| Table 1: Pressure Distribution
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<td>Notes</td>
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*Note: This is a sample table for demonstration purposes. The actual table content is not visible in the image.*
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THE SEVEN-HOLE PRESSURE PROBE

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This Technical Note is approved for publication.

[Signature]

THOMAS E. McCANN, Lt Colonel, USAF
Director of Research and Computer Based Education
This technical note is the final report covering the period 1 Oct 83 to 30 Sep 84 in response to NASA A 10498C sponsored by the NASA Ames Research Center and administered by Mr. W.P. Nelms, and in response to AMD/RDS 84-16 sponsored by AFAMRL/CC (CREST) and administered by Major A.M. Higgins. Majors Frederick M. Jonas and Jack D. Mattingly were the principal investigators for this effort, aided by the faculty and cadets at the Air Force Academy. Of particular note is the contribution of 2Lt Lawrence Reed, former student and now Air Force officer, who is the principal author for this paper. The paper documents the current status of the seven-hole probe developed at the Air Force Academy under the sponsorship of the NASA Ames Research Center.
THE SEVEN-HOLE PRESSURE PROBE

Lawrence Reed*, Jack D. Mattingly**, and Frederick Jonas**

Abstract

This paper documents recent and past developments with respect to the seven-hole pressure probe. Included are discussions on probe design, construction, calibration, and measurement capabilities. The effects on probe measurements in shear flows, as well as methods of correction, are also included. Finally, research applications in the measurement of unknown flowfields are presented. As shown, the seven-hole pressure probe is a valuable and highly accurate device for qualitatively or quantitatively documenting unknown flowfields.

Introduction

The purpose of this paper is to document recent and past developments with respect to the seven-hole pressure probe developed at the United States Air Force Academy in conjunction with the NASA Ames Research Center. As with all pressure probes, the seven-hole probe can be used to map unknown internal or external flowfields, giving valuable information to the aerodynamicist. The advantage of the seven-hole probe over other flowfield measuring devices is its widely expanded measurement range and flexibility. To document this effort, Section II presents brief background material about the probe's design and use. Section III includes an overview of probe calibration in uniform flows, both compressible and incompressible. In Section IV, recent results of investigations into probe measurement capabilities in shear flow environments are presented. Measurement capabilities as well as measurement sources of error

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We discussed in Section V. Finally, recent research efforts on the application of seven hole probe measurements in unknown flows are presented in Section VI.

II. Background

Numerous techniques are available to measure unknown flow fields. While methods such as tufts, streamers, and vanes are primarily used for flow visualization, they are insufficient for quantitative information. Detailed data on flow size, direction, and pressure usually require direct intrusive flow measurement.

One of the oldest known quantitative techniques may be found in the pressure probe. Although other techniques, such as hot wire anemometers and laser doppler anemometers, have been developed, pressure probes are desirable from a flight vehicle standpoint because of their simplicity and ruggedness (Ref 1). Coupled with relatively low cost, pressure probes are an excellent measurement device for research, development, and industrial applications.

One of the classic problems in intrusive flow measurement is the disturbance caused by a probe to the flow field it is measuring. For certain applications, such as aircraft pitot-static tubes, this disturbance or perturbation is relatively unimportant, but for other applications it can be significant. Nevertheless, pressure probes and techniques have been proven to be of great utility.
was developed at the United States Air Force Academy to increase the measurement range of a non-nulling pressure probe while minimizing probe size.

A. Nulling Versus Stationary Probes

The increased measurement range of a multi-hole probe revolves on its ability to sense more pressures at the face of the probe. Two procedures, nulling and stationary, may be used to acquire the pressure data. A nulling probe is rotated in one or two planes until opposing peripheral ports measure equal pressures. The corresponding angle of rotation determines the flow angle. For the non-nulling approach, the probe is held stationary as pressures in opposing peripheral ports are recorded. The pressure differences are then transformed to flow angles through previously known calibration relationships. Although the nulling procedure allows analysis of high angle flow, it is mechanically complex, time consuming, and hence, may not be cost effective. On the other hand, stationary probes with up to five holes are incapable of accurate measurements at high local flow angles (greater than 40 degrees) because of flow separation around the probe tip. Explained in detail in a later section, the seven hole probe is the only non-nulling probe that can determine flow angles up to 70 degrees relative to its axis. When combined with a computerized data acquisition system, the probe may record nearly two data points per second. This rate is significantly faster than current nulling devices, which require considerable time for the balancing of probe tip pressures before each measurement can be taken (Ref. 4).

3. Analytical Model of Pressure Probe

4
Pressure probes used for measurement of flow direction and magnitude normally consist of an aerodynamic body with a symmetrical arrangement of sensing holes (Figure 1). Some typical geometries for pressure probes are reviewed in References 3 through 17 and pressure probes are treated in more general terms in References 18 and 19.

Figure 1. TYPICAL FLOW DIRECTION PROBE
(EACH UNIT ON SCALE REPRESENTS 0.1 CM)

The pressure probes of interest are to be used without correction, the ambient flow regularity is extremely...
While numerically solving the three-dimensional potential flow equations for the flow around an aerodynamic probe may be more accurate than slender body theory under some conditions (Ref. 11 and 22), it does not lend itself to synthesis of probe shapes. The slender body approach to the analysis of the probe provides a basis for formulating analytic calibration relations and considerable insight into the physical process.

Figure 2. SLENDER BODY OF REVOLUTION IN A CROSS-FLOW (REF. 18)

Huffman modeled the probe as a yawed body of revolution as shown in Figure 2. For a slender body, the equation of motion is linear and Huffman wrote the velocity potential as the sum of the axial flow and the cross flow components (this follows a method specified by Liepmann and Roshko, Ref. 23). Solution of the equation of motion yielded the following velocity field in cylindrical coordinates:
\[ \frac{v_r}{V_\infty} = R \frac{R}{r} (\cos \alpha \cos \beta) + (1 - \frac{R^2}{r^2}) (\sin \beta \cos \theta + \sin \alpha \cos \beta \sin \theta) \]
\[ \frac{v_\theta}{V_\infty} = (1 + \frac{R^2}{r^2}) (\sin \alpha \cos \beta \cos \theta - \sin \beta \sin \theta) \]
\[ \frac{v_z}{V_\infty} = (1 + \frac{f}{2}) (\cos \alpha \cos \beta) + 2 R \frac{R}{r} (\sin \beta \cos \theta + \sin \alpha \cos \beta \sin \theta) \]

where \( f \) denotes the body-geometry function given by

\[ f = \frac{1}{2} (R')' \left( \frac{1}{\sqrt{z^2 + \delta^2 r^2}} - \frac{1}{\sqrt{(\ell - z)^2 + \delta^2 r^2}} \right) \]
\[ + \frac{1}{2} (R')'' \left[ \frac{z}{\sqrt{(\ell - z)^2 + \delta^2 r^2}} + \frac{z}{\sqrt{z^2 + \delta^2 r^2}} + \sin \left( \sqrt{(\ell - z)^2 + \delta^2 r^2} \right) \right] \]
\[ - \frac{1}{4} (R^2)''' \left( \frac{z}{\sqrt{z^2 + \delta^2 r^2}} - \frac{(\ell - z)^2}{\sqrt{(\ell - z)^2 + \delta^2 r^2}} + 2 \sqrt{(\ell - z)^2 + \delta^2 r^2} \right) + \ldots \]

The superscript prime (') denotes the derivative with respect to the variable \( z \), and \( \delta = \alpha - M \alpha \).

The pressure coefficient is defined as

\[ C_p = \frac{P - P_\infty}{\frac{1}{2} \rho V_\infty^2} = 1 - \frac{V^2}{V_\infty^2} \]

And when evaluated at the body the following expression results:

\[ C_p = -\frac{1}{2} \cdot \frac{\alpha}{\delta} \cos \alpha \cos \beta \sin \theta \sin \beta - (R') \sin \alpha \sin \beta \sin \theta - \alpha (R^2) \cos \alpha \cos \beta \]

\[ \sin \theta \sin \beta - (R') \sin \alpha \sin \beta \sin \theta - \alpha (R^2) \cos \alpha \cos \beta \]
where \( f \) is to be evaluated on the body surface, i.e. \( r=R \), Equation 4 represents the major result of Huffman's analysis and can be used to determine probe angular and static pressure sensitivity for arbitrary geometries.

The pressure difference between ports on opposite sides of the probe is used to determine the flow angularity. Since the side ports of the probe are at the same \( z \) and \( r \) locations, the pressure difference is proportional to the difference of \( C_p \) values at different \( \theta \) locations and

\[
P_{i} - P_{j} = \sin \theta \left[ \sin^2 \theta - \sin^2 \theta \right] + (R')^2 \left( \cos^2 \theta - \cos^2 \theta \right)
+ \sin^2 \theta \left[ \cos^2 \theta - \cos^2 \theta \right] + (R')^2 \left( \sin^2 \theta - \sin^2 \theta \right)
+ \sin \theta \left[ \cos^2 \theta - \cos^2 \theta \right] \left( \sin^2 \theta - \sin^2 \theta \right)
- 2R' \left( 1 + f/2 \right) \cos \theta \sin 2\theta \left( \cos^2 \theta - \cos^2 \theta \right) + \sin 2\theta \cos^2 \theta \left( \sin^2 \theta - \sin^2 \theta \right)
\]

Figure 3. SEVEN-HOLE PROBE
For the seven-hole probe shown in Figure 3, the six holes on the side face are located at $\theta_1=90^0$, $\theta_2=150^0$, $\theta_3=210^0$, $\theta_4=270^0$, $\theta_5=330^0$, and $\theta_6=30^0$. The resulting three pressure coefficient differences for opposite side holes are

\[
\Delta C_1 = \frac{C_{p4} - C_{p2}}{p_4} = \frac{4R'(1+f/2) \sin 2a \cos \beta}{3}
\]

\[
\Delta C_2 = \frac{C_{p3} - C_{p5}}{p_3} = \frac{2R'(1+f/2)[3 \cos a \sin 2\beta + \sin 2a \cos \beta]}{3}
\]

\[
\Delta C_3 = \frac{C_{p3} - C_{p6}}{p_5} = \frac{2R'(1+f/2)[3 \cos a \sin 2\beta - \sin 2a \cos \beta]}{3}
\]

These three pressure coefficients can be combined into a coefficient that is mainly dependent on the flow angle $a$ and another coefficient that is mainly dependent on the flow angle $\beta$ as follows:

\[
\Delta C_a = \frac{2 \Delta C_1 + \Delta C_2 - \Delta C_3}{3} = \frac{4R'(1+f/2) \sin 2a \cos \beta}{3}
\]

\[
\Delta C_\beta = \frac{\Delta C_2 + \Delta C_3}{3} = \frac{4R'(1+f/2) \cos a \sin 2\beta}{3}
\]

The sensitivity to changes in flow angle is defined as $\frac{\partial \Delta C_a}{\partial a}$ or $\frac{\partial \Delta C_\beta}{\partial \beta}$. The sensitivities are:

\[
\frac{\partial \Delta C_a}{\partial a} = \frac{4R'(1+f/2) \cos 2a \cos \beta}{3}
\]

\[
\frac{\partial \Delta C_\beta}{\partial \beta} = \frac{4R'(1+f/2) \cos a \sin 2\beta}{3}
\]
Also, the sensitivity is proportional to the slope of the probe's surface, \( R' \). Equation 8 is plotted in Figure 4 for \( f = 0 \). It is quite apparent from the figure that the probe angularity sensitivity -- regardless of shape -- depends on \( \alpha \) and \( \beta \).

Approximately a 10% reduction in sensitivity occurs for \( \alpha \) and \( \beta \) values of 10°. Note that \( \frac{\partial \Delta C_\alpha}{\partial \alpha} \) is the same for both positive and negative values of \( \alpha \) and \( \beta \).

**Figure 4. PROBE ANGULAR SENSITIVITY AS A FUNCTION OF \( \alpha \) AND \( \beta \) (REF. 18)**
The average pressure coefficient can be used to determine the flow-field static pressure, \( P_m \). \( C_p \) is evaluated at a number of theta values and the results summed and divided by the number of theta values. For the seven-hole probe of Figure 3, this process yields

\[
\langle C_p \rangle = \left[ 1 - 4(R')^2 \right] \cos^2 \alpha \cos^2 \beta - [1 + 2(R')^2]
\]

Huffman developed an expression for a quasi total pressure from his analysis by integrating the pressure over the body surface, resolving this force into an axial component, and dividing by the cross sectional area. His relationship is

\[
\langle C_p \rangle = \frac{\cos^2 \alpha \cos^2 \beta}{R'} \int_0^t \left[ f + f^2/4 + (R')^2 \right] dz \sin \gamma \sin \alpha \cos \gamma.
\]

where \( t \) denotes the integration length.

For a conical shaped body, \( R' \) is constant and Equation 11 reduces to

\[
\langle C_p \rangle = \left[ 1 - 4(R')^2 \right] \cos^2 \alpha \cos^2 \beta - 1
\]

The total pressure coefficient that is proportional to the dynamic pressure can be obtained by subtracting Equation 12 from Equation 11:

\[
\langle C_p \rangle - \langle C_p \rangle = 2(R')^2 [1 - \cos^2 \alpha \cos \beta]
\]
Some insight into probe calibration can be obtained by studying the plot of $\alpha$ and $\beta$ versus $C_\alpha$ and $C_\beta$ in Figure 5, the plot of $\langle C_p \rangle$ versus $\alpha$ and $\beta$ in Figure 6, the plot of $\langle C_p \rangle$ versus $\alpha$ and $\beta$ in Figure 7, and the plot of $\left[\langle C_p \rangle^2 - \langle C_p \rangle \right]$ versus $\alpha$ and $\beta$ in Figure 8. Figures 5 through 8 were calculated for $R'=0.268$ and $f=0$. Note the near linear relation between each angle and its respective pressure coefficient as shown in Figure 5. Also, the nearly circular contours of constant $\langle C_p \rangle$, $\langle C_p \rangle^2$, and $\left[\langle C_p \rangle^2 - \langle C_p \rangle \right]$ as shown in Figures 6, 7, and 8, respectively. Probe manufacturing imperfection and angular misalignment of the pressure ports will cause these curves to become somewhat distorted.
Figure 5. $C_2$ VERSUS $C_0$ FOR A TYPICAL FLOW DIRECTION PROBE (REF 18)
Figure 6. CONTOURS OF CONSTANT $<C_p>$
Figure 7. CONTOURS OF CONSTANT $\langle C_p \rangle_z$.
Figure 8. CONTOURS OF CONSTANT $\langle C_p \rangle_z - \langle C_p \rangle$
C. Seven-Hole Probe Design

The seven-hole probe is characterized by six periphery ports surrounding one central port (Fig. 9). The probe is constructed by packing seven properly sized stainless steel tubes into a stainless steel casing. For the current probe used at the Air Force Academy, the inner seven tubes have an outside diameter of .023 inches with a .005-inch wall thickness. Once assembled in the order shown below, the tubes are soldered together and machined to the desired half angle $\epsilon$ (usually 25 or 30 degrees) at the tip (Ref. 2). It should be noted that the manufacture of seven-hole probes (versus four/five hole probes) is much simpler due to this packing arrangement.

Figure 9. PROBE GEOMETRY
III. Seven-Hole Probe Calibration Theory

Because of their small size and individual construction, all probes have inherent manufacturing defects that require the unique calibration of each probe. Gallington (Ref. 24) developed the calibration theory required for incompressible, uniform flows. His power series method produces explicit polynomial expressions for the desired aerodynamic properties and is easily programmed. The following section presents a synopsis of Gallington's scheme for incompressible, uniform flow calibration as presented by Gallington (Ref. 24) and by Gerner and Maurer (Ref. 2).

A. Low Versus High Flow Angles, Incompressible Flow

In this calibration technique for incompressible, uniform flow, the probe face is divided into two sectors. The inner flow sector deals with low flow angles in which the angle between the probe's axis and the freestream velocity vector is less than 30 degrees. The other or outer sector deals with high flow angles of over 30 degrees. Thirty degrees is the dividing point because flow typically begins to detach over the top surface of the probe tip around this local flow angle. The hole numbering system used for both sectors and the remainder of the report is as follows:

![Diagram of probe face and side view](image)

**Figure 10. PORT NUMBERING CONVENTION AND PRINCIPAL AXES**
The x-axis is defined to be positive in the unperturbed freestream flow direction. The origin is a point at the tip of the cone formed by the probe (Fig. 10).

3. Probe Axis System for Low Flow Angles

The axis system for low flow angles is the tangential alpha-beta system depicted below (Fig. 11). The angle of attack, alpha (αT), is measured directly as the projection on the x-plane. To preserve symmetry, the angle of sideslip (βT) is measured directly as the projection on the y-plane.

![Diagram of probe axis system](image)

<table>
<thead>
<tr>
<th>CONVENTIONAL</th>
<th>TANGENT</th>
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<tbody>
<tr>
<td>u = V cos α cos β</td>
<td>( α_T = \arctan \frac{w}{u} )</td>
</tr>
<tr>
<td>v = V sin β</td>
<td>( β_T = \arctan \frac{v}{u} )</td>
</tr>
<tr>
<td>w = V sin α cos β</td>
<td></td>
</tr>
</tbody>
</table>

Figure 11. LOW ANGLE REFERENCE SYSTEM

4. Pressure Coefficients for Low Flow Angles

The flow angle is determined as a function of dimensionless pressure coefficients. Three pairs of opposing peripheral ports measure the differences in pressure from one side of the probe to the other in four of the following three relationships:

\[
C_1 = \frac{P_{2} - P_{1}}{P_{2} - P_{1-e}}, \quad C_{12} = \frac{P_{3} - P_{1}}{P_{3} - P_{1-e}}, \quad C_{2} = \frac{P_{4} - P_{1}}{P_{4} - P_{1-e}}
\]
As the numerator measures changes in flow angularity, the denominator nondimensionalizes the expression with the apparent dynamic pressure. The center port pressure, \( P_7 \), approximates the local total pressure while the average of the circumferential port pressures \( P_{1-6} \) approximates the local static pressure. To transform these pressure coefficients to the tangential reference system, Gallington (Ref. 24) formulates the following relationships:

\[
C_a = \frac{1}{3} \left( 2C_{a_1} + C_{a_2} - C_{a_3} \right), \quad C_b = \frac{1}{\sqrt{3}} \left( C_{a_2} + C_{a_3} \right)
\]

It is important to realize that \( C_a \) and \( C_b \) are not independent of each other; that is, \( C_a \) is a function of all six peripheral ports while \( C_b \) is a function of all but ports 1 and 4.

Besides the two angular pressure coefficients, two other low angle pressure coefficients, \( C_0 \) and \( C_q \), are defined in Gallington's work (Ref. 24):

\[
C_0 = \frac{P_7 - P_{oL}}{P_7 - P_{1-6}}, \quad C_q = \frac{P_7 - P_{1-6}}{P_{oL} - P_{oL}}
\]

\( C_0 \) is the apparent total pressure coefficient with respect to each hole and is a means to convert actual pressures measured by the probe to accurate values of local total pressure. The numerator measures the difference between the approximate total pressure measured by the center port \( P_7 \) and the actual local total pressure \( P_{oL} \). As with the previous coefficients, the denominator nondimensionalizes the expression with the apparent local dynamic pressure.

The velocity pressure coefficient, \( C_q \), serves a similar conversion function as \( C_0 \) except that it relates the probe
pressures to the actual dynamic pressure. The numerator in this coefficient represents the probe's approximation of the local dynamic pressure while the denominator represents the actual dynamic pressure of the freestream test conditions.

II. Probe Axis System for High Flow Angles

The real advantage of using the non-nulling seven-hole probe over other multi-hole probes appears in the ability to measure high-angle flows. At high angles of attack (greater than 30 degrees), the flow detaches over the upper surface of the probe, and pressure ports in the separated wake are insensitive to small changes in flow angularity (Fig. 12).

Figure 12. FLOW PATTERN OVER SEVEN-HOLE PROBE AT HIGH ANGLE OF ATTACK
Consequently, high angle measurements must only be made from ports in the attached flow. Five-hole and other pressure probes have been ineffective in this regime due to this flow separation and lack of sensing ports in the attached flow region. The seven-hole probe avoids this problem because at least three ports remain in the attached flow region, allowing sufficient data to be recorded to document the flow angle.

According to Gallington, the tangential reference system is inappropriate at high flow angles because of indeterminate angles and singularities (Ref. 24). Instead, the polar reference system is used (Fig. 13) where \( \theta \) represents the pitch angle and \( \phi \) represents the roll angle.

![Figure 13. HIGH ANGLE REFERENCE SYSTEM](image)

Here specifically, \( \phi \) denotes the angle that the velocity vector forms with respect to the probe's x-axis, and \( \phi \) signifies the azimuthal orientation of the velocity vector in the y-z plane. We should note that a positive \( \phi \) is measured counterclockwise from the negative z-axis when the probe is viewed from the front.
Based on Hallington's original work with the seven-hole probe at high angles of attack, Gerner and Mauer accomplished the following analysis (Ref 2). Kuethe and Chow's Foundations of Aerodynamics: Bases of Aerodynamic Design, explains that the separation points of a cylinder in turbulent flow are over 100 degrees from the frontal stagnation point. Flow around a conical body such as a probe tip was likely to remain attached longer, and the u-velocity component was prone to extend the separation points downstream even further. Therefore, for the high angle flow shown in Figure 14, ports 3, 4, 5, and 7 lie in attached flow, and port 1 lies in separated flow. The flow over ports 2 and 6 is unpredictable, and their readings are discarded (Ref. 2).

Figure 14. FLOW OVER PROBE AT HIGH ANGLE OF ATTACK
pressure differences. Unlike low flow angles, however, the center port pressure is now the most dependent port on local flow angles. Consequently, a pitch angular pressure coefficient should measure the difference between the center port and a new stagnation port. In the example below, the pressure at \( P_4 \) approximates the local total pressure.

\[
C_{D_4} = \frac{P_4 - P_T}{P_T + P_2} \quad \frac{P_T - \frac{P_1 + P_5}{2}}{2}
\] 

The expression is again nondimensionalized by the apparent dynamic pressure. The average of \( P_3 \) and \( P_5 \) approximates the static pressure and is relatively independent to changes in roll.

Although the average is independent of \( \phi \), the difference is sensitive to roll angle. As the probe's azimuthal orientation changes, the windward pressure rises and the leeward pressure falls. The result is a roll angular pressure coefficient (example for port 4 is Equation 8 below) that is also nondimensionalized by the apparent dynamic pressure.

\[
C_{D_4} = \frac{P_4 - P_T}{P_T + P_2}
\] 

The high flow-angle \( C_o \) and \( C_q \) coefficients are translated from their low flow angle counterparts using the same rationale found in the development of \( C_o \) and \( C_\phi \). The low flow angle \( C_o \) and \( C_q \) coefficients are changed to account for the different ports that represent total and static pressures in the high-angle regime. The equations for \( C_o, C_\phi, C_o, \) and \( C_q \) are as follows:
\[ C_{\Theta_1} = \frac{P_1 - P_7}{P_1 - \frac{P_2 + P_6}{2}} \quad , \quad C_{\phi_1} = \frac{P_6 - P_2}{P_1 - \frac{P_6 + P_2}{2}} \]

\[ C_{\Theta_2} = \frac{P_2 - P_7}{P_2 - \frac{P_1 + P_3}{2}} \quad , \quad C_{\phi_2} = \frac{P_1 - P_3}{P_2 - \frac{P_1 + P_3}{2}} \]

\[ C_{\Theta_3} = \frac{P_3 - P_7}{P_3 - \frac{P_2 + P_4}{2}} \quad , \quad C_{\phi_3} = \frac{P_2 - P_4}{P_3 - \frac{P_2 + P_4}{2}} \] \quad (19)

\[ C_{\Theta_4} = \frac{P_4 - P_7}{P_4 - \frac{P_1 + P_5}{2}} \quad , \quad C_{\phi_4} = \frac{P_1 - P_5}{P_4 - \frac{P_1 + P_5}{2}} \]

\[ C_{\Theta_5} = \frac{P_5 - P_7}{P_5 - \frac{P_4 + P_6}{2}} \quad , \quad C_{\phi_5} = \frac{P_4 - P_6}{P_5 - \frac{P_4 + P_6}{2}} \]

\[ C_{\Theta_6} = \frac{P_6 - P_7}{P_6 - \frac{P_5 + P_1}{2}} \quad , \quad C_{\phi_6} = \frac{P_5 - P_1}{P_6 - \frac{P_5 + P_1}{2}} \]
\[
\begin{align*}
C_{o_1} &= \frac{P_1 - P_{oL}}{P_1 - \frac{P_2 + P_6}{2}}, \\
C_{o_2} &= \frac{P_2 - P_{oL}}{P_2 - \frac{P_1 + P_3}{2}}, \\
C_{o_3} &= \frac{P_3 - P_{oL}}{P_3 - \frac{P_4 + P_2}{2}}, \\
C_{o_4} &= \frac{P_4 - P_{oL}}{P_4 - \frac{P_5 + P_1}{2}}, \\
C_{o_5} &= \frac{P_5 - P_{oL}}{P_5 - \frac{P_6 + P_4}{2}}, \\
C_{o_6} &= \frac{P_6 - P_{oL}}{P_6 - \frac{P_1 + P_5}{2}}, \\
C_{q_1} &= \frac{P_1 - P_2 + P_6}{P_{oL} - P_{oL}}, \\
C_{q_2} &= \frac{P_2 - P_3 + P_1}{P_{oL} - P_{oL}}, \\
C_{q_3} &= \frac{P_3 - P_4 + P_2}{P_{oL} - P_{oL}}, \\
C_{q_4} &= \frac{P_4 - P_5 + P_3}{P_{oL} - P_{oL}}, \\
C_{q_5} &= \frac{P_5 - P_6 + P_4}{P_{oL} - P_{oL}}, \\
C_{q_6} &= \frac{P_6 - P_1 + P_5}{P_{oL} - P_{oL}}.
\end{align*}
\]
P. Division of Angular Space

The most obvious difference between low and high pressure coefficients is that six high coefficients are needed for the high angles. This fact leads to the question of what factors determine when a certain set of coefficients should be employed. Gallington proposes the "division of angular space" shown in Figure 15 (Ref. 24).

![Division of Angular Space Diagram]

**Figure 15. DIVISION OF ANGULAR SPACE**
This method separates probe measurements into seven sectors—a central low angle sector and six circumferential high angle sectors. Data points are placed in a given sector based on the highest port pressure measured on the probe.

G. Polynomial Power Series Expansion

Once the pressure coefficients are calculated, they must be converted to $u', \beta_T, \gamma_0$, or $C_q$ for low angle flows and $\theta, \phi, C_0$, or $C_1$ for high angle flows. This conversion is accomplished by solving the following fourth order power series:

\[
A_i = K_1^A + K_{12}^A a_i + K_{13}^A b_i + K_{14}^A c_i + K_{15}^A d_i + K_{16}^A e_i + K_{17}^A f_i + K_{18}^A g_i
\]

Order of Terms

- 0th
- 1st
- 2nd
- 3rd
- 4th

"A" signifies the desired output quantity with the subscript denoting the $i$th such quantity. "K" denotes the presently unknown calibration coefficients. The calibration process entails finding these calibration coefficients with the A's as known conditions and the C's as measured pressure coefficients at these known conditions.
II. Determination of Calibration Coefficients

Rewriting in matrix notation for \( n \) terms, the power series becomes

\[
\begin{bmatrix}
A_1 \\
A_2 \\
A_3 \\
\vdots \\
A_n
\end{bmatrix} = 
\begin{bmatrix}
ant_1 & \cdots & \cdots & \cdots & t_n \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
ant_1 & \cdots & \cdots & \cdots & t_n
\end{bmatrix}
\begin{bmatrix}
C_{\beta_1} & \cdots & \cdots & \cdots & C_{\beta_1}^n \\
C_{\beta_2} & \cdots & \cdots & \cdots & C_{\beta_2}^n \\
C_{\beta_3} & \cdots & \cdots & \cdots & C_{\beta_3}^n \\
\vdots & \cdots & \cdots & \cdots & \cdots \\
C_{\beta_n} & \cdots & \cdots & \cdots & C_{\beta_n}^n
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2 \\
A_3 \\
\vdots \\
A_n
\end{bmatrix}
\]

or simply

\[
[A] = [C] [K]
\] (23)

Solving for \([K]\) by the least squares curve fit outlined by Netter and Winterman in Callington (Ref. 24), the following is obtained:

\[
[K] = (C^T C)^{-1} C^T [A]
\] (24)

With known calibration coefficients, the above can be used to determine output quantities \( \theta_1, \theta_2, \ldots, \theta_n, \phi \).

It is worth noting that in this point the flow field has been uniform and constant, the pressure or velocity gradients have not been measured. Finally, while the local total and dynamic pressure have not been determined explicitly from the plenum pressure, the...
easily determined by substituting $C_0$ and $C_q$ into Equation 15 (low angles) or Equation 10 (high angles).

1. Extension to Compressible Flows

Gerner and Maurer (Ref. 2) expanded the technique to subsonic compressible flows with the introduction of a nondimensional pressure coefficient representative of compressibility effects. This coefficient, $C_M$, had to become insignificant at very low Mach numbers; thus, any terms bearing $C_M$ in the power series expansion would have to approach zero as Mach approached zero. This would leave essentially an incompressible power series with the regular two angular pressure coefficients as previously described. Because pressure probes are unable to determine Mach number in the hypersonic range, $C_M$ had to approach a finite limit at very high Mach numbers. Consequently, large changes of Mach numbers in this region would have negligible effect on the compressibility coefficient.

Gerner and Maurer (Ref. 2) found that these requirements were satisfied by the apparent dynamic-to-total-pressure ratio (low angles).

$$C_M = \frac{P_7 - \bar{P}_1 - \bar{P}_6}{P_7}$$

Similarly, for high angle flow, the compressibility coefficient in each sector is:

$$C_{M_1} = \frac{P_1 - \frac{P_6 + P_2}{2}}{P_1}$$

$$C_{M_2} = \frac{P_2 - \frac{P_1 + P_3}{2}}{P_2}$$

$$C_{M_3} = \frac{P_3 - \frac{P_2 + P_4}{2}}{P_3}$$

$$C_{M_4} = \frac{P_4 - \frac{P_1 + P_2}{2}}{P_4}$$

$$C_{M_5} = \frac{P_5 - \frac{P_3 + P_6}{2}}{P_5}$$

$$C_{M_6} = \frac{P_6 - \frac{P_5 + P_4}{2}}{P_6}$$
One of the primary problems of extending the seven-hole probe's capabilities into compressible flow lies in the mathematics. First, by adding a third coefficient to the fourth order power series, the number of terms and hence calibration coefficients jumps from 15 to 35. Gerner and Mauer say that approximately 80 data points in two variables (Ca and Cα) for each of the seven sectors are needed for a complete incompressible calibration. That fact results in 560 data points, and the addition of a Mach number compressibility coefficient makes the data set unwieldy (Ref. 2). These two factors create complex and time-consuming matrix operations. Additionally, the amount of run time needed to operate the wind tunnel for all of the calibration data points is prohibitive as well as costly.

Gerner and Mauer suggest a two-part solution to make the addition of Cm feasible. By reducing the fourth order power series to a third order, the number of calibration coefficients is reduced to 20. Gerner and Mauer also decrease the number of data points required for calibration by employing a 6x6 Latin Square technique (Ref. 2) for purposes of obtaining calibration data. The Latin Square is a numerical method that ensures a non-repetitive, uniform sampling of a three-dimensional parameter space.

Seven-hole probe work incorporating these two changes results in the following conclusions for compressible flow. The third order polynomial flow accurately represents the parameter of interest in the range of 0 to 70 degrees of pitch. Next, based on previous work on correlations between the standard deviations of coefficients and the angle of the resulting calibration, Latin
Squares produce a sample which accurately represents a very large three-dimensional parameter space.

With these changes, the seven-hole probe may now be used in an unknown uniform, compressible flow. Since the seven-hole probe has never been subjected to supersonic applications, no shocks will lie ahead of the probe, and the isentropic flow relationship is employed to find Mach number (Ref. 2).

\[
M_n = \frac{\sqrt{\frac{2}{\gamma}}}{1 - \frac{1 - \frac{P_o - P_{in}}{P_o}}{(1 - \frac{P_o}{P})}^\frac{(1-\gamma)}{\gamma}}
\]

Knowing \( C_o \), \( C_q \), and the seven port pressures, the local Mach number may be determined explicitly from the dynamic to total pressure ratio. This ratio for the inner sector is represented in Equation 28 below (Ref. 2).

\[
\frac{P_{oL} - P_{oL}}{P_{oL}} = [C_q \left( \frac{P_o}{P_{7} - P_{81-6}} - C_o \right)]^{-1}
\]

Similar equations for the outer sectors may be derived, using the appropriate port pressures for approximate total and dynamic pressures.

IV. Seven-Hole Probe in Shear Flow

Under the present calibration scheme, the existence of a velocity gradient in the flow will cause the probe to measure an erroneous uniform flow at fictitious flow angle. For example, a flow having the properties \( M = 0.3, \alpha = 0^\circ, \beta = 0^\circ \), with a velocity gradient (Fig. 16) might cause the probe to "think" that it sees a flow with a certain angularity, say \( \beta = 10^\circ \) (see Fig. 17).
\[ \alpha = 0^\circ \]
\[ \beta = 0^\circ \]
\[ M = 0.3 \]

Figure 16. ACTUAL FLOW CONDITION

\[ \beta_c = 10\% \]

Figure 17. APPARENT FLOW CONDITION

different pressure coefficient, all seven values compute a single
flow condition. The use of these data to describe flow
-flow condition is a

determined. Then using the "backstepping" technique developed in the following sections, the apparent measured values can be corrected to the actual flow values.

A. Slender Body Theory

The slender body theory developed by Huffman is utilized in this section to model flow around the probe (Ref. 20). With this model, the probe's port pressure coefficients can be estimated as a function of flow angularity. Using Huffman's method of viewing the probe as a small perturbation to an otherwise uniform stream, the pressure coefficients are determined by analytic means. Further, the model determines the variation of pressure coefficients with flow direction. Not included are viscous or flow separation effects as the theory is based on potential flow.

The body-geometry function, \( f \), is a parameter used to define the geometric characteristics of a slender body and is given in Equation 2. Huffman incorporates the body shape function into calculations for the pressure coefficient as given by Equation 4. A simplified form of Equation 4 was used in this analysis as given below.

\[
C_{p}^{c} = \cos^{2} \alpha \cos^{2} \theta [-f - (R')^{2}] + \sin^{2} \theta [1 - 4 \sin^{2} \theta] \\
+ \sin^{2} \alpha \cos^{2} \theta [1 - 4 \cos^{2} \theta] + \cos \alpha \sin 2 \theta [-2R' \cos \theta] \\
+ \sin 2 \alpha \cos^{2} \theta [-2R' \sin \theta] + \sin 2 \alpha \cos \theta [4 \sin \theta \cos \theta]
\]

where \( R' \) is \( \frac{dR}{dX} \) with \( R \) as the radius of the probe, and \( \theta \) is the angle measured from the \( y \)-axis to the pressure port in question. The subscript "c" signifies that the approximation is valid for uniform incompressible flow.
The function that Huffman develops for the pressure coefficient at point 7 simplifies to Equation 30.

$$C_{p7} = \cos^2 \alpha \cos^2 \beta - \sin^2 \beta - \sin^2 \alpha \cos^2 \beta$$

We must realize that slender body theory is only an approximate model of flow around the seven-hole probe. The reason for this stems from the conditions that the body radius is much smaller than the body length (slender body), and that it is easily applied to compressible flows. The only problem with this approach is that it assumes that the rate of change of body radius with respect to body length must be small. The seven-hole probe does not completely conform to this assumption for two reasons. First, the probe has a blunt tip with a slope discontinuity, while slender body theory assumes an aerodynamically smooth body coming to a point. Because the tip is blunt, flow disturbances originate at that tip. Second, due to the orientation of the peripheral hole surfaces, separation occurs on the tip even at zero degrees of attack. Slender body theory assumes non-separated flow over the body for all flow angles. Even with these discrepancies, slender body theory remains valuable in predicting basic trends for coefficient $C_{p7}$ and equation (30) is used in this work.
a. The Shear Gradient

In order to work with a two-dimensional shear gradient, an
alpha-beta plane is defined normal to the direction of the actual
velocity and through the center point of port number 7 (Fig. 19a).
The velocity at port 7 is used as a reference velocity; hence,
by the time gradients of...
by determining this gradient distance, the following general equation can be used to define the gradient’s effect on the velocity.

\[ V = V_7 + \frac{dV}{dy} y + \frac{dV}{dz} z \]

where \( y \) and \( z \) are the components of the gradient distance and \( \frac{dV}{dy} \) and \( \frac{dV}{dz} \) are indicators of the magnitude of the alpha and beta gradients respectively. To put this equation in a more usable form, the gradients are non-dimensionalized:

\[ V = V_7 \left( 1 + \frac{s}{V_7} \frac{dV}{dy} y + \frac{s}{V_7} \frac{dV}{dz} z \right) \]

where \( s \) is the distance between the centers of opposite ports.

With this new equation, the terms \( \frac{s}{V_7} \frac{dV}{dy} \) and \( \frac{s}{V_7} \frac{dV}{dz} \) can be assigned a magnitude of relative gradient.
Gradient Effects on Flow Pressure Coefficients

For a uniform calibration the pressure at the peripheral holes may be found by the following expression.

\[
C_p = C_{p_1} \left( 1 + \frac{a}{V_f} \frac{dV}{dy} (y) + \frac{b}{V_f} \frac{dV}{dz} (z) \right)^2
\]

(33)

Since \( C_{p_1} \) is a uniform coefficient based on the geometry of the slender body theory, \( V_1 \) is the only term that accounts for gradient effects. Consequently, \( V_1 \) in shear flow differs for each peripheral hole. If the velocity term is kept constant at \( V_f \) and the gradient effects are accounted for by the coefficients of pressure, the following expression results:

\[
P_1 = P_m + C_{p_1} \left( \frac{V_1}{V_f} \right)^2
\]

(34)

Solving Equations 33 and 34 for \( C_p \), it is evident that pressure is proportional to the square of velocity in incompressible flow.

\[
P_1 = P_m + C_{p_1} \left( \frac{V_1}{V_f} \right)^2
\]

(35)
the determination of point \((x, y, z)\). The vector analysis behind the location of point \((x, y, z)\) is explained in detail in Johnson and Reed (Ref. 25). The results are as follows:

\[
x = x_1 + (x_7 - x_1) \cos \alpha \cos \beta - \cos \alpha \cos \beta (y_1 \sin \beta + z_1 \sin \alpha \cos \beta)
\]
\[
y = y_1 + (x - x_1) \tan \beta / \cos \alpha
\]
\[
z = z_1 + (x - x_1) \tan \alpha
\]

<table>
<thead>
<tr>
<th>Port (1)</th>
<th>(x_1)</th>
<th>(y_1)</th>
<th>(z_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(s/2\tan \epsilon)</td>
<td>((s/2)\cos \epsilon)</td>
<td>((s/2)\sin \epsilon)</td>
</tr>
<tr>
<td>2</td>
<td>(s/2\tan \epsilon)</td>
<td>((s/2)\cos \epsilon)</td>
<td>((s/2)\sin \epsilon)</td>
</tr>
<tr>
<td>3</td>
<td>(s/2\tan \epsilon)</td>
<td>((s/2)\cos \epsilon)</td>
<td>((s/2)\sin \epsilon)</td>
</tr>
<tr>
<td>4</td>
<td>(s/2\tan \epsilon)</td>
<td>((s/2)\cos \epsilon)</td>
<td>((s/2)\sin \epsilon)</td>
</tr>
<tr>
<td>5</td>
<td>(s/2\tan \epsilon)</td>
<td>((s/2)\cos \epsilon)</td>
<td>((s/2)\sin \epsilon)</td>
</tr>
<tr>
<td>6</td>
<td>(s/2\tan \epsilon)</td>
<td>((s/2)\cos \epsilon)</td>
<td>((s/2)\sin \epsilon)</td>
</tr>
<tr>
<td>7</td>
<td>(d/2\tan \epsilon)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since \(x_1, y_1, z_1, \) and \(x_7\) are known values, the knowledge of \(\alpha\) and \(\beta\) will locate the point \((x, y, z)\).

E. Shear Flow Measurement Corrections

At this point, the mathematical relationships necessary to compare the effects of velocity gradients are available. Equation 15 provides the ability to calculate \(C_{a_1}, C_{a_2}, C_a,\) and \(C_{\beta},\) and \(C_{\theta},\) realizing that

\[
C_{a_1} = \frac{P_4 - P_1}{P_7 - P_{1-6}} = \frac{C_{P_4} - C_{P_1}}{C_{P_7} - C_{P_{1-6}}}
\]
\[
C_a = \frac{P_3 - P_5}{P_7 - P_{1-6}} = \frac{C_{P_3} - C_{P_5}}{C_{P_7} - C_{P_{1-6}}}
\]
\[
C_{a_2} = \frac{P_4 - P_6}{P_7 - P_{1-6}} = \frac{C_{P_4} - C_{P_6}}{C_{P_7} - C_{P_{1-6}}}
\]
\[
C_{\beta} = \frac{P_3 - P_6}{P_7 - P_{1-6}} = \frac{C_{P_3} - C_{P_6}}{C_{P_7} - C_{P_{1-6}}}
\]
In order to determine the effects of different shear gradients at varying alphas and betas, Johnson and Reed (Ref. 25) calculated the apparent flow angles resulting from a span of actual flow angles and shear gradients. The calculations show the errors between the apparent angles and actual input values. Consequently, the actual flow angles can be determined by backstepping from the apparent angles. Although these calculations only allow manual corrections to apparent measurements, the theory of this section can be used to generate a family of data points. These points may then be input into a computer surface fitting scheme for highly accurate, near real-time correction to the apparent data.

6. Seven-Hole Probe Measurement Capabilities

Three key elements of the seven-hole probe's measurement effectiveness in unknown flows are discussed further in order to provide an understanding of the basics behind its actual and potential applications. The three elements to be discussed are the inherent flow data, the static probe, and the flow prober.

A. Static Probe

The probe must be uniquely calibrated due to manufacturing imperfections. The specifics of the seven-hole probe's static probe calibration and information on how it is calibrated are given in Section 6.1.
probes, seven-hole probe calibration allows for the relative flow angles, total pressure, and static pressure to be determined explicitly. With computer processing, these calculations are performed in near real time.

As discussed earlier, problems with the present calibration arise when attempting to make measurements in shear flow. This is because the present calibration and associated calibration coefficients are determined in a uniform flow. Jonas recorded measurement discrepancies in his attempts to use the seven-hole probe to map unknown flow fields with suspected shear (Ref. 26). Specifically, he compared seven-hole and total pressure probe measurements in a vortex wake created by the wing leading-edge extensions of a Northrop VATOL model.
TOP VIEW

10 in

SIDE VIEW

1/40th Scale Northrop VATOL Model

Figure 20. NORTHROP TOP-MOUNTED INLET VATOL CONCEPT
In a vertical flow, local total pressure should decrease through the wake because of viscous effects. This requires the coefficient \( \frac{p_{w1} - p_0}{p_f - p_0} \) to be a negative value. The results of Jones’ pressure measurement comparison are shown in Table 1 below (Ref. 261).

<table>
<thead>
<tr>
<th>Pressure Probe</th>
<th>Seven-Hole</th>
<th>Added with Inlet Flow</th>
<th>Added with Exit Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Minimum Value</td>
<td>-0.107</td>
<td>-0.150</td>
<td>-0.031</td>
</tr>
<tr>
<td>Local Maximum Value</td>
<td>-0.035</td>
<td>-0.127</td>
<td>-0.061</td>
</tr>
<tr>
<td>Static Value</td>
<td>-0.030</td>
<td>-0.157</td>
<td>-0.087</td>
</tr>
<tr>
<td>Static Pressure Difference</td>
<td>4.04</td>
<td>3.44</td>
<td>4.37</td>
</tr>
<tr>
<td>Static Pressure Difference</td>
<td>1.39</td>
<td>3.44</td>
<td>4.37</td>
</tr>
</tbody>
</table>

Note: These values are calculated based on data from Table 1. As can be seen from the data, the total pressure measurement represents the difference between the exit flow and the freestream. For the total pressure probe, since there were no measurements made when the total pressure probe was used, no conclusion can be drawn from the data. However, these values may not be an accurate measure of local total pressure because flow angularity may have caused effects at the probe tip. For the total pressure probe, it can be seen that the local flow, 90 percent of the measurements resulted in a local total pressure greater than that of the freestream value. These measurements, however, are lower than those obtained with...
the seven-hole probe. One must realize that the total pressure probe was bent at an angle based on measurements made using the seven-hole pressure probe. Since the seven-hole measurements were made in a region of high shear flow, the local flow angularity reading may also be erroneous. Consequently, the flow around the tip of the total pressure probe supposedly aligned with the local flow might also be separated, again leading to false pressure readings. The average value for \( C_{TOTAL} \) of the bent total pressure probe, however, remains negative. This evidence seems to confirm our theoretical expectations of viscous effects.

Although the contradictory probe readings seem to indicate incorrect probe measurements in the high shear regime, the possibility of positive \( C_{TOTAL} \) regions may not be totally excluded. A vortex flow is very complex and not completely understood.

Comparisons in a low shear flow environment (airfoil wake) of seven-hole probe measurements and hot-wire anemometer measurements are, however, very favorable and no discrepancies or positive \( C_{TOTAL} \) measurements exist (Ref. 26). This seems to indicate that the problem of positive \( C_{TOTAL} \)'s are only associated with high shear flows, and not due to the probe (manufacture, calibration, etc.) but due to the environment flow conditions.

A pressure probe should be as small as possible to keep the flow disturbance to a minimum; however, this is accompanied by several construction disadvantages. For example, smaller probe diameter results in lower pressure signal quality. As size decreases,
... and other manufacturing imperfections cause fluctuations in construction errors, and the part tolerances are more limited.

Although the five-hole probe may seem simpler to construct than the seven-hole, the exact opposite is true. The construction of the seven-hole probe is greatly simplified since the part tolerances are arranged in the only geometric fitting, i.e., those of the stainless and soldering (Ref. 21) explain these. The stainless steel-based geometric arrangement evermore, while not exist in the five-hole probe.

Figure 21. SEVEN-HOLE PROBE CONSTRUCTION
First, it is difficult to hold five-hole tubing parallel with the center tube and in perfect azimuthal position for soldering.

Second, a high-powered magnifier is required to assure equal alignment on the four side tubes. Finally, there is no guarantee that the four solder fillets shown below will be equal or even nearly equal.
As a quick note, a simpler geometric arrangement for five tubes is shown below. This order, however, lacks the required center tube.

![Diagram of five tubes in the smallest circumscribed circle]

**Figure 24. FIVE TUBES IN THE SMALLEST CIRCUMSCRIBED CIRCLE**

Over the years, multi-hole probe construction has uncovered certain features that increase the accuracy of pressure measurement. Total pressure measurement can be largely desensitized to alpha and beta by flaring the stagnation port at an angle of 30 to 60 degrees. Yet, the probe's overall sensitivity to flow angularity can be increased by decreasing the tip half-angle (α). The drawback to a small half-angle is that the flow will separate from the leeward surface when αT exceeds (Ref. 1).

C. Measurement Error Sources

Once smaller probes are built, they are more sensitive to measurement errors caused by flow debris and damage. These effects can be minimized prior to use by reverse airflow through clogged ports, by dust covers over port entrances, and by simple...
Huffman identifies three other error sources that adversely affect a probe measurement of pressure, orientation, and velocity (Ref 1). These sources include the time lag between sensed and actual pressures, the pressure transducer resolution and frequency response, and the resolution of the analog-to-digital converter. The pressure transducer resolution and analog-to-digital resolution are both beyond the scope of this report; however, the other error sources are discussed in the following paragraphs (Ref. 1).

As flow field conditions change, the probe surface pressures change nearly instantaneously. These pressure changes are transmitted to the pressure transducers by a finite amount of air through a connection tube. Huffman discusses the fact that the pressure lag caused by this finite travel time is related to the speed of pressure propagation and the pressure drop due to the friction effects of the tubing (Ref 1). Three physical characteristics directly affect this time lag:

1. Tubing diameter
2. Tubing length
3. Transducer cavity

The pressure delay is given by:

\[ \tau = \frac{\sqrt{2L}}{c} \]

where \( L \) is the tubing length and \( c \) is the speed of propagation of the pressure pulse to propagate from the tubing exit.
Schanzei and Russell (Ref 28) define frequency response as the inverse of the maximum dwell time necessary for the probe system to react to the maximum pressure differential expected where the pressure sensed by the pressure transducer reaches 99 percent of the actual (surface) pressure.

Although pressure time lag is the major factor in the probe's frequency response, it is not the sole contributor. The frequency response of the transducer, the computational time required to convert the measured pressure to the desired output, the pressure differential between the probe tip and the transducer face, and the fluid density all affect the probe's overall frequency response (Ref 9).

Probe frequency response rates are generally of a few Hertz. Because modern transducers and microprocessors require only milliseconds to operate, their operation is essentially instantaneous, and little can be accomplished in these areas to increase the overall frequency response. The final two factors are associated with the fluid dynamic properties of the flow. The difference between the fluid pressure at a port entrance and the pressure at the transducer face acts as a driving potential, and a greater differential increases the frequency response (Ref. 28).

Since a higher density increases the collision rate between molecules, and therefore, increases the propagation rate of a pressure pulse, greater density is synonymous with greater frequency response. In considering the effect of density, one must consider the assumption of a constant static pressure across the face of the probe. Remembering Ref 8, one should realize that the density of the flow is dependent upon pressure.


\textbf{Measurements of unknown Flows}

With the seven-hole probe center on two

stabilizers, the capabilities and limitations of the

device are mapped unknown flow fields. The following

explains how these objectives are accomplished in the

aforementioned wind tunnel.


\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{wing_vortex_test}
\caption{WING VORTEX TEST}
\end{figure}

In the present study, a finite wing section of a NACA

0012 airfoil is used in the two by three foot supersonic wind

tunnel at the United States Air Force Academy (Figure 26). The

wind tunnel nozzle is pointed at the wing at a angle of attack of 8 degrees and

a freestream flow velocity of 150 ft/sec.
Computer graphics techniques were used to map out the flow velocities and directions for all the points in a series of parallel two-dimensional planes, all of which were perpendicular to the uniform flow.

Figure 28. WING VORTEX DATA PLANES
The two-dimensional planes or test grids, started twelve inches aft of the wing's trailing edge and moved forward (toward the wing) by two inch increments for every data set. From these test runs, capabilities and limitations of the seven-hole probe under mild gradient conditions are examined and a wingtip vortex is mapped.

vi. Crossflow Velocities

The first step in vortex analysis is to examine the crossflow velocities in each plane. Crossflow velocity plots, which are divided by a factor of two are shown in Figures 27a to 27h (Ref. 29). With past errors in gradient regime, one would expect errors in seven-hole readings. Suspect data of this nature (Ref. 29) are identified by inconsistencies in the crossflow plots, such as an inordinately large or small arrow (Fig. 28) or an arrow in the wrong direction. However, the actual crossflow plots do not confirm these expectations. The velocities are well-behaved, and no inconsistencies are seen near the vortex core where high shear exists.
Figure 27. CROSSFLOW VELOCITIES OF WING TIP VORTEX
Figure 28. CROSSFLOW DATA ERRORS

Although complete velocity data may be shown on a velocity vector contour plot for ease of analysis (Figure 29).
Figure 29. MAGNITUDE OF CROSSFLOW VELOCITY
This new representation of the velocity data shows how the vortex expands and decays as it moves downstream. Represented by the velocity magnitude, the vortex strength decreases as the vortex expands. The vortex velocity gradient also decreases since the vortex is diffusing. Discontinuities do occur, however, at the eight-inch plane. As seen in Figure 29, the velocity magnitudes are smaller than those for the six-inch plane, but they are also smaller than those in the ten-inch plane. This data must be incorrect, for the vortex cannot be weaker at eight inches downstream than it is at ten inches. Because of the suspect data, the eight-inch plane is removed from future analysis.

In order to find a point at which the velocity gradient in the flow is great enough to induce noticeable error, additional measurements were taken at planes one-quarter and one-half inch behind the trailing edge. Again, the seven-hole probe exceeds expectations by revealing no such noticeable error. The crossflow plots for the two additional planes are shown in Figures 30a and 30b and are scaled by a factor of ten.

![Crossflow Velocity of Wing Tip Vortex](image-url)
C. Definition of Flow Field Pressure Coefficients

Because pressure measurements are affected by slight shifts in wind tunnel velocity and temperature, pressure coefficients are used when the local and total tunnel pressures are compared and non-dimensionalized by the tunnel dynamic pressure. These three coefficients are defined below:

\[ C_{\text{STATIC}} = \frac{P_{\text{L}} - P_\infty}{P_0 - P_\infty} \]
\[ C_{\text{TOTAL}} = \frac{P_{\text{L}} - P_0}{P_0 - P_\infty} \]
\[ C_{\text{DYN}} = C_{\text{TOTAL}} - C_{\text{STATIC}} = \frac{P_{\text{L}} - P_{\text{ML}}}{P_0 - P_\infty} - 1 \]

Probe measurement errors may be found by determining the correct values for these three pressure coefficients, either theoretically or experimentally with another device, and comparing these values with actual seven-hole probe data. Vortex pressure coefficient trends may be found through analysis of a two-dimensional vortex.

D. \( C_{\text{TOTAL}} \)

For an ideal two-dimensional vortex, angular velocity increases inversely as the distance to the vortex center decreases (Figure 31). Real vortices, however, are subject to viscous effects. Outside of point A, the flow is essentially inviscid. Inside that point, viscous forces reduce angular velocity until it reaches zero at the vortex center.
These two flow regions directly affect $C_{TOTAL}$ values. Because total pressure is constant in incompressible, inviscid flow, $C_{TOTAL}$ is zero in the inviscid region. On the other hand, the viscous forces found within the vortex core decrease the flow's fluid-mechanical energy. $P_0$ decreases causing $C_{TOTAL}$ to become lower. Figure 31 diagrams $C_{TOTAL}$'s expected behavior.
The expected shape was derived by Jonas (Ref. 26) from theoretical considerations of a 2-D vortex decaying with time (Equation 40).

\[ \frac{\partial \psi}{\partial r} = \frac{1}{r} \left( \frac{R}{2\pi} \right)^2 \frac{1}{2\nu t} e^{-r^2/4\nu t} (1-e^{-r^2/4\nu t}) \]  

As discussed, the bucket-shape phenomenon is present in the experimental vortex data shown in Figure 33.
As the flow progresses downstream, lower velocity gradients produce smaller viscous forces, smaller deficits of fluid-mechanical energy, and small bucket-shaped area plots. The two areas of positive $C_{TOT}$ in each of the centerline plots is unexpected. The fact that the magnitude of the positive $C_{TOT}$ values is always greater on the outboard side of the wing than on the inboard side is inexplicable. The sets of contour plots and axonometric plots (Appendix A) show that $C_{TOT}$ is positive only on the two sides of the vortex parallel to the wing. The vortex data behaved normally above and below the wing. In duplicating Jonas's previous positive $C_{TOT}$ results the vortex data seem to indicate a limitation in the seven-hole probe’s measurement capabilities. It is still possible, however, that the positive $C_{TOT}$ values may be the result of a transfer mechanism that is not yet understood.

E. $C_{STAT}$

Like $C_{TOT}$, the presence or absence of viscosity affects the values of $C_{STAT}$. At freestream conditions, $C_{STAT}$ equals zero. Progressing inward from the freestream conditions to point A, the angular velocity increases, static pressure drops, and $C_{STAT}$ decreases from zero to a negative value. To determine what happens in the viscous region, the following mathematical analysis is required.

\[
\frac{dp}{dr} = \frac{\rho u^2}{r}
\]

For a free vortex $u = \frac{c}{r}$ so, \[
\frac{dp}{dr} = \frac{\rho c^2}{r^2}
\]

For solid body rotation $u = \omega r$ so, \[
\frac{dp}{dr} = \rho \omega^2 r
\]
Integrating for $p$ results in the following:

$$P = C_1 - \frac{\mu r^2}{2r^2}$$

(414)

$$P = C_2 + \frac{\omega^2 r^2}{2}$$

(415)

Equations 414 and 415 when superimposed reveal a bucket-shape like that of $C_{TOTAL}$. The entire $C_{STATIC}$ curve is shown in Figure 34.

![Figure 34. VORTEX C_{STATIC}](image)

**Figure 34. VORTEX C_{STATIC}**

**C. DYNAMIC**

$C_{DYNAMIC}$ is simply the subtraction of $C_{STATIC}$ from $C_{TOTAL}$ as shown in Section 2B. Again, at the freestream, $C_{DYNAMIC}$ is simply zero. $C_{DYNAMIC}$ equals the absolute value of $C_{STATIC}$ from the freestream conditions inward to point A since $C_{TOTAL}$ is zero. At the center of the vortex, local velocity and therefore local dynamic pressure are zero causing $C_{DYNAMIC}$ to equal -1. From point A to the center, $C_{DYNAMIC}$ changes as in Figure 35.
Experimental data supports the theoretical analysis of $C_{DYNAMIC}$. 

Figure 35. VORTEX $C_{DYNAMIC}$
This seems somewhat surprising in lieu of the questionable positive C TOTAL values. The important factor missing from the coefficient plots are the relative magnitudes. Because C TOTAL values are very close to zero, the contribution of C TOTAL is not very significant in the calculation of C DYNAMIC.

1. Research Applications

Today's flow mapping efforts are much more complex than the vortex example. Besides Jonas' VATOL flow field measurements, seven-hole probe work at the Air Force Academy has examined canard wakes, lifting surface wakes of canard/swept wing aircraft, and flow field characteristics of square cross-sectional missile bodies. Probe calibration and measurement of unknown flowfields have also been conducted at the NASA Ames Research Center in the 3X4 foot and 14 foot transonic wind tunnels (Ref. 30).

Measurements of wing and canard jet-flap effects as well as the effects of prop fan installations have been made with multiple and single probe installations. Yet, these more complex research efforts are founded on the basic analysis and data presentation explained in the vortex example.

Griffin (Ref. 31) used cross velocity and pressure contours in the study of canard/forward swept wing aircraft. Figures 37 and 38 show the two types of data plots in the same spatial relationship that their data points have to the model.
Jenns (Ref 26) superimposed these two types of plots in his investigation of $C_{\text{TOTAL}}$ (see Figure 39).
Figure 39. REGIONS OF POSITIVE CTOTAL, DATA PLANE ZMP75

Jonas also used a series of axonometric projections to analyze the growth (diffusion) and decay (dissipation) of vortices. Evidence of mixing is found in the relative flattening of the CTOTAL values (negative CTOTAL being up or out of the plane of projection).
Figure 40. AXONNMETRIC PROJECTIONS, C TOTAL, DATA PLANES ZP6 TO ZMP5
Gerner and Durston at NASA Ames took Jonas' data and produced a series of 12 color contour photographs that span the development through decay of the VATOL vortices. The study was limited to the examination of local total pressure or $C_{TOTAL}$. The color contours allow more detail in data representations by making pressure differences easier to see. Regions of positive $C_{TOTAL}$ are clearly distinguished from other points in the flow field. As seen in the other data schemes, the color contours reveal that the VATOL is in a slight sideslip. This causes the right vortex to burst prior to the left vortex.

VII. Conclusions

As shown, the seven-hole pressure probe remains a valuable measurement tool for the documentation of unknown flowfields. The device has a greater measurement range and flexibility than other similar intrusive flow measuring devices. The seven-hole probes themselves are easily constructed and calibrated for use in subsonic compressible flows. Measurements in adverse shear flows can be corrected to give actual flow conditions based on the methodology developed in this paper. Finally, as shown in the application section, the probe is not only a valuable research device but educational as well in demonstrating fundamental flow properties.

VIII. Acknowledgement

The authors would like to acknowledge Lt. Col. Roger Gallington, who developed the methodology for the seven-hole pressure probe. The authors would also like to thank Mr. Don Durston of the NASA Ames Research Center and the faculty and cadets of the USAF Academy who aided in the completion of this effort.
Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i$</td>
<td>the $i$th value of a particular data point either known or determined from calibration equations</td>
</tr>
<tr>
<td>$C_D$</td>
<td>local dynamic pressure coefficient</td>
</tr>
<tr>
<td>$C_{ARM}$</td>
<td>coefficient representative of compressibility effects</td>
</tr>
<tr>
<td>$C_p$</td>
<td>apparent total pressure coefficient</td>
</tr>
<tr>
<td>$C_p'$</td>
<td>coefficient of pressure</td>
</tr>
<tr>
<td>$C_{ij}$</td>
<td>apparent dynamic pressure coefficient</td>
</tr>
<tr>
<td>$C_{STATIC}$</td>
<td>local static pressure coefficient</td>
</tr>
<tr>
<td>$C_{TOTAL}$</td>
<td>local total pressure coefficient</td>
</tr>
<tr>
<td>$C_{\alpha}$</td>
<td>angle of attack pressure coefficient</td>
</tr>
<tr>
<td>$C_{\beta}$</td>
<td>angle of sideslip pressure coefficient</td>
</tr>
<tr>
<td>$C_{\gamma}$</td>
<td>roll angle pressure coefficient</td>
</tr>
<tr>
<td>$C_{\delta}$</td>
<td>pitch angle pressure coefficient</td>
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<tr>
<td>$F$</td>
<td>body geometry function</td>
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<tr>
<td>$K_i$</td>
<td>calibration coefficients</td>
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<td>length</td>
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<tr>
<td>$M$</td>
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<tr>
<td>$P$</td>
<td>pressure</td>
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<tr>
<td>$q$</td>
<td>dynamic pressure</td>
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<tr>
<td>$(r,s,z)$</td>
<td>cylindrical coordinates</td>
</tr>
<tr>
<td>$R$</td>
<td>radius</td>
</tr>
<tr>
<td>$s$</td>
<td>distance between centers of opposing holes</td>
</tr>
<tr>
<td>$u$</td>
<td>local velocity, perturbation velocity component</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity</td>
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<tr>
<td>$(x,y,z)$</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle of attack</td>
</tr>
<tr>
<td>$\beta$</td>
<td>angle of sideslip</td>
</tr>
</tbody>
</table>
\[ \delta = \sqrt{1-M^2} \]

\( \theta \) probe tip half angle

\( \gamma \) ratio of specific heats

\( \varphi \) roll angle

\( \varrho \) density

\( \upsilon \) pitch angle

subscripts

\( u \) uniform incompressible flow

\( (i,j) \) local property or condition

\( o \) total or stagnation condition

freestream conditions

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APPENDIX A

Wing Tip Vortex CTOTAL Plots
2 INCH PLANE