\kappa_0 + b_2^2/(k_0 y_0)^3].$$  \hspace{1cm} (18)

If we neglect waveguide losses, index-antiguided modes have negative $\Delta \kappa$ values. The magnitude of the negative step influences both astigmatism and attenuation of the mode. Consider the fundamental $s = 0$ mode when $-1/4 < k_0^2 y_0^2 \Delta \kappa < 0$, $b_0$ is real, and thus there is no phase variation of the field along the facet. Astigmatic leaky modes occur only when $k_0^2 y_0^2 \Delta \kappa < -1/4$. In most instances the attenuation of leaky modes is a direct reflection on power loss in the lateral directions so that in active devices a mode having a zero net attenuation occurs when power supplied by the active regions offsets the losses.

Let us now turn to waveguides in laser structures which have gain/losses. The lateral profile must be modified to include the dielectric variations occurring in the active layer. From (9) the profile is

$$k(y) = k_0(y) + \Gamma_2(y) \kappa(y).$$  \hspace{1cm} (19)

Neglecting temperature and strain effects

$$\kappa_2(0) = \Gamma_2(0) n_2/k_0$$ \hspace{1cm} (20a)

and

$$\kappa_2(\infty) = -i\alpha_2 n_2/k_0$$ \hspace{1cm} (20b)

where it is assumed that $g(0)$ is the gain at $y = 0$ and $\alpha_2 = -g(\infty)$ is the absorption constant of layer 2 at large lateral distances. In the unpumped active layer, the absorption coefficient $\alpha_2 \approx 100 - 500$ cm$^{-1}$. If we make the ad hoc assumption that $\Gamma_2(y) \kappa_2(y)$ has a lateral dependence similar to $k_0(y)$, then $k(y)$ can be approximated to exhibit the hyperbolic cosine variation. The dielectric step becomes

$$\Delta \kappa = n_2^2 - n_0^2 + i n_2 [\Gamma_2(0) g(0) + \Gamma_2(\infty) \alpha_2]/k_0$$  \hspace{1cm} (21)
the stripe contact. Other planar active-region devices such as
in the channel-substrate-planar laser [5] have only trapped lateral
modes which lase; the structure is designed so that the real
part of the lateral dielectric step is positive. In the CDH laser
discussed here, the structure is fabricated with heterojunctions
forming a concave waveguide [6] that has an active region
thickness of \(a = 0.16 \mu m\) at the lasing spot \((y = 0)\) and
\(d_s = 0.28 \mu m\) at large lateral distances, defined in our formu-
lation as \(y = \infty\). Using the Al concentrations in the various
layers, we have \(n_a = 3.4866\) and \(n_p = 3.5213\); calculations
give \(\Gamma(0) = 0.445\), \(\Gamma(\infty) = 0.71\), \(\Delta' = 0.243\), and \(\Delta'' = 
\left(4.54 J_{eff}/24\right) \times 10^{-4}\), where the gain distribution is es-
imated from the effective current density which is almost uni-
form over the waveguide vicinity. Assuming a value \(y_0 = 6 \mu m\),
the condition for a proper mode \(\Delta'' = (\Delta')^{1/2}/k_0 y_0^n\)
(crossing from the curve bounding the cross-hatched region of
Fig. 9) gives \(J_{eff} = 20 \text{ kA/cm}^2\) which corresponds to gain
values \(g(0) \approx 5000 \text{ cm}^{-1}\). Thus, it is not possible for this
particular leaky device to reach the condition of a fundamental
proper mode. On the other hand, using \(18\), the leaky mode
propagating in the guide will reach threshold (assuming end
losses \(\sim 30 \text{ cm}^{-1}\)) at \(g(0) \approx 600 \text{ cm}^{-1}\) and \(g(\infty) \approx 260 \text{ cm}^{-1}\).
at these threshold gain values \(b_0 = -0.33 + i22\) which would
respond to a very astigmatic mode; for \(b_0\) the real part of \(b_0\)
means the intensity increases without limit at large \(y\) values.
However, experimental results showed that such devices lased
with a very narrow spot \((\sim 2 \mu m)\) at the facet. There are two
possible explanations for the discrepancy between our calcula-
tions and the experimental observations: 1) in the waveguide
region at \(y = 0\) the actual guide was relatively flat so that local
omnic heating in the region \(|y| < 2 \mu m\) produced an index
increment, thus introducing a new waveguide parameter \(y_0\)
(the resulting structure would in fact be a W-guide), and 2) the
possibility of “almost” total internal reflection of the mode at
the waveguide facet. The problem with the first explanation is
that the near field spread should be much larger than \(2 \mu m\)
and, in fact, the laser should behave more like a stripe geometry
device. The possibility of total internal reflection is more
likely because of the large \(b_0\) values which are associated with the
ray directions in the active layer. In particular, at large \(y\) values
\(g(y) \sim e^{-b_0 y/\gamma} s = e^{-b_0 y/\gamma} s e^{i\beta_0 y/\gamma}. \)

The two plane wave components in region 2 which form the
optical mode with respect to the \(x\) direction, are propagating
at an angle with respect to the \(z\) axis. Thus, the net propa-
gation vector \(s \hat{\beta}_y + \hat{\beta}_x + \hat{\beta}_z\) of a single “plane wave” in the
active layer has \(\beta_0 = k_0(n_a^2 - n_p^2 \gamma)1/2\), \(\beta_x = b_0^x y_0^n\), and
\(\beta_z = \text{Im}(\gamma), \text{ determined from } 18\). Combining the above
components, rays in the active layer region with \(d_s = 0.28 \mu m\)
propagate at an angle (with respect to the \(z\) axis) \(\theta \sim 14.2^\circ\),
which is near the critical angle for total internal reflection at
the facet. Further, if these rays escape, it is doubtful that they
entered the microscope. This explains why no radiation leak-
age was observed for the “leaky-guide” CDH device [6]. Only
radiation propagating along the \(z\) axis could be detected, and
that provided a relatively narrow, centered beam (\(\theta_0 \approx 20^\circ\)).
As the width \(d_s\) drops from \(0.28 \mu m\), the value of \(\beta_z\) increases
and the corresponding ray angles increase. But near \(y = 0\) the

with
\[
\Delta' = n_a^2 - n_p^2 \tag{22a}
\]

and
\[
\Delta'' = n_2 \left[\Gamma_3(0) g(0) + \Gamma_1(\infty) \alpha_s\right]/k_0. \tag{22b}
\]

In (22b) the total value of the imaginary dielectric step is the
sum of two positive terms because the gain is opposite in
sign to the absorption so that the effective gain changes from
\(\Gamma_3(0) g(0)\) to \(-\Gamma_1(\infty) \alpha_s\). In some devices the current distribu-
tion might tend to be uniform in the region containing the
dielectric waveguide so that the gain in the active layer varies
according to the thickness of the active layer \(d_s\). If the current
has inconsequential lateral variations along the lateral direc-
tions, the gain \(g(\text{cm}^{-1})\) is written as [37] (neglecting lateral
carrier diffusion)
\[
g = 45(J_{eff}/d_s) \sim 190
\]
where \(J_{eff} = \eta J\) and \(\eta\) is the internal efficiency, \(J(\text{mA/cm}^2)\) is
the injection current density, and \(d_s\) is dimensioned in
microns. Now the imaginary part of the dielectric step becomes
\[
\Delta'' = n_2 \left[\Gamma_3(0) g(0) - \Gamma_1(\infty) g(\infty)\right]/k_0. \tag{22c}
\]

Devices exhibiting steps of the form (22c) are commonly
found in constricted double-heterojunction structures unless
tight current confinement is incorporated in the laser design.

VI. DEVICE CALCULATIONS

Waveguide parameters will now be calculated for index-
antiguized structures (CDH and CDH-LOC lasers) to illustrate
the utility of the model.


It is well known that stripe geometry devices have modes
supported by index antinwaveguides, and that the modes are
essentially trapped because of the lateral gain distribution.
The modes are not leaky, due to the fact that the optical field
in the active layer is confined more or less to the region below
the stripe contact. Other planar active-region devices such as

Fig. 9. Cutoff curves for the lateral modes as a function of the complex
dielectric step and waveguide “width” \(y_0\). In the cross-hatched region
the fundamental mode is leaky or improper.
plane waves change directions, thus increasing the possibility for light escaping from the facet.

B. CDH-LOC Lasers

The geometry of CDH-LOC devices is given in Fig. 5; we will use the structure with 9.7 percent Al in layer 3. This is an index-antiguided structure that can operate in a high-order transverse mode, but the fundamental is assumed. At \( y = 0 \), 
\[ d_3 = 0.2 \mu m \text{ and } d_4 = 1.3 \mu m \]  
whereas at large \( y \), 
\[ d_3 = 0.15 \mu m \text{ and } d_4 = 2.5 \mu m \]. The confinement factors are \( \Gamma_3(0) = 0.157 \) and \( \Gamma_4(\infty) = 0.023 \). The refractive indexes \( n_4 = 3.5363 \) and \( n_3 = 3.5373 \) are used to get \( \Delta' = -0.007 \). The value \( y_0 = 5.3 \mu m \) is estimated from lateral points where the refractive index changes go through one half of their total change, i.e., if \( y_0 \) represents the point of half variation, \( y_0 = 0.759 y_n \).

Assuming a uniform current distribution over the lasing portion of the waveguide, we find for a trapped or proper mode to exist, \( J_{eff} = -2.5 \) kA/cm\(^2\) which corresponds to \( g(0) = 370 \) cm\(^{-1}\) and \( g(\infty) = 560 \) cm\(^{-1}\). Even though \( g(\infty) > g(0) \), the effective gains are modified by the confinement factors of the various lateral points. At \( J_{eff} = 2.5 \) kA/cm\(^2\), the fundamental mode attenuation coefficient \( \alpha_0 = 13 \) cm\(^{-1}\), but at threshold, \( \alpha_0 = 30 \) cm\(^{-1}\) and \( J_{eff} = 3 \) kA/cm\(^2\), which give \( g(0) = 485 \) cm\(^{-1}\) and \( g(\infty) = 710 \) cm\(^{-1}\), and \( b_0 = 0.16 + 13.4 \). At these small \( b''_o \) values, the rays in the active layer are determined predominantly by \( \beta_x \) and \( \beta_y \).

As a final example, we consider a CDH-LOC laser [15] that supports index-guided modes. This structure has \( n_1 = 3.418 \), \( n_2 = 3.6 \), \( n_3 = 3.502 \), and \( n_4 = 3.44 \) and \( d_2 \sim 0.27 \mu m \), \( d_3 \sim 1.8 \mu m \) at \( y = 0 \) and \( d_4 \sim 2.1 \mu m \) at \( y = 10 \). Computations give \( n_3 = 3.5297 \), \( n_4 = 3.5153 \), \( \Gamma(0) = 0.6157 \) and \( \Gamma(\infty) = 0.4282 \). From geometrical estimates, \( y_n \sim 7 \mu m \) giving \( y_0 \sim 5.3 \mu m \). Again assuming a uniform current distribution, threshold occurs when \( g(0) \sim 60 \) cm\(^{-1}\) and \( g(\infty) \sim 150 \) cm\(^{-1}\). At these gain values, \( b_0 = 12.3 - i 0.076 \).

Actually, the gain distribution in this structure plays little or no role in the lateral mode shape. Nevertheless, the large charge density differential between the points \( y = 0 \) and \( y = 10 \) \( \mu m \) will cause a relatively large diffusion current directed toward \( y = 0 \). It is interesting to note that our model holds for cases such as when the gain is suppressed at \( y = 0 \) and increases with \( y \); this is opposite to the CDH structure discussed above.

VII. Radiation Patterns

The optical radiation pattern can be determined once the near field distribution is known. The most elementary approach and the one given here is to calculate the Fourier transform of the near field mode. This method is reasonably accurate for proper modes where there is little internal mode conversion at the facet. In the case of the "leaky" CDH device [6] discussed earlier, portions of the fundamental mode in the regions \( y > y_0 \) might see a very high reflection coefficient at the facet. Thus, little light would escape from the device, with most coming from the region around \( y = 0 \). Under these conditions, much of the reflected light would be distributed in the waveguide radiation modes and be dissipated internally.

Since the near field is separated into \( f(x, y) \) and \( g(y) \) via the effective index method, the far field can be similarly divided into independent perpendicular angle dependencies. We concentrate here on the lateral pattern \( F(\theta) \) given by

\[
F(\theta) = \int_{-\infty}^{\infty} g(y) e^{ixy} dy \tag{24}
\]

where \( x = k_0 \sin \theta \) and \( \theta \) defines the lateral angle from the facet normal. (The obliquity factor has been dropped.) For the sake of brevity, we consider only the fundamental mode pattern in the lateral direction. Using \( g(y) = \cosh^{-2b_2} (y/y_0) \)

\[
F(\theta) = \frac{2b_0^{-3} y_0^{3} \Gamma(b_0/2 + i\pi/2)}{\Gamma(b_0)} \tag{25}
\]

where \( \Gamma(w) \) is the gamma function [not the confinement factor in (6), (9), (12a), and (19)]. Contour and relief maps [38] of the gamma function with complex arguments can be used to analyze the behavior of the far field patterns. The gamma function \( \Gamma(w) \) has a pole at \( w = 0 \). If we put \( w = u + iv \) with \( u \) constant and vary \( v \), the gamma function peaks at \( v = 0 \) and varies monotonically from \( v = 0 \). In the far field pattern of the fundamental mode, the two gamma functions actually form different peaks at symmetrical positions about \( k_0 \sin \theta = 0 \). If the value \( b_0'' \) is sufficiently small the two peaks combine to form a single peak at \( \theta = 0 \). But in the case when the two peaks can be separated and \( b_0'' \) is large, peaks occur at

\[
\sin \theta = \pm \frac{b_0''}{k_0 y_0} \tag{26}
\]

As the value of \( b_0'' \to 0 \) from the positive side, the two peaks increase without limit. The value of \( b_0'' \) is dependent upon the dielectric step \( \Delta x \) and \( k_0 y_0 \). If \( \Re[\Delta x] \) is positive and large, \( b_0'' \sim 0 \), but if \( \Re[\Delta x] \) is large and negative, then for \( k_0 y_0 \gg 0 \), \( b_0'' \sim k_0 y_0 (n_3^2 - n_4^2)^{1/2} \) so that far field peaks occur at

\[
\sin \theta = \pm (n_3^2 - n_4^2)^{1/2} \tag{27}
\]

which allows us to estimate the index step for index-antiguided modes. For large \( b_0'' \) values, Stirling's approximation for the gamma function can be employed.
To illustrate the use of (25), we calculate the radiation pattern of the index-guided CDH-LOC [15] discussed earlier. Using the index calculation, Fig. 10 shows the half-power beamwidth of the fundamental lateral mode as a function of $2\gamma_0$. At $2\gamma_0 \sim 14 \mu m$ we find that $\theta_y \sim 8^\circ$, which is in good agreement with the experimental patterns. In Fig. 11 we plot both theory and experiment.

VIII. CONCLUSION

The effective index method has been applied to both CDH and CDH-LOC lasers to analyze their lateral mode behavior. The LOC devices have shown that lateral mode behavior is strongly influenced by transverse mode operation when the optical cavity contains AlGaAs layers with relatively large aluminum concentrations. This occurs in geometries where the optical cavity increases in thickness laterally. In devices with relatively small active layer thicknesses all transverse modes are index-antiguided in the lateral direction.

Lateral modes are analyzed using the smoothly varying hyperbolic cosine distribution. The effects of the active region gain are included in the model to indicate when leaky waveguide modes become proper.

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REFERENCES


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Dan Botez (S'71-M'79), for a photograph and biography, see p. 870 of the May 1982 issue of this JOURNAL.
Prediction of transverse-mode selection in double heterojunction lasers by an ambipolar excess carrier diffusion solution

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Transverse-mode selection is characterized for GaAs/AlGaAs double heterojunction lasers from optical field and electron/hole interaction. The electron/hole distribution determined from a solution of the ambipolar diffusion equation provides the necessary information about gain/mode coupling to predict the current at threshold. Lasing power out versus current solutions provide information about internal differential quantum efficiency. Theory is matched to experiment for a multimode laser with one heterojunction having a very small index step. It is found that the laser's characteristics over a temperature and current range are predicted by adjusting the active-layer refractive index as determined from far-field measurements.

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I. INTRODUCTION

In recent years, there has been considerable effort spent fabricating various semiconductor laser-device geometries for control of mode operation. In GaAs/AlGaAs devices, the transverse-mode operation is usually governed by the growth of various layer thicknesses and index steps at the grown heterojunctions. Double heterojunction lasers which confine the optical field to the thin active layers usually operate in the fundamental mode. Furthermore, the lasing mode is independent of drive because small index changes due to injection have negligible effects on mode selection. While transverse-mode lasing can be well controlled by epitaxial heterojunctions, lateral-mode operation in contemporary lasers is drive sensitive. Stripe-contact devices have lateral guides defined only by the current density which affects the gain distribution while more sophisticated structures have grown layers whose thicknesses are functions of lateral positions. Thus, in these devices the lateral modes are shaped by the lateral effective index as well as gain variations.

Because the gain distribution plays a major role in defining mode stability in lasers, it is important to understand the mechanisms of electron/hole transport in active layers. In this paper, we discuss the transverse-mode operation of lasers which are affected by the electron/hole distribution. For the first time the ambipolar diffusion equation is solved for the active layer in an injection laser. The direct consequence of this solution is the ability to predict crossover of competing modes in the active layer. In contrast to ambipolar diffusion, assuming simple electron injection does not explain this phenomena in sufficient detail.

In this work we investigate mode selection from the standpoint of the exact selection process. This includes mode gain coupling and differential quantum efficiency. We concentrate on the transverse (perpendicular to the junction plane) modes in a double heterojunction laser. The excess carrier distribution is assumed ambipolar and a complete solution is obtained. From mode/gain coupling, it is possible to predict the threshold, and from the slope efficiency, it is possible to predict mode crossover.

The laser we apply this to is a broad-area double heterojunction GaAs device with a small index step at one of the heterojunctions. The laser is heated over a temperature range and the first two modes are investigated. It is found that cavity-mode preference is very sensitive to temperature and current level. Application of the diffusion equation is used to describe the crossover of the first two modes and it is established that the mode with the highest slope efficiency will dominate. Far-field patterns are described by standard multilayer waveguide techniques. Finally, a calculation is made to estimate the effect of free carriers on the refractive index in the material.

II. THEORY

A. Approach

Excess carrier movement in the active layer of an injection laser is described using the ambipolar diffusion equation. Quasi-Fermi level continuity as well as electron and hole current continuity is imposed at each interface. The quasi-Fermi level location is calculated using a nonparabolic extremum, and parabolic extremum. The boundary condition is set up in such a way as to eliminate the need for a solution within the space charge layer surrounding each interface. The spontaneous recombination lifetime is calculated without imposing quasimomentum conservation for parabolic bands. Above threshold, a stimulated recombination term is included in the ambipolar diffusion equation and the gain is assumed to be pinned at the threshold value. The lasing modal shape for the transverse modes (perpendicular to the metallurgical junctions) is calculated by solving Maxwell's equations for a multilayer dielectric wave guide.

B. The excess carrier distribution

The diffusion equation evolves along the standard route. We demand, as is customary, that each carrier type
the internal field in such a way as to satisfy Poisson's equation and must then diffuse at a natural mean recombination time. We further demand that Poisson's equation be satisfied:

$$\nabla E = (4\pi \rho /\epsilon),$$

where $\rho$ is the space charge density and $\epsilon$ is the dielectric constant.

Rather than solve Eqs. (1) and (2) exactly, we take an approximate route where we assume that the internal field between the electron and hole charge densities is strong enough to guarantee charge neutrality. Although this condition is violated in the space charge layer surrounding the heterojunctions, we can formulate the problem in such a way that only diffusion processes outside the heterojunctions need be considered. Under this assumption, the excess electron and hole densities, $\delta n$ and $\delta p$, respectively, must be equal:

$$\delta n = n - n_0 = p - p_0 = \delta p.$$

Here, $n_0, p_0$ designate the thermal-equilibrium carrier densities. If we take $(\partial \delta n/\partial x) - (\partial \delta p/\partial x) = 0$, we can substitute Eq. (3) into Eq. (1) to obtain two equations for the two unknowns $\delta p$ and $E$. The terms involving $\nabla E$ can be eliminated by multiplying Eq. (1a) by $\mu_n D_n$ and Eq. (1b) by $\mu_p D_p$ and adding to obtain

$$D^* \nabla^2 (\delta p) - \mu^* E \nabla (\delta p) + g' - \frac{\delta p}{\tau} = \frac{\delta p}{\partial t},$$

where

$$\delta p = \text{excess carrier density},$$

$$D^* = \frac{\mu_n D_n + \mu_p D_p}{\mu_n + \mu_p},$$

$$\mu^* = \frac{\mu_n \mu_p (n_0 - p_0)}{\mu_n + \mu_p},$$

$$g' = g'' = g,'$$

$$\frac{\delta p}{\tau} = \frac{p_0 + \delta n - p_0}{\tau},$$

$$\frac{\delta p}{\tau} = \frac{n_0 + \delta n - n_0}{\tau},$$

$$g' = g'' = \text{excess carrier generation rates},$$

$$\tau = \text{excess carrier recombination lifetime}.$$  

We have implicitly assumed that the net recombination rate ($\delta p/\tau$) is the same for electrons as for holes, as it must be. We have further assumed that the equilibrium generation rate is equal to the equilibrium recombination rate for both electrons and holes.

For the ambipolar diffusion Eq. (4), observe that the ambipolar diffusion rate for excess carriers is determined by the average value of $D_n$ and $D_p$ weighted by $\mu_n$ and $\mu_p$. In other words, the two oppositely charged clouds interact with each other through the internal field in such a way as to satisfy Poisson's equation and must then diffuse at a naturally acceptable rate. This is crucial because of the very large injected charge densities encountered in injection lasers ($-2 \times 10^{18}$ cm$^{-3}$).

The electric field in Eq. (4) refers to both the internal and applied fields. Under the conditions encountered in an injection laser, these fields exert only a small direct influence on carrier distribution, and as such can be ignored in Eq. (4).

In other words, the electric field does not alter the actual excess carrier spatial distribution, but rather only causes carriers to drift. This is the case if the drift length is much less than the diffusion length, i.e.,

$$\mu^* E r < (D^* \tau)^{1/2}. (5)$$

This condition is met in the solutions ultimately obtained. The major effect of the internal field, that of charge neutrality, is still accounted.

If we assume the only excess carrier drift, other than spontaneous recombination $\delta p/\tau$, is stimulated recombination, then for an infinitely long cavity and time-averaged values

$$r = \frac{\delta p}{\partial t} = \frac{1}{\eta} \frac{g P(x)}{\eta}.$$  

where $P(x)$ is time-averaged power density, $g = $ gain, $\eta = $ stimulated quantum efficiency, and $\eta = $ photon energy. For 100% carrier confinement, $\eta = $ internal differential quantum efficiency. In these calculations we put $\eta = 1.$

Equation (4) becomes

$$D^* \nabla^2 (\delta p) - \frac{\delta p}{\partial t} = r,$$

where $D^* = $ ambipolar diffusion constant, $r = $ spontaneous recombination lifetime, and $\tau = $ stimulated recombination rate. The value $\tau$ is equated to zero in the region where $g < 0.$

The spontaneous recombination lifetime is

$$\tau = 1/[B (n_0 + p_0 + \delta p)].$$

where $B = 1.3 \times 10^{-10}$ cm$^3$/sec.$^{14,12}$

C. The boundary conditions

At each boundary within the device there exists a space charge layer with a width severely reduced under conditions of forward bias. Because quasicharge neutrality is violated within this layer, we match across the layer under the assumptions it is thin enough and internal fields are strong enough that carriers are swept through without significant recombination. Hence, we demand that the electron and hole current densities be the same on each side of the space charge layer. These densities are written

$$J_n = q \mu_n E + q D_n \nabla (\delta p),$$

$$J_p = q \mu_p E - q D_p \nabla (\delta p).$$

Here, $\overline{E}$ is the total field and $q$ is the electron charge. Although $\overline{E}$ can be ignored in Eq. (7), where it has a small effect, it must be included in the boundary conditions because both drift and diffusion terms are significant.

In addition to continuity of current densities, we also impose continuity of hole and electron quasi-Fermi levels across each layer interface. Although it is clear this alignment of Fermi levels implies band bending, i.e., space charge, we assume an abrupt heterojunction and ignore such effects.
D. The electromagnetic field distribution

The standard multilayer slab dielectric waveguide model is used to describe the transverse far-field patterns.\textsuperscript{13,14} Standard calculations yield propagation constants \( \gamma = \alpha + j \beta \) and fields. For instance, for a TE mode, we can express the \( m \)-th mode field distribution for the \( i \)-th layer as

\[
E_m^i(x) = E_m^i \cos(h_m x + \phi_m),
\]

where \( h_m \) is the complex eigenvalue and \( x \) is the coordinate across the active region. The overall field solution is obtained by matching the field components at each interface. The far-field pattern then becomes proportional to the Fourier transform of the near field on a lasing facet.

E. The diffusion-equation solution

Assume a three-layer device with the central layer active (see Fig. 3). Also, assume the mode is propagating down the \( z \) axis with \( x \) perpendicular to the guide. The excess carrier diffusion equations for this case become

\[
\frac{D}{dx^2} - \frac{\delta p}{(L_*)^2} = 0, \quad i = 1,3,
\]

\[
\frac{D}{dx^2} - \frac{\delta p}{(L_2*)^2} = g(x)P_m(x), \quad i = 2,
\]

where \( L_* = (D^*)^{1/2} \), \( g(x) \) is the localized gain and \( P_m(x) \) is the optical Poynting vector component along the propagation direction.

These equations are linearized by replacing \( g(x) \) with its weighted average within the active region\textsuperscript{10}

\[
g = \frac{\int_{-\infty}^{\infty} g(x)E_m^2(x) \, dx}{\int_{-\infty}^{\infty} E_m^2(x) \, dx} = \frac{g_m N_m}{T_m},
\]

where \( N_m \) is the field normalization constant and \( T_m \) is the mode intensity integrated over the active layer. It should be noted that \( g(x) = 0 \) outside the active region. Further, assuming no losses in layers external to the active region, \( g_m \) represents the propagation gain of the \( m \)-th mode. With \( D^* \) and \( r \) assumed to be piecewise constant layer-to-layer, Eq. (11b) becomes

\[
\frac{D}{dx^2} - \frac{\delta p}{(L_2*)^2} = \beta_m \cos^2(h_m x + \phi_m),
\]

where

\[
\beta_m = \frac{g_m P_m E_m^2}{D^* T_m \eta \nu},
\]

\[
P_m = \int_{-\infty}^{\infty} P_m(x) \, dx.
\]

Ignoring the minor effect of the layer 3-4 interface on the carrier distribution, the solution to the diffusion equation in the various layers becomes

\[
\delta p_1(x) = G_1 \cos(h_1 x/L_1^*) + F_1 \sinh(x/L_1^*),
\]

\[
\delta p_2 = G_1 \cosh(x/L_1^*) + F_2 \sinh(x/L_1^*) - \beta_m (L_2^*)^2/2 \left( 1 + \frac{\cos^2(h_2^* x + \phi_2^*)}{1 - (2h_2^* L_2^*)^2} \right),
\]

\[
\delta p_3(x) = G_1 \cosh(x/L_1^*) + F_1 \sinh(x/L_1^*).
\]

With these eigenfunctions, the solution follows directly from matching the current density and quasi-Fermi levels at each interface. The Fermi-level continuity condition is applied using Kane's nonparabolicity for the \( E_{\text{FC}} \) minimum and a parabolic expression for both \( E_{\text{FC}} \) and the \( X_{\text{FC}} \) minima. The effect of aluminum concentration on carrier mobility, effective mass, and band gap is included. The volume recombinaction rate is related to the optical gain of the laser in linear fashion using coefficients obtained by integrating over parabolic bands without quasimomentum conservation. The recombinaction lifetime is found in a similar manner. The diffusion coefficients are calculated using the generalized Einstein equation. The complete formulation of the problem is presented elsewhere.\textsuperscript{15} The necessary roots are calculated on a computer using a real-root searching routine. The final solution provides current-voltage curves, power output-cur- rent curves, threshold current density of each mode, and finally electron and hole injection efficiencies into the active layer.

F. Validation of numerical results

This theory has been applied to double heterojunction lasers, single heterojunction lasers, and separate electron- and hole-injection lasers. Close agreement has been found with experimental values. Two typical results for a double heterojunction laser are shown in Figs. 1 and 2. The first shows threshold current density as the active region thickness is reduced, and the second shows threshold as a function of the aluminum concentration in the passive regions. The slope of Fig. 1 is 5.2 kA/cm\textsuperscript{2}/\mu m, a value which compares closely with available data.\textsuperscript{16,17}

![Graph](image-url)

**FIG. 1.** Calculated hole injection efficiency, relative peak power, and threshold current density of a symmetric DH laser as a function of cavity width. The active layer doping is \( p \times 10^{13} \) cm\textsuperscript{-2} and the passive n-layer doping is \( 10^{14} \) cm\textsuperscript{-2}. The passive p layer has a 5 \times 10^{18} cm\textsuperscript{-3} doping level. The AlAs percentage in the outer layers is 20%.
FIG. 2. Calculated threshold current density as a function of AlAs percentage in the GaAlAs passive p layer of a symmetric DH laser with a 1-μm wide GaAs (p-10^{18} cm^{-3}) active layer. Three passive p-layer doping levels are shown. The passive n-layer doping level is 1 x 10^{18} cm^{-3}.

III. EXPERIMENT

The lasers characterized in this experiment are broad-area double heterojunction lasers grown by liquid phase epitaxy methods. Figure 3 gives the geometry of one such laser, PL174-1-31. Other lasers of this type were characterized, but PL174-1-31 was selected for its transverse modal behavior with temperature due to the small refractive index step at one heterojunction. Estimating the refractive index of the aluminum layers is accomplished using experimental data^18,19 at a lasing wavelength \( \lambda = 9000 \) Å. This gives \( \Delta n = 0.62x \), where \( x \) is the fraction of aluminum in the solid. The laser has a cross section of 214 x 356 μm, where the latter is the cavity length. The chip thickness is 77 μm. The laser is mounted p-side up on the header and has no reflective coating on the facets.

The device is driven with 250-nsec pulses with a 0.01% duty cycle to minimize internal heating. While driven, the laser is heated over temperature range of 20–40 °C. The temperature/current-drive characteristics of the laser are characterized by its far- and near-field dependence.

To measure the far field, the laser is placed on a rotating platter with an axis turned by a 2-rph timing motor. The laser's output is collected by monochromator and amplified by an RCA 7102 photomultiplier tube (PMT). Typical measurement parameters are 2-Å slit width and 750-V bias for the PMT. The output of the PMT is fed to a lock-in amplifier triggered by a reference signal from the pulser. An HP 7560 A log converter and HP 2470 A x 10 dc multiplier process the signal for the y axis of a chart recorder. An impedance matching network allows the y axis to be calibrated to three decibels per inch deflection. The calibration is good for over five decades of response. The speed of the x axis, combined with the 2-rph timing motor presents 24° rotation per inch deflection for the x axis. An alternative presentation is to load the PMT with a 50-Ω terminator and display the output pulse on a sampling oscilloscope with a 50-Ω input. This provides a time decomposition of the light-output pulse for ready comparison to the current-input pulse. Patterns were typically made with \( J = 1.1J_{\text{th}} \).

The measurement scheme for the near field is accomplished by attaching the laser mount to a micropositioner which has a three-dimensional adjustment. A microscope objective with 160 magnification is mounted opposite the laser facet and a video camera collects the output of the objective lens and displays the image on a monitor.

Peak threshold current values for laser PL174-1-31 are shown in Fig. 4. Triangles denote the fundamental mode whereas crosses mark the second. In the heat-sink temperature range 20–28 °C, only the second mode is present; for 28–42 °C, both fundamental and second modes are present; and above 42 °C, only the fundamental mode propagates. The lasing wavelengths for the two modes are shown in Fig. 5.

The multilayer waveguide model is used to describe the transverse far-field patterns of the modes. Figure 6 is an example of a match of the second mode at 20 °C. The solid curve is the measured pattern, and the dashed curve is the match. The bump on one shoulder at 21.6° of Fig. 6 is the result of light coupling to the lossy substrate. This bump can be accurately positioned to match the experimental pattern by applying Snell's law at the substrate-air interface. The internal angle from the normal to the facet \( \theta' \) is given by \( \tan \theta' = \tan \theta = h_x \gamma / \beta \gamma \eta_s \sin \theta \). Here, \( h_x \) is the imaginary part of \( h_x \), the real propagation constant \( \beta = \text{Im}(\gamma) \), \( \theta \) is the external angle from the normal to the facet, and \( \eta_s \) is the refractive index of the substrate. The relative intensity or amplitude of the bump is adjusted by the separating width \( d_2 \).

\[ \theta_{mp} \]

\[ \text{transverse direction } x \]

FIG. 3. Geometry of PL174-1-31. For the excess carrier calculation, only layers 1, 2, and 3 are used.
is $\delta n = -0.002$. The fundamental mode also is marked by this refractive index decrease over the temperature range. The half-power full-width of the fundamental transverse mode $\theta$, decreased over 28-42 °C in a manner consistent with the $\delta n$ calculated for the second mode. The second-mode lobe-separation angle $\theta_{pp}$ was chosen for calculations for better accuracy.

An interesting feature of this laser is the cavity selection of modes at an intermediate temperature of 38 °C, where Fig. 4 suggests both lower-order modes may exist during a current pulse. In Figs. 8(d) and 8(e) the far field is filtered with the monochromator and the output of the photomultiplier is fed into the sampling scope with a 50-Ω termination. The top pulse of each picture is the current pulse while the lower is the optical pulse of the laser. It was established during the experiment, from output pulses as in Fig. 8 and from near-field pictures, that the laser switched modes during the current pulse. The discussion to follow explains this multimode switching with current magnitude at an intermediate temperature. Theoretical results accompanying the experiment.
show the cavity selection is sensitive to gain/mode profile coupling and index change with excess carrier injection.

IV. DISCUSSION

The data from Fig. 3 were used with the data from Table I for a multilayer waveguide analysis of the transverse modes. This method was used first to check the accuracy of material parameters. In Fig. 6, the dashed line shows a close fit with the solid measured curve of the far field. The pip on one shoulder is a leaky wave coupling to the substrate layer. As previously mentioned, Snell's law dictates a fixed refractive index for the substrate. It was noticed in the experiment that the pip remained stationary in angular displacement with temperature change 20-40 °C and with current drive. Also, the pip remains about 4 dB below the main lobes over temperature and current variations. This means the thickness and refractive index of layer 3 in Fig. 2 should be fixed in order to hold the pip stationary. The only feature to change for the second mode is the angular separation of the major lobes designated as $\theta_m$. These data are represented in Fig. 7 and will be discussed subsequently.

The only way to match the $\theta_m$ change with temperature is to slightly decrease the active-layer refractive index with increased current drive. The increase in threshold current of a mode with temperature occurs due to a modal confinement change, internal quantum-efficiency decrease, or a carrier-lifetime or absorption-coefficient change. Near-field measurements did not show a shift of the transverse mode with temperature. The refractive-index change of each layer with temperature $\delta n/\delta T$ can be shown to be $1.5 \times 10^{-4}$/°K. For a twenty-degree range, $20-40 °C$, $\delta n \approx 0.003$. On the other hand, the index change due to dispersion for a layer is $\delta n/\delta E \approx 0.8$ eV. For $\delta E = 20 \AA$ from Fig. 5 (the first high-order mode), it can be shown that $\delta n \approx -0.003/\AA$.

The dispersion and heating effects on refractive index of each layer tend to cancel. Therefore, with the evidence that the near field remains constant with temperature, the pip on the shoulder of the far field remains stationary and dispersion and heating effects tend to cancel, it is assumed that the dominant refractive-index change is that of the active layer.

| TABLE I. Typical values for the physical constants used in the ambipolar diffusion solution. |
|---|---|---|
| Layer 1 | Layer 2 | Layer 3 |
| Doping | $p \times 10^{-18}$ | $p \times 5 \times 10^{-17}$ | $n \times 5 \times 10^{-17}$ |
| Al concentration | 20% | 0 | 2% |
| $\eta$ | 3.48 | variable | 3.5865 |
| Absorption | 5 | 20.8 | 2.4 |
| Width | ... | $2.65 \times 10^{-4}$ | ... |
| $\mu_n$ | 3132 | 3649 | 3649 |
| $\mu_p$ | 138 | 167 | 167 |
| $D_e$ | 81.0 | 198 | 128 |
| $D_p$ | 3.67 | 4.65 | 4.33 |
| $D_e$ | $7.69 \times 10^{-9}$ | $3.39 \times 10^{-9}$ | $8.46 \times 10^{-9}$ |
| $D_p$ | 80.6 | 15.6 | 6.8 |
| $L^*$ | $7.87 \times 10^{-4}$ | $2.37 \times 10^{-4}$ | $2.4 \times 10^{-4}$ |

*All quantities are in cgs units, with the exception of mobility which is in the mixed units cm$^2$/V sec and absorption which is in cm$^{-1}$. 

FIG. 8. Time display of input laser current and optical output. In (a) an input current pulse is drawn. $J_{th1}$ and $J_{th2}$ are the threshold current densities of the first two modes, respectively. In (b), optical flux $\Phi_1$ at the fundamental mode wavelength is shown to exist in regions I and III. In (c), the second mode optical flux $\Phi_2$ is detected in region II at the second mode wavelength. In (d), the experimental output pulse is shown in the lower trace at the fundamental mode wavelength. The top pulse is the input current. In (e), the lower trace is the second-mode output at its wavelength.
due to injection level. The data of Tables I and II and Fig. 3 are used to find the solution to the ambipolar diffusion equation. A rise in temperature is represented by a reduction in refractive index as required in matching the far-field pattern of Fig. 6.

With the relevant material parameters listed in Fig. 3, a solution was found for the ambipolar diffusion equation. In Fig. 9 the threshold current density for the first two modes is plotted as a function of the active-region refractive index. For larger \( \eta_2 \) values (corresponding to a larger index step) the second mode has the lowest threshold due to superior coupling to the excess carrier distribution. But as the active-region index is lowered and the second mode approaches cutoff, its coupling efficiency diminishes and the fundamental mode emerges with the lowest threshold.

Typical material parameters calculated (fundamental mode at \( \eta_2 = 3.5948 \)) are shown in Table I. Since many of the parameters depend on injection levels, the quantities in Table I vary slightly over the graphs presented. With these calculated values, the usefulness of the ambipolar equation can be shown for the active layer. The calculated hole-injection efficiency is 99.4% and the electron-injection efficiency is 86% for this case. This calculation predicts a crossover point at a slightly lower \( \eta_2 \) value than the multilayer model range in Fig. 7. This is because in the multilayer model, the presence of the substrate with a relatively high refractive index increases the active-region eigenvalue. For instance, if the buffer-layer thickness \( d_b \) were decreased, the active-layer eigenvalue would shift even more.

<table>
<thead>
<tr>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold current density (kA/cm²)</td>
<td>15.58</td>
</tr>
<tr>
<td>Modal absorption (cm⁻¹)</td>
<td>19.74</td>
</tr>
<tr>
<td>Internal differential quantum efficiency (%)</td>
<td>66.8</td>
</tr>
</tbody>
</table>

Figure 9 is useful in interpreting the phenomena shown in Fig. 4. The fundamental mode does not occur below 30°C. At 20°C, the second mode has a threshold current density of about 12 kA/cm². However, abruptly above 30°C, the fundamental mode appears. From Fig. 8, a slight change in \( \eta_2 \) on the order of \( \delta \eta_2 = -0.002 \) is sufficient to cause a crossover in threshold currents between the two modes. Both Figs. 4 and 9 show that the second-mode threshold increases sharply after the crossover. Threshold current densities predicted by the ambipolar solution are in fair agreement with experimental values.

Modal output power as a function of current density is plotted in Fig. 10. Because the linear theory of gain is assumed, only one mode at a time is calculated. At power levels above threshold, the modal gain is assumed pinned at its threshold value. Note that the second mode, despite higher threshold current density and reduced coupling efficiency, has a higher internal differential quantum efficiency than the fundamental mode. Thus, for a sufficient overdrive beyond threshold, the second mode will eventually be the more efficient overall and will dominate. An explanation for this higher internal differential quantum efficiency can be seen in Table II. The modal absorption of the second mode is low because a significant portion of its energy is propagating outside the laser active region whereas most of the fundamental mode is well confined to the active region. Since modal loss dominates slope efficiency, the second mode has the higher value. This higher value, in conjunction with a higher threshold due to poorer coupling efficiency, causes the mode efficiency crossover point.

With these results, the phenomena of mode crossover in this laser can be understood. At 20°C the second mode has the lower threshold and higher efficiency and consequently only the second mode appears. At an intermediate temperature, the second mode will appear at high-enough current overdrive due to better slope efficiency. Above 42°C, the threshold for the second mode becomes prohibitive. Figure 8 depicts a time resolution between the two modes at an intermediate temperature. Figure 8(a) represents an input drive pulse. The two threshold current densities for the two modes are labeled \( J_1 \) and \( J_2 \). As the current pulse rises above \( J_1 \), the first mode turns on in region I. Output flux \( \Phi_1 \) for the first mode is represented in Fig. 8(b). As the input current progresses to \( J_2 \), region II is entered. The second mode turns on, as shown in Fig. 8(c), as \( \Phi_2 \). Finally, with the drop of input current, region III is entered and first mode reappears. As shown in Figs. 8(d) and 8(e), this situation is observed experimentally. In Fig. 8(d), the monochromator is set on the first-mode wavelength and the lower pulse is the output power. Reviewing the two output pulses, it can be seen the two modes have a time separation during a current pulse. This is explained by the fact that, from Fig. 9, the second-mode threshold can still be reached slightly after crossover. From Fig. 10, the second mode clearly is predicted to dominate the first mode if threshold can be reached, due to better efficiency.

Theoretical mode and excess carrier coupling for the laser is shown in Fig. 11, where the excess carrier profile is plotted across the active layer for the two modes operating at...
FIG. 10. Ambipolar diffusion solution. Output power is plotted against threshold current density for the first two modes. The slight curvature in the mode 1 curve is caused by carrier redistribution. The index \(\eta_2 = 3.5948\).

two power-output levels (threshold and 0.67 w/mil.). Included on the carrier plot is the normalized modal distribution. Several factors should be noted. First, the second-mode distribution is clearly penetrating the passive \(n\) layer, indicating a near cutoff situation and a modal loss dominated by the \(n\)-layer absorption. With all curves normalized, a simple product of \(\delta \eta\) and \(E^2\), integrated over the active region, shows the fundamental mode better coupled than the second, as expected. Second, it can be seen that the carrier profile peaks on the side where holes are injected. This profile contradicts the notion that electron injection dominates and the peak occurs on the \(n\) side. Since the charge cloud is ambipolar, it must accommodate the low-hole mobility to maintain quasi-charge neutrality. Hence, the profile is skewed toward the hole injection or \(p\) side of the active layer. As expected, carrier profiles change shape with current density.

As a final note, we use the refractive-index dependency of the active region on temperature and hence current density to estimate the free-carrier effect. The influence of free carriers on the refractive index in a material is determined from the classical theory of dispersion. The refractive index in the presence of free carriers \(\eta\) is related to the refractive index in the absence of carriers \(\tilde{\eta}\) by

\[
\eta^2 = \tilde{\eta}^2 \left(1 - \frac{4\pi Nq^2}{m^*\omega^2\tilde{\eta}^2}\right). \tag{15}
\]

Here, \(m^*\) is the effective mass of the carrier, \(q\) is the charge of an electron, \(N\) is the carrier density, \(\omega = 2\pi c/\lambda\), and \(\epsilon_0\) is the vacuum permittivity. Since the change in index is much smaller than \(\tilde{\eta}\), Eq. (15) can be rearranged to

\[
\delta \eta_\text{le} = \frac{2\pi Nq^2}{m^*\omega^2\tilde{\eta}^2}. \tag{16}
\]

In other words, the change in index due to free carriers is directly proportional to the concentration of free carriers \(N\), where \(K\) is a constant we wish to determine. By plotting refractive-index change versus carrier increase, the constant \(K\) would be the slope at a particular point

\[
d (\delta \eta_\text{le}) = K d (N) \tag{17}
\]

or

\[
K = \frac{d (\delta \eta_\text{le})}{d (N)}. \tag{19}
\]

A change in a particular parameter, say lifetime \(\tau\), with injection would be reflected in Eq. (17) having a nonzero second derivative. We take \(\delta \eta_\text{le}\) from Fig. 7 and conclude that this index change is very nearly a linear function of temperature. However, we find from Fig. 4, carrier concentration \(N\), related to the peak threshold current, is not quite a linear function with temperature. Therefore, under the assumption the free-carrier effect is the dominant cause of observed pattern change, the constant \(K\) would be precisely

\[
K = \frac{d (N_j)/dT}{d (N)/dT}. \tag{18}
\]

This expression contains the information that \(K\) is not a true "constant" but is itself a slight function of injection level. Nevertheless, it is instructive to estimate \(K\) using Eq. (19) to compare with conventional theory. Using

\[N = \frac{J\tau}{qd_d}\]

and a straight-line approximation to \(d (N)\) over the temperature range 20–42 °C we get

\[K = -3.1 \times 10^{-21} \text{ cm}^3, \quad \tau = 3 \text{ nsec}\]

and

\[K = -6.2 \times 10^{-21} \text{ cm}^3, \quad \tau = 2 \text{ nsec}\]
These results can be compared to \( K = -4 \times 10^{-21} \text{ cm}^3 \) from Thompson\(^2\) and \( K = -(5-11) \times 10^{-21} \text{ cm}^3 \) from Selway.\(^3\) Thompson used \( \tau = 3 \) nsec for his calculation. Several other values for \( K \) have recently been reported. Olsson and Tang\(^4\) report \( K = -(4.9 \pm 0.4) \times 10^{-21} \) for \( \tau = 2.2 \) nsec. Henry et al.\(^5\) report \( K = -(1.8 \pm 0.4) \times 10^{-21} \) for lasers with carrier densities estimated at \( 1.02 \times 10^{18} \text{ cm}^{-3} \).

V. CONCLUSION

A solution for the ambipolar diffusion was presented for the active region of a double heterojunction laser. Along with the solution of modes from Maxwell's equations in a multilayer waveguide, this combined description was applied to a broad-area heterojunction laser. The refractive-index change of the active region with temperature was estimated from far-field pattern matching of the transverse modes. Transverse-mode-selection conditions were predicted with the diffusion equation and compared with the transverse-mode selection of the sample laser.

Mode threshold was shown to depend on coupling efficiency between excess carriers and modal distribution in the active layer. The differential internal quantum efficiency depends primarily on mode absorption and injection efficiency. Mode crossover occurs when the mode with the highest threshold also has the highest slope efficiency.

Carrier transport in a semiconductor laser is an ambipolar diffusion process which can be described using the relaxation time approximation. In bulk regions, quasi-charge
neutrality is satisfied because of the strong internal field between electrons and holes. The active region of a semiconductor laser undergoes double injection: hole injection from the $p$ side and electron injection from the $n$ side. In devices with good injection efficiencies, the injected carrier profile is higher at the $p$-$p$, rather than at the $p$-$n$, interface because of low hole mobility.

The device modeled here shows a distinct peak of carriers on the $p$-$p$ interface. The slope of the carrier profile increases with increased current density. Since these carrier profiles demonstrate significant skewing, it is clear that this can affect mode selection if the modal shape couples poorly with the carrier concentration.

ACKNOWLEDGMENTS

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Abstract

A numerical method and the effective-index method are applied to a three-layer, constant thickness dielectric waveguide with smoothly varying dielectric constant inside the active layer and constant permittivity in the confining layers. The results of the two methods are compared in terms of the propagation constant calculated by each method. Application of the effective-index method facilitates a physical understanding of dielectric waveguide modes as well as providing an efficient approximate method for calculating mode behavior.

*Supported by the U.S. Army Research Office.

Introduction

Several papers [1-3] have analyzed mode propagation in dielectric waveguides with spatially varying refractive index, usually approximated by parabolic or tanh² functions, which go to infinity at large distances from the reference point x = 0. We will use the approximation [4]

\[ \kappa = \kappa_0 + \delta \cosh^2 \left( \frac{x}{x_0} \right) \]  

(1)

to describe the variation of \( \kappa \). This is in closer correspondence with the physical situation and leads to equations with known solutions. The disadvantage in this case is the fact that the field solutions consist of a finite (possibly empty) set of confined (trapped) modes, an infinite set of diverging "leaky" modes and a continuum of solutions that will be designated as "radiation" modes.

The 2-dimensional character of the problem due to the variation of the refractive index in the lateral (x) and transverse (y) directions requires the use of approximate methods or numerical solutions. Among the former, the effective index method is the most popular ([2], [4] - [8]). Direct numerical integration is possible, but the computation time required is much larger than that needed for other methods like the one used in [1], which we will apply in this paper.

The following sections describe the class of waveguides considered, the numerical method and the application of the effective-index method to our problem, ending with conclusions.

Description of Structures

Figure 1 shows the structure considered in this paper. The confining layers A and C are assumed identical, their refractive indices being described by

\[ \kappa_A = \kappa_C = \kappa_0 - i \delta \kappa_A / \kappa_0 \]  

(2)

\( \delta \kappa_A \) describes the power loss in these layers and is constant with distance. The active layer has a constant thickness d, with a refractive index

\[ \kappa(x) = \kappa^n(x) - i \delta(x) n(x) / \kappa_0 \]  

(3)

whose dependence with x is considered to be reasonably well-approximated by (1). In that equation,

\[ \Delta \kappa = \kappa_0 - \kappa_A \]  

(4a)

\[ \kappa_0 = \kappa(0) = \frac{\kappa_0^2}{\kappa_0^2 + i \delta_0 \kappa_0 / \kappa_0} \]  

(4b)

\[ \kappa_A = \kappa_s^2 - i a_s \delta_0 / \kappa_0 \]  

(4c)

\( \kappa_0 \) is a parameter related to the width of the stripe in the case of a semiconductor laser. The values of power attenuation coefficient and refractive index inside the active region far away from the stripe are \( \kappa_0 \) and \( \kappa_A \), respectively. The quantity \( \delta_0 \) represents the peak power gain under the stripe (\( x = 0 \)), where the refractive index is \( \kappa_0 \). For \( \delta_0 = \kappa_0^2 - \kappa_A > 0 \) the mode will be index-guided, whereas for \( \delta_0 < 0 \) it will be index anti-guided, and this latter effect can eventually offset the guiding effect of the gain distribution.

Numerical Solution

We follow the method used in [1]. Maxwell's equations are solved for both the active and the confining layers, subject to boundary conditions that the general solutions inside and outside the active layer and their normal derivatives match at the interfaces \( y = \pm d/2 \). We also demand that these solutions vanish at \( x = \pm \infty, y = \pm \infty \). Application of the method of separation of variables results in a vertical field solution inside the active region of the form

\[ \phi(y) = \left[ \sin \left( \frac{\sin \theta y}{\cos \theta y} \right) \right] \]  

(5)

and a differential equation

\[ \frac{d^2 \phi}{dx^2} + \left[ k_0^2 \kappa_s^2 - q^2 + k_0^2 \frac{\delta_0}{\cosh^2 \left( \frac{x}{x_0} \right)} \right] \phi = 0 \]  

(6)

where \( q \) is the separation constant. Equation (6) possesses solutions of the form

\[ \psi_{\kappa}(x) = \left[ \cos \left( \frac{\cos \theta x}{\cos \theta x_0} \right) \right] C_{\kappa}^{\theta}(\tanh \left[ x / x_0 \right]) \]  

(7)

where \( C_{\kappa}^{\theta}(x) \) are Gegenbauer polynomials [9] and

\[ b_0 = - k_0^2 \left( k_0^2 + \kappa_0^2 \right) \frac{\delta_0}{\kappa_0^2} \]  

(8)

To satisfy the boundary conditions we require

\[ \text{Re}(b_0 - \kappa) > 0 \]  

(9)

For \( \text{Re}(b_0) > 0 \), the fundamental mode \(( \kappa = 0 \) is trapped. Modes of order \( \kappa \) such that \( \text{Re}(b_0 - \kappa) < 0 \) are still solutions of (6) but diverge as \( |x| \to \infty \) and will be designated as "leaky". Radiation modes would be described by other solutions of (6) for arbitrary (non-integer) eigenvalues [9]. The general solution will be a linear combination of the few discrete trapped
modes (9) satisfying (9), plus an integral over the continuum, and the leaky modes must be excluded if the field has to vanish for \( |x| = \infty \). The importance of the continuum can be expected to decrease as the number of trapped modes increases. Since this number is relatively large for structures with modal gains that are high and not very sensitive to the refractive index step \( n_0 \neq n_g \). This continuum will be neglected. Hence, the general even solution inside the active region is approximated by

\[
\psi_I(x, y) = \begin{cases} 
\sum_{l=0,2,\ldots} \sqrt{q_0} \gamma_l(x) \cos(q_l y) & \text{if } y \leq d/2 \\
0 & \text{if } y > d/2
\end{cases}
\]

where \( \sqrt{q_0} \) is the maximum even value of \( q \) for which \( \gamma_l(x) \) is confined. Solution of the wave equation for the confining layers proceeds as in [1]. Matching the functions and their derivatives with respect to \( y \) at the boundary \( y = d/2 \) and applying the orthogonality relation for the trapped modes results in a finite system of homogeneous linear equations

\[
(A - I)Q = 0
\]

where \( A^T = (A_0, A_2, \ldots, A_T) \), \( I \) is the unit matrix, \( Q \) is the column vector of \( q_0, q_2, \ldots, q_T \), and the matrix elements of \( A \) are given by

\[
A_{i,j} = \begin{cases} 
\cos(q_j d/2) & \text{if } i = j \neq 0  \\
\gamma_i(x) \sin(q_j d/2) - \gamma_j(x) \sin(q_i d/2) & \text{if } i = j = 0  \\
\gamma_i(x) \cos(q_j d/2) - \gamma_j(x) \cos(q_i d/2) & \text{if } i = j \\
0 & \text{otherwise}
\end{cases}
\]

where \( q_j(x) \) being the normalized Fourier cosine transform of \( \psi(x) \), and \( i, j = 0, 2, \ldots, T_{\text{max}} \). The \( q_k \) satisfy

\[
\frac{d^2 q_k}{dx^2} + \left[ \gamma^2 + k_0^2 - q_k^2 \right] q_k = 0
\]

The system of equations (11) has a non-trivial solution only if \( \det(A-T) = 0 \). Numerical computation of the roots yields the possible values of the propagation constant \( \gamma \).

**Numerical Results**

The method was applied to a structure described by the following parameters:

\[
\begin{align*}
n_A &= 3.38, \\
n_S &= 50 \text{ cm}^{-1}, \\
n_0 &= 50 \text{ cm}^{-1}, \\
n_g &= 200 \text{ cm}^{-1}, \\
n_0 &= 3.595, \\
\gamma &= 6 \text{ um}.
\end{align*}
\]

Direct solution of (11) involves computation of the \( A_{i,j} \). In general, \( A_{i,j} = A_{j,i} \), but \( I_{i,i} = T_{i,i} \). Values of the modal loss \( \text{Re}(\gamma) \) were computed by evaluating \( I_{i,i} \) using the exact expression (13). This has to be done once for each value of \( \gamma \). Instead of solving the complete system (11), we start with a 2x2 matrix, go on to a 2x2 matrix, etc., and observe that the result converges relatively fast for a 2x2 case, which is considered sufficient for our approximation. In spite of this, the computation time is still impractically high. An increase in speed by nearly two orders of magnitude while still maintaining good accuracy is achieved expanding the radical in (13) using the binomial theorem (which results in integrals that do not depend on \( \gamma \)) and retaining the first three terms.

Figure 2 shows plots of the modal loss \( \text{Re}(\gamma) \) as a function of \( \gamma \) with \( n_0 \), the gain under the stripe, as a parameter. We notice that, for each value of \( n_0 \), \( \gamma \) can be decreased up to a certain value beyond which the fundamental mode becomes leaky.

**Effective-Index Calculation**

The effective-index method consists basically of reducing a two-dimensional problem to an equivalent one-dimensional one. In our case, the two-dimensional character of the problem is given by the dependence of the dielectric constant on \( x \) and \( y \). As a first approximation, the variation in one direction (in our case, \( x \)) is neglected; this is justified if this variation is much less than that in the \( y \) direction. This is equivalent to approximating the waveguide with a simple 3-layer guide whose dielectric constants do not vary with \( x \). The solution of this problem yields the transverse or vertical variation of the field. Next, the original equation describing the 2-dimensional equation in \( x \) can be solved for the lateral variation of the field, and the overall solution is approximated by the product of this lateral field and the vertical field found from the 3-layer problem. We start with the wave equation in 2 dimensions:

\[
\frac{d^2 \psi(x,y)}{dx^2} + \left[ \gamma^2 + k_0^2 - q_k^2 \right] \psi(x,y) = 0
\]

For the simple 3-layer guide we assume \( \psi(x) = \psi_0 \) inside the active layer. Now, we transform \( \psi(x,y) = \psi(x) \psi(y) \), multiplying it by \( \psi(x) \) and integrating it over \( y \) from \( -d/2 \) to \( d/2 \), and obtain

\[
\frac{d^2 \psi_0}{dx^2} + \left[ \gamma^2 - q_k^2 \right] \psi_0 = 0
\]

with

\[
\begin{align*}
\Gamma &- \Gamma', \\
\Gamma' &- \Gamma
\end{align*}
\]

where \( \Gamma \) is the filling or confinement factor.

Equation (16) has the same form as eq. (6). It will also have polynomial solutions similar to (7) that represent trapped modes:

\[
\psi_k(x) = [\text{cosh}(k_0 x_0)] \gamma_k (\tanh \frac{x}{x_0})
\]

\( i = 0, 1, 2, \ldots \)

where \( b_0 \) is defined as in (8) with \( A \) replaced by \( \Gamma \) \( A \) and \( p \) is the eigenvalue of the simple 1-layer problem for the fundamental mode, \( i = 0 \), and we obtain

\[
\gamma^2 - k_0^2 r_A - (\Gamma_0 b_0^2 k_0^2 - (p^2 - (\gamma_0 b_0^2 \Gamma_0) k_0^2)) = 0
\]

for the propagation constant.

**Discussion**

Values of \( \gamma \) obtained using (19) are compared with those obtained with the numerical method in fig. 3. Solutions are very close for all values of \( \Delta n \) for which the mode exhibits a gain which is relatively high and low sensitivity to \( \Delta n \). The results differ most in the range of \( \Delta n \) for which the mode has a net loss or has a relatively low gain with higher sensitivity to \( \Delta n \).

Figure 4 gives the required value of the peak power gain \( n_0 \) under the stripe to obtain a given modal gain for a function of \( \Delta n \), using the effective-index method.
Also included is the region for which the fundamental mode becomes leaky. The vertical confinement factor \( r \) did not vary appreciably with \( \Delta n \); a typical value for the case considered was \( r = 0.4963 \). We see that lateral variations in the refractive index affect the gain much more by altering the lateral field distribution than by affecting the vertical variation.

The effective-index method is seen to be a fast and relatively accurate way to obtain the field distributions for the class of waveguides considered, for which the numerical method we used is not practical for extensive modeling due to its long computation time in spite of all approximations. The remarkable agreement between the effective-index method and experimental results found by other workers \((10)\) increases our confidence in this powerful approximate method.

References


Figures

1. Waveguide structure considered

2. Modal loss as a function of \( \Delta n \) with peak power gain \( R_0 \) as a parameter. Regions at left of vertical lines correspond to leaky fundamental mode.

3. Modal loss \( \text{Re}(\gamma) \) as a function of \( \Delta n \) for numeric (3x1 matrix) and effective-index methods.

4. Peak power gain under contact stripe (\( R_0 \)) as a function of \( \Delta n \) with modal gain as a parameter. Shaded area shows region corresponding to leaky fundamental mode.
Modal Solutions of Active Dielectric Waveguides by Approximate Methods

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Abstract—Approximate methods are used to obtain the modal properties of stripe-contact semiconductor injection lasers using a planar three-layer waveguide model. The central active layer has a dielectric constant that varies smoothly along the direction parallel to the heterojunction boundaries. The complex dielectric constant under the stripe contact is dependent on the gain and approaches a constant value at large lateral distances. The two methods are compared in terms of their modal propagation constants. An application of the effective index method facilitates a physical understanding of dielectric waveguide modes as well as providing an efficient calculation procedure.

I. INTRODUCTION

A NALYSIS OF mode propagation in dielectric waveguides with a spatially varying refractive index has been the subject of several papers [1]–[3]. Typically, the variation of the dielectric constant with distance has been approximated with a parabolic profile [1], [2] or a function of the form $\kappa = \kappa_0 + \kappa_1 \tanh^2(x/x_0)$ [3]. Both approximations have the disadvantage that the value of $\kappa$ goes to infinity at large distances from the point $x = 0$, which corresponds to the axis of lateral symmetry of the structure. In the case of a semiconductor laser, this corresponds to the region below the center of the contact stripe. Another approximation that eliminates this disadvantage is the use of a function of the form [4]

$$\kappa = \kappa_0 + \Delta \kappa / \cosh^2(x/x_0)$$

(1)

In describing the variation of $\kappa$, this is in closer correspondence with the physical situation, since $\kappa$ now acquires the value $\kappa_0$ for $x \gg x_0$. Even if this particular form of variation of $\kappa$ does not describe the actual variation very closely, it retains the most important features, and leads to equations with known solutions. The disadvantage in this case is the fact that the field solutions consist of a finite (possibly empty) set of confined trapped modes, an infinite set of discrete, diverging “leaky” modes, and a continuum of solutions that will be designated as “radiation” modes, as opposed to an infinite set of discrete trapped modes only, as in the parabolic and $\tanh^2(x/x_0)$ profiles. Mode analysis is a two-dimensional problem, since the refractive index varies in both lateral ($x$) and transverse ($y$) directions. Therefore, numerical or approximate methods need to be applied. The most popular and effective approximation method is the “effective-index” solution, whereby the two-dimensional problem is reduced to an equivalent, one-dimensional one [2], [4]–[8]. Numerical methods have also been developed. For example, in [1] the parabolic variation is used. Maxwell’s equations are solved both for the active layer and the confining layers, and then superposition is applied to both types of solutions to form a general expression for the field. These solutions and their derivatives are matched at the boundaries of the active layer, yielding an infinite system of linear homogeneous equations, whose solutions, numerically obtained, are the expansion coefficients for the mode in terms of the eigenfunctions of the active layer problem. Of course, direct numerical integration of the two-dimensional wave equation is possible, but the computation times are long compared to those required by the algorithm discussed in this paper.

For the type of variation considered here, a general field in the active layer must be expressed as a superposition of the few confined discrete modes plus an integral over the continuum. Leaky modes cannot be included in the expansion if the field is to decrease to zero for large distances from the stripe.

Direct application of the numerical method used in [1] results in a finite set of linear equations (due to the finite number of trapped modes) coupled with an integral equation (due to integral over the continuum). For the case in which only one trapped mode exists (the fundamental mode), an integral equation results, which can in principle be solved. However, these cases will be seen to correspond to structures with net modal loss or low gains very sensitive...
Leaky modes ($l > l_{\text{max}}$) cannot be used when expressing a general confined field as a superposition of modes. This results in a finite discrete eigenvalue spectrum, and the need arises for a continuum in order to have a complete orthogonal set of functions. We will seek an approximate solution neglecting the continuum. For the trapped modes, the product solutions will be of the form

$$\psi_i(x) \cos(q_y y)$$

and the general solution will be approximated by

$$\Psi_h(x, y) = \sum_{l=0}^{l_{\text{max}}} A_l \psi_l(x) \cos(q_y y)$$

for a field even in $x$, where $l_{\text{max}}$ is the maximum even value of $l$ for which $\psi_i$ is confined. The trapped modes satisfy the orthogonality relation

$$\frac{1}{x_0} \int_{-x_0}^{x_0} \psi_i(x) \psi_j(x) \, dx = N_l \delta_{l,l'}.$$  

(26)

The first few normalization constants are

$$N_0 = \frac{\sqrt{\pi} \Gamma(b_0)}{\Gamma(b_0 + \frac{1}{2})}$$

(27a)

$$N_1 = \frac{2(b_0 - \frac{1}{2})^{1/2}}{(b_0 - 1)} \frac{\sqrt{\pi} \Gamma(b_0)}{\Gamma(b_0 + \frac{1}{2})}$$

(27b)

$$N_2 = \frac{2\sqrt{\pi} (b_0 - 3/2)^{1/2}}{(b_0 - \frac{3}{2})} \frac{\Gamma(b_0)}{\Gamma(b_0 + \frac{1}{2})}$$

(27c)

where $\Gamma(z)$ is the factorial function. An analytic but lengthy expression for the $N_l$ exists but is not given here.

Solution of (8) for the confining layers proceeds as in [1]. Separation of variables is applied to (8); the lateral solution is of the form $\cos(q_y y)$ whereas the vertical solution is a decreasing exponential. Superposition is then applied; the results in an integral since no boundary conditions are available that would result in a discrete spectrum. The general solution will be

$$\Psi_v(x, y) = \int_{\gamma}^{\gamma} B(x) \cos(q_y y)$$

$$\exp \left[ (x^2 - \gamma^2 - k^2 y^2)^{1/2} / 2 \right] \, dx$$

(28)

where

$$k^2 = k_0^2 \kappa_A.$$  

Now, (25) and (28) and their normal derivatives have to be matched at $y = \pm d/2$, the boundaries of the active layer. Matching $\Psi_v$ and $\Psi_h$ results in an expression for $B(x)$

$$B(x) = \frac{2}{\pi} \sum_{l=0,2, \ldots}^{l_{\text{max}}} A_l \sqrt{N_l} \cos \left( q_d^d / 2 \right) \psi_l(x)$$

(29a)

where

$$\psi_l(x) = \int_{0}^{\infty} \frac{\psi_i(x)}{\sqrt{N_l}} \cos(q_y y) \, dx.$$  

(29b)

Matching the derivatives respect to $y$ at $y = d/2$ and applying the orthogonality relation (26) yields, after using (29a)

$$\frac{4}{\pi x_0} \sum_{l=0,2, \ldots}^{l_{\text{max}}} A_l \sqrt{N_l} \cos \left( q_d^d / 2 \right) I_{l', l}(y)$$

$$= A_{l'} q_d^d \sin \left( q_d^d / 2 \right) \sqrt{N_l}$$

(30)

with $l, l' = 0, 2, \ldots, l_{\text{max}}$ and

$$I_{l', l}(y) = \int_{0}^{\infty} \tilde{\psi}_l(x) \tilde{\psi}_l(x) (x^2 - \gamma^2 - k^2 y^2)^{1/2} \, dx$$

(31)

$$\tilde{\psi}_l(x) = \frac{1}{\sqrt{N_l}} \sum_{m=0}^{l} (-1)^m \frac{\Gamma(b_0 - l - 1)}{\Gamma(b_0 - l + m) \Gamma(b_0 + l - 1 - m) \Gamma(b_0 + m)}$$

$$\sum_{k=0}^{l - m} \left( \frac{1}{2} + m - k \right) k! \left( \frac{1}{2} + m - k \right) k!$$

(32)

where

$$r(x, \gamma, x_0) = \frac{2^{l+2} \pi x_0 \Gamma \left( \lambda + \frac{2 x_0}{2} \right) \Gamma \left( \lambda - \frac{x_0}{2} \right)}{\Gamma (\lambda)}. \quad (33)$$

This is a finite system of linear homogeneous equations of the form

$$(\Omega^T - I) \bar{x} = 0$$

(34)

where $\bar{x} = (A_0, A_2, \ldots, A_{l_{\text{max}}})$. $I$ is the unit matrix, and the matrix elements of $\Omega$ are given by

$$\Omega_{l', l}(y) = \frac{4 \cos \left( q_d^d / 2 \right) \sqrt{N_l}}{\pi x_0 q_d^d \sin \left( q_d^d / 2 \right) \sqrt{N_l}} I_{l', l}(y).$$

(35)

The $q_i$ satisfy, from (12a) and (12b)

$$q_i^2 = \gamma^2 + k_0^2 \kappa_A + \frac{(b_0 - l)^2}{x_0^2}.$$  

(36)

The system of equations (34) has a nontrivial solution only if det($\Omega^T - I$) = 0. Numerical computation of the roots yields the possible values of the propagation constant $\gamma$.

IV. NUMERICAL RESULTS

The method was applied to a structure described by the following parameters:

$$n_A = 3.38 \quad \quad g_0 = 200 \text{ cm}^{-1} \quad \quad n_0 = 3.595 \quad \quad \alpha_A = 50 \text{ cm}^{-1} \quad \quad \alpha_S = 50 \text{ cm}^{-1} \quad \quad x_0 = 6 \mu \text{m}.$$  

Direct solution of (34) involves computation of the $\Omega_{l', l}$. In general, $\Omega_{l'} = \Omega_{l', l}$, but $I_{l'} = I_{l'}$. Values of the modal loss at $\text{Re(}\gamma)$ were computed first by evaluating $I_{l', l}(y)$ using the exact expression (31). This required one computation of this integral for every value of $\gamma$.
boundaries $y = \pm d/2$, and demand that $\Psi_z$ and $\Psi_y$ decrease to zero as $x \to \pm \infty$, and also that $\Psi_z \to 0$ as $|y| \to \infty$. These boundary conditions are homogeneous, so we can apply separation of variables.

First we consider the solution inside the active layer making

$$\Psi_z(x, y) = \phi(x) \psi(y).$$

Substitution in (7) results in a vertical solution of the form

$$\psi(y) = \left( \frac{\sin qy}{\cos qy} \right)$$

and a differential equation

$$\frac{d^2 \psi}{dx^2} + \left[ k_0^2 \kappa_5 + \gamma^2 - q^2 + k_0^2 \frac{\Delta \kappa}{\cosh^2(x/x_0)} \right] \psi = 0$$

(10b)

where $q$ is the separation constant. The substitutions

$$\xi = \tanh(x/x_0)$$

(11a)

$$\psi = (1 - \xi^2)^{-1/2} W(\xi)$$

(11b)

$$b_0(x_0 + 1) + k_0^2 x_0^2 \Delta \kappa$$

(12a)

$$B^2 = (q^2 - \gamma^2 - k_0^2 \kappa_5) x_0^2$$

(12b)

in (10b) result in

$$(1 - \xi^2) \psi'' - (2\lambda + 1) \xi \psi' + \alpha(\alpha + 2\lambda) W = 0$$

(13)

with

$$\lambda = B + \frac{1}{2}$$

(14a)

$$\alpha = b_0 - \lambda + \frac{1}{2}.$$  

(14b)

Several solutions exist for (13). When $\alpha$ is an integer $l = 0, 1, 2, \ldots$, we have polynomial solutions, which are the ultraspherical or Gegenbauer polynomials $C_l(\xi)$. Physically, they give rise to an infinite number of discrete modes, of which some may be trapped. The radiation modes would be described by other solutions that satisfy (13) for arbitrary $\alpha$ (see [9]).

We will now examine the polynomial solutions $W(\xi) = C_l(\xi)$ in detail. For $\alpha$ an integer $l$

$$\lambda = b_0 - l + \frac{1}{2}$$

(15a)

$$B_i = b_0 - l, \quad l = 0, 1, 2, \ldots$$

(15b)

The $C_l(\xi)$ are defined by

$$C_0(\xi) = 1$$

(16a)

$$C_1(\xi) = 2\lambda \xi$$

(16b)

$$(l + 1)C_{l+1}(\xi) = 2(l + \lambda) \xi C_l(\xi) - (l + 2\lambda - 1) C_{l-1}(\xi).$$

(16c)

Using (11a) and (11b) we obtain the expression for the modes

$$\psi_l(x) = [\cosh(x/x_0)]^{-b_0} C_l^{\Delta \kappa - l + \frac{1}{2}}(\tanh[x/x_0])$$

(17)

where, from (12a)

$$b_0 = -\frac{1}{2} + \left( \frac{1}{2} + k_0^2 x_0^2 \Delta \kappa \right)^{1/2}.$$  

(18)

For $x \gg x_0$, (17) becomes

$$\psi_l(x) = 1/2 \exp \left[-(x/x_0)(b_0 - l)\right] C_l^{b_0 - l + 1/2}(1).$$

(19)

So to satisfy the boundary conditions, we require

$$\text{Re}(b_0 - l) > 0.$$  

(20)

For a given $b_0$, if $\text{Re}(b_0) > 0$ there exists at least one mode that decreases as $x \to \pm \infty$, for $l = 0$ (the fundamental mode). Modes of order $l$ such that $\text{Re}(b_0 - l) < 0$ are still solutions of (10b) but diverge as $|x| \to \infty$, and will be designated as "leaky". Notice that, if $\text{Re}(b_0) < 0$, even the fundamental mode becomes leaky. Condition (20) can be expressed in terms of the real and imaginary parts of $\Delta \kappa$, defining the quantities

$$\delta = \Delta' + \Delta''$$

(21a)

$$\Phi = 2k_0 x_0 (\Delta'')^{1/2}$$

(21b)

where $\Delta \kappa = \Delta' + i \Delta''$. We obtain

$$\delta > \frac{(2l + 1)^2(l + 1) - \Phi^4}{\Phi^2(2l + 1)^2}$$

(22)

as the region for lateral confinement. Fig. 4 shows $\Phi$ as a function of $\delta$ for several values of $l$. Fig. 5 shows the behavior of $|\psi_l|$ for sets of parameters resulting in the $(\delta, \Phi)$ points marked in Fig. 4. In particular, notice the behavior of $|\psi_l|$ when we go from "1" to "2" to "3".

The $\psi_l(x)$ will diverge for $l > l_{\text{max}}$, where

$$l_{\text{max}} = INT[\text{Re}(b_0)].$$

(23)
to the dielectric step size. For structures that exhibit higher, stable gain, several discrete trapped modes exist, and we achieve reasonable convergence with the first few modes, so that considering the continuum is not necessary. We also apply the effective-index method and compare it with the approximate numerical method in terms of the propagation constant \( \gamma \), which is calculated as a function of the dielectric step size in the active layer \( \Delta n = n_0 - n_s \), where \( n_0 \) and \( n_s \) are the values of refractive index under the stripe and far away from it. The results of the two methods practically coincide for the cases in which several trapped modes exist. They differ appreciably only for that range of \( \Delta n \) for which only one trapped mode exists. The discrepancy may be possibly due to the continuum, but this paper does not investigate this matter further. The following sections will consist of a description of the class of waveguides considered, followed by a description of the approximate numerical method, ending with the application of the effective-index method to this problem and conclusions.

II. DESCRIPTION OF STRUCTURES

Fig. 1 shows the structure considered in this paper. The confining layers \( A \) and \( C \) are assumed identical, their refractive indices being described by

\[
\kappa_A = \kappa_C = n^2 - i \alpha_A n_A/k_0. \tag{2}
\]

\( \alpha_A \) describes the power loss in these layers, and is constant with distance. The active layer has a constant thickness \( d \), with a refractive index

\[
\kappa(x) = n^2(x) - i \alpha(x)n(x)/k_0 \tag{3}
\]

whose dependence with \( x \) is considered to be reasonably well approximated by

\[
\kappa(x) = \kappa_S + \Delta \kappa/\cosh^2(x/x_0) \tag{4}
\]

where

\[
\Delta \kappa = \kappa_0 - \kappa_S \tag{5a}
\]

\[
\kappa_0 = \kappa(0) = n_0^2 + i \gamma_0 n_0/k_0 \tag{5b}
\]

\[
\kappa_S = n_s^2 - i \alpha_A n_s/k_0. \tag{5c}
\]

\( x_0 \) is a parameter related to the width of the stripe. The values of power attenuation coefficient and refractive index inside the active region far away from the stripe are \( \alpha_S \) and \( n_S \), respectively. The quantity \( \gamma_0 \) represents power gain under the stripe, where the refractive index is \( n_0 \), and \( k_0 = 2\pi/\lambda \).

Using (4) and (5), we can obtain expressions for the variation of the refractive index \( n \) and the loss \( \alpha \) (or gain \( -\alpha \)) as a function of distance. Figs. 2 and 3 show \( n(x) \) versus \( x/x_0 \) and \( \alpha(x) \) versus \( x/x_0 \). For \( \Delta n = n_0 - n_s > 0 \), the mode will be index-guided, while for \( \Delta n < 0 \), it will be index-antiguided. In this latter case, it will still be confined because of the gain distribution, but the field will be more spread and the modal gain will be low (eventually we may have a net power loss). If \( \Delta n \) is negative enough, the guiding effect is lost and the modes become leaky. The condition \( \Delta n > 0 \) results in strong confinement and high stable values of modal gain.

III. NUMERICAL SOLUTION

We assume an electric field of the form

\[
E_x = \Psi(x, y). \tag{6}
\]

Following [1], we apply Maxwell's equations to the structure in Fig. 1, and obtain inside the active layer

\[
\nabla_x^2 \Psi_A + \left[ \gamma^2 + k_0^2 \kappa(x) \right] \Psi_A = 0 \tag{7}
\]

and outside the active layer

\[
\nabla_x^2 \Psi_A + \left[ \gamma^2 + k_0^2 \kappa_A \right] \Psi_A = 0 \tag{8}
\]

where \( \kappa(x) \) is given by (4). We require the functions \( \Psi_A \) and \( \Psi_A \) and their normal derivatives to be continuous at the
For common values of \( n \), this resulted in unacceptably high computation times, in part because \( \psi \) involves repeated use of a gamma-function routine and also because the integrand is an oscillating function of \( \chi \).

In order to improve speed, instead of solving the complete system (34) we start with a \( 1 \times 1 \) matrix, go on to a \( 2 \times 2 \) matrix, etc., and observe the convergence of the result. We see that the result converges relatively fast, so a \( 2 \times 2 \) matrix is sufficient for our approximation. This holds for the case \( l_{\max} \geq 2 \). For \( l_{\max} = 0 \), we are limited to the \( 1 \times 1 \) case. In Fig. 6, for \( \Delta n < -0.01, l_{\max} = 0 \). For \( \Delta n > -0.01, l_{\max} \) increases as shown in the figure.

In spite of this, even for \( l_{\max} = 0 \), the computation time is impractically high. An increase in speed is achieved recognizing that (31) can be written as

\[
I_{\mu}(\gamma) = (\gamma^2 - k_0^2)^{1/2} \int_{-\infty}^{\infty} \left[ 1 + \frac{X^2}{\gamma^2 - k_0^2} \right]^{1/2} \psi_\mu(\chi) \bar{\psi}(\chi) \, d\chi.
\]

For common values of \( k_0^2 \) and \( \gamma \), \( |X^2/(\gamma^2 - k_0^2)| \ll 1 \) in the range of values of \( \chi \) for which \( \psi_\mu(\chi) \) and \( \bar{\psi}(\chi) \) are appreciably different from zero. This allows us to expand the radical in (37) using the binomial theorem and retaining only a finite number of terms. This results in

\[
I_{\mu}(\gamma) = (\gamma^2 - k_0^2)^{1/2} \left\{ \int_{-\infty}^{\infty} X^2 \psi_\mu \bar{\psi} \, dX + \frac{1}{2(\gamma^2 - k_0^2)} \right. \]
\[
- \frac{1}{8(\gamma^2 - k_0^2)^2} \left( \int_{-\infty}^{\infty} X^4 \psi_\mu \bar{\psi} \, dX + \cdots \right). \tag{38}
\]

The integrals do not depend on \( \gamma \), so they need to be computed only once for a given set of material parameters, and not for every value of \( \gamma \), as (31) would require. Furthermore, the properties of the Fourier transform guarantee that

\[
\int_{-\infty}^{\infty} \psi_\mu \bar{\psi} \, dX = \delta_{\mu}. \tag{39}
\]

Agreement with exact computation of (31) is excellent with the first three terms in the expansion (38) and the computation time is reduced substantially, by nearly two orders of magnitude. This allows us to perform more extensive modeling using this method.

Fig. 7 shows plots of the modal loss \( \text{Re}(\gamma) \) as a function of \( \Delta n \), with \( g_0 \), the gain under the stripe, as a parameter. We notice that for each value of \( g_0 \), \( \Delta n \) can be decreased up to a certain value beyond which the fundamental mode becomes leaky, i.e., the antiguiding effect of the (negative) \( \Delta n \) offsets the guiding effect of the gain distribution.

More negative values of \( \Delta n \) are required to offset higher values of \( g_0 \). For any given \( g_0 \), using (5), (18), and (20) with \( l = 0 \), it can be shown that the boundary value of \( \Delta n \) is

\[
\Delta n = -\frac{1}{2} x_0^2 \Gamma^2 (\alpha_0 + \alpha_2)^2. \tag{40}
\]

These values are marked with vertical lines in Fig. 7. This relation shows that the guiding effect of the gain distribution does not depend on the individual values of \( g_0 \) (gain under the stripe) and \(-\alpha_2 \) (gain far away from the stripe), but only on their difference \( g_0 - (-\alpha_2) = g_0 + \alpha_2 \).

V. EFFECTIVE-INDEX CALCULATION

The effective-index method consists basically in reducing a two-dimensional problem to an equivalent one-dimensional one. In our case, the two-dimensional character of the problem is given by the dependence of the dielectric constant on \( x \) and \( y \). As a first approximation, the variation in one direction (in our case: \( x \)) is neglected; this is justified if this variation is much less than that in the \( y \) direction. This is equivalent to approximating the waveguide with a simple three-layer guide whose dielectric constants do not vary with \( x \). The solution of this problem yields the transverse or vertical variation of the field. Next, the original equation describing the two-dimensional equation in \( x \) can be solved for the lateral variation of the field, and the overall solution is approximated by the product of this lateral field and the vertical field found from the
three-layer problem. We start with the wave equation in two dimensions, which is obtained merging (7) and (8)

$$\nabla^2 \Psi + \left[ \gamma^2 + k_0^2 \kappa(x, y) \right] \Psi = 0 \quad (41)$$

where

$$\kappa(x, y) = \begin{cases} \kappa_A, & |y| > d/2 \\ \kappa_S + \Delta \kappa \cosh^2(x/x_0), & |y| < d/2 \end{cases} \quad (41')$$

For the simple three-layer guide we assume $$\kappa(x) = \kappa_0$$ inside the active layer. We then have

$$\frac{d^2 \phi}{dy^2} = \begin{cases} -q_0^2 \phi, & |y| \leq d/2 \\ p^2 \phi, & |y| > d/2 \end{cases} \quad (43a)$$

with

$$p^2 + q_0^2 = k_0^2 \kappa_0 - k_A^2 \quad (44a)$$

$$p = q \tan(qd/2). \quad (44b)$$

Now, we transform (41) making $$\Psi(x, y) = \phi(x) \psi(y)$$, multiplying it by $$\phi(x)$$, and integrating it over $$y$$ from $$-\infty$$ to $$\infty$$, taking into account (42) and (43) and defining the confinement factor

$$\Gamma = \frac{\int_{-d/2}^{d/2} \phi(x) \psi(y) dy}{\int_{-\infty}^{d/2} \phi(x) \psi(y) dy}. \quad (45)$$

We obtain

$$\frac{d^2 \psi}{dx^2} + \left[ \gamma^2 - q_{\text{eff}}^2 + k_0^2 \kappa_{\text{eff}} + k_0^2 \frac{\Delta \kappa_{\text{eff}}}{\cosh^2(x/x_0)} \right] \psi = 0 \quad (46)$$

with

$$q_{\text{eff}}^2 = \Gamma k_0^2 - p^2 - k_A^2 \quad (47a)$$

$$\kappa_{\text{eff}} = \Gamma k_S \quad (47b)$$

$$\Delta \kappa_{\text{eff}} = \Gamma \Delta \kappa. \quad (47c)$$

Equation (46) has the same form as (10b). It will also have polynomial solutions similar to (17) that represent trapped modes

$$\psi(x) = \left[ \cosh(x/x_0) \right]^{-1} b_{\text{eff}} \tan x/x_0 \quad (48)$$

where $$b_{\text{eff}}, \Delta \kappa_{\text{eff}}$$, and $$\kappa_{\text{eff}}$$ satisfy relations identical to (12b), (15b), and (18). For the fundamental mode, $$l = 0, B = b_{\text{eff}}$$, and using (44b), we obtain

$$\gamma^2 = -k_0^2 \kappa_A + \frac{b_{\text{eff}}^2}{x_0^2} - \left( p^2 - k_0^2 \Gamma \Delta \kappa \right) \quad (49)$$

for the propagation constant.

VI. DISCUSSION

Values of $$\gamma$$ obtained using (49) are compared with those obtained with the numerical method in Fig. 8. Solutions are very close for all values of $$\Delta n$$ for which the mode exhibits a gain which is relatively high and with low sensitivity to $$\Delta n$$. The results differ most in the range of $$\Delta n$$ for which the mode has a net loss or has a relatively low gain with higher sensitivity to $$\Delta n$$. This is the same region for which only one trapped mode exists, so the error in the numerical method is the greatest because the neglected continuum is more important.

Fig. 9 gives the required value of the peak power gain $$g_0$$ under the stripe to obtain a given modal gain $$G$$, as a function of $$\Delta n$$, using the effective-index method. Also included is the region for which the fundamental mode becomes leaky. Figs. 10-12 show the normalized lateral field distributions for different values of $$\Delta n$$. The increasing antiguiding effect of decreasing $$\Delta n$$ is apparent. The distance $$x$$ at which the gain is zero (loss/gain boundary) is shown in these figures with vertical dashed lines. It can be shown to be given by

$$x = x_0 \cosh^{-1} \left[ 1 + \frac{g_0}{a_S n_S} \right]^{1/2}. \quad (50)$$

For the set of parameters considered, $$x = 1.45 x_0$$. This allows us to qualitatively understand why the modes have the loss (gain) indicated. The vertical confinement factor $$\Gamma$$ did not vary appreciably with $$\Delta n$$; a typical value for the case considered was $$\Gamma = 0.4963$$. We see that lateral variations in the refractive index affect the gain much more by altering the lateral field distribution than by affecting the vertical variation.
LATERAL DISPLACEMENT \( x/x_0 \)

Fig. 10. Normalized lateral field distribution for \( \Delta n = -0.0156 \). The resulting modal loss is 20 cm\(^{-1}\). Vertical dashed lines indicate the distance at which the gain inside the active layer is zero.

LATERAL DISPLACEMENT \( x/x_0 \)

Fig. 11. Normalized lateral field distribution for \( \Delta n = -0.006 \). \( \mathrm{Re}(\psi) = -4.46 \mathrm{~cm}^{-1} \) (gain). Vertical dashed lines have the same meaning as in Fig. 10.

LATERAL DISPLACEMENT \( x/x_0 \)

Fig. 12. Normalized lateral field distribution for \( \Delta n = 0.002 \). The modal gain, \(-\mathrm{Re}(\psi)\), is now 3.23 cm\(^{-1}\).

The effective-index method is seen to be a fast and relatively accurate way to obtain the field distributions for the class of waveguides considered, for which the numerical method we used is not practical for extensive modeling due to its long computation time in spite of all approximations. The remarkable agreement between the effective-index method and experimental results found by other workers ([10]) increases our confidence in this powerful approximate method.

REFERENCES


Alfredo Lina was born in 1951 in Quito, Ecuador. He received the degree of Ingeniero en Electrónica y Telecomunicaciones from Escuela Politécnica Nacional, Quito, in 1977, summa cum laude. From 1977 to 1979 he served as Instructor at EPN. Through the Fulbright Program he was awarded as Assistantship at Southern Methodist University, Dallas, TX, where he obtained the M.S.E.E. degree in 1981, and is currently working toward a Ph.D. degree.

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APPLICATION OF A GENERAL PURPOSE CIRCUIT SIMULATION PROGRAM TO SEMICONDUCTOR LASER MODELING - PART I: ELECTRICAL ANALYSIS

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Abstract

This article describes the application of SPICE to the electrical modeling of modern semiconductor laser structures which cannot be described analytically. Given the geometrical structure of the device and some material characteristics such as impurity concentration and type of semiconductor (p or n), we must obtain the distribution of potential and current density. We are mainly interested in knowing the current density at the edge of the active layer, since it determines the distribution of injected carriers inside it, together with the diffusion equation. The carrier concentration, in turn, determines the optical gain in the active layer. The gain profile will have a strong influence on the lateral dependence of the optical field. In this article we start analyzing a very simple structure which nevertheless contains many features found in real lasers. Then we present a discrete model which can be applied to virtually any geometry and we use it to explore some laser structures with nonlinear elements (p-n junctions). We confine ourselves to purely electrical effects; the interaction between carrier diffusion and electrical parameters is left for a subsequent article.

* Supported in part by the U. S. Army Research Office and RCA Laboratories.
APPLICATION OF A GENERAL PURPOSE CIRCUIT SIMULATION PROGRAM TO
SEMICONDUCTOR LASER MODELING - PART II: DIFFUSION ANALYSIS

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Abstract

This article is the continuation of a previous one in which the modeling of electrical characteristics of semiconductor laser devices using SPICE was described. Here we consider the application of SPICE to the modeling of diffusion and the integration of the electrical and the diffusion models. The model is then applied to a particular structure and the results are discussed. The model shows that the junction current density distribution remains virtually unchanged for a wide range of diffusivities.

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AN INTEGRATED ELECTRICAL/DIFFUSION MODEL
FOR THE P-N JUNCTION

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Abstract

A lumped parameter circuit model is presented, which solves for the
coupled electrical/diffusion problem for a p-n junction. Externally, the
model yields the electron and hole current components separately. Internally,
an adjoint "diffusion network" which is an electrical equivalent of the one-
dimensional diffusion equation, yields the values of excess carrier
concentration in both the n and p regions. The model is then used to obtain
the transient response of a diode. The effect of the material parameters
(lifetime, diffusivity, resistivity) on the electrical response of the device
can be readily modeled. When used with SPICE, the model can easily be used as
a subcircuit to incorporate it into networks containing diodes.

* Supported in part by the U. S. Army Research Office and RCA Laboratories.
APPENDIX C

DISSERTATION ABSTRACTS
The lateral dielectric constant in stripe geometry injection lasers is modeled with a step and a parabolic dependence. For the step profile a set of cutoff curves for the lateral modes is used to compare various laser designs. As the vertical geometry of a laser is varied a contour is described on the cutoff curves which is a measure of lateral mode stability. Two laser designs examined are the simple stripe geometry and channeled-substrate planar lasers. The parabolic model of the lateral dielectric constant is extended to include matched fields at the vertical heterojunctions. A result is a deformed fundamental mode shape in the case of small normalized frequencies and depression of the refractive index under the stripe. This presents an alternative explanation for the shoulders that have heretofore been attributed to leaky waves.

Lateral mode stability with drive current above threshold is complicated by an uncertain dielectric profile perturbed by free carrier effects and spatial hole burning. An experimental scheme is devised to measure the free carrier effect on the refractive
index in GaAs active regions of injection lasers. Transverse field and modal selection mechanisms of the active cavity are characterized for a broad area high power laser as a function of temperature and injection level. The amount of index depression as a result of carrier increase in the active region is measured to be $(3 - 9) \times 10^{-27} \text{ m}^3$. 

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ABSTRACT

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B.S.E.E., Escuela Politecnica Nacional, Quito, Ecuador, August 1977
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MODELING OF SEMICONDUCTOR INJECTION LASERS
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A set of programs to model the operation of semiconductor laser devices below and above threshold is presented, together with the design and construction of an automatic measurement system used to obtain far field radiation patterns. The programs incorporate electrical, diffusion and optical modeling.

The program SPICE is used to model electrical and diffusion phenomena, while direct numerical techniques are used to solve the optical and diffusion problems.
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