AN INDIRECT MEASURE OF BELOW-GROUND ELECTRIC FIELD CONDUCTIVITY AND DIELECTRIC CONSTANT(U) HARRY DIAMOND LABS ADELPHI MD R P MANRIQUEZ ET AL. SEP 84

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An Indirect Measure of Below-Ground Electric Field, Conductivity, and Dielectric Constant

Technical Report

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This paper uses experimental data obtained on a buried, conducting, parallel-plate sensor to evaluate the electric field beneath the air/ground interface. The sensor is described with a simple equivalent circuit, and the characteristic response is evaluated both in time and frequency. The agreement between the calculated and measured transmitted electric field is quite good for reasonable values of constant and frequency-dependent electrical conductivity and dielectric permittivity of the ground. The electrical properties of the ground were the only parameters that needed to be adjusted to provide agreement between theory and experiment. Those values taken were considered within reasonable bounds.
FOREWORD

The National Communications System (NCS) in response to Presidential Directive/NSC-53, "National Security Telecommunications Policy," is funding a comprehensive program on the effects of nuclear weapons on selected telecommunications systems. A portion of this effort is directed at determining the high-altitude electromagnetic pulse (EMP) vulnerability of the commercial Bell Telephone T1 Carrier systems, and at developing a T1 Carrier system specifically engineered to be EMP hard. The work described in this report was performed in support of these efforts.
CONTENTS

FOREWORD ......................................................... 3

1. INTRODUCTION .................................................. 9

2. ANALYTICAL CALCULATION OF TRANSMITTED E FIELD ............. 10

3. EXPERIMENTALLY MEASURED FIELDS .................................. 12

4. EFFECTS OF CONSTANT GROUND PARAMETERS--SENSOR CHARACTERIZATION .... 15

5. EFFECTS OF FREQUENCY-DEPENDENT GROUND PARAMETERS--SENSOR CALIBRATION .......... 21

6. CONCLUSION AND RECOMMENDATIONS ................................ 29

LITERATURE CITED ................................................. 32

SELECTED BIBLIOGRAPHY ........................................... 33

DISTRIBUTION ...................................................... 35

FIGURES

1. Flow chart of an indirect measure of below-ground E field,
conductivity, and dielectric constant ..................................... 10

2. Waves and fields .................................................. 11

3. Isometric view of buried E-field sensor, measuring instruments,
and REPS .............................................................. 13

4. Measured $H_x(t)$ fields at TP1 and TP4 .......................... 14

5. Measured sensor voltage $V_o(t)$ at TP1 and TP4 .................... 14

6. Equivalent circuit of E-field sensor .................................. 15

7. Results of first term and second term of equation (8) at TP1
with $\sigma = 0.007\; \text{mho/m}$ and $\varepsilon_r = 15$ ......................... 16

8. Results of first term and second term of equation (8) at TP4
with $\sigma = 0.007\; \text{mho/m}$ and $\varepsilon_r = 15$ ......................... 17

9. Comparison between calculated and measured transmitted electric
fields at TP4, with $\sigma = 0.001\; \text{mho/m}$ and $\varepsilon_r = 15$ .......... 18
10. Comparison between calculated and measured transmitted electric fields at TP4, with \( \sigma = 0.007 \) mho/m and \( \varepsilon_r = 15 \) ...................... 18

11. Comparison between calculated and measured transmitted electric fields at TP4, with \( \sigma = 0.02 \) mho/m and \( \varepsilon_r = 15 \) ...................... 19

12. Comparison between calculated and measured transmitted electric fields at TP4, with \( \sigma = 0.007 \) mho/m and \( \varepsilon_r = 1 \) ...................... 19

13. Comparison between calculated and measured transmitted electric fields at TP4, with \( \sigma = 0.007 \) mho/m and \( \varepsilon_r = 80 \) ...................... 20

14. Comparison between calculated and measured transmitted electric fields at TP1, with \( \sigma = 0.007 \) mho/m and \( \varepsilon_r = 15 \) ...................... 20

15. Magnitude of transfer function \( A(\omega) \), measured sensor voltage \( V_o(\omega) \), and "unfolded" sensor voltage \( V(\omega) \) at TP4, with \( \sigma = 0.007 \) mho/m and \( \varepsilon_r = 15 \) ...................... 22

16. Phase shift function \( \phi(\omega) \) response of sensor at TP4, with \( \sigma = 0.007 \) mho/m and \( \varepsilon_r = 15 \) ...................... 23

17. Conductivity versus frequency for various volume percentages of water .......................................................... 25

18. Dielectric constant versus frequency for various volume percentages of water .......................................................... 26

19. Magnitude of transfer function \( A(\omega) \), measured sensor voltage \( V_o(\omega) \), and "unfolded" sensor voltage \( V(\omega) \) at TP4, using Longmire's soil data (10-percent moisture content) ...................... 26

20. Phase shift function \( \phi(\omega) \) response of sensor at TP4, using Longmire's soil data (10-percent moisture content) ...................... 27

21. Comparison between calculated and measured transmitted electric fields at TP4, using Longmire's soil data (10-percent moisture content) .......................................................... 27

22. Comparison between calculated and measured transmitted electric fields at TP4, using Longmire's soil data (25-percent moisture content) .......................................................... 28
FIGURES (Cont'd)

23. Comparison between calculated and measured transmitted electric fields at TP1, using Longmire's soil data (10-percent moisture content) .................................................. 28

24. Comparison between calculated and measured transmitted electric fields at TP1, using Longmire's soil data (25-percent moisture content) .................................................. 29

Table 1. Coefficient $a_n$ for Universal Soil ................................. 24
1. INTRODUCTION

Analytical techniques exist for determining the electromagnetic (EMP) fields that are transmitted into the ground for given parameters of the ground and the incident field. The main objective of this paper is to indirectly measure the conductivity \( \sigma \), dielectric constant \( \varepsilon_r \), and electric \( E \) field below ground due to an incident EMP field as produced by the repetitive EMP simulator (REPS). REPS is a horizontal dipole radiator driven by a 1-MV repetitive pulse generator. The measurements were taken at the Harry Diamond Laboratories (HDL), Woodbridge Research Facility (WRF), Woodbridge, VA.

There are two methods to accomplish this objective:

(1) The determination of the "calculated" transmitted \( E \) field, \( E_t \), from the measured magnetic field, \( H_x \), at 1 m above ground, and the associated Maxwell equations and Fresnel coefficients in a continuous air/ground interface. The ground can be treated as a good conductor whose most important electrical parameters are conductivity and dielectric permittivity, \( \varepsilon_r \).

(2) The determination of the "measured" transmitted \( E \) field from the induced voltage, \( V \), across a buried capacitive parallel-plate \( E \)-field sensor with a plate separation \( l \). This \( V \) is "unfolded" from the measured voltage, \( V_0 \), at the sensor load through the use of the time-domain and frequency-domain solution techniques. The time-domain solution is derived from an equivalent circuit model of the \( E \)-field sensor. From this solution, the sensor can be characterized as an \( E \)-field sensor and an \( E \) (first derivative of \( E \)) field sensor. The frequency-domain solution technique depicts the behavior of the \( E \) field below ground through parametric variations of frequency-independent (constant) ground parameters. The frequency-domain solution of the same equivalent circuit model of the \( E \)-field sensor uses ground parameters that are either frequency independent (constant) or frequency dependent to describe the transfer function, \( A \), or sensor calibration of the sensor.

These two methods independently arrive at the solution of the transmitted \( E \) field below ground but both depend on \( \sigma \) and \( \varepsilon_r \). The parameters \( \sigma \) and \( \varepsilon_r \) were the only ones adjusted to provide agreement between the calculated and measured \( E \) fields transmitted below ground. When the calculated and measured \( E \) fields are in good agreement for given \( \sigma \) and \( \varepsilon_r \), a conclusion can be drawn from the results.

A flow chart of an indirect measure of below-ground \( E \) field, conductivity, and dielectric constant is shown in figure 1.

This report presents comparisons between calculated and measured transmitted \( E \) fields using both constant and frequency-dependent ground parameters.
### Analytical Method

<table>
<thead>
<tr>
<th>$H_d(0)$</th>
<th>Measured Above Ground Using SRI Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_d(0) - \mathcal{F}{H_d(0)}$</td>
<td></td>
</tr>
<tr>
<td>Maxwell Equations and Fresnel Coefficients to Calculate $E_d(\omega t)$</td>
<td></td>
</tr>
</tbody>
</table>

$E_d(\omega t) - \mathcal{F}^{-1}\{E_d(\omega)}$ |

---

### Measurement Method

<table>
<thead>
<tr>
<th>$V_d(0)$</th>
<th>Measured Below Ground Using Parallel Plate E-Field Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_d(0) - \mathcal{F}{V_d(0)}$</td>
<td></td>
</tr>
<tr>
<td>$V(\omega t) = A(\omega t)V_d(0)$</td>
<td></td>
</tr>
<tr>
<td>$E_d(\omega t) - \mathcal{F}^{-1}{V(\omega t)}$</td>
<td></td>
</tr>
</tbody>
</table>

$E_d(\omega t) = \frac{C_1 V_d(0)}{f} + \frac{C_2 e^{-C_3 t}}{f} \int_0^t e^{-C_4 t'} V_d(t')dt'$

---

Figure 1. Flow chart of an indirect measure of below-ground electric field, conductivity, and dielectric constant.

2. **Analytical Calculation of Transmitted E Field**

When an EMP is incident at the plane boundary of a linear, homogeneous, isotropic, and conducting medium, some of it is reflected and some is transmitted into the medium. The transmitted E field can be found with the use of Fresnel reflection and transmission coefficients.\(^1\) - 3\(^3\) The Fresnel coefficients are a function of the electrical properties of the ground and the incident angle of the electromagnetic wave. It is assumed in this study that the incident wave is a linearly polarized plane wave (of constant amplitude and phase) and the air/ground boundary is a semi-infinite plane.

The pertinent equations involve plane monochromatic waves (i.e., with single frequency) as directly derived from Maxwell's equations. Detailed derivations governing these equations can be found in works cited in the Selected Bibliography. Figure 2(a) diagrams the wave vectors of the incident, reflected, and transmitted waves used in this study. Figure 2(b) shows the conventional directions of electric and magnetic fields for horizontal polarization.

Figure 2. Waves and fields: (a) wave vectors of incident reflected and transmitted waves and (b) conventional directions of electric and magnetic fields for horizontal polarization.

The solution for the incident electric field above ground treated by Marx\(^4\) is

\[
E_0(\omega) = \frac{Z_0 H_x(\omega)}{\sin \psi \left( 1 - R_h(\omega) e^{-j\omega t_d} \right)} \frac{|\sin(\omega t_d)/(\omega t_d)|^2}{1}.
\]  

(1)

\(E_0(\omega)\) is a function of the free-space wave impedance \(Z_0\), the magnetic field \(H_x(\omega)\), incident angle \(\psi\), Fourier transform variable \(\omega\) (\(\omega = 2\pi f\)), the Fresnel coefficient for horizontal polarization \(R_h(\omega)\), and time delay \(t_d\). The time delay describes the difference of arrival time between the incident and reflected pulses at the field measurement point above ground as shown in figure 2(a). The last term of equation (1) is a filter function that removes the singularities at

\[\omega t_d = k\pi, \quad k = 1, 3, 5, \ldots\]

\(R_h(\omega)\) is found to be

\[
R_h(\omega) = \frac{\sin \psi - \left[ \varepsilon_r - j(\sigma/\varepsilon_0 \omega) - \cos^2 \psi \right]^{1/2}}{\sin \psi + \left[ \varepsilon_r - j(\sigma/\varepsilon_0 \omega) - \cos^2 \psi \right]^{1/2}}.
\]

(2)

where \(\varepsilon_0\) is the dielectric permittivity of free space. The time delay is

\[t_d = \frac{2h \sin \psi}{c},\]

where \(h\) is the height of the \(H\)-field sensor above ground (1 m) and \(c\) is the speed of light.

The transmitted electric field \( E_t(\omega) \) is

\[
E_t(\omega) = T_h(\omega)e^{-\gamma d}E_o(\omega),
\]

where \( d \) is the distance from the interface in the soil and \( \gamma \) is the propagation constant, defined as

\[
\gamma = (-\omega^2\mu\varepsilon + j\omega\sigma)^{1/2}.
\]

The transmission coefficient\(^5\) \( T_h(\omega) \) is

\[
T_h(\omega) = \frac{2 \sin \psi}{\left| \sin \psi + [\varepsilon_x - j(\sigma/\omega\varepsilon_0) - \cos^2 \psi]^{1/2} \right|}.
\]

The time-domain electric field \( E_t(t) \) is numerically computed through an inverse Fourier transform\(^6\) of equation (4). Finally, \( E_t(t) \) is averaged over a 12-in. depth from 1 to 13 in. below the surface; the result is taken as the E field at 7 in. below the ground.

3. EXPERIMENTALLY MEASURED FIELDS

Field measurements were made at the REPS facility at the following locations (see fig. 3):

(a) test point 4 (TP4), close to the centerline at \( x = 800 \) ft and \( y = 82.5 \) ft south of the centerline, and

(b) test point 1 (TP1), off the centerline at \( x = 800 \) ft and \( y = 609 \) ft north of the centerline.

At each test point, two field measurements were taken: (1) the total magnetic field, \( H_x(t) \), at 1 m above ground and (2) the transmitted component of the tangential \( E \) field averaged over a 12-in. depth from 1 to 13 in. below the surface.

The \( H_x(t) \) was measured with a conventional Stanford Research Institute (SRI) cubical sensor box.\(^7\) Figure 4 shows the measured \( H_x(t) \) at TP1 and TP4.

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The transmitted E field was measured by the use of two buried parallel aluminum plates. These plates are 12 in. long x 12 in. wide x 1/4 in. thick. They were separated a distance \( l \) of 1.75 in. at TP4 and 2.5 in. at TP1. In both cases the plates were inserted to achieve intimate ground contact. The voltage \( V(t) \) generated across the plates by the field was measured by the use of a fiber-optic transmitter attached to the plate by an RG214, 50-Ω coaxial double-shielded cable. This cable was 1.75 ft long at TP4 and 3 ft long at TP1. The 50-Ω fiber-optic transmitter was connected to the instrumentation van remote-reading equipment by a fiber-optic cable and a 50-Ω fiber-optic receiver. The fiber-optic transmitter and receiver data link were designed and built by Jim Blackburn of HDL. Figure 5 shows the measured sensor voltage \( V(t) \) at TP1 and TP4.
Figure 4. Measured $H_x(t)$ fields at TP1 (dashed line) and TP4 (solid line).

Figure 5. Measured sensor voltage $V_o(t)$ at TP1 (dashed line) and TP4 (solid line).
4. EFFECTS OF CONSTANT GROUND PARAMETERS--SENSOR CHARACTERIZATION

The E-field data are obtained from the measured voltage by the use of the equivalent circuit model as shown in figure 6. This model is described by Baum\(^8\) as a short dipole antenna model. When time variations are slow enough that the short antenna approximation is valid, and assuming that the edge effects of the plates are not a significant factor, the equivalent circuit is used to represent the following relationship between the voltage through the load, \(V_o(t)\), and the magnitude of the electric field, \(E(t)\):

\[
\left( \frac{C_s}{d} + G \right) [V(t) - V_o(t)] = \frac{V_o(t)}{R_L} + C_c \frac{dV_o(t)}{dt}, \quad \text{for } t \geq 0. \tag{7}
\]

Here \(V(t) = E(t) \lambda\), \(G = 1/R_s = C_s \sigma / \epsilon\), \(R_L\) is the load resistance, \(C_s = \text{area}\), \(\epsilon / \lambda\) is the sensor capacitance, and \(C_c\) is the cable capacitance. This is the same model for the E-field sensors used in Aurora with time-varying air conductivity.\(^9\)

\[V = IE_i\]

\[+V_o\]

\[C_s\]

\[\frac{G = C_s \sigma}{\epsilon}\]

\[C_c\]

\[R_L\]

\[\text{Figure 6. Equivalent circuit of E-field sensor.}\]

The solution of differential equation (7) is composed of a complementary and a particular integral. In network terminology, these are also referred to as the natural, source-free, or transient response and the forced or steady-state response, respectively. Examination of equation (7) reveals that when \(R_L\) is large and \(C_c\) is zero, the source terms are zero and the solution is only complementary. Solving equation (7) by the method of variation of parameters, the solution for \(E(t)\) is

---

\(^8\)C. E. Baum, Electromagnetic Pulse Sensor and Simulation Notes, Vol. 1, Air Force Weapons Laboratory, Note 13 (June 1970).

\[ E_t(t) = \frac{C_s + C_c}{\xi c_s} v_o(t) + \frac{C_s - G R_L C_c}{\xi R_L C_s^2} e^{-t/R_s C_s} \]

\[ \times \int_0^t e^{-t'/R_s C_s} v_o(t') \, dt', \quad \text{for } t \geq 0. \]  

(8)

Several important observations may be made from equation (8). When \( R_L \) is large and \( C_c \) is zero, and the transit time \((\xi/c)\) of the antenna model is long compared to the rise time \( \tau \) of the incident pulse (i.e., \( R_L C_s \gg \tau \)), the first term on the right-hand side of equation (8) dominates. In this case, the sensor can be regarded as an E-field sensor. On the other hand, when \( R_L C_s \ll \tau \), the second term of equation (8) dominates and the sensor can be regarded as an E-field sensor. Otherwise, the sensor can be described as a combination of an E- and E-field sensor. The transmitted electric field \( E_t(t) \) can be numerically computed from equation (8) or, alternatively, the voltage \( V(t) = \xi E_t(t) \) can be obtained by solving the differential equation (7) by a Runge-Kutta or Gear method. The solutions of the first and second terms of equation (8) are shown in figure 7 at TP1 and figure 8 at TP4, respectively, with a constant \( \sigma = 0.007 \text{ mho/m} \) and \( \varepsilon_r = 15 \).

Figure 7. Results of first term (solid line) and second term (dashed line) of equation (8) at TP1 with \( \sigma = 0.007 \text{ mho/m} \) and \( \varepsilon_r = 15 \).
Actual measurements of $\sigma$ and $\varepsilon_\infty$ were not available for the time this test was performed. However, previous data collected by the National Bureau of Standards (NBS) show that the ground conductivity is approximately 0.007 mho/m and the $\varepsilon_\infty$ is 15 at 1 MHz. These data are discussed elsewhere\textsuperscript{10} and measured for a limited frequency range. The results from equation (8) and the time-domain Fourier transform of equation (4) are shown in figures 9 to 13, at TP4, for $\sigma$ varied with 0.001, 0.007, and 0.02 mho/m at $\varepsilon_\infty = 15$, and $\varepsilon_\infty$ varied with 1, 15, and 80 at $\sigma = 0.007$ mho/m. Figure 14 shows the comparison between the results of equation (8) and the time-domain Fourier transform of equation (4), at TP1, for $\sigma = 0.007$ mho/m and $\varepsilon_\infty = 15$. The significance of the parametric variational effects to the expected values at the extreme is apparent. As $\sigma$ and $\varepsilon_\infty$ increase, the amplitude of the electric field decreases. The waveshapes at late times and low frequencies are somewhat altered at higher conductivities. The peak amplitude is particularly sensitive to the changes of the dielectric constant at higher frequencies.

\textsuperscript{10}Norman V. Hill, Effect of Frequency-Dependent Soil Parameters on Reflection Coefficients, Harry Diamond Laboratories, HDL-TR-2004 (December 1982).
Figure 9. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP4, with $\sigma = 0.001$ mho/m and $\varepsilon_r = 15$.

Figure 10. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP4, with $\sigma = 0.007$ mho/m and $\varepsilon_r = 15$. 
Figure 11. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP4, with $\sigma = 0.02 \text{ mho/m}$ and $\varepsilon_r = 15$.

Figure 12. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP4, with $\sigma = 0.007 \text{ mho/m}$ and $\varepsilon_r = 1$. 
Figure 13. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP4, with $\sigma = 0.007$ mho/m and $\varepsilon_r = 80$.

Figure 14. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP1, with $\sigma = 0.007$ mho/m and $\varepsilon_r = 15$. 

20
One of the unique features of the measuring system, depicted in figure 3, is the fiber-optic system. The advantage of implementing this system is to electrically isolate the E-field sensor from the instrumentation van, thereby eliminating the need for a long cable between the sensor and the van.

5. EFFECTS OF FREQUENCY-DEPENDENT GROUND PARAMETERS--SENSOR CALIBRATION

The use of constant (frequency-independent) values for \( \sigma \) and \( \epsilon_r \) results in a sensor calibration (or transfer function) \( A \) that is a constant. This \( A \) can be used to determine the transmitted E field across the parallel-plate sensor as

\[
E_t(\omega) = \frac{V_o(\omega)}{\xi} A .
\]  

(9)

However, in reality, \( \sigma \) and \( \epsilon_r \) are frequency dependent, and for larger variations of frequency, a more accurate calibration of the buried E-field sensor must include a frequency-dependent transfer function.

Let \( A(\omega) \) be the transfer function of the buried E-field sensor as determined by taking the Laplace transform of equation (7) in the s-domain \((s = j\omega)\). In general, the transfer function is a complex quantity and can be written as

\[
A(\omega) = \frac{V(\omega)}{V_o(\omega)} .
\]  

(10)

\( A(\omega) \) is also stated in terms of magnitude and phase as

\[
A(\omega) = |A(\omega)| e^{j\phi(\omega)} ,
\]  

(11)

where \( |A(\omega)| \) is the amplitude-response function and \( \phi(\omega) \) is the phase-shift function of the sensor. The transfer function depends on the circuit parameters as

\[
A(\omega) = \frac{M + sB}{G + sC_s} ,
\]  

(12)

where \( M = (1 + R_L G)/R_L \) and \( B = C_s + C_c \). The amplitude-response function \(|A(\omega)|\) is

\[
|A(\omega)| = \frac{1}{[G^2 + \omega^2C_s^2][MG + \omega^2BC_s]^2 + \omega^2(BG - C_sM)^2]^{1/2} ,
\]  

(13)

and the phase-shift function \( \phi(\omega) \) is

\[
\phi(\omega) = \tan^{-1} \frac{\omega(BG - C_sM)}{MG + \omega^2BC_s} .
\]  

(14)
The results for $|V_o(\omega)|$, $|V(\omega)|$, $|A(\omega)|$, and $\Phi(\omega)$ as a function of frequency are shown in figures 15 and 16 at TP4, respectively, for a constant $\sigma = 0.007$ mho/m and $\varepsilon_r = 15$. Ideally, this sensor should produce an amplitude frequency response that looks "flat" in the frequency band of interest and a phase shift that is a linear function of frequency. In other words, the spectrum of the measured input voltage $V(\omega)$ is identical to the spectrum of the output voltage $V_o(\omega)$ as expressed in equation (10). This means that the input voltage is passed undistorted by the measuring system. But for some cases, when the amplitude and phase frequency response are functions of $\sigma$ and $\varepsilon_r$ that vary with frequency and moisture content, the output voltage may be substantially different from the input. From these viewpoints, depending on the ground parameters, the sensor's transfer function could appreciably alter or distort the output voltage.

Finally, the "unfolded" measured transmitted electric field is

$$E_T(\omega) = A(\omega)V_o(\omega)/\ell \ .$$

(15)
Several studies have been conducted concerning the measurements and theoretical formulations of the electrical properties of the soil, namely, \( \sigma \) and \( \varepsilon_r \) as functions of frequency and moisture content. Longmire and Smith\(^{11}\) developed a universal formula for \( \sigma \) and \( \varepsilon_r \) over the frequency range of 5 Hz to \( 3 \times 10^{12} \) Hz, based on Scott's data\(^{12}\) for soils and Wilkenfeld's data for some concrete and grout samples (Wilkenfeld's data can be found in Longmire and Smith\(^{11}\)).


The $\sigma$ and $\varepsilon_r$ derived from Longmire and Smith's "universal RC network model" are

$$\varepsilon_r = \varepsilon_\infty + \sum_{n=1}^{N} \frac{a_n}{1 + (f/f_n)^2} \quad \text{(relative)} \quad (16)$$

$$\sigma = \sigma_0 + 2\pi\varepsilon_\infty \sum_{n=1}^{N} a_n f_n \frac{(f/f_n)^2}{1 + (f/f_n)^2} \quad \text{(mho/m)} \quad (17)$$

where

- $N = 13$,
- $\varepsilon_\infty = 5$,
- $f$ = frequency (Hz),
- $a_n$ = the constant coefficients (see table 1),
- $f_n = F(P)f_n(10\%)$,
- $f_n(10\%) = 10^{n-1}$ Hz,
- $F(P) = (P/10)^{1.28}$,
- $P$ = water content (percent), and
- $\sigma_0 = 8.0 \times 10^{-3} \ (P/10)^{1.54}$ (mho/m).

Figure 17 shows the ground conductivity $\sigma$ versus frequency for various volume percentages of water. Figure 18 shows the dielectric constant $\varepsilon_r$ versus frequency for various volume percentages of water.

**TABLE 1. COEFFICIENT $a_n$ FOR UNIVERSAL SOIL**

(see eq (16) and (17))

<table>
<thead>
<tr>
<th>$n$</th>
<th>$a_n$</th>
<th>$n$</th>
<th>$a_n$</th>
<th>$n$</th>
<th>$a_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.4 \times 10^6$</td>
<td>6</td>
<td>$1.33 \times 10^2$</td>
<td>11</td>
<td>$9.80 \times 10^{-1}$</td>
</tr>
<tr>
<td>2</td>
<td>$2.75 \times 10^5$</td>
<td>7</td>
<td>$2.72 \times 10^1$</td>
<td>12</td>
<td>$3.92 \times 10^{-1}$</td>
</tr>
<tr>
<td>3</td>
<td>$2.58 \times 10^4$</td>
<td>8</td>
<td>$1.25 \times 10^1$</td>
<td>13</td>
<td>$1.73 \times 10^{-1}$</td>
</tr>
<tr>
<td>4</td>
<td>$3.38 \times 10^3$</td>
<td>9</td>
<td>$4.80$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$5.26 \times 10^2$</td>
<td>10</td>
<td>$2.17$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Because of the unavailability of Woodbridge's soil data over a wide range of frequency, it was necessary to implement Longmire and Smith's universal formula for \( \sigma \) and \( \varepsilon_r \) in the program. However, some old data taken by NBS for Woodbridge's soil show relatively low \( \sigma \) and \( \varepsilon_r \); Hill\textsuperscript{10} discusses these data. In the analysis, 10-percent soil moisture content, \( \sigma = 0.007 \) mho/m, and \( \varepsilon_r = 15 \) provided close agreement between calculated and measured transmitted E fields below ground. The results for \(| V_0(\omega)|, |V(\omega)|, |A(\omega)|, \) and \( \Phi(\omega) \) with 10-percent moisture content at TP4 are shown in figures 19 and 20. The comparison between the inverse Fourier transform of the calculated transmitted electric field (eq (4)) and the "unfolded" measured transmitted electric field (eq (15)) using \( \sigma \) and \( \varepsilon_r \) dependent with frequency at 10- and 25-percent moisture content are shown in figures 21 and 22 at TP4, and figures 23 and 24 at TP1. Indeed, the transfer function of the sensor is highly sensitive to the electrical parameters of the soil.

![Figure 17. Conductivity versus frequency for various volume percentages of water.](image)

\textsuperscript{10}Norman V. Hill, Effect of Frequency-Dependent Soil Parameters on Reflection Coefficients, \textit{J. Harry Diamond Laboratories, HDL-TR-2004} (December 1982).
Figure 18. Dielectric constant versus frequency for various volume percentages of water.

Figure 19. Magnitude of transfer function $A(\omega)$ (solid line), measured sensor voltage $V_o(\omega)$ (dashed line), and "unfolded" sensor voltage $V(\omega)$ (dash-dot line) at TP4, using Longmire's soil data (10-percent moisture content).
Figure 20. Phase shift function $\Phi(\omega)$ response of sensor at TP4, using Longmire's soil data (10-percent moisture content).

Figure 21. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP4, using Longmire's soil data (10-percent moisture content).
Figure 22. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP4, using Longmire’s soil data (25-percent moisture content).

Figure 23. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP1, using Longmire’s soil data (10-percent moisture content).
Figure 24. Comparison between calculated (dashed line) and measured (solid line) transmitted electric fields at TP1, using Longmire's soil data (25-percent moisture content).

6. CONCLUSION AND RECOMMENDATIONS

This paper documents an attempt to experimentally measure the E-field component of an EMP below ground and to compare the results to an analytical calculation. The results of equations (4) and (15) showed good agreement for 10-percent moisture content, constant $\sigma = 0.007$ mho/m, and $\varepsilon_r = 15$.

The measurements can be improved by the use of a differential-mode voltage probe to measure the transmitted E field. The RG214 cable can be removed and the sensor directly connected to the fiber-optic transmitter. Another improvement would be to accurately determine $\sigma$ and $\varepsilon_r$ over a wide range of frequency and depth at the same location where the fields were measured, and at about the same time. The availability of more soil data would reduce the uncertainty in the sensor calibration.

The assumption that the EMP was a plane wave over a homogeneous plane semi-infinite ground in the far-field radiation zone may be justified by the quality of the results. The transmitted E field vanishes at late times (>1 $\mu$s) but the transmitted H fields may not. Also not taken into account were the multiple reflection of the fields and the effects of dispersion due to the existence of different layers of strata below ground. These uncertainties can be resolved by measurement of the E and H fields at different depths below the air/ground interface.

29
Analysis has shown that it may be possible to indirectly measure \( \sigma \) and \( \varepsilon_r \) as a function of moisture content and frequency with the parallel-plate E-field sensor. The sensitivity of the sensor is demonstrated through the sensitivity \( S \) analysis of the equivalent circuit model of the sensor's response to varying moisture content \( P \), i.e., \( S = 3V/3P \) (see fig. 21 to 24).

Perhaps the most significant aspect of this effort is that, through an adequate calibration of the buried E-field sensor, a method now exists for the immediate and relatively easy indirect determination of ground conductivity and dielectric constant. This method, through the Fourier transform, could then be made available as frequency-domain data and applied as derived to all EMP coupling programs.

The curve-fitting equations (eq (16) and (17)) used by Longmire and Smith\(^1\) that determined the \( \varepsilon_r \) and \( \sigma \) based on Scott and Wilkinfeld's data can be further modified by adjusting the necessary coefficients in the equations to obtain a closer correlation between the calculated and measured transmitted electric fields. A computer-aided optimization procedure\(^2\) is needed to accomplish this task. This curve-fitting method can analytically improve the determination of the \( \sigma \) and \( \varepsilon_r \) for Woodbridge's soil.

The use of calibrated, shallow, buried parallel plates should be made part of all field-test system programs because it is a simple, inexpensive method of determining the soil conductivity at the same time that the experimental coupling data are collected on the system. Thus, a conductivity measurement made at the beginning of each test day can be used to predict the signal levels expected. And, in addition, an accurate evaluation of experimentally collected data can then be used by the analyst to predict the levels of induced signals for any conditions of soil.

Future efforts will be to explore ways of improving the measurement method by (1) the determination of the effects of RG214 cable on the measurement, (2) an independent direct measurement of \( \sigma \), \( \varepsilon_r \), and \( P \) by whatever means, (3) the use of a different buried sensor (dipole, magnetic loop, two parallel cylinders, two parallel spheres, etc), and (4) the measurement of conduction current density \( \sigma E_t \) below the ground using the parallel-plate sensor. This last measurement can be performed by connecting a large resistor between the sensor and a short RG214 cable. In series with the cable will be an impedance-matching device. This device will match the high-impedance sensor system (sensor and resistor) to the low-impedance data-link system (cable and fiber-optic system). The use of an impedance-matching device makes it possible to directly measure the induced sensor voltage. The signal propagated below the ground will be produced by the REPS.


A fifth way to improve the method is by the measurement of displacement current density--\(\varepsilon(dE_t/dt)\)--below the ground. This measurement can be performed by covering one of the plates with a thin insulator (e.g., plastic) and burying these plates below the ground. The results of all these measurements will further validate the results obtained from the existing analytical techniques employed in this report.

A conclusion reached as a result of this study is that the equivalent circuit model of the sensor is an adequate model for REPS field rise times and typical ground parameters. The modeled sensor system performed well and provided physical insight to the problem. The transfer functions were evaluated directly from the circuit model and showed the characteristic response of the sensor.

The determination of the E field below ground due to an incident EMP field is summarized as follows:

(1) The H field above ground was measured and used to calculate the transmitted E field with the aid of Maxwell's equations and the Fresnel coefficients.

(2) The induced voltage across the parallel-plate sensor was measured and "unfolded" in two ways:

(a) time-domain formulation of the equivalent circuit model of the E-field sensor using constant ground parameters, and

(b) frequency-domain formulation of the same equivalent circuit model using ground parameters dependent on frequency and moisture content.

Finally, the applied conceptual and measurement scheme showed satisfactory results and provided vital information about EMP field sensors and the electrical properties of the conducting ground.
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(8) C. E. Baum, Electromagnetic Pulse Sensor and Simulation Notes, Vol. 1, Air Force Weapons Laboratory, Note 13 (June 1970).


(10) Norman V. Hill, Effect of Frequency-Dependent Soil Parameters on Reflection Coefficients, Harry Diamond Laboratories, HDL-TR-2004 (December 1982).


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