Passive-synthetic apertures were formed with experimental data from the coherent summation of subapertures formed from 1 to 4 hydrophone groups at successive time samples. The results show that the increase in coherent signal gain for synthetic apertures with lengths up to 95λ, when compared to the conventional result, was a function of the number of successive time samples (N_{ts}) described by a \( 10 \log(N_{ts}) \) with \( \alpha \approx 0.7 \). The hydrophones were towed with a speed-to-wavelength ratio of 0.168s^{-1} at a depth of 26λ. A calibrated
moored source was at a depth of $37\lambda$ in 3200 m of water at distance-to-wavelength ratios between 20 and $30 \cdot 10^3$ from the receiver. The stable environment was characterized by a sound speed profile with a negative gradient of $-0.130s^{-1}$ to a sound channel axis of 150 m and a positive gradient of $+0.016s^{-1}$ to a critical depth of 1825 m. The resulting propagation was RR and RSR with losses of between 90 and 100 dB. These results show that for these environmental-acoustic conditions that synthetic apertures can be formed by the coherent summation of phase corrected subaperture beams over successive time samples as long as the synthetic aperture length is less than the single path coherence length and the synthetic aperture processing time is less than the signal's temporal coherence length. The results presented represent an extension of earlier work (R. Fitzgerald, J. Acoust. Soc. Am., 60(3), 752-753 (1976), R. Williams, J. Acoust. Soc. Am., 60(1), 60-73, (1976)) by the demonstration that coherent gain can actually be achieved with resolution such that multipath vertical array angle differences can be resolved.
THE FORMATION OF A SYNTHETIC APERTURE WITH TOWED HYDROPHONES

William M. Carey
Code 113
Naval Ocean Research and Development Activity

N. Yen
Naval Research Laboratory


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The formation of a synthetic aperture with towed hydrophones, W. Carey, NORDA, Arlington, Virginia 22217, N. Yen, NRL, Washington, DC 20375

Passive-synthetic apertures were formed with experimental data from the coherent summation of subapertures formed from 1 to 4 hydrophone groups at successive time samples. The results show that the increase in coherent signal gain for synthetic apertures with lengths up to 95λ when compared to conventional results was a function of the number of time samples (Nts) described by $10 \log(N_{ts})$ with $\alpha = 0.7$. The hydrophones were towed with a speed-to-wavelength ratio of $0.168 \text{s}^{-1}$ at a depth of $26\lambda$. A calibrated moored source was at a depth of $37\lambda$ in 3200 m of water at distance-to-wavelength ratios between 20 and 30 $10^3$ from the receiver. The stable environment was characterized by a sound speed profile with a negative gradient of $-0.130 \text{s}^{-1}$ to a sound channel axis of 150 m and a positive gradient of $+0.016 \text{s}^{-1}$ to a critical depth of 1825 m. The resulting propagation was RR and RSR with losses of between 90 and 100 dB. These results show that for these environmental-acoustic conditions, that synthetic apertures can be formed by the coherent summation of phase corrected subaperture beams over successive time samples as long as the synthetic aperture length is less than the single path coherence length and the synthetic aperture processing time is less than the signal's temporal coherence length. The results presented represent an extension of earlier work (R. Fitzgerald, J. Acoust. Soc. Am., 60(3), 752-753 (1976), R. Williams, J. Acoust. Soc. Am., 60(1), 60-73, (1976)) by the demonstration that coherent gain can actually be achieved with resolution such that multipath vertical arrival angle differences can be resolved.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>TITLE SLIDE</td>
<td>2</td>
</tr>
<tr>
<td>OVERVIEW</td>
<td>3</td>
</tr>
<tr>
<td>BASIC TECHNICAL ISSUES</td>
<td>4</td>
</tr>
<tr>
<td>DIRECTIONAL SPATIAL CHARACTERISTICS</td>
<td>5</td>
</tr>
<tr>
<td>SUPERDIRECTIVE SOUND SOURCE DETERMINATION</td>
<td>6</td>
</tr>
<tr>
<td>HIGH RESOLUTION SPATIAL PROCESSING TECHNIQUES</td>
<td>7</td>
</tr>
<tr>
<td>THE PROBLEM OF PHASE DETERMINATION</td>
<td>8</td>
</tr>
<tr>
<td>THE BEAM PATTERN OF A MOVING ARRAY</td>
<td>9</td>
</tr>
<tr>
<td>THE DETERMINISTIC PLANE WAVE RESPONSE OF A SYNTHETIC APERTURE</td>
<td>10</td>
</tr>
<tr>
<td>PROCESSING OF ARRAY DATA</td>
<td>11</td>
</tr>
<tr>
<td>PROCESSING PARAMETERS AND DATA (ARRAY DEPTH VARIATIONS)</td>
<td>12</td>
</tr>
<tr>
<td>SIGNAL AND NOISE VERSUS TIME</td>
<td>13</td>
</tr>
<tr>
<td>MEASURED DATA, SIGNAL HYDROPHONE GROUP RESULTS</td>
<td>14</td>
</tr>
<tr>
<td>DUAL HYDROPHONE GROUP RESULTS</td>
<td>15</td>
</tr>
<tr>
<td>FOUR HYDROPHONE GROUP RESULTS</td>
<td>16</td>
</tr>
<tr>
<td>EIGHT HYDROPHONE GROUP RESULTS</td>
<td>17</td>
</tr>
<tr>
<td>SIXTEEN HYDROPHONE GROUP RESULTS</td>
<td>18</td>
</tr>
<tr>
<td>BEARING ERROR FOR THE SYNTHETIC APERTURE RESULTS</td>
<td>19</td>
</tr>
<tr>
<td>A BASIS FOR SYNTHETIC APERTURE COMPARISON</td>
<td>20</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>21</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>22</td>
</tr>
</tbody>
</table>
THE FORMATION OF A SYNTHETIC APERTURE
WITH TOWED HYDROPHONES

WILLIAM M. CAREY       NORDA

N. YEN                  NRL
OVERVIEW

This paper presents experimental results obtained in the deep ocean which demonstrate the formation of synthetic apertures when the source-receiver motion is known or inferred by a velocity filter, when the synthetic aperture length is less than the signal coherence length, and when the processing time is less than the signal decorrelation time.

Synthetic apertures have been used in the field of radar since 1953. A monograph of classic papers in radar has been published by Kovaly. Early attempts at the formation of synthetics in underwater acoustics concentrated on kHz frequencies and active systems. Sam Hanish reviewed this field in 1975 and Louis J. Cutrona published a paper in August of 1975 which discussed active sonar system design. In 1976 observations by Ross Williams and R. Fitzgerald based on single element phase records led to the speculation and conclusions that passive synthetic apertures could be formed. The Fitzgerald letter, Vol 60, #3, 1976, focused on a 10 Hz (150 m wavelength) signal over a distance of 65 km ($R/\lambda = 6.5 \times 10^3$) and the inference that apertures up to 3300 m ($L/\lambda = 22$) could be formed. This conclusion was not demonstrated by coherent summation of the signals or the measurement of signal gain.

Ross Williams in his 1976 paper (Vol 60, #1) discussed phase versus time records obtained by a moored sensor and a moving source for distances up to 508 km ($R/\lambda = 10^4-10^5$) at a frequency of 400 Hz. He was able to find a linear trend between the average phase and time for several intervals during which he speculated that apertures up to 900 m (240 $L/\lambda$) could be formed. The effects of phase fluctuation and sensitivity of the technique to source-receiver motion were discussed by the author but were not incorporated into an estimate of coherent gain.

The motivation behind this paper was to investigate the use of synthetic aperture processing in the lower frequency range (<500 Hz) as a high resolution processing technique. The genesis of this work was in the earlier work by Yen on ambient noise directionality (1977, 62, #5) and on experimental data which I obtained at sea. This paper, although it confirms and demonstrates the conclusion of Fitzgerald and Williams, has the additional feature insofar as the synthetic aperture approach acts as a velocity filter and can resolve sources proportional to the number of time samples. The paper will present some simplified vugraphs on high resolution techniques to orient us on the same basis. Then the implementation of the technique will be presented, followed by experimental realization of the synthetic aperture.
ISSUES ADDRESSED IN THIS PAPER:

- CAN THE COHERENCE LENGTH OF A SOUND FIELD BE WHOLLY UTILIZED WITH A TOWED ARRAY?

- CAN A "SHORT" LOW FREQUENCY TOWED ARRAY BE OPERATED WITH HIGH ANGULAR RESOLUTION?

- CAN A SIMPLE PROCESSING TECHNIQUE BE UTILIZED TO IMPROVE CONVENTIONAL ARRAY PROCESSING GAIN AND RESOLUTION?
Basic issues addressed in this paper

The ability to form a synthetic aperture in the deep ocean depends on the spatial coherence and stationarity of the signal field. Thus key questions can be posed concerning the formation of a synthetic aperture. Can the horizontal coherence length of the sound field be used to form a synthetic aperture with a towed hydrophone? Does such a synthetic aperture produce high angular resolution in a multipath field? Can a simple processing technique be utilized to form a synthetic aperture? These are the basic questions discussed in this paper.
OBSERVATION OF SPATIAL CHARACTERISTICS
OF AN ACOUSTIC FIELD

MEASUREMENTS OF DIRECTION WITH A LINE ARRAY:
- Require more than one sensor
- Depend on sensors' separation distance
- Have ambiguity problems

\[ d = \frac{\lambda}{4} \quad d = \frac{\lambda}{2} \quad d = \lambda \]
Directional spatial characteristics

Before I present our experimental results I would like to present, in the next several slides, a few basic concepts in order that this work may be placed in perspective. Directionality may be obtained by a pair of sensors by changing spacing, $d$, from the case $\lambda > d$, omnidirectional, to $d \sim \lambda$ directional. Further these sensors may be placed in an array such that $L = (n-1)\lambda/2$ and the resolution is given by $\Delta \theta \sim \pi (L/\lambda)$. This conventional approach for obtaining angular resolution requires more than one sensor, depends on the separation distance between the sensors, the array length and has a left/right ambiguity problem. The performance of a line array whether discrete or continuous can be improved by employing high resolution techniques which will be discussed later. However an important consideration is that with $N$ sensors one may only resolve $N$ discrete sources, at fundamental limitation.
reviewed by Harris\textsuperscript{11} for discrete Fourier Transforms for the cases of harmonic signals in broadband noise and in the presence of interfering harmonic interference. Kay and Marple have also considered the case of windows and their application to spectrum analysis. An alternative approach to the problem of noise rejection is to minimize the noise interference while keeping the maximum response in the signal direction, thereby reducing the noise and enhancing the desired signal.\textsuperscript{13} Several techniques such as optimum beamforming, adaptive beamforming, superdirective arrays,\textsuperscript{14} minimum variance beamforming, maximum likelihood estimation have also been developed.\textsuperscript{8} For the application to line arrays these techniques have the drawback that they are sensitive to the type of noise interference and are computationally extensive, such as found in experimental data.

An elegant approach adopted to describe the array's output is to express the measured results by a series of coherent components, eigenvalues or eigenvectors.\textsuperscript{8} These techniques are placed in the general category of orthogonal function representation. These techniques have a problem, in some instances, in the interpretation of the results. In particular, the interpretation of the eigenvalues may be very difficult when one has a partially coherent noise field or a multipath signal field such as found in experimental data.

High resolution techniques used to improve spectral estimates\textsuperscript{12} are analogous to the line array problem and have direct applicability. The Fourier transformations and shading techniques have been previously discussed. We now turn attention to those remaining techniques which are classified as Analytic Continuation Techniques, such as maximum entropy and linear predictive, which have successfully been applied to line array processing.\textsuperscript{8} The analytic continuation techniques basically extrapolate the estimate of noise outside the temporal or spatial processing window. The primary differences among these techniques is the method employed in the extrapolation of the knowledge of the noise. The minimum cross-spectral estimates use prior information or knowledge of the noise field and its characteristics concerning its stationarity. The techniques are applicable in those instances when one has a noise field which is wide sense stationary.

The synthetic aperture technique presented in this paper represents what we call a time-space expansion technique which requires a stationary signal. The advantage of this technique is the determination of the temporal and spatial stationarity of the signal plus the use of $N$ time frames means the resolution of capability $(N-1)$ discrete sources.
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SOURCE \[ S_0 = s_0 e^{i(\omega t - k r)} \]

SENSOR 1 \[ R_1 = s_1 e^{i(\omega t - k r_1)} + n_1 \]

SENSOR 2 \[ R_2 = s_2 e^{i(\omega t - k r_2)} + n_2 \]

CROSS PRODUCT
\[ R_1 R_2^* = S e^{i k d \cos \theta} + N e^{i \phi} \]

WHERE \[ A = S \left[ 1 + (N/S)^2 + 2(N/S) \cos (\psi - \phi) \right]^5 \]

\[ \tan \psi + (N/S) \sin \phi \cos \psi \]

\[ \alpha = \arctan \left( \frac{1 + \tan \psi (N/S) \cos \phi \sin \psi}{\tan \phi} \right) \]

\[ \psi = k d \cos \theta \]

\[ d = \text{SENSOR SEPARATION DISTANCE} \]

A. Fundamental parameters for the determination of source direction.

B. Phase angle probability density distribution for a sine wave mixed with Gaussian noise.

The problem of the determination of source direction from the measure of phase of a sinusoidal signal in a Gaussian background.
The problem of phase determination

The success of the synthetic aperture techniques will depend on the utilization of phase information. Shown here are curves taken from DeVilbiss, Martin and Yen\textsuperscript{15} which indicate the degree of uncertainty in the measurement of phase of a sine wave in a Gaussian noise background by either a single sensor or pairwise sensors as a function of time. Observe that the uncertainty decreases as a function of $(S/N)$, however between 0 and 3 dB, the width of the probability density curves are on the order of 30°. This variance represents a high frequency jitter as the measured phase versus time record such as observed by Ross Williams and R. Fitzgerald.
THE BEAM PATTERN OF A MOVING ARRAY

sensor spacing = $1/3 \lambda$, aperture length = $2 1/3 \lambda$

ARRAY
(8 ELEMENT ARRAY)

SYNTHETIC
SAMPLED AT $2\lambda/\nu$ sec

COMPOSITE

NOTE ALIASED RESPONSE
The beam pattern of a moving array

This slide illustrates the sub-aperture approach adopted in this paper. The sub-aperture arrays are composed of 1/3 wavelength hydrophone spacings. In this example, eight hydrophones have been used to create a sub-aperture 2 1/3 wavelengths long. The beam response is shown for this eight element array. At successive time samples, approximately \( 2\lambda = \frac{V}{R} \Delta t = \frac{V}{R} (t_1 - t_2) \), each sub-array is beamformed. The beam response for an array with omnidirectional sensors at \( t_1, t_2, t_3 \) etc., is that of an array with \( 2\lambda \) element spacings as shown under the heading synthetic. The composite of the two responses is via the product theorem shown at the right. Note that aliasing lobes appear in the sparse-2\( \lambda \)-array beam response pattern.
THE DETERMINISTIC PLANE WAVE RESPONSE OF A SYNTHETIC APERTURE

LINEAR ARRAY: \[ b(k, \theta, t_0) = \sum_{n=1}^{N} s_n e^{j(\omega t_0 - k \cdot r_n)} \cdot A_n e^{jk\pi n d \cos(\theta)} \]

SYNTHETIC APERTURE: \[ B(k, t_0, t_0 + M \Delta t) = \left[ b(k, \theta, t_0) \right] \sum_{m=0}^{M-1} e^{j(\omega m \Delta t - k \cdot V m \Delta t - j \phi_m)} \]

LIMIT ON PERFORMANCE: \[ \lim_{M \to \infty} \sum_{m=0}^{M} e^{j(\omega m \Delta t - k \cdot V m \Delta t - j \phi_m)} \approx \int_{0}^{\infty} d \mu e^{j(\mu)^m} \]

\[ B(\omega, \theta, k) \approx A \cdot S(\omega, \theta) \]

VELOCITY DEPENDENCE: \[ \cos \theta_s = \cos \theta_o \pm \frac{\Delta V}{V} \cos \theta_o \pm \frac{\Delta \omega}{W_s} \cos \theta_o \mp \frac{\Delta \omega}{W_s} c / V_s \]
The deterministic plane wave response of a synthetic aperture

A rigorous treatment of the effect of statical fluctuations on the response of antennas may be found in Shifrin.\textsuperscript{24} For the purposes of this paper we choose not to discuss the importance of phase, amplitude, and system shape fluctuations but rather to consider the deterministic plane wave response of a synthetic aperture. Shown in this slide are the classical expressions for the plane wave response of the sub-aperture for a plane wave of amplitude $s_0$, angular frequency $\omega$ and wave number $K$. The first equation shows the amplitude shading and phase steering of the array to $\theta_s$. By changing the phase steering as a function $\theta_s$ one obtains the azimuthal response of the array. The formation of the synthetic aperture is accomplished by adjusting the beamformed output of the linear array by a factor $\phi_m$ which if all values are known is equal to $\omega_m\Delta t - \mathbf{K} \cdot \mathbf{V}_m\Delta t$. Where $m\cdot\Delta t$ is the number of time intervals. $\mathbf{K}$ is the wave number vector and $\mathbf{V}$ is the relative velocity. The term $\omega_m\Delta t$ accounts for the phase due to the time difference between $t_1$ and $t_{m+1}$. The term $\mathbf{K} \cdot \mathbf{V}_m\Delta t$ accounts for the spatial difference $V_m\Delta t = \Delta L$ and the direction of the wave vector $\mathbf{K}$. In the limit of perfect stationarity the plane wave response is the weighted source function. Also shown is the effect of velocity and frequency errors.

The formation of the synthetic aperture is thus seen to be straightforward provided the signal is stationary in space and time and that the relative velocity be known or measured. Narrowband analysis results in $\Delta \omega \to 0$ hence we are going to be driven by the uncertainty in velocity $\Delta V$ and our estimation of phase $\phi_m$. The form of the velocity error equation shows that the uncertainty in the velocity results in a missteering of the beam response and a decrease in $|B|$, hence a velocity filter characteristic is expected.
PROCESSING OF ARRAY DATA

CONVENTIONAL

SYNTHETIC

ARRAY OUTPUT

INTEGRATION AVERAGE

MAGNITUDE SQUARE

COMPLEX SUM BEAMFORMING

FFT

SENSORS OUTPUT

ARRAY OUTPUT

INTEGRATION AVERAGE

MAGNITUDE SQUARE

COMPLEX SUM BEAMFORMING

FFT

SENSORS OUTPUT

TIME SEQ. COH. SUM

PHASE CORRECTION

TOWING SPEED
Processing of array data

The implementation of the foregoing analysis is shown on the flow diagram in this slide. A comparison between the conventional and synthetic aperture diagrams shows that the computational difficulty or load is the same. The determination of tow speed, measured or inferred from phase versus time plots or from the subaperture; is shown as the input to coherent synthetic array beamformer.
A. Depth versus time.

B. Depth differential versus time.

Array depth and depth differential between fore and aft sensors versus time.
Processing parameters and data

The next several slides contain information concerning the conditions under which this experiment was conducted. The parameters have been expressed in dimensionless parameters but should be representative for those cases where sea surface scattering and bottom scattering are not dominant. The following slide shows the depth variations of the whole seismic streamer. The depth is seen to vary with a high-frequency RMS variation on the order of 1 m and a lower frequency modulation up to 8 m. During periods of time when motion and deformation are important, the synthetic aperture technique obviously will not work.

PROCESSING PARAMETERS FOR THE DATA MEASURED IN THE OCEAN ENVIRONMENT

- **ARRAY**
  
  HYDROPHONE SPACING: \( d = \frac{1}{3} \lambda \)
  
  SUB-APERTURE LENGTHS: \( \frac{1}{3} \leq \frac{L}{\lambda} \leq 6 \)
  
  ARRAY DEPTH: \( \frac{D}{\lambda} = 26 \)

- **TOW SHIP**
  
  CONSTANT SPEED: \( V \Delta \tau = 2 \lambda \)
  
  \( V/\lambda = 0.168 \ S^{-1} \)

- **SOURCE**
  
  CW MOORED SOURCE DEPTH: \( \frac{D}{\lambda} = 37 \)

- **NARROW BAND ANALYSIS**: \( BW \times \Delta \tau = 1 \)

- **RANGE**: \( \frac{R}{\lambda} = 20 - 30 \times 10^3 \)
The mean hydrophone signal plus noise and noise level (dB re $\mu$Pa) versus time in a 0.125 Hz band.
Signal and noise versus time

The data obtained in this experiment covered a four hour period. The received signal versus time for a hydrophone group is shown in this slide. Both high and low signal to noise ratios were employed and comparisons made over the four hour period. The data shown in this paper as examples are taken from the beginning of the record.
A. Synthetic aperture (synthetic aperture length \( L_e/\lambda = 94 \)) formed with a single hydrophone group (subaperture length \( L_{SA}/\lambda = 0.30 \)) from 42 consecutive time frames. The differential array signal gain (DASG) is -0.2 dB.

B. Synthetic aperture \( (L_e/\lambda = 125) \) formed with a single hydrophone group \( (L_{SA}/\lambda = 0.30) \) from 56 consecutive time frames. The differential array signal gain is -0.2 dB.

Results are shown for synthetic apertures formed with a single hydrophone group from a) 42 and b) 56 consecutive time frames 8 seconds in duration at 13 second intervals. (Relative levels may be converted to band levels (dB re \( 1\mu Pa \)) by the addition of 109.)
**Measured data**

The next five slides show results for synthetic apertures formed with sub-aperture containing 1, 2, 4, 8, and 16 hydrophone groups. These slides are self-explanatory and I will simply point out the interesting features.

The first slide shows the results of a synthetic aperture formed from a single hydrophone group over 42 and 56 consecutive time frames using phase corrections determined from the array relative velocity and time interval data. The top result shows multipath resolution and the expected aliased response. The longer aperture does not show the same response as the increase in time length from 42-56 has caused us to exceed the decorrelation time. For this reason we restrict ourselves to 42 time frames or less.

The next several slides will show a comparison of the conventional beam formed results and the synthetic aperture results. The 2-group and 4-group results still show aliased lobes but show a clear resolution of multipaths and a better peak to side lobe level. The 8-group results show a reduction in the aliased lobe level due to the side lobe level response of the 8-group sub-aperture and a very clear resolution of the multipath as well as an improved peak-to-side-lobe level ratio. Finally this result is further enhanced by the use of 16 groups to form a sub-aperture. The impressive point to be made is that these results are obtained on ocean-acoustic data by simply using a good estimate of tow velocity and by driving the ship carefully.
A. The response of an array formed from two hydrophone groups over a single time frame shows little directionality.

B. The results of a synthetic aperture \( \left( \frac{L_S}{\lambda} = 94 \right) \) formed from sub-apertures \( \left( \frac{L_{SA}}{\lambda} = 0.6 \right) \) with two hydrophone groups over 42 successive time frames show multipath resolution and aliased response.

Results of a synthetic aperture formed from sub-apertures with two hydrophone groups.
A. The response of an array formed from four hydrophone groups ($L_{SA}/\lambda = 1.2$) shows no pronounced directional response.

B. The response of a synthetic aperture ($L_{S}/\lambda = 23$) formed from sub-apertures with four hydrophone groups over 10 successive time frames shows source resolution.

C. The response of a synthetic aperture ($L_{S}/\lambda = 95$) formed from sub-apertures with four hydrophone groups over 42 successive time frames shows both source direction and resolution of multipath arrivals. The aliased response is also observed at 75°.

Results of a synthetic aperture formed from sub-apertures with four hydrophone groups.
A. The response of a synthetic array formed from sub-apertures with eight hydrophone groups ($L_s/\lambda = 2.33$) over a single time frame shows no pronounced directional response.

B. The response of a synthetic aperture ($L_s/\lambda = 25$) formed from sub-apertures with eight hydrophone groups over ten successive time intervals shows resolution of source direction. The aliased response is not observed.

C. The response of a synthetic aperture ($L_s/\lambda = 96$) formed from sub-apertures with eight hydrophone groups over forty-two successive time intervals shows resolution of source directions and multipath arrivals. The aliased response is not observed.

Results for a synthetic aperture formed from sub-apertures with eight hydrophone groups.
A. The response of an array formed from sixteen hydrophone groups \((L_{SA}/\lambda = 5)\) over a single time frame. A broad maximum is observed in the direction of the source.

B. The response of a synthetic aperture \((L_{S}/\lambda = 27)\) formed from sub-apertures with sixteen hydrophone groups over ten successive time intervals shows source resolution and no aliased response, but does not resolve multipaths.

C. The response of a synthetic aperture \((L_{S}/\lambda = 98)\) formed from sub-apertures with sixteen hydrophone groups over forty-two successive time intervals shows source and multipath resolution.

Results for a synthetic aperture formed from sub-apertures with sixteen hydrophone groups.
Bearing error for the Synthetic Results
Bearing error for the synthetic aperture results

We have mentioned previously that an error in velocity causes a missteering and reduction in response to a given source. These effects are shown in this slide. The ±10% error in velocity is seen to produce a reduced beam response which is missteered and reduced in amplitude. These results are obtained on the previously shown data set with a deliberate error introduced. Thus the velocity determines the response and acts as a filter. We have tuned this filter to our moored source based on the relative velocity and in so doing have "detuned" our system to other sources of acoustic energy with different velocities. This is the reason that we observe an increase in the peak-to-side-lobe ratio as aliasing lobes were reduced.
A BASIS FOR SYNTHETIC APERTURE COMPARISON

CONVENTIONAL: \[ BSL_c = 10 \log \left( \sum_{i=1}^{N_{TF}} b_i^2 \right) + CF - 20 \log (N_{SA}) - 10 \log (N_{TF}) \]

SYNTHETIC: \[ BSL_s = 10 \log \left( \left( \sum_{i=1}^{N_{TF}} b_i \right)^2 \right) + CF - 20 \log (N_{SA}) - 10 \log (N_{TF}) \]

THE DIFFERENCE: \[ \Delta = BSL_s - BSL_c = \alpha \cdot 10 \cdot \log (N_{TF}) \]

THE RESULTS: \[
\begin{array}{cccc}
N_{SA} & N_{TF} & 10 / (L/\lambda)_{10} & N_{TF} = 42 / (L_s/\lambda)_{42} \\
1 & - & 0.8 & 94 \\
2 & - & 0.7 & 94 \\
4 & 0.8 & 23 & 0.7 & 95 \\
8 & 0.7 & 25 & 0.7 & 96 \\
10 & 0.7 & 27 & 0.75 & 98 \\
\end{array}
\]

AVERAGE VALUES: \[ \alpha = 0.75 \pm 0.3, \ BW \sim 58^\circ / L_s / \lambda \]
A basis for a synthetic-conventional array comparison

The attractiveness of synthetic aperture processing can be determined by a comparison of the coherent gain achieved with a conventional and synthetic array. To perform this comparison we basically determine the beam-signal-level (BSL_c) for the conventional processing and with the equivalent space and time data the synthetic BSLs. As shown on this slide, the processing simply represents an incoherent average for the BSL_c and a coherent average for the BSLs. If coherent gain resulted in synthetic approach then the difference between the two beam signal levels would be 10 Log (N_{tf}). As shown on the slide we simply determine this difference (Δ) from measured data which when divided by the 10 Log (N_{tf}) produces a coefficient α, a measure of the actual gain. We find the average difference to be 7.5 Log (N_{tf}) or an α factor of 0.75 ±0.3 over a four hour period. The ±0.3 uncertainty results from errors introduced by using the average relative velocity. We feel this uncertainty could be reduced by using a more reliable velocity estimate. The average measured beam response widths were found to be proportionate to 58°/(L/λ).
CONCLUSIONS:

- FOR THOSE CASES WHEN $L_s \leq L_c$ AND $t_s \leq t_c$ SYNTHETIC APERTURE PROCESSING IS POSSIBLE UP TO RANGES OF $R/\lambda = 10^4$ AND $L/\lambda = 95$.

- THE SYNTHETIC APERTURE ACTS AS A VELOCITY FILTER.

- SYNTHETIC APERTURES ARE BASED ON THE COHERENT SUMMATION OF $M$ INDEPENDENT SAMPLES AND THUS CAN RESOLVE $M-1$ SOURCES.

- THE RESOLUTION OBTAINED WITH A SYNTHETIC APERTURE IS $\sim \Delta \theta = 58^\circ / L/\lambda$ WHEN $L_s \leq L_c$ AND $\Delta t_s \leq t_c$. 
Conclusions

This slide enumerates the conclusions which we have drawn from the experimental information presented in this paper. When the synthetic aperture processing time and equivalent-spatial length are less than the temporal and spatial coherence lengths determined by the source-receiver motion and propagation effects then a synthetic aperture may be formed.

The synthetic aperture discussed here is a strong function of the source-receiver relative velocity. The phase correction required to form the synthetic aperture requires an estimate of this velocity. The error in the estimate or the output of the synthetic aperture versus relative velocity represents a filtering effect. We term this as a velocity filter. The velocity filter characteristics allow one to tune in on a specific source of a given relative velocity at the expense of other sources at different velocities.

The key feature which a synthetic aperture possesses is the resolution of M-1 sources based on an aperture formed from M time frames or samples. This is true insofar as each successive time sample represents an independent sample of the acoustic field. This feature contrasts with the results obtained with other high resolution techniques. This paper has measured the "coherent" signal gain achievable with a synthetic aperture by comparison of beam signal levels between a conventional and synthetic beamformer. The resolution, however, was simply determined from the synthetic-beam response to be ~58°/(L/λ) for the previously mentioned stationarity requirements.
References


