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RADIO TELEMETRY FORMULA APPLICATIONS,
A PRACTICAL USERS GUIDE

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RADIO TELEMETRY FORMULA APPLICATIONS, A PRACTICAL USERS GUIDE

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The relationship between the various parameters of a frequency-modulated (FM) or double frequency-modulated (FM/FM) radio telemetry link and the resulting output signal-to-noise ratios are presented. Most of the relationships have been presented in varying degrees of applicability, but the purpose of this report is to present formulas that can be used as a quick reference for telemetry system designers. The mathematical derivation of all equations can be found in various radio telemetry and communications textbooks and papers.
20a. (continued)

The basic radio frequency link transmission formula with a sample calculation is also presented.
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I. INTRODUCTION

Frequently a need exists to calculate the required radio-frequency (RF) carrier deviation, receiver intermediate-frequency (IF) bandwidth, and the postdetection signal-to-noise (SNR) ratios in a given frequency modulation (FM) or double frequency-modulation (FM/FM) transmission link. Also, when certain postdetection SNR requirements are established, the link parameters must be correspondingly specified.

The elements of a typical FM or FM/FM transmission link are shown in Figure 1. An FM link would begin with the transmitter modulator input and terminate at the receiver-demodulator output after postdetection filtering. An FM/FM link would begin at the subcarrier oscillator input and terminate at the subcarrier discriminator low-pass filter output. The modulating data signals considered will be assumed sinusoidal and periodic; however, this is not a limitation, since the postdetection SNR derived can be considered to apply to aperiodic signals during their time of occurrence. The general and special case formulas for calculating the elements of Figure 1 are given in Section II of this report. The special case formulas are for the Inter-range Instrumentation Group (IRIG)¹ proportional bandwidth (PBW) and constant-bandwidth (CBW) channels. Sample calculations are made where appropriate. Section III presents the formulas required for calculating the parameters involved in the radio-frequency transmission link.

The symbols used in Section II are defined in the List of Symbols.

II. SIGNAL-TO-NOISE RATIO IMPROVEMENT FORMULAS FOR FREQUENCY-MODULATED AND DOUBLE FREQUENCY-MODULATED RADIO LINKS

A. General Formulas for Frequency-Modulated Radio Links

1. **Single FM System** - The formula used for calculating the output SNR versus the input SNR (same as the second detection system in an FM/FM link) is

   \[
   \text{SNR}_{\text{out}} = \text{SNR}_{\text{in}} \left[ \frac{\sqrt{1.5}}{(B_{\text{IF}})^{1/2}} \frac{\Delta F}{(P_{\text{m}})^{3/2}} \right].
   \]  

   \[\text{(1)}\]

2. **First Detection Process in an FM/FM System** - To calculate the subcarrier filter output SNR versus the carrier input SNR,

---
Figure 1. Elements of an FM or FM/FM Transmission Link
\[ \text{SNR}_{sc} = \text{SNR}_c \left[ \frac{\sqrt{1.5} (B_{\text{if}})^{1/2} \Delta F_c}{(F_u - F_l)^{3/2}} \right] \] (2)

is used.

For subcarrier peak deviations small in comparison to the subcarrier center frequency, (i.e., the IRIG channels) Eq. (2) can be simplified to

\[ \text{SNR}_{sc} = \text{SNR}_c \left[ \frac{B_{\text{if}}^{1/2} \Delta F_c}{(2B_{sc})^{1/2} F_{sc}} \right]. \] (3)

3. **Second Detection Process in an FM/FM System** - The equation used for calculating the data output SNR versus the subcarrier filter output SNR is given by

\[ \text{SNR}_d = \text{SNR}_{sc} \left[ \frac{(1.5)^{1/2} B_{sc}^{1/2} \Delta F_{sc}}{(F_d)^{3/2}} \right]. \] (4)

4. **Overall FM/FM System** - The data output SNR versus the carrier input SNR is calculated from

\[ \text{SNR}_d = \text{SNR}_c \left[ \frac{(0.75)^{1/2} B_{\text{if}}^{1/2} \Delta F_c \Delta F_{sc}}{(F_d)^{1/2} F_{sc} F_d} \right]. \] (5)


These equations are used for calculating the subcarrier filter output SNR versus the carrier input SNR using a constant K derived from the IRIG standard subcarrier bandwidths and deviations. This is represented by a special case of Eq. (3).

1. **Proportional Bandwidth Channels**
Using the term \((2 \, B_{sc})^{1/2}\) from Eq. (3), where

\[(2 \, B_{sc})^{1/2} = (2 \times 2 \times \frac{\% \text{ deviation}}{100} \times F_{sc})^{1/2},\]

and introducing

\[K = \left(\frac{100}{4 \times \% \text{ deviation}}\right)^{1/2},\]

Eq. (3) then becomes

\[\text{SNR}_{sc} = \text{SNR}_c \left[\frac{K \,(B_{1f})^{1/2} \, \Delta F}{(F_{sc})^{3/2}}\right].\] (6)

The constant \(K\) is 1.826 for IRIG Channels 1-25 (± 7\(\frac{1}{2}\) % deviation) and 1.291 for IRIG Channels A-L (± 15% deviation).

2. Constant Bandwidth Channels

Again examining the term \((2 \, B_{sc})^{1/2}\) from Eq. (3) for this case,

\[(2 \, B_{sc})^{1/2} = (2 \times 2 \times \Delta F_{sc})^{1/2} = 1/K.\]

Eq. (3) then becomes

\[\text{SNR}_{sc} = \text{SNR}_c \left[\frac{K \,(B_{1f})^{1/2} \, \Delta F}{F_{sc}}\right].\] (7)
The magnitude of \( K \) is

\[
11.18 \times 10^{-3} \text{ for } \pm 2 \text{ kHz deviation,} \\
7.9 \times 10^{-3} \text{ for } \pm 4 \text{ kHz deviation,} \\
5.59 \times 10^{-3} \text{ for } \pm 8 \text{ kHz deviation,} \\
3.95 \times 10^{-3} \text{ for } \pm 16 \text{ kHz deviation,} \text{ or} \\
2.79 \times 10^{-3} \text{ for } \pm 32 \text{ kHz deviation.}
\]

C. Second (Subcarrier) Detection Process for the Standard Interrange
Instrumentation Group Channels

These equations are used for calculating the data output SNR versus the
subcarrier filter output SNR using a constant \( K \) derived from the IRIG standard
subcarrier bandwidths and deviations. These are represented by a special case
of Eq. (4).

1. Proportional Bandwidth Channels

Using the terms \((1.5)^{1/2} \left( \frac{B_{sc}}{F_{sc}} \right)^{1/2} \Delta F_{sc}\) from Eq. (4),

\[
(1.5)^{1/2} \left( \frac{B_{sc}}{F_{sc}} \right)^{1/2} \Delta F_{sc} = (1.5)^{1/2} \left( 1.05 \times 2x \frac{\% \text{ deviation}}{100} \right) F_{sc}^{1/2} \left( \frac{\% \text{ deviation}}{100} \right) F_{sc},
\]

and

\[
K = (3.15)^{1/2} \left( \frac{\% \text{ deviation}}{100} \right)^{3/2}.
\]

Eq. (4) then becomes

\[
\text{SNR}_{d} = \text{SNR}_{sc} \left[ \frac{K \left( F_{sc} \right)^{3/2}}{\left( F_{d} \right)^{3/2}} \right], \tag{8}
\]

where the values for \( K \) are \(3.65 \times 10^{-2}\) for IRIG Channels 1-25 (\( \pm 7 1/2 \% \text{ deviation}\)) and \(10.31 \times 10^{-2}\) for IRIG Channels A-L (\( \pm 15\% \text{ deviation}\)).
2. **Constant Bandwidth Channels**

Using the terms \((1.5)^{1/2} \frac{B_{sc}}{F_{sc}} \Delta F_{sc}\) from Eq. (4),

\[
(1.5)^{1/2} \frac{B_{sc}}{F_{sc}} \Delta F_{sc} = (1.5)^{1/2} (1.05 \times 2 \times \Delta F_{sc})^{1/2} \Delta F_{sc} = K,
\]

where

\[
K = (3.15)^{1/2} (\Delta F_{sc})^{3/2},
\]

Eq. (4) then becomes

\[
\text{SNR}_d = \text{SNR}_{sc} \left[ \frac{K}{(F_d)^{3/2}} \right],
\]

where
- \(K\) is \(1.59 \times 10^5\) for \(\pm 2\) kHz deviation,
- \(4.49 \times 10^5\) for \(\pm 4\) kHz deviation,
- \(12.70 \times 10^5\) for \(\pm 8\) kHz deviation,
- \(35.92 \times 10^5\) for \(\pm 16\) kHz deviation, or
- \(101.60 \times 10^5\) for \(\pm 32\) kHz deviation.

D. **Composite (Overall) Signal-to-Noise Ratio for the Standard Intergroup Channels**

These equations are used for calculating the data output SNR versus the carrier input SNR using a constant \(K\) derived from the IRIG standard subcarrier bandwidths and deviations. (Special case of Eq. (5)).

1. **Proportional Bandwidth Channels**

Using the terms \(\frac{(0.75)^{1/2} \Delta F_{sc}}{F_{sc}}\) from Eq. (5),

it can be shown that
\[
\frac{(0.75)^{1/2} \Delta F_{sc}}{F_{sc}} = \frac{(0.75)^{1/2} \left( \text{% Deviation} \times F_{sc} \right)}{100 F_{sc}} = K.
\]

Simplifying all terms,

\[ K = 0.866 \times 10^{-2} \text{ (% deviation)}, \]

and Eq. (5) then becomes

\[
\text{SNR}_d = \text{SNR}_c \left[ \frac{K \left( B_{if} \right)^{1/2} \Delta F_c}{(F_d)^{3/2}} \right],
\]

(10)

where \( K \) is 6.495 \times 10^{-2} for IRIG Channels 1-25 (\( \pm 7 \frac{1}{2} \text{% deviation} \)) or 12.99 \times 10^{-2} for IRIG Channels A-L (\( \pm 15 \text{% deviation} \)).

The formula for calculating the RF carrier deviation (\( \Delta F_c \)) when \( \text{SNR}_d \) is specified as 40 db (100:1) at a receiver threshold of \( \text{SNR}_c = 10 \text{ db (3.162:1)} \) is

\[
\Delta F_c = \frac{31.63 (F_d)^{3/2}}{K \left( B_{if} \right)^{1/2}}.
\]

(11)

From the relationships

\[
\text{DR} = \frac{\Delta F_{sc}}{F_d},
\]

and

\[
\Delta F_{sc} = PF_{sc},
\]

where \( P \), a constant, is either 0.075 or 0.15,

Eq. (11) becomes
\[ \Delta F_c = \frac{31.63 \times (P)^{3/2}}{K(B_{if})^{1/2} \times (DR)^{3/2}} \times (F_{sc})^{3/2} \]  

(12)

In a proportional bandwidth system, \(P, K, B_{if},\) and \(DR\) are constant and it can be seen that the RF carrier deviation \(\Delta F_c\) varies with respect to \((F_{sc})^{3/2}\). This fact illustrates the classical "3/2" power pre-emphasis characteristic.

2. **Constant Bandwidth Channels**

Using the terms \((0.75)^{1/2} \Delta F_{sc}\) for the value of \(K,\) Eq. (5) becomes

\[ \text{SNR}_d = \text{SNR}_c \left[ \frac{K \times (B_{if})^{1/2} \times \Delta F_c}{F_{sc} (F_d)^{3/2}} \right] \]  

(13)

where \(K = 1.732 \times 10^3\) for \(\pm 2\) kHz deviation,
\(3.464 \times 10^3\) for \(\pm 4\) kHz deviation,
\(6.928 \times 10^3\) for \(\pm 8\) kHz deviation,
\(13.856 \times 10^3\) for \(\pm 16\) kHz deviation, or
\(27.712 \times 10^3\) for \(\pm 32\) kHz deviation.

The formula for calculating the RF carrier deviation \(\Delta F_c\) when \(\text{SNR}_d\) is specified as 40 db \((100:1)\) at a receiver threshold of \(\text{SNR}_c = 10\) db \((3.162:1)\) is

\[ \Delta F_c = \frac{31.63 \times F_{sc} \times (F_d)^{3/2}}{K \times (B_{if})^{1/2}} \]  

(14)

From the relationship

\[ \text{DR} = \frac{\Delta F_{sc}}{F_d} \]

Eq. (14) then becomes

\[ \Delta F_c = \frac{31.63 \times (\Delta F_{sc})^{3/2}}{K(B_{if})^{1/2} \times (DR)^{3/2}} \times F_{sc} \]  

(15)
In a constant bandwidth system, $\Delta F_{sc}$, $K$, $B_if$ and DR are constant, and it can be seen that the RF carrier deviation ($\Delta F_c$) varies proportionately with respect to $F_{sc}$.

3. Sample Calculation of RF Carrier Deviation

The requirement for this example is to find the required RF carrier deviations for five standard proportional bandwidth channels in order to have a data output SNR of 40 db at receiver threshold. An FM/FM link requires an $\text{SNR}_c = 10$ db. The parameters given include a receiver IF bandwidth of 500 kHz, the IRIG channels selected are 14 thru 18, and the deviation ratio (DR) is 5.

From Eq. (12),

$$\Delta F_c = \frac{31.63 \times (0.075)^{3/2} \times (F_{sc})^{3/2}}{6.495 \times 10^{-2} \times (500 \times 10^{-3})^{3/2} \times (5)^{3/2}} \quad \text{and}$$

$$\Delta F_c = 1.265 \times 10^{-3} \times (F_{sc})^{3/2}.$$

**Table 1. Sample Calculation**

| Channel No. | $F_{sc}$ (kHz) | $\Delta F_c$ (kHz) | M  
|-------------|----------------|--------------------|-----
| 14          | 22             | 4.13               | 0.19|
| 15          | 30             | 6.57               | 0.22|
| 16          | 40             | 10.12              | 0.25|
| 17          | 52.5           | 15.22              | 0.29|
| 18          | 70             | 23.43              | 0.33|

**E. Receiver Intermediate-Frequency Bandwidth Considerations**

There seems to be a number of schools of thought for calculating the required receiver IF bandwidth for an FM/FM multiplex system. Schwartz,²

Gruenberg, and Stein state that a general rule of thumb equation based on a single-frequency sinusoidal modulating signal is,

\[ B_{if} = 2 (\Delta F_c + F_{sc}) \quad (16) \]

where \( F_{sc} \) is taken as the highest subcarrier frequency in the multiplex system. This equation designates a bandwidth that is wide enough to include all sideband current pairs that are greater than 10% of the amplitude of the unmodulated carrier. If the unmodulated carrier and sideband current amplitudes were converted to power levels, then Eq. (16) would include all sidebands having power levels greater than 1% of the unmodulated carrier power.

The Radio Engineers Handbook states that an estimate of the IF bandwidth required for transmission of a complex modulation signal is given by

\[ B_{if} = 2(\Delta F_c + 2 F_{sc}) \quad (17) \]

This equation, based on a single-frequency sinusoidal modulating signal, designates a bandwidth that includes all sideband current pairs greater than 4% of the amplitude of the unmodulated carrier. This corresponds to 0.16% for power levels.

Eqs. (16) and (17) plus a table of Bessel functions can be plotted in convenient form as shown in Figures 2 and 3. The Bessel functions are plotted to include sideband current pairs that are greater than 1% of the unmodulated carrier. The IF bandwidth on the plots has been normalized to the subcarrier frequency for convenience.

---


Figure 2. Receiver Bandwidth vs Modulation Index, Large M
Figure 3. Receiver Bandwidth vs Modulation Index, Small M
F. Derivation of Optimum Intermediate-Frequency Bandwidth and Radio-Frequency Deviation for a Single Subcarrier Frequency

The optimum IF bandwidth and RF carrier deviation for a single subcarrier frequency, (usually the highest SCO frequency in a multiplex) is derived from Eq. (5),

\[
\text{SNR}_d = \text{SNR}_c \left[ \frac{\sqrt{0.75} \ (B_{If})^{\frac{1}{2}} \ \Delta F_c \ \Delta F_{sc}}{(F_d)^{\frac{1}{2}} \ F_{sc} \ F_d} \right].
\]

Assuming that

\[
C = \frac{\text{SNR}_d}{\text{SNR}_c \sqrt{0.75}}, \quad \text{(18)}
\]

\[
\Delta F_{sc} = R F_{sc}, \quad \text{and} \quad \text{(19)}
\]

\[
F_d = K F_{sc}, \quad \text{(20)}
\]

and substituting Eqs. (17), (18), (19), and (20) in Eq. (5),

\[
C = \left( 2 \Delta F_c + 4 F_{sc} \right)^{\frac{1}{2}} \Delta F_c \ R F_{sc} \left( K F_{sc} \right)^{\frac{1}{2}} F_{sc} K F_{sc}.
\]

Simplifying and squaring all terms,
\[
C^2 = \frac{(2 \frac{\Delta F}{F_c} + 4 \frac{\Delta F}{F_{sc}}) \frac{\Delta F}{F_c} \frac{R^2}{F_{sc}^2}}{\left(K \frac{F_{sc}}{F_c} \frac{R^2}{F_{sc}^2} \right)}.
\]

Multiplying all terms and separating,

\[
C^2 = \frac{2 \frac{\Delta F}{F_c} \frac{R^2}{F_{sc}^3}}{K^3 F_{sc}^3} + \frac{4 \frac{\Delta F}{F_{sc}} \frac{R^2}{F_c^3}}{K^3 F_{sc}^3}.
\]

Simplifying and substituting \(M\) for \(\frac{\Delta F}{F_{sc}}\),

\[
C^2 = \frac{2M^2 R^2}{K^3} + \frac{4M^2 R^2}{K^3},
\]

and we arrive at the result

\[
M^3 + 2M^2 = \frac{C^2 K^3}{2 R^2}.
\]  \(21\)

A sample problem, given that \(\text{SNR}_d = 100\) (40 db), \(\Delta F_{sc} = 32\) kHz, \(\text{SNR}_c = 3.162\) (10 db), \(F_d = 8\) kHz, and \(F_{sc} = 256\) kHz results in the following:

\[
C = \frac{100}{3.162 \times 0.866} = 36.518 \text{ (from Eq. (18)),}
\]

\[
R = \frac{32 \times 10^3}{256 \times 10^3} = 0.125 \text{ (from Eq. (19)), and}
\]

\[
K = \frac{8 \times 10^3}{256 \times 10^3} = 3.125 \times 10^{-2} \text{ (from Eq. (20)).}
\]
Solving Eq. (21),

\[ M^3 + 2M^2 = 1.302 \, . \]

Using the solution for solving a cubic equation,

\[ M = 0.695 \, (A \, positive \, real \, root) . \]

Since \( M = \frac{\Delta F_c}{F_{sc}} \) then

\[ \Delta F_c = 177.92 \, kHz, \]

and from Eq. (17),

\[ B_{1f} = 1379.84 \, kHz. \]

The value of \( M \) will vary for each of the IRIG constant bandwidth channels but will remain constant for each set of the proportional bandwidth channels. The values of \( M \) for the PBW channels are \( M = 0.4076 \) for \( \pm 7 \frac{1}{2} \% \) deviation channels and \( M = 0.559 \) for \( \pm 15\% \) deviation channels.

III. RADIO-FREQUENCY LINK TRANSMISSION FORMULA

The radio-frequency link transmission formula is a very useful tool for calculating a safety factor when all link parameters are defined, or determining any of the individual factors in the link. The basic formula can be stated in terms of power levels relative to a fixed reference. For this discussion a reference of 1 milliwatt into 50 ohms (0 dbm) will be used. This formula is a convenient tool for link calculations since all parameters are expressed in the same terms instead of meters, seconds, Hertz, etc.

A. Basic Transmission Formula

The basic equation for transmission is

\[ P_t = R_n + SNR - G_t - G_r + L + PL + SF , \]  \hspace{1cm} (22)
where $P_t$ is the transmitter power (dbm), $R_h$ is the equivalent noise input of receiving system (dbm) (this term will be discussed in Part B), SNR is the signal-to-noise ratio required for a particular type of transmission (normally 9–12 db is required for an FM/FM Telemetry link), $G_t$ is the transmitter antenna gain (db), $G_r$ is the receiving antenna gain (db), $L$ represents the Miscellaneous losses (polarization fade, cable losses, vswr, etc., 10 db is normally used), and PL is the Path loss or attenuation (db). The latter term is represented by

$$ PL = C + 20 \log f + 20 \log d^5, \quad (23) $$

where $f$ is the transmitter frequency (MHz), $d$ is the distance, and $C = 36.58$ when $d$ is expressed in miles (statute), $-37.87$ when $d$ is given feet, and $-27.55$ when $d$ is cited in meters. The term SF in Eq. (22) is the Safety factor (db).

**B. Equivalent Noise Input of Receiving System ($R_n$)**

The formula for the term $R_n$ of Eq. (22) is,

$$ R_n = K T_e B_{if} \text{ (Watts)}^{(5)} \quad (24) $$

where $K$ is equal to $1.38 \times 10^{-23}$ joules/$^0$K (Boltzmanns constant), $T_e$ is the effective receiving system noise temperature ($^0$K ), and $B_{if}$ is the Receiver IF bandwidth (Hz). Eq. (24) can be expressed in Logarithmic notation referenced to 0 dbm,

$$ R_n = -198.6 + 10 \log T_e + 10 \log B_{if}. \quad (25) $$

The term $T_e$ in Eq. (24) and Eq. (25) can be calculated from

$$ T_e = T_R + TA/L + T_L (1 - 1/L), \quad (26) $$
where \( T_R \) is the receiving system noise temperature referred to its input \((^0K)\), \( T_A \) is the effective antenna temperature \((^0K)\), \( T_L \) is the temperature of losses between antenna and receiving system (normally \(290^0K\)), and \( L \) is the power-loss ratio between antenna and receiving system. In most cases the term \( T_A/L \) can be neglected for frequencies above 20 MHz.

Some receiving systems are composed of a preamplifier located near the antenna and a receiver located at a distance in a building or van. For this case the formula for the system noise temperature of networks in cascade is

\[
T_R = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots ,
\]

(27)

where \( T_1 \) is the noise temperature of first network \((^0K)\), \( T_2 \) is the noise temperature of second network \((^0K)\), \( G_1 \) is the effective gain (power ratio) between first and second networks \((\text{i.e. gain minus losses})\), and \( G_2 \) is the effective gain (power ratio) between second and third networks.

Most specifications for preamplifiers and receivers state the system noise temperature in terms of a db noise figure. This noise figure can be converted to noise temperature by use of the following equation,

\[
T = (Nf - 1),
\]

(28)

where \( Nf \) is the noise factor (power ratio of the noise figure) and \( T_0 = 290^0K \).

A useful formula for finding the overall noise figure of a preamplifier and receiver in cascade can be found by substituting Eq. (28) in Eq. (27) which yields

\[
(Nf_R - 1) T_0 = (Nf_1 - 1) T_0 + \frac{(Nf_2 - 1) T_0}{G_1} .
\]

Simplifying all terms,

\[
Nf_R = Nf_1 + \frac{(Nf_2 - 1)}{G_1} .
\]

(29)
The cascaded noise figure is then

\[ \text{NF}_R = 10 \log \text{NF}_R \]  \hspace{1cm} (30)

C. Sample Calculation for the Safety Factor in a Radio-Frequency Transmission Link

Presuming that the following parameters are given for a transmission link,

- transmitter power \( (P_t) = 250 \text{ milliwatts (} + 24 \text{ dbm}) \),
- transmitter antenna gain \( (G_t) = +4 \text{ db} \),
- receiver antenna gain \( (G_r) = +16 \text{ db} \),
- distance \( (d) = 12 \text{ miles} \),
- frequency \( (f) = 1500 \text{ MHz} \),
- receiver IF bandwidth \( (B_{IF}) = 500 \text{ kHz} \),
- preamplifier gain = 18 db (63:1) with a noise temperature = 600 \( ^0K \) (\( \text{NF} = 4.87 \text{ db} \)),
- receiver noise temperature = 3000 \( ^0K \) (\( \text{NF} = 10.55 \text{ db} \)), and
- cable losses, for antenna to preamplifier = 5 db (3.16:1) and for preamplifier to receiver = 6 db (3.98:1),

all parameters of Eq. (22) are defined except \( R_n \) and \( PL \). They may be calculated as follows:

\[ \text{TR} = 600 + \frac{3000}{(63-3.98)} \]  \hspace{1cm} (from Eq. (27)), yielding

\[ \text{TR} = 651^0K, \text{ and} \]

\[ T_e = 651 + 290 \left( 1 - \frac{1}{3.16} \right) \]  \hspace{1cm} (from Eq. (26)), or

\[ T_e = 849^0K. \]

From Eq. (25)
\[ R_n = -198.6 + 10 \log 849 + 10 \log (500 \times 10^3) \], which gives

\[ R_n = -112 \text{ dbm (rounded off)}. \]

From Eq. (23)

\[ PL = 36.58 + 20 \log 1500 + 20 \log 12, \text{ or} \]

\[ PL = 122 \text{ db (rounded off)}. \]

The safety factor (SF) is calculated by inserting all given and computed values into Eq. (22),

\[ P_t = R_n + SNR - G_t - G_r + L + PL + SF, \text{ which gives} \]

\[ 24 = -112 + 12 - 4 - 16 + 10 + 122 + SF, \text{ and the result} \]

\[ SF = 12 \text{ db}. \]

The significance of the safety factor is that the signal at the input of the receiver will be 12 db greater than that required for the FM threshold of the receiver at a maximum range of 12 miles.

The radio link parameters can be expressed in tabular form, but the use of a level diagram permits easy visualization and helps to prevent errors of sign or omission during link calculations. Figure 4 presents a convenient form using the parameters from the sample calculation.
IV. SUMMARY

The relationships and formulas presented in this report should give the FM/FM telemetry system designer enough information to design a practical operating telemetry system. Further information on the derivation of the formulas and additional theory can be found in the list of references.
REFERENCES


APPENDIX A

COMPUTER PROGRAM FOR THE CALCULATION OF TELEMETRY RF DEVIATIONS AND RECEIVER IF BANDWIDTH (WRITTEN IN BASIC LANGUAGE FOR THE HEWLETT PACKARD MODEL 9845 COMPUTER)
CALCULATION OF TELEMETRY R.F. DEVIATIONS AND I.F. BANDWIDTH (3/28/75)
--- OPTIONS: 1=DEV FOR ACTUAL BW, 2=OPTIMUM DEV VS BW, 3=BOOTH ---
*****************************************************************************

10 OPTION BASE 1
20 PRINT IS 0
30 PRINT "CALCULATION OF TELEMETRY R.F. DEVIATIONS AND I.F. BANDWIDTH (3/28/75)"
40 PRINT "--- OPTIONS: 1=DEV FOR ACTUAL BW, 2=OPTIMUM DEV VS BW, 3=BOOTH ---"
50 PRINT "*****************************************************************************"
60 PRINT LINC(3)
70 INPUT "WHAT IS INPUT OPTION =",O
80 IF O=1 THEN 110
90 IF O=2 THEN 130
100 IF O=3 THEN 150
110 PRINT "*** CALC OF DEVIATION FOR ACTUAL BANDWIDTH ***"
120 GOTO 160
130 PRINT "*** CALCULATION OF OPTIMUM DEVIATION VS BANDWIDTH ***"
140 GOTO 160
150 PRINT "*** CALCULATION OF OPTIMUM DEV VS BW AND DEV FOR ACTUAL BW ***"
160 PRINT LINC(2)
170 INPUT "WHAT IS INPUT CARRIER SNR = (DB)",N1
180 INPUT "WHAT IS OUTPUT DATA SNR = (DB)",N3
190 PRINT "INPUT CARRIER SNR = ";N1;" DB";" OUTPUT DATA SNR = ";N3;" DB"
200 PRINT
210 N2=10^(N1/20)
220 N4=10^(N3/20)
230 C9=N4/(N2+SNR(.75))
240 INPUT "WHAT IS SCO CENTER FREQ (KHz)";F
250 INPUT "WHAT IS SCO FREQ DEV (KHz)";F1
260 INPUT "WHAT IS SCO DEVIATION RATIO",D
270 PRINT "SCO CENTER FREQ = ";F;" KHz SCO FREQ DEV = ";F1;" KHz DEV RATIO = ";D"
280 PRINT
290 F=F*1000
300 F1=F1*1000
310 F3=F1/D
310 IF O=1 THEN 1170
320 PRINT "C9 = ";C9
330 PRINT
340 P=1
350 Q=0
360 R=-(C9^2/(2*P^3))*F1/F
370 A=1/3*(3*Q-P^2)
380 B=1/27*(2*P^3-9*P*Q+27*R)
390 C=B^2/4+A^3/27
400 PRINT IS 0
410 PRINT "P = ";P;" Q = ";Q;" R = ";R
420 PRINT "A = ";A;" B = ";B;" C = ";C
430 PRINT
440 IF C<0 THEN Unequal
450 IF C>0 THEN Imag
460 PRINT "THERE ARE THREE REAL ROOTS, TWO ARE EQUAL"
470 PRINT
480 S1=1
490 B1=-B/2
500 IF B1>=0 THEN 520
510 S1=-1
520 M1=S1*ABS(B1)^(1/3)*2-P/3
530 M2=-S1*ABS(B1)^(1/3)-P/3

29
M=M1
560 Rep;PRINT "M1= ";M1;" M2= ";M2;" M3= ";M3
570 PRINT
580 PRINT "M= ";M
590 PRINT
600 S1=M1^3+2*M1^2+R
610 S2=M2^3+2*M2^2+R
620 S3=M3^3+2*M3^2+R
630 PRINT "S1= ";S1;" S2= ";S2;" S3= ";S3
640 GOTO Quit
650 Imag: A1=-(B/2)-SQR(C)
660 PRINT "THERE IS ONE REAL & TWO CONJUGATE IMAGINARY ROOTS"
670 PRINT
680 B1=-(B/2)+SQR(C)
690 S1=1
700 S2=1
710 IF A1>0 THEN 730
720 S1=-1
730 IF B1>0 THEN 750
740 S2=-1
750 A1=S1*ABS(A1)^(1/3)
760 B1=S2*ABS(B1)^(1/3)
770 PRINT "A1= ";A1;" B1= ";B1
780 PRINT
790 M1=A1+B1-P/3
800 M=M1
810 M2=-(A1+B1)/2-P/3
820 I2=(A1-B1)/2*SQR(3)
830 M3=M2
840 I3=-I2
850 PRINT "M1= ";M1
860 PRINT "M2= ";M2;" I2= ";I2
870 PRINT "M3= ";M3;" I3= ";I3
880 PRINT
890 PRINT "M= ";M
900 PRINT
910 S1=M1^3+2*M1^2-C9^2/(2*D^3)*F1/F
920 PRINT "S1= ";S1
930 GOTO Quit
940 Unequal: T2=-(B/2)+SQR(-A^3/27)
950 PRINT "THERE ARE THREE REAL UNEQUAL ROOTS"
960 PRINT
970 T3=SQR(1-T2^2)
980 T=ATN(T3/T2)
990 IF T2>0 THEN 1010
1000 T=-T+PI
1010 T1=2*SQR(-A/3)
1020 M1=T1*COS(T/3)-P/3
1030 M2=T1*COS(T/3+2*PI/3)-P/3
1040 M3=T1*COS(T/3+4*PI/3)-P/3
1050 M=M1
1060 IF M1<M THEN 1080
1070 M=M2
1080 IF M1<M THEN 1100
1090 M=M3
1100 GOTO Rep
1110 Quit: F2=M*F/1000
1120 W=2*(F2+1000+2*F)/1000
1130 PRINT
1140 Imag1:IMAGE "OPT CARRIER DEV= ";DDD," KHz
1150 PRINT "OPT I.F. BW= ";DDDD," KHz"
1170 INPUT "WHAT IS ACTUAL I.F. BW (Hz)"
1180 PRINT
1190  F2=C9+F+F3^1.5/SQR(W1)+F1
1200  PRINT USING Imag2;W1/1000;F2/1000
1210  PRINT
1220  W2=2*(F2+F)
1221  Imag3:IMAGE "REQUIRED BW FOR R.F. DEV= ",DDDD," KHz"
1230  PRINT USING Imag3;W2/1000
1240  Blank:PRINT LIN(3)
1250  PRINTER IS 16
1260  STOP
1270  END
APPENDIX B

COMPUTER PROGRAM FOR THE CALCULATION OF THE SAFETY FACTOR FOR AN RF
TRANSMISSION LINK (WRITTEN IN BASIC LANGUAGE FOR THE HEWLETT PACKARD MODEL
9845 COMPUTER)
CALCULATION OF SAFETY FACTOR FOR RF TRANSMISSION LINK

10 OPTION BASE 1 ! RFLINK
20 PRINTER IS 0
30 PRINT " CALCULATION OF SAFETY FACTOR FOR RF TRANSMISSION LINK"
40 PRINT " ******************************************
50 PRINT LIN(2)
60 INPUT "ENTER TRANSMITTER POWER=(MW)", P1
70 PRINT PAGE
80 INPUT "ENTER TRANSMITTER ANTENNA GAIN=(DB)", G1
90 PRINT PAGE
100 INPUT "ENTER RECEIVER ANTENNA GAIN MINUS CABLE LOSS TO PRE-AMP OR RCVR=(DB)")", G2
110 INPUT "ENTER TRANSMISSION RANGE=(FEET)", D
120 INPUT "ENTER TRANSMISSION FREQUENCY=(MHZ)", F
130 INPUT "ENTER RECEIVER I.F. BANDWIDTH=(HZ)", B
140 INPUT "ENTER PREAMPLIFIER GAIN=(DB), ENTER 0 WITHOUT PREAMPLIFIER", G3
150 INPUT "ENTER PREAMPLIFIER NOISE FIGURE=(DB), ENTER 0 WITHOUT PREAMPLIFIER" , N1
160 INPUT "ENTER RECEIVER NOISE FIGURE=(DB)", N2
170 INPUT "ENTER CABLE LOSS(PREAMP-RCVR)=(DB) ENTER 0 IF NO PRE-AMP USED", L2
180 INPUT "ENTER REQUIRED SIGNAL TO NOISE RATIO=(DB)", S
190 INPUT "ENTER SYSTEM MISCL LOSSES=(DB)", L
200 P=10^LGT(P1)
210 A=-37.87+20*LGT(F)+20*LGT(D)
220 N4=10^((N1*N2)/10) ! RCVR NF
230 IF (G3=0) AND (N1=0) THEN 330
240 N3=10^((N1/10) ! P.A. NF
250 N4=10^((G3/10) ! P.A. GAIN
260 L4=10^((L2/10) ! P.A. RCVR
270 N=N3+(N4-1)/(G4-L4)
280 T=(N-1)*290
290 GOTO 340
300 R=198.6+10*LGT(T)+10*LGT(B)
310 S1=P+R+G1+G2-S-L-A
320 PRINT "INPUT PARAMETERS"
330 PRINT " TRANSMITTING POWER OUTPUT= "; P1; " MILLIWATTS"
340 PRINT " SYSTEM ANTENNA GAIN= "; G1; " DB"
350 PRINT " RECEIVING ANTENNA GAIN= "; G2; " DB"
360 PRINT " SYSTEM PREAMPLIFIER NOISE FIGURE= "; N1; " DB,(0=NO PRE-
370 PRINT " PREAMPLIFIER GAIN= "; G3; " DB,(0=NO PRE-AMP)
380 PRINT " RECEIVER NOISE FIGURE= "; N2; " DB"
390 PRINT " I.F. BANDWIDTH= "; B/1000; " KHz"
400 PRINT " REQUIRED SIGNAL TO NOISE RATIO= "; S; " DB"
410 PRINT " SYSTEM LOSSES PREAMPLIFIER TO RECEIVER= "; L2; " DB"
420 PRINT " MISCELLANEOUS= "; L; " DB"
430 PRINT " TRANSMISSION RANGE= "; D; " FEET"
440 PRINT " FREQUENCY= "; F; " MHZ"
450 PRINT LIN(2)
460 PRINT "OUTPUT DATA"
550 PRINT
551 Image1: IMAGE S X, "PATH LOSS=",DDD," DB"
560 PRINT USING Image1;A
570 PRINT
571 Image2: IMAGE S X, "EQUIVALENT NOISE OF RECEIVER=",MDDD," DBM"
580 PRINT USING Image2;R
590 PRINT
591 Image3: IMAGE S X, "OVERALL SAFETY FACTOR=",DDD," DB"
600 PRINT USING Image3;S1
610 PRINT LIN(2)
620 PRINTER IS 16
630 STOP
640 END
LIST OF SYMBOLS

\( B_{1f} \)  
Receiver intermediate frequency equivalent noise* bandwidth (Hz)

\( B_{sc} \)  
Sub-carrier filter equivalent noise* bandwidth (Hz)

\( DR \)  
Deviation ratio = \( \Delta F_{sc}/F_d \)

\( \Delta F_c \)  
Carrier peak deviation produced by the sub-carrier output voltage (Hz)

\( \Delta F_{sc} \)  
Sub-carrier peak deviation produced by the input data voltage (Hz)

\( F_c \)  
Carrier radio frequency (Hz)

\( F_d \)  
Discriminator output low pass filter frequency or maximum data frequency (Hz)

\( F_k \)  
Output filter-lower band edge (Hz)

\( F_m \)  
Modulating frequency (Hz) - in most cases \( F_m = F_d \)

\( F_{sc} \)  
Sub-carrier center frequency (Hz)

\( F_u \)  
Output filter-upper band edge (Hz)

\( M \)  
Modulation index = \( \Delta F_c/F_{sc} \)

\( SNR_c \)  
Carrier pre-detection RMS signal-to-noise ratio (db)

\( SNR_d \)  
Data or discriminator output RMS signal-to-noise ratio (db)

\( SNR_{sc} \)  
Sub-carrier pre-detection RMS signal-to-noise ratio (db)

* The noise bandwidth is assumed equal to \( \sqrt{1.05} \times (-3 \text{ db bandwidth}) \)

NOTE: All signal-to-noise ratios used in the formulas of Section II are RMS voltage ratios. Power ratios can be found by squaring all terms within the brackets.
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