The Electron Momentum Transfer Cross Sections in N₂ and O₂

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**Abstract:** The electron momentum transfer cross section in nitrogen (\( \text{N}_2 \)) and oxygen (\( \text{O}_2 \)) are reviewed and data are presented for electron energies from 0.01 eV to 1000 eV.
16. SUPPLEMENTARY NOTATION

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THE ELECTRON MOMENTUM TRANSFER CROSS SECTIONS IN $N_2$ AND $O_2$

1. INTRODUCTION

High energy and high current electron beams traversing air dissipate their energies by collisions and by ohmic heating of the plasma electrons. The ohmic heating depends on the channel conductivity. The conductivity, on the other hand, depends on the electron momentum transfer collision frequency which in turn depends on the electron ion and electron neutral momentum transfer cross sections. These cross sections are essential for the calculations of the air conductivity and ohmic energy dissipation.

In this report we review the current status of the data for the electron momentum transfer cross section in $N_2$ and $O_2$. From these one can obtain the momentum transfer cross section in air. In Section 2 we review the momentum transfer cross section in $N_2$ for the electron energy range of 0.1 eV to 1 KeV. Section 3 provides the review in the same electron energy range for the momentum transfer cross section in $O_2$.

2. THE MOMENTUM TRANSFER CROSS SECTION IN $N_2$

The momentum transfer cross section obtained by Englehardt, et al\textsuperscript{1} for the electron energy range of 0.01 eV to 40 eV is shown in Figure 1. This cross section obtained by the Swarm experiments has been utilized exclusively in various discharge studies. Other studies have since followed to obtain the momentum transfer cross section using other techniques, mainly through the measurements or calculations of the differential elastic scattering cross sections. Additionally, the electron energy range was increased beyond the 40 eV limit of the Swarm experiments. Some discrepancy between these new data and the measurements of Englehardt, et al\textsuperscript{1} arises and will be discussed in this section.

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The momentum transfer cross section as derived by Shyn, et al\textsuperscript{2} from the measurements of the differential cross section for elastic scattering in \textit{N}_2 is also shown in Figure 1 for comparison with the measurements of Ref. 1. The data by Shyn, et al\textsuperscript{2} is for electrons in the energy range of 5 eV to 90 eV. The agreement between the results of Refs. 1 and 2 is good only at an electron energy of $E = 10$ eV. Furthermore, there is a drastic difference between them at $E > 10$ eV. Whereas Shyn, et al\textsuperscript{2} data show a decrease in the momentum transfer cross section with increasing electron energy for $E > 10$ eV, the earlier data of Englehardt, et al\textsuperscript{1} show a rise and a flat behavior for the cross section as a function of $E$. This discrepancy may well be due to two effects (a) the results of Ref. 1 includes both the elastic and inelastic collision contributions to the momentum transfer cross section (b) the results of Ref. 1 is based on Swarm data which assumes a set of cross sections varied to be compatible with the measured transport coefficients. Obviously with the availability of better cross sections, the Swarm data can provide more reliable data on the momentum transfer cross sections.

The experimental momentum transfer cross section in \textit{N}_2 obtained from the differential scattering cross section by Srivastava, et al\textsuperscript{3} is also shown in Figure 1. These results are somewhat higher than those of Shyn, et al\textsuperscript{2} presumably due to lower differential cross sections at higher scattering angles in the latter measurements. Cartwright\textsuperscript{4} has calculated the contributions of the inelastic processes in \textit{N}_2 to the momentum transfer cross section. However, this contribution does not include two important inelastic channels, i.e. the dissociation and ionization. Cartwright\textsuperscript{4} adds the inelastic contribution to the elastic component obtained by Srivastava, et al\textsuperscript{3} to obtain the momentum transfer cross section for $E$ up to 60 eV. His results are also shown in Figure 1. An observation is made in Ref. 4 that the contribution of the inelastic processes amounts to 25\% of the total momentum transfer cross
section for electrons in the energy range of 10 eV to 40 eV. The momentum transfer cross section obtained by Cartwright is still lower than those of Ref. 1. However, they are higher than those by Shyn, et al., presumably because of the inelastic contribution included by Cartwright. It is possible that the contribution from the dissociation and ionization of nitrogen, if included into the Cartwright calculation, would bring the total momentum transfer cross section closer to those of Englehardt, et al. for higher electron energies.

As for the momentum transfer cross section for electrons above \( E > 100 \) eV no direct data is derived or available. However, we shall use a limited information on the differential scattering cross sections to obtain the momentum transfer cross section in this region. To proceed, however, let us review the available data on the total elastic cross section where information is available for electron energies above 100 eV. Figure 2 shows the elastic scattering cross section measured by Shyn, et al., Srivastava, et al., Finn and Doering. These measurements are all for electron energies below 100 eV. However, one measurement of the elastic cross section at 500 eV is due to Romberg. Furthermore, Finn and Doering have used the differential scattering cross sections of Romberg at \( E = 300 \) eV and 400 eV to obtain the elastic cross sections at 300 eV and 400 eV. These results are shown on Figure 2. It is obvious from this Figure that one can easily extend the experimental results below \( E = 100 \) eV right into the data at 500 eV. The theoretical calculations of the elastic cross section of Wedde and Strand which should be valid for high energy electrons is also shown in Figure 2 where the agreement is very reasonable compared with the measurements of Romberg.

To obtain the momentum transfer cross section above 100 eV we utilize the differential scattering cross section of Romberg and Wedde and Strand shown
These differential cross sections are utilized into Equation (1)

\[ \sigma_m = 2\pi \int \frac{d\sigma}{d\theta} (1 - \cos\theta) \sin\theta \, d\theta \]  

(1)

and the integration is carried out piecewise in \( \theta \) to obtain the momentum transfer cross section. This approximate calculation yields a value of

\( 6 \times 10^{-17} \text{cm}^2 \)

at \( E = 500 \) eV and is shown in Figure 1. Thus a nice fit from \( E = 10 \) eV to \( 1000 \) eV can be obtained to provide the momentum transfer cross section. As for \( E < 10 \) eV, the data of Englehardt, et al\(^1\) is recommended.

3. THE MOMENTUM TRANSFER CROSS SECTION IN \( \text{O}_2 \)

The differential scattering cross section of electrons of 2.0 - 200 eV in \( \text{O}_2 \) has been measured by Shyn and Sharp\(^9\). From these, both the total elastic and momentum transfer cross sections are obtained through the appropriate integrations. Figure 4 shows the momentum transfer cross section in \( \text{O}_2 \) as derived by Shyn and Sharp\(^9\) for electrons from 2 eV to 200 eV. This cross section is compared with the most often utilized momentum transfer cross section obtained by Hake and Phelps\(^{10}\), also shown in Figure 4. It should be noted again that there is a wide disagreement between these two measurements. This disagreement may well be due to the Swarm data technique as discussed earlier and that the momentum transfer cross section of Ref. 10 includes various other contributions to the momentum transfer beyond the elastic component.

The theoretical calculations for the total elastic scattering cross section of electrons in \( \text{O}_2 \) by Wedde and Strand\(^7\), which is valid for high
energy electrons, is utilized to obtain the momentum transfer cross section. This cross section is also shown in Figure 4 for electron energy of up to 1 keV. The method used to obtain the momentum transfer cross section from the total elastic $\sigma_e$ is to use the ratios of $\sigma_e/\sigma_m$ from the data of Shyn and Sharp$^9$ and apply it to the calculation of $\sigma_e$ of Wedde and Strand$^7$ then extrapolate this ratio beyond $E = 200$ eV.

Thus the momentum transfer cross section for electrons with energy from thermal to 1 keV can be constructed from the data given in Figure 4, where above $E = 10$ eV a best fit can be obtained using the data of Refs. 7 and 9. Below 10 eV the data of Refs. 9 and 10 are to be utilized.
Fig. 1 The momentum transfer cross section in N$_2$ as a function of the electron energy.
Fig. 2  The elastic cross section in $N_2$ as a function of the electron energy.
Fig. 3 The differential elastic scattering cross section in $N_2$ as a function of the scattering angle.
Fig. 4 The momentum transfer cross section in $O_2$ as a function of the electron energy.
REFERENCES


