FLUID LAYER BETWEEN INFINITE ELASTIC PLATES II
DISTRIBUTION OF POWER FLOW

J. H. JAMES
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II. DISTRIBUTION OF POWER FLOW

BY

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Summary
Formulae are given which, together with formulae contained in Part I, enable numerical evaluation of the separate power flows in the plates and fluid due to line-force and line-source excitation. Plots of power flow show significant interchange of energy between plates and fluid. They are complimentary to the acoustic intensity vector plots of Part I.
1. INTRODUCTION

A previous report [1] has given the formulae necessary for evaluating numerically the pressure and displacements, due to line-force and line-source excitation, of a system which comprises of two identical steel plates separated by a layer of fluid. Plots of intensity vectors, in a 20cm layer of water bounded by 1cm steel plates, showed significant interchange of energy between the fluid and the driven plate when the excitation was a line-force, but not when the excitation was a source. One of the recommendations of the report was that the distribution of the power flow between plates and fluid should be evaluated as a function of the horizontal distance from the excitation. This is the main aim of the present report.

The motivation behind this suite of reports is the need to understand the physics of the interchange of energy between a pipe's wall and its contained fluid. James [2] gives intensity vector plots which show this effect in a steel pipe containing water, and Fuller [3] gives some formulae which enable the distribution of power flow between the pipe's wall and the fluid to be calculated, at large distances from the source. The distribution of power varies with distance due to the non-orthogonality of the system modes over the fluid's cross-section. At large distances from the source, it is the interaction of predominantly 'fluid' modes with predominantly 'elastic' modes which is responsible for the interchange of energy, while close to the source, interaction among evanescent wave-fields may cause intense energy circulation between the fluid and wall.

2. MATHEMATICS OF PROBLEM

The pressure and displacements of the system shown in Figure 1 are represented by the Fourier integrals

\[
\begin{bmatrix}
W_1(x) \\
W_2(x) \\
p(x, z) \\
w_x(x, z)
\end{bmatrix} = \frac{1}{1/2\pi} \int \begin{bmatrix}
\bar{W}_1(\alpha) \\
\bar{W}_2(\alpha) \\
\bar{P}(\alpha, z) \\
\bar{W}_x(\alpha, z)
\end{bmatrix} \exp(\text{i} \omega x) d\alpha
\]

(1)

in which \(W_1(x)\) and \(W_2(x)\) are the displacements of the upper and lower plates respectively, \(p(x, z)\) is the acoustic pressure in the fluid layer, \(w_x(x, z)\) is the horizontal acoustic particle displacement. The time-harmonic factor, \(\exp(-\text{i} \omega t)\), is omitted throughout. Formulae for the transforms \(\bar{W}_1(\alpha)\) etc. are given elsewhere [1].

With reference to Figure 1B, which shows a sign convention for the positive directions of shear forces and moments, the power per unit length flowing in the positive x-direction of each plate is the sum of the two terms.
\[ P_N(x) = (1/2) \text{Real}[W(x) - \partial W^*(x)/\partial x] \]

and
\[ P_S(x) = (1/2) \text{Real}[-S(x) W^*(x)] \]

in each plate. \( P_N(x) \) and \( P_S(x) \) are the powers transmitted by the bending moments and shearing forces, respectively; \( W(x) \) is the normal velocity of the plate, \(-iW(x)\); and the symbol \( * \) denotes complex conjugate. The bending moment and the shearing stress per unit length are simply

\[ W(x) = D \cdot d^2 W(x)/dx^2 \]
\[ S(x) = -D \cdot d^3 W(x)/dx^3 \]

where \( D \) is the plate's flexural rigidity. The following Fourier integrals are obtained from equations (1) and (3)

\[
\begin{bmatrix}
W(x) \\
dW(x)/dx \\
W(x) \\
S(x)
\end{bmatrix} = (1/2\pi) \int_{-\infty}^{\infty} \begin{bmatrix}
\hat{W}(\alpha) \\
1D\alpha \hat{W}(\alpha) \\
-D\alpha^2 \hat{W}(\alpha) \\
1D\alpha^3 \hat{W}(\alpha)
\end{bmatrix} \exp(i\alpha x) d\alpha
\]

The power flow per unit length in the fluid, in the positive \( x \)-direction, is the integral of the horizontal component of the acoustic intensity over the fluid's cross-section. It is

\[ P_F(x) = (1/2) \int_0^\infty \text{Real}[p(x,z) \tilde{W}_x(x,z)] dz \]

In the case of line-force excitation, \( P_S(x) \), a check on the numerical results is possible by use of the formula

\[ P_{M1}(x) + P_{S1}(x) + P_{M2}(x) + P_{S2}(x) + P_F(x) = (1/4) \text{Real}[P_2 W^*(0)] \]

which is only valid close to \( x=0 \), because dissipation is included in the system. Equation (6) reflects the fact that the input power of the line-force excited plate flows equally in the positive and negative directions.

3. NUMERICAL RESULTS

The Fourier integrals of equation (4), with upper limits of integration set to approximately twice the highest free-wavenumber at the selected frequency, were evaluated by a simple adaptive Gaussian quadrature scheme. Because the integrals must be evaluated in the principal value sense, it is necessary to introduce damping into the system via a complex Young's modulus, \( E(1-i\eta) \), and a complex sound velocity, \( c(1-i\eta) \). The
integral in equation (5) was evaluated by Simpson's rule, from the 21
values of \( p(x,z) \) and \( W_x(x,z) \) that were obtained at each of 60 horizontal
x-stations. At \( x=0 \), the power transmitted by the shearing force, \( S(x) \),
cannot be evaluated numerically, but evaluation proceeds with increasing
accuracy as \( x \) increases; the power transmitted by the bending moment is
zero by virtue of \( dW/dx \) vanishing; and, the acoustic power flow must be
zero because \( W_x \) vanishes. Subject to the accuracy of the computations,
the consistency test of equation (6) was satisfied by the numerical results
shown herein. The following constants in SI units were used to obtain
Figures 2-5:

\[
\begin{align*}
    E &= 19.5 \times 10^3, \\
    \sigma &= 0.29, \\
    \rho_s &= 7700.0, \\
    \rho &= 1000.0, \\
    c &= 1500.0, \\
    \eta_x &= 0.02, \\
    \eta_f &= 0.001.
\end{align*}
\]

The dispersion plots of the dissipation-free system are shown in
Figure 2. The near identical branches labelled 1 and 2 are symmetric and
antisymmetric 'plate' waves whose energy is mostly confined to the plates,
their group velocity being approximately twice their phase velocity. The
branches labelled 3-5 are predominantly 'fluid' waves in which the phase
and group velocities at cut-on are infinity and zero, respectively.

In Figures 3-5 the percentages of the total power flow, in the
positive x-direction, contained in the plates and fluid is given as a
function of distance from the excitation. Power is not conserved because
of finite values of the loss-factors, but the attenuation is small -
in fact, not more than 1dB in the range \( x=0.4 \) to 0.8m. The power in the
top plate is very small compared with the power in the bottom plate, as
can be inferred from the intensity vector plots [1].

The power flow due to a line-force excitation is shown in Figures
3 and 4. At 1kHz the individual power flows settle down to constant values
due to the absence of 'fluid' waves, the power in the fluid being less than
10% of the total. At 4, 7 and 10kHz the power levels away from the source
oscillate considerably due to interaction between 'plate' waves (1,2) whose
wavelengths are almost equal, and 'fluid' waves (3,4,5) of differing wave-
lengths. At 4kHz the power levels oscillate almost sinusoidally because
there is only a single 'fluid' wave, the level in the fluid ranging from a
minimum of 10% to a maximum of 30%; at 7kHz the presence of two 'fluid'
waves causes complex beating in which the level varies from 15% to 45%;
and, at 10kHz, where three 'fluid' waves have cut-on, the power in the
fluid ranges from 15% to 50%. In the aforementioned plots, both the fluid
and plate power flows are positive everywhere. However, the high-amplitude
evanescent wave-fields near to cut-on frequencies sometimes superpose local
circulatory energy flows between fluid and plate which result in net local
power flows in the fluid which may be negative. Some plots obtained at
2 and 5kHz, but not included here, show this effect.

Intense circulating energy flows between plates and fluid have been
illustrated elsewhere [1] in a plot of intensity vectors due to a line-
source excitation at 1kHz. The power flow is shown in the top of Figure 5.
The local power flow in the fluid ranges from flow in the negative direction
of 180% to flow in the positive direction of 66%, while the power in the
plate ranges from 280% to 34% of the net power flow. At distances greater
than 0.3m from the source, the levels have settled down to the same constant percentages as were obtained from line-force excitation. At frequencies of 4, 7 and 10kHz, the power in the plates due to line-source excitation is very small; hence, only the plot at 4kHz is shown, in the lower half of Figure 5.

4. CONCLUDING REMARKS

Formulæ have been given which enable numerical evaluation of the separate power flows in the plates and fluid. Plots of power flow have been presented for the case of a 20cm layer of water bounded by 1cm steel plates. They add quantitative information to the qualitative information contained in the intensity vector plots given elsewhere [1]. When the excitation is a line-force the plots (a) show that most of the power flow resides in the plates; (b) help to confirm that the energy interchange is caused by an interaction between 'plate' and 'fluid' waves; (c) show that energy interchange may be significant — up to 35% of the total at 10kHz. When the excitation is a line-source, the plots (a) show negative power flow close to the source, due to intense circulating energy flow caused by interaction among evanescent waves; (b) show negligible power in the plates when 'fluid' waves have cut-on.

Future reports will extend the range of material and geometric constants, and, in particular will (a) show plots at frequencies close to the cut-on frequencies of 'fluid' waves and (b) investigate energy interchange when the distinction between fluid and plate waves is less clear. Also to be investigated is the effect of line-constraints.

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REFERENCES


3. FULLER C.R., Monopole excitation of vibrations in an infinite cylindrical elastic shell filled with fluid. Private communication from author. To be published.
FIG. 1 GEOMETRY AND SIGN CONVENTION
Two 1 cm steel plates
Separated by 20 cm water

Algorithm is deficient when roots are very close

Figure 2: Dispersion plots for layer with 1 cm plates
FIG. 3 % OF NET POWER FLOW IN POSITIVE X-DIRECTION
FORCE EXCITATION. 1kHz AND 4kHz
**FIG. 4** % OF NET POWER FLOW IN POSITIVE X-DIRECTION
FORCE EXCITATION, 7kHz AND 10kHz
FIG. 5 % OF NET POWER FLOW IN POSITIVE X-DIRECTION
SOURCE EXCITATION, 1kHz AND 4kHz

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