A PRELIMINARY STUDY OF USING A STRAIN-GAUGED BALANCE AND PARAMETER ESTIMATION
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A PRELIMINARY STUDY OF USING A STRAIN-GAUGED
BALANCE AND PARAMETER ESTIMATION TECHNIQUES
FOR THE DETERMINATION OF AERODYNAMIC FORCES
ON A MODEL IN A VERY SHORT DURATION WIND TUNNEL.

by

A. P. BROWN and R. A. FEIX

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A PRELIMINARY STUDY OF USING A STRAIN-GAUGED BALANCE AND PARAMETER ESTIMATION TECHNIQUES FOR THE DETERMINATION OF AERODYNAMIC FORCES ON A MODEL IN A VERY SHORT DURATION WIND TUNNEL

A.P. BROWN and R.A. FEIK

SUMMARY

This memo presents a preliminary study of a proposed method of measuring the aerodynamic forces on a supported model in an intermittent very short duration wind tunnel with a relatively high airflow dynamic pressure (of the orders of 200 usec and 1/3 atmosphere respectively). A semiconductor strain gauged cantilever beam balance is used to record strain time histories associated with model displacement in response to aerodynamic force. The practical feasibility of obtaining sufficiently resolvable strains for the prescribed tunnel conditions with the given strain gauge configuration is established. The proposed method uses a system identification procedure to determine the system dynamic response characteristics using a known calibration force input. Subsequently, aerodynamic forces during a tunnel run follow from the recorded strain gauge time histories. The procedure has been demonstrated successfully using simulated data. However, the experimental situation did not lead to a successful analysis in the way proposed. Reasons for this are discussed and recommendations made for improvements. A brief series of shots in the ANU free piston shock tunnel also highlights the need to isolate as much as possible the model/balance from external vibrations.
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<tr>
<td>A</td>
<td>Cross-sectional area of balance beam.</td>
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<tr>
<td>b</td>
<td>Balance beam width.</td>
</tr>
<tr>
<td>c</td>
<td>Damping coefficient.</td>
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<tr>
<td>Cz</td>
<td>Aerodynamic normal force coefficient.</td>
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<td>d</td>
<td>Balance beam depth.</td>
</tr>
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<td>Strain gauge longitudinal separation distance, non-dimensionalised by 1.</td>
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<td>E</td>
<td>Young's Modulus.</td>
</tr>
<tr>
<td>F</td>
<td>Force/Moment vector.</td>
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<tr>
<td>I</td>
<td>Second moment of area.</td>
</tr>
<tr>
<td>Iy</td>
<td>Moment of inertia.</td>
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<tr>
<td>k</td>
<td>Stiffness coefficient.</td>
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<tr>
<td>l</td>
<td>Balance beam length.</td>
</tr>
<tr>
<td>M</td>
<td>Pitching or bending moment.</td>
</tr>
<tr>
<td>m</td>
<td>Mass.</td>
</tr>
<tr>
<td>n</td>
<td>&quot;noise&quot; strain.</td>
</tr>
<tr>
<td>n</td>
<td>Noise vector.</td>
</tr>
<tr>
<td>p</td>
<td>Top and bottom strain gauge pair placement point, non-dimensionalised by 1.</td>
</tr>
<tr>
<td>R</td>
<td>Strain gauge voltage change/reactive force sensitivity matrix.</td>
</tr>
<tr>
<td>S</td>
<td>Strain/reactive force sensitivity matrix.</td>
</tr>
<tr>
<td>t</td>
<td>Time.</td>
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<tr>
<td>u</td>
<td>Control vector.</td>
</tr>
<tr>
<td>V</td>
<td>Strain gauge voltage change vector.</td>
</tr>
<tr>
<td>ΔV</td>
<td>Change in strain gauge potentiometer circuit series resistance voltage.</td>
</tr>
<tr>
<td>x</td>
<td>Displacement of model centre of gravity in the axial direction, forward is positive.</td>
</tr>
<tr>
<td>xB</td>
<td>Deflection of the point B in the axial direction of the balance.</td>
</tr>
<tr>
<td>xC</td>
<td>Distance in the axial direction from model centre of gravity to balance reference centre.</td>
</tr>
</tbody>
</table>
NOTATION CONT'D

x  State vector.

X  Axial force.

y  Calculated response vector.

z  Displacement in the normal direction, positive downwards.

z_B  Deflection of the point B in the direction normal to the balance.

z_G  Distance in the normal direction from model centre of gravity to balance reference centre.

z  Measured response vector.

Z  Normal force.

ξ  Axial deflection of the balance reference centre in the state vector.

ε  Rotational deflection of the model in the state vector.

ɛ  Strain.

Łɛ  Vector of appropriate differences and summation of strains.

ζ  Normal deflection of the balance reference centre in the state vector.

SUBSCRIPTS

a  aerodynamic

x  in the longitudinal direction

y  about the lateral axis

z  in the normal direction

1,2,3, pertaining to a particular strain gauge

A  balance/sting junction point

B  balance/model junction point (balance reference centre)
1. INTRODUCTION

The measurement of aerodynamic forces and moments on models in intermittent short duration wind tunnels has been accomplished for a number of years using piezoelectric crystal and semiconductor strain gauge force balances (developed during the early to mid 1960's, see References 1 and 2) and using accelerometer balances (developed during the late 1960's, see Reference 3). A thorough review of these and free flight methods is given in Reference 4. Inertia-compensated force balances are only suitable for flow durations of above several milliseconds. This is because the principle of operation is reduction of the force transducer's output to a quasi-steady signal by compensating for inertial accelerations. More recently, Bernstein and Stott (Reference 5) have devised a method of obtaining the aerodynamic forces using a laser interferometer to measure the model's motion in response to the shock tunnel airflow. Such a system has been designed to operate successfully in shock tunnel flows of under 1 millisecond duration and low dynamic pressure. For the testing of lifting vehicles, the Australian National University's T3 Free Piston Shock Tunnel is often operated in the nozzle airflow enthalpy range 20-40 MJ/kg achieving clean airflow duration times of about 400-100 microseconds, and corresponding test section dynamic pressures of the order of 35 kilo Pascals (References 6 and 7). Over such short time intervals a restrained model of moderate mass will be displaced several micro-metres (microns) by aerodynamic force.

This paper proposes a method whereby the aerodynamic forces and moments can be resolved under such conditions. Time histories describing the model response to applied force or moment are obtained using a simple cantilever beam between model and sting, the beam being gauged with semiconductor strain gauges connected in quarter bridge potentiometer circuits to a d.c. supply. Because the forcing duration relative to the period of vibration of the system is relatively small, the model/balance deflection is small so that balance reactive forces and moments will, except for an extremely stiff balance, be small i.e. the model will accelerate in response to the aerodynamic force almost as if it were unconstrained. Under these circumstances the conventional force balance opposing design criteria, stiffness versus sensitivity, are no longer in opposition - up to a certain point the stiffer the balance the more sensitive i.e. the more strain for a given balance deflection of the balance end-point. This is illustrated further in Section 2.

Resolution of the aerodynamic forces and moments from the balance/model motion is dependent on the dynamic calibration of the system. A crucial part of the method to be described is the employment of a system identification program for obtaining the system's stiffness, inertial and damping terms completely. Once these are obtained the mathematical model thus established can be used to analyse actual tunnel run oscillograms to obtain the forces and moments that resulted in the observed strain gauge circuit voltage changes. The complete procedure is illustrated using simulated noisy data in Section 4, and application to real data is discussed in Section 5. Prior to that the model and balance system characteristics used in this study and their mathematical description are described in Sections 2 and 3.
This study also serves as one example of the possible application of parameters identification techniques, widely used in flight test data analysis, to the determination of wind tunnel aerodynamic forces from dynamic response measurements.

2. BALANCE AND MODEL

2.1 Feasibility

To illustrate the feasibility of the concept, the following numerical example is used. Consider the model to be supported by a simple rectangular section cantilever balance with a sting and support system stiffness such that the balance/sting junction is effectively rigid. For the simplest case consider the model centre of gravity to be coincident with the balance/model junction (Figure 1). From linear beam theory the slope and deflection of the balance beam at the point B are:

- to normal force $Z$:
  \[ \frac{dz}{dx} \bigg|_B = \frac{\ell^3}{2EI} Z \]
  \[ z_B = \frac{\ell^3}{3EI} Z \]

- to pitching moment $M$:
  \[ \frac{dz}{dx} \bigg|_B = -\frac{\ell}{EI} M \]
  \[ z_B = -\frac{\ell^2}{2EI} M \]

- to axial force $X$:
  \[ \frac{dz}{dx} \bigg|_B = 0 \]
  \[ z_B = 0, \quad x_B = \frac{\ell}{AE} X \]

Considering normal force only, at the point C:

\[ \frac{dz}{dx} \bigg|_C = \frac{3\ell^2}{8EI} Z \]

\[ z_C = \frac{5\ell^3}{48EI} Z \]

and the strain on the top and bottom of the balance beam at C is:

\[ \varepsilon_{z,C} = \frac{f_d}{4EI} Z \]

Substituting for $Z$ from Equation (1) gives the following relationship:

\[ \varepsilon_{z,C} = \frac{3}{4} \frac{d}{\ell^2} z_B \]
i.e. for any defined balance beam length \( l \), \( \varepsilon \), for any deflection \( z_B \) depends only on \( d \), the depth of the beam. For a practical estimate of \( z_B \) and hence of \( \varepsilon \), consider the following example. Assume that the centre of gravity is coincident with the model centre of pressure i.e. no aerodynamic moment is present. Initially also the reactive normal force and moment caused by beam deflection is negligible compared to the aerodynamic force.

For a 150 mm length model of a re-entrant flight vehicle design being tested in the ANU T3 free piston shock tunnel, representative values of \( Z \) (using a high-inclination value of \( C \) from Reference 8) are 116N with the conical nozzle and 633N with the contoured nozzle. (The contoured nozzle leads to a much higher nozzle exit dynamic pressure).

For this typical model (see Section 3 for a description of the model under study), a suitable cantilever balance would have \( b = 3 \) mm, \( d = 12 \) mm and \( \ell = 100 \) mm, giving a balance stiffness \( k = 90.7 \) N/mm. If the model mass was 0.2 kg, then a 200 ms period of negligibly restrained acceleration from rest under the action of the lower of the above values of \( Z \) results in a deflection \( z \) of 11.6 \( \mu \)m which would give a ratio of reactive to aerodynamic force \( (kz/Z) \) of approximately 0.01. The corresponding strain is \( |\varepsilon| = 10.4 \mu \varepsilon \). With typical semiconductor strain gauges (see the next section) recording the strain and connected in potentiometer circuits with 10 volts stabilised power source, such a strain amounts to a voltage change across the series resistance of 1.2 millivolts, easily recordable. The strain in the contoured nozzle would be much greater due to the higher forces.

2.2 Strain Gauging

On a simple rectangular section balance the three reactive force/moment components i.e. normal and axial forces and end-point moment can be resolved by using three semiconductor strain gauges as positioned in Figure 2 and connected in quarter-bridge potentiometer circuits. Variables in the placement process are then the fractions \( p \) and \( e \). For simplicity in this example \( p \) has been chosen as 1/2.

With the model/balance junction compressed or extended and deflected, the strains at points 1, 2 and 3 shown in Figure 2 can be expressed as:

\[
\varepsilon_1 = \frac{X_B}{AE} + \frac{1}{2EI} + \frac{M_d}{2EI} + n_1
\]

\[
\varepsilon_2 = \frac{X_B}{AE} + \frac{2}{2EI} + \frac{M_d}{2EI} + n_2
\]

\[
\varepsilon_3 = \frac{X_B}{AE} - \frac{3}{2EI} + \frac{M_d}{2EI} + n_3
\]

\( \text{..(4)}\)
where tensile strain is positive (and \( n \) is due to random structural vibrations). The problem then remains of reducing \( \varepsilon \) to the point where adequate sensitivity is maintained whilst stress wave propagation time between the two points can be neglected. Under this condition, leaving out additive random noise,

\[
\Delta \varepsilon_{1-2} = \varepsilon_1 - \varepsilon_2 = \frac{d}{2EI} (M_1 - M_2) = -\frac{dM}{2EI} Z_B \quad \text{as} \quad M_1 = M_2 - dZ_B
\]

\[
\Delta \varepsilon_{2-3} = \varepsilon_2 - \varepsilon_3 = \frac{dM}{EI}
\]

\[
\Delta \varepsilon_{2+3} = \varepsilon_2 + \varepsilon_3 = \frac{2X}{AE}
\]

Thus these three combinations of \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) uniquely define \( Z_B, M_B \) and \( X_B \) with \( M_B \) being obtained from the relation:

\[
M_B = M_0, Z_B = (1-p)Z
\]

In practice interaction terms due to Poisson's effect, or non-linear effects due to d/\( \ell \) ratio or to one or more of the strain gauges being placed near either end of the balance beam, may be appreciable. In that case off-diagonal terms in the sensitivity matrix, \( S \), will not be zero, where \( S \) relates the strain vector \( \Delta \varepsilon \) whose components are given by Equation 5 to the force/moment vector, \( F \), with components \( X_B, Z_B, M_B \), i.e.:

\[
\Delta \varepsilon = [S] F
\]

Considering the resolution of the normal force strain difference, \( \Delta \varepsilon_{1-2} \), in particular, Equation (3) can be written for strain gauge No. 2 at any point \( C \) located by the factor \( p \) of Figure 2; i.e.

\[
\varepsilon_{z_2} = \frac{3}{2} \frac{pd}{\varepsilon^2} Z_B
\]

Thus, as a ratio,

\[
\frac{\Delta \varepsilon_{1-2}}{\varepsilon_{z_2}} = \frac{-e}{p}
\]

For the numerical example of Section 2.1, if \( e = 0.15 \) (a spacing of 15 mm), then as \( p = 0.5 \),

\[
\Delta \varepsilon_{1-2} = 3.12 \mu E
\]

If, on the other hand the strain gauges were placed internally, say at \( p = 0.75 \), then \( \varepsilon_{z_2} = 5.2 \mu E \) and
\[ \Delta t_{1-2} = 1.04 \mu \text{s} \]

which is still a satisfactory strain level.

2.3 Model Description

The model under consideration is depicted in Figure 3 and was used for a brief series of shots in the T3 tunnel to be described in Section 5. It consists of a cone of 70° included angle with a cylindrical afterbody of diameter 80 mm and length 125 mm. Overall length is 180 mm. Its balance beam was 12.7 mm deep but was of varying width, with a cut-out in it for the placement of an accelerometer in the rear of the model. The beam was attached by screws to the model, internally, 115 mm aft of the model cone. Placement of the strain gauges was similar to that depicted in Figure 2, but was considerably closer to the model attachment point (to remain internal). The strain gauges used were p-type semiconductor gauges of resistance 120 \( \Omega \) and nominal gauge factor 110. Each of the three strain gauges was connected in a quarter bridge potentiometer circuit (Figure 4) with a d.c. power supply of 9.8 volts. Series resistance was 1k\( \Omega \). The series resistance voltage changes were measured using a 0.1 \( \mu \text{F} \) capacitor in series with pre-amplifiers (100X) and displayed on a pair of oscilloscopes. Pre-amplifiers were used as they resulted in higher quality traces. Summing/differencing pre-amplifiers are preferable but were not used in this experimental set-up for reasons of availability. The electrical coupling time constant of 100 \( \mu \text{sec} \) introduced into the voltage measurements was neglected in the simulated study (Section 4) and may need to be taken into account when considering real data.

3. MATHEMATICAL REPRESENTATION OF THE MODEL/BALANCE SYSTEM

An integral part of the presently-discussed method is a suitable representation of the model/balance system dynamics by a mathematical model and identifying the unknown elements of the system dynamic transfer function which relates the output, e.g. the strain gauge readings or a function thereof, to the input i.e. the applied forces and moment.

Without the precise strain gauge factors being known, series resistance voltage changes, \( V \), can be related to balance beam reactive loads, \( F \), by the matrix equation (see also equations 5, 6):

\[ V = [R]F \]

where

\[ F = [X_B^T, Z_B^T, M_B^T] \]

\[ V = [\Delta v_3, \Delta v_2, \Delta v_1 - \Delta v_2, \Delta v_3 - \Delta v_2] \]

and the elements of \( R \) obtained by dead weight loading as is carried out for the calibration of conventional force balances (see also Reference 1). In the case of the present model/balance system placement of the strain gauges 2 and 3 close to the front end of the balance beam resulted in significant \( M \) and \( X \) interactions of \( Z \), and \( Z \) and \( M \) interactions on \( X \), due mostly to strain gauge 2 being near a stress rising corner, which is sensitive to \( M \) and \( X \) but not \( Z \).
The problem of obtaining sufficient $R_{zz}$ sensitivity compared to $R_{xx}$ sensitivity (by increasing the factor $e$ - in this strain gauge layout $e$ is 0.15 and is limited by physical constraints of the model/balance system) is also highlighted. Another parameter for further investigation is variation of the factor $p$ (see Figure 2). For this layout $p = 0.90$. Reducing $p$ places the gauges in the more linear region of the beam (thereby reducing interactions) and in a dynamic event increases the ratio of stress level due to $Z$ to noise (environmental) stress level, but also increases propagation time of stress waves from the model/balance junction.

In this balance beam also $R_{xx}$ sensitivity is limited by cross-sectional area of the beam. To reduce the latter by cutting out the centre of the beam for a certain depth either side of the neutral axis would probably be more desirable than reducing $b$, which would reduce column stability to any slight yaw/sideforce, but the detrimental increment to stress wave propagation times would probably be similar in each case. However, the sensitivity and linearity obtained with these particular strain gauges and their location is encouraging. Some typical dead weight calibration curves are shown in Figure 5.

The system identification program used is a modified version of that described in Reference 9. To apply the program, the model/balance system is represented by the following set of equations:

\[
P \dot{x} = A x + B u \\
y = F x + G u \\
z = y + n
\]  
\( \text{(8)} \)

where 
- $x$ is the state vector
- $u$ is the control vector
- $y$ is the calculated response vector
- $z$ is the measured response vector
- $n$ is the noise vector.

In particular, $z$ can be taken as the strain gauge voltages referred to above (Equation (7)). $P$, $A$, $B$, $F$ and $G$ are matrices describing the system.

For the model/balance system used, the model centre of gravity lies in the model vertical plane of symmetry but is displaced from the balance centreline end point. Using the nomenclature of Figure 6 the equations describing angular pitching motion and vertical and axial displacement dynamics, assuming a rigid sting, fit into the general structure of Equation (8) with:
state \( x = [x_1, x_2, x_3, \xi, \theta, \zeta]^T \)
where \( x_1 = \dot{\xi}, x_2 = \dot{\theta}, x_3 = \dot{\zeta} \)
control \( u = [X_a, Z_a, M_a] \), \( M_a \) referred to model c.g.

\[
P = \begin{bmatrix}
m & -m x_G & 0 \\
0 & I_y & 0 \\
0 & m z_G & m \\
0 (3 \times 3) & I (3 \times 3)
\end{bmatrix}
= \text{inertia matrix}
\]

\[
A = \begin{bmatrix}
-c_{11} & -c_{12} & 0 & -k_{11} & -k_{11} & 0 \\
0 & -c_{22} & 0 & -k_{21} & -k_{22} & z_G k_{33} \\
0 & -c_{32} & -c_{33} & 0 & 0 & -k_{33} \\
I (3 \times 3) & 0 (3 \times 3)
\end{bmatrix}
\]

where \( c_{ij} \) are the damping terms, \( k_{ij} \) are stiffness terms

\[
B = \begin{bmatrix}
I (3 \times 3) \\
\end{bmatrix}
\]

For the response, \( y \), take the strain gauge voltages,
giving

\[
P = R. \begin{bmatrix}
k_{11} & k_{12} & 0 \\
k_{21} & k_{22} & -z_G k_{33} \\
0 & 0 & k_{33}
\end{bmatrix}
; \ G = [0]
\]

where \( R \) is defined in Equation (7).
The stiffness terms, \( k_{ij} \), which relate balance beam reactive loads, \( F \), to the displacements \( \zeta, \theta, \xi \) may be identified as unknown system parameters. In the latter case, the vector of unknown parameters, assuming \( x_G, z_G \) are known, is

\[
[m, I_y, c_{11}, c_{12}, c_{22}, c_{33}, k_{11}, k_{12}, k_{21}, k_{22}, k_{33}]^T
\]

4. CALIBRATION AND RESULTS WITH SIMULATED DATA

In order to check the feasibility of identifying the unknown parameters, a simulated dynamic calibration was carried out. A mathematical model of the system was set up using values of mass, inertia, stiffness and c.g. offsets based on a priori knowledge of the current model/balance system partly obtained from previous static calibrations. Values for the damping terms were guessed. The resulting 6th order system had three complex natural modes of period 2, 10 and 20 ms approximately and damping ratios of 0.1, 0.1 and 0.2 respectively. The response of the system to simulated force and moment pulses of 1 ms duration was calculated. This represents the response to a blow from a hammer instrumented with a force transducer. A typical set of responses over 19 ms is shown in Figure 7.

Strain gauge measurements of the responses were assumed to be corrupted by white Gaussian noise with standard deviation of up to 1% of the peak values. Measurements of input pulses were assumed noise free, although this assumption can be removed. Given the input and output measurements, the identification program extracted the unknown parameters starting from a priori values substantially offset from their true values. Results with 0.5% measurement noise are shown in the table below, obtained using 19 ms of record sampled at 10 samples per second. (CPU time on a DECSYSTEM-10 computer was about 2 minutes).

<table>
<thead>
<tr>
<th>TRUE</th>
<th>CALCULATED</th>
<th>TRUE</th>
<th>CALCULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>1.65</td>
<td>1.64</td>
<td>(0.003)</td>
</tr>
<tr>
<td>( I_y )</td>
<td>0.0062</td>
<td>0.0063</td>
<td>(0.000004)</td>
</tr>
<tr>
<td>( c_{11} )</td>
<td>218</td>
<td>220</td>
<td>(1.8)</td>
</tr>
<tr>
<td>( c_{12} )</td>
<td>-3.7</td>
<td>-3.5</td>
<td>(0.1)</td>
</tr>
<tr>
<td>( c_{22} )</td>
<td>0.82</td>
<td>0.79</td>
<td>(0.003)</td>
</tr>
<tr>
<td>( c_{32} )</td>
<td>7.7</td>
<td>7.4</td>
<td>(0.15)</td>
</tr>
<tr>
<td>( c_{33} )</td>
<td></td>
<td></td>
<td>1073</td>
</tr>
<tr>
<td>( k_{11} )</td>
<td></td>
<td>264</td>
<td>262 (0.8)</td>
</tr>
<tr>
<td>( k_{12} )</td>
<td></td>
<td>15.4</td>
<td>15.4 (0.1)</td>
</tr>
<tr>
<td>( k_{21} )</td>
<td></td>
<td>8.7</td>
<td>9.1 (0.08)</td>
</tr>
<tr>
<td>( k_{22} )</td>
<td></td>
<td>3.1</td>
<td>3.2 (0.03)</td>
</tr>
<tr>
<td>( k_{33} )</td>
<td></td>
<td>5512</td>
<td>5474 (11.6)</td>
</tr>
</tbody>
</table>

The values in brackets are estimated standard deviations as calculated by the program. Note that the stiffness coefficients are scaled values. The results demonstrate that all coefficients can be extracted to good accuracy. Increasing noise levels lead to a gradual degradation in accuracy.
With complete knowledge of the system it is possible to infer forces on the model from the response time histories, by an inverse process. However this requires first and second time derivatives of the displacement histories. For the current simulation, a moving least squares fit was found to give good results provided the order of the fit was optimised and sampling rate was adequate.

To take a specific example, a simulated response to step inputs in forces and moments was calculated and measurements of $\zeta$, $\theta$ and $\xi$ assumed to be available with 1% noise. It is required to infer the forces and moments using the previously identified system parameters. The results shown in Figure 8 were obtained with a sampling rate of 200/sec using a 3rd order Least Squares fit over 200 points at a time for the $\zeta$, $\theta$ derivatives and a 4th order fit over 100 points at a time for the $\xi$ derivatives. The results for $Z_a$, $M$, and $X_a$ are within a few percent of the true values except for the first and last 0.25 ms of the 2 ms period considered. This is probably related to increasing inaccuracy of the second derivatives at each end of the record. For comparison, a low noise case (0.1% noise) requiring smaller sampling rates, is also shown in Figure 8 indicating the improvement which can be obtained.

Although $Z_a$, $M$, and $X_a$ are constant over most of the period shown in Figure 8 (this is not necessary), the balance between inertial, stiffness and damping terms is continuously varying. Initially, the inertial terms are dominant and, for $Z_a$ and $M$, even at 1 ms account for around 90% of the total, and at 2 ms are still half to two thirds of the total. On the other hand, the axial mode period is much shorter (2 ms) and consequently the balance between inertial, stiffness and damping contributions to $X_a$ varies rapidly over that period.

5. APPLICATION TO REAL DATA

A limited, but unsuccessful, effort was made to apply the foregoing method to calibration of the real model/balance system. Effort in this area has now ceased but possible reasons for the lack of success are discussed below. In addition, an example of a shock tunnel run with the model is shown and considered in the light of the proposed method.

5.1 Balance Dynamic Calibration

Figure 9 shows plots of a typical set of strain gauge responses to an impulsive blow applied to the model by an instrumented hammer. The inputs and responses were recorded on an oscilloscope and subsequently read off manually at a rate of 10 samples per millisecond. Clearly the accuracy of such a process is not ideal and automatic digital recording oscilloscopes would be highly desirable. This is especially obvious in the early part of the records where small amplitude perturbations are superimposed on the responses. Small errors can lead to large discrepancies when the sum and differences of the strain gauge outputs are taken in order to obtain the reactive loads (see Equation (9)).

Another possible source of experimental error is the interaction on the vector $V$ of lateral/directional model motion. Although the model centre of gravity is vertically co-planar with the balance centreline
and the line of action of the impulsive blow, any misalignment may result in slight lateral-directional oscillations. Whilst the strain gauges are on the beam yawing neutral axis, a combination of any yawing and rolling moments at the lower surface/mounting block corner, near which strain gauge No. 2 is placed, could lead to rotation of the neutral axis, thereby leading to apparent strains.

A more serious problem would be an inadequate mathematical model of the model/balance system. In particular, the sting has been assumed rigid and stress wave propagation times insignificant. The difficulty encountered in matching the records, especially the high frequency component in the early part of the records may be due to lack of validity of the assumptions and would require modification of the mathematical model or redesign of the experimental apparatus. Another possibility is that of the electrical coupling as mentioned in Section 2.3. By comparison, the force transducer in the instrumented hammer has a resonant period of 14.6 sec.

5.2 Shock Tunnel

Shock tunnel shots with the model described in Section 3 at incidences of 22° and 35° were conducted in the ANL3 tunnel fitted with a conical nozzle. Copies of strain gauge circuit voltage fluctuation oscillograms for model angle of incidence of 22° are shown in Figure 10a. The voltage fluctuations are much larger than would be expected under the action of aerodynamic force only. With the aid of an accelerometer fixed to a stand next to the dump tank it is seen (Figure 10b) that the strain gauges response is due to structural vibrations transmitted through the floor of the laboratory coincident with the shock wave reflection from the end of the shock tube. The magnitude of this vibration highlights the need to isolate the model sting support system from the dump tank probably in a manner similar to that described in Reference 10. Whilst inclusion of the dynamics of the sting support system may be possible in the mathematical model, the vibrations transmitted to the model through the dump tank are less accountable and can thereby lead to significant degradation of accuracy in resolution of forces by the proposed method.

6. CONCLUDING REMARKS

A method has been proposed for the measurement of three-component aerodynamic forces on a model in a very short duration intermittent wind tunnel. The method relies on (i) a strain-gauged cantilever beam sensing the bending and axial strains associated with vertical, rotational and axial deflection of the model/beam junction due to the motion of the model stimulated by aerodynamic forces, and (ii) parameter estimation techniques to resolve, from a digital record of strain gauge potentiometer circuit series resistance voltage changes, a time history of the model displacement and the aerodynamic forcing function.

A simple strain gauge placement pattern and resolution procedure is outlined for a balance beam which utilises three semiconductor gauges. The positioning of the gauges on the beam in this particular case resulted in large M on Z and Z, M on X interactions, due to the beam dimensions being constrained by design to be clear of an accelerometer mounted in the test model.
In a high dynamic pressure shock tunnel such as the ANU T3 tunnel, calculations show that for representative nozzle flow and test-time conditions, a typically sized model would be displaced in the normal direction by at least 10 microns, resulting in bending strains at suitable gauge positions of the order of several microstrain and a Z-channel strain difference of about one microstrain. However, whilst such strains are by themselves easily discernible, the presence of mechanical noise (shock vibrations transmitted through the balance support system) would degrade their resolution. This was observed in the few shots in the T3 tunnel with this model and highlights the need for the balance support system to be vibrationally isolated from its surroundings.

A modified system parameter identification program is shown to identify successfully system inertial, stiffness and damping terms, using a suitable mathematical model and simulated data. However, initial attempts to apply the program to a dynamic calibration of the actual model/balance system was unsuccessful. Possible reasons for this include the manual method of digitising response records, impulsive loading misalignment, balance response to possible lateral oscillations, and inadequacies in the mathematical model (which did not include sting terms which may alter the free vibration behaviour of the system). Recommendations for further study are digital recording of strain gauge response, development of a calibration rig to eliminate possible loading errors and extension of the mathematical model to include the dynamics of the sting. The effect of relative measurement delays due to transducer dynamics and electrical coupling in the strain gauge circuits may also be worth investigating.

In the present application, the applied Moment and Normal force are dominated initially by the inertial terms which require the double differentiation of measured data. One way of reducing the significance of the inertial terms would be to increase the system stiffness, (basically by having a deeper beam) thereby increasing the natural frequency in relation to the duration of the forcing term. The method of analysis would apply equally well as is clear from the axial force results in Section 4. A further way would be to reduce the model mass as much as practical. With the high sensitivity strain gauges used in the present work, the limit to increasing beam stiffness appears to be the degree to which the model/balance system can be completely isolated from random vibrations.
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FIG. 1 REPRESENTATIVE MODEL/BALANCE SYSTEM
FIG. 2 PLACEMENT OF STRAIN GAUGES
FIG. 3 SHOCK TUNNEL MODEL
FIG. 4 STRAIN GAUGE CIRCUITS
\[ V_Z = [\Delta V_1 - \Delta V_2] \times 10^4 \text{ (V)} \]

\[ \frac{\partial V_Z}{\partial Z} = 0.000275 \text{ V/N} \]

\[ \frac{\partial V_Z}{\partial Z} = 0.0000506 \]

\[ \frac{\partial V_Z}{\partial Z} = -0.000152 \]

\[ \frac{\partial V_Z}{\partial Z} = -0.000371 \]

\[ V_M = [\Delta V_3 - \Delta V_2] \times 10^4 \text{ Volts} \]

\[ \frac{\partial V_M}{\partial Z} = 0.00116 \text{ V/N} \]

\[ \frac{\partial V_M}{\partial Z} = 0.000421 \]

\[ \frac{\partial V_M}{\partial Z} = -0.000324 \]

\[ \frac{\partial V_M}{\partial Z} = -0.00108 \]

\[ \frac{\partial V_Z}{\partial x} = -\frac{\partial V_Z}{\partial M} = -R_{ZM} \]

\[ \frac{\partial V_Z}{\partial x} \bigg|_{x=0} = R_{ZZ} \]

\[ R_{ZM} = -0.005550 \]

\[ R_{ZZ} = -0.0000660 \]

FIG. 5 STATIC CALIBRATION CURVES FOR THE CANTILEVER BALANCE
\[ V_X = (3v_3 + 2v_2) \times 10^4 \]

\[ \frac{3V_X}{3Z} = 0.000090 \]
\[ \frac{3V_X}{3Z} = 0.000330 \]
\[ \frac{3V_X}{3Z} = -0.000027 \]
\[ \frac{3V_X}{3Z} = -0.000076 \]

\[ R_{XZ} = 0.0000094 \]
\[ \frac{3V_X}{3Z} = \frac{3V_X}{3M} \]
\[ R_{XM} = 0.001414 \]

\[ V_X = (3v_3 + 2v_2) \times 10^4 \]

Volts

\[ \frac{3V_M}{3X} = -0.000162 \] V/N

Volts

\[ \frac{3V_Z}{3X} = -0.0000573 \]

FIG. 5 (CONT.)
FIG. 6 DISTURBED MOTION OF THE MODEL CENTRE OF GRAVITY AND BALANCE REFERENCE CENTRE (Directions shown are positive and forces and moments are correspondingly so)
FIG. 7 MODEL DYNAMIC RESPONSE TO PULSE INPUTS
FIG. 8 INFERRED FORCE AND MOMENT HISTORIES
FIG. 9 STRAIN GAUGE RESPONSES TO IMPULSIVE BLOW
Microphone-Room
Accelerometer 10 mV/g
Stag. pressure 0.21 mV/bar

(b) Illustrating Structural Vibration transmitted through Laboratory floor

FIG. 10 STRAIN GAUGE POTentiOMETER CIRCUIT SERIES RESISTANCE VOLTAGE TRACES
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This memo presents a preliminary study of a proposed method of measuring the aerodynamic forces on a supported model in an intermittent very short duration wind tunnel with a relatively high airflow dynamic pressure (of the orders of 200 l/sec and 1/3 atmosphere respectively). A semiconductor strain gauged cantilever beam balance is used to record strain time histories associated with model displacement in response to aerodynamic force. The practical feasibility of obtaining sufficiently resolvable strains for the prescribed tunnel conditions with the given strain gauge configuration is established. The proposed method uses a system identification procedure to determine the system dynamic response characteristics using a known calibration force input. Subsequently, aerodynamic forces during a tunnel run follow from the recorded strain gauge
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16 Abstract (Contd)

time histories. The procedure has been demonstrated successfully using simulated data. However, the experimental situation did not lead to a successful analysis in the way proposed. Reasons for this are discussed and recommendations made for improvements. A brief series of shots in the ANU free piston shock tunnel also highlights the need to isolate as much as possible the model/balance system from external vibrations.

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