Report No. 6

White Light Optical Information Processing

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Methods for optical processing and holography with light of reduced coherence are described. Specifically described are: (a) a method for making holographic optical elements in light of reduced spatial coherence; (b) a method for doing real time phase conjugation in light of reduced spatial coherence; and (c) a method for doing off-axis Fourier transform holography in spatially incoherent light.
1. Research Objectives

Our research objectives, as stated in the work statement of our proposal, were:

1. Demonstrate white light grating interferometry method of phase-amplitude recording using phase and phase amplitude objects from various areas, including engineering and biological.

2. Extend the white light and incoherent optical processing concepts into such areas as robotic vision and phase conjugation, as well as into other areas of image processing.

3. Investigate the SNR for incoherent optical processing systems in the presence of artifact noise, detector noise, and bias build up, and determine the significant parameters, such as space-band width product of the system impulse response.

4. Compare the SNR for the coherent case, polychromatic achromatic coherent case, and the extended source incoherent case, and characterize the situation when the coherent (monochromatic or polychromatic) is advantageous and when the incoherent is advantageous.

5. Examine the trade off between resolution and SNR.

6. Conduct experimental comparisons between coherent and incoherent processing to verify the analytic conclusions.

2. Accomplishments

We gave attention to all of the above areas, except #5, with stress on item 2, 3, 4 and 6. In the context of carrying out these investigations, we developed ideas that were not in the original proposal.

Our principal accomplishments for the year have been:
1. We adapted our method for doing reduced-coherence phase conjugation to real time processes, and we conducted experiments to verify the ideas.

2. We carried out an extensive analysis, development and experimental program on the method for recording in spatially incoherent light the Fourier transform of an object. This method, conceived in the closing days of the previous year, works as if the light were coherent, even though the light is in fact coherent. For example, it works on the light amplitude and it preserves phase.

3. We discovered, about a year ago, a method for making holographic optical elements in spatially incoherent light. Much of our effort for this past year has been in developing and extending this idea. In the process, we investigated the SNR achievement in making HOES with incoherent light as opposed to coherent light. In short, we carried out much of the work suggested in items 3, 4 and 6 of our work statement within the context of HOE construction.

4. The white light spatially incoherent grating interferometry method for recording phase amplitude image was applied to image plane holography with successful results.

3. Real Time Aberration Compensation (Phase Conjugation)

with Partially Coherent Light

The phase conjugation imaging technique has the capability for producing diffraction-limited resolution even in severely aberrated imaging systems. In this technique, various beams are mixed in a non-linear medium. An object-bearing beam, passing through the medium, generates a conjugate beam $u^*$, which retraces the path of the original beam $u$, and the aberrations in $u^*$ are then compensated by the aberrations of the imaging system. This process, carried out in real time is similar to a process developed in static holography in 1965.1-3
The process in a basic way requires coherent light, which is poor for imaging. First, coherent light, when interacting with a diffuse material, either by reflection from it or transmission through it, acquires a speckly appearance. Second, any scattering or perturbing structure along the optical path results in noise that overlays the image.

There appears to be basically two alternatives in imaging through an aberrating system. First, we can carry out the imaging process in a conventional way, using incoherent light, thereby achieving a good signal to noise ratio, but degraded resolution. Second, we can carry out the process in a phase conjugation process with coherent light, thereby achieving improved resolution but a poorer signal to noise ratio.

One solution to this problem was given by Huignard et al. In this method, one in effect makes a sequence of hundreds of exposures, each with a different noise function. The recording medium adds these, giving a resultant where the noise is reduced by the averaging process.

Our method achieves a comparable result using an essentially different principle. Instead of making a large number of coherent operations in sequence, we perform a single operation with light of reduced spatial coherence, and possibly also of reduced temporal coherence. This method is an adaptation of a method described in last year's report for static holographic imaging through a phase-distorting medium. Here, we modify the earlier result by adapting it to real time operation, principally by the use of a Mach Zehnder interferometer in place of the grating interferometer previously used. The light loss is thereby reduced by an order of magnitude.

The method, in the embodiment we have used for its demonstration, is shown in Fig. 1. Light from a laser source is passed through an interferometer that splits the incident light into two beams, then recombines them. The first element is a beam splitter, and the remaining elements are mirrors that redirect the beams so that they
Fig. 3.1. The Basic System
are brought together again at the plane of the phase conjugating medium. One of the two beams is the object beam, which contains an object and an aberrated imaging system, as well as some noise, resulting from such various defects as dust, digs and scratches on the lenses, etc. Lenses $L_3$ and $L_4$, arranged in the telescopic configuration, image the object $O$ onto the phase conjugating medium $P$. Mirror $M_4$ reflects the reference beam back on itself and as it passes again through the conjugating medium, a conjugate beam $u^*$ is generated. The conjugate beam travels through the imaging system in reverse and forms an image.

The object beam carries the image, the aberrations and the noise. Ideally, the image information should be perfectly passed, so that a high resolution image can be formed. The phase error, too, should be preserved, so that it can be cancelled in the conjugation process, thus permitting the high resolution that we seek. The noise, however, should be reduced, i.e., smoothed out, so that it no longer degrades the image. We can, for example, reduce the noise by reducing the spatial coherence of the light; indeed, complete spatial incoherence would essentially eliminate the noise entirely. However, the reduction in coherence would also partially destroy the image and phase error information that is to be transferred onto the conjugate beam.

However, by forming an image on the conjugation plane, the coherence requirements for preserving the image information become nil—perfectly incoherent light would do. The aberrations, on the other hand, originate at a plane other than the object plane; indeed, the aberration sources are distributed, arising from entire regions rather than from single plane. Thus, the aberration sources are not imaged at the conjugation plane.

The aberrations are slowly varying; therefore, they require only a low degree of coherence for their preservation. The noise, however, is generally of higher spatial frequency. Therefore, we expect that the spatial coherence can be reduced to a degree
such that the noise is significantly reduced, but that the aberration is essentially preserved. Indeed, if the aberrations are varying sufficiently slowly, considerable reduction in coherence is permissible, with good aberration preservation along with enormous noise reduction.

We can reduce the coherence of the source in one of the many ways; the simplest is to place a rotating ground glass in the beam near the source point. The degree of coherence reduction depends on how far the ground glass is from the source. Another point that is important in the present context is that this method supposes the field to change rapidly compared to the time constant of the conjugation process. Otherwise, the process will be a sequence of coherent recordings, instead of a single recording with partially coherent light.

The reduced coherence will tend to destroy the interference fringes produced at the conjugation plane by the superposition of the object and reference beams. However, the interferometer of Fig. 1, which is basically a modified Mach Zehnder, is capable of broad source operation. The production of fringes under broad source illumination is an important aspect of classical interferometry, and is discussed in texts on interferometry.\(^6\) If the beams are adjusted so that the corresponding rays in the two beams, i.e., rays that come from a single ray impinging on the beam splitter, are brought back together, then the two beams will interfere in this plane even when the source has reduced coherence. It has been shown that, for the Mach Zehnder interferometer, the number of fringes obtained from a monochromatic extended source is approximately\(^7\)

\[
N = \frac{1}{(2\Delta \theta)^2}
\]

where \(\Delta \theta\) is the angle subtended by the source at the collimating lens \(L\). This relation allows a large number of fringes, perhaps, several thousand, even for a source that is many times larger than what the system would see as a point source.
To perform the experiment, we first construct a controlled aberration by coating a portion of a glass slide with optical cement to a thickness of about 100-200 microns. The coating, rather irregular in thickness, provided an aberration that varied from region to region. The object beam, a few mm in diameter, intercepted only a portion of the aberration plate, and by moving the plate, a suitable aberration could be injected into the beam.

The system was first tested using conventional, static holography, that is, using photographic plate instead of a phase conjugation material. The object was a grid of fine slits of width 1 mm. The hologram was developed and replaced in its original position, the object beam was blocked, and the reference beam was reflected by a mirror \( M_4 \) back through the system, providing the required conjugate wave. Photographs of the reconstruction (Fig. 2) show the process works as expected. The aberrated image is shown in 2a, just as it appears at the plane of hologram formation. Figures 2b and 2c show the reconstruction with light of reduced coherence in both the making and readout steps. Figure 2b differs from 2c in that the hologram was used in a liquid gate, that is, a glass plate was fluid-coupled to the emulsion side of the hologram in the reconstruction process. This greatly reduced the noise from surface defects on the emulsion. Finally, Fig. 2d shows the result when the process was carried out with coherent light. The phase error compensation is very good, but the noise level is high.

By rotating the mirror, \( M_4 \), the reference beam could be aimed in various directions, so that the conjugate beam path could be shifted, thus making the phase error compensation imperfect. When this was done, the principal effect was to produce curvature in the slit images, sometimes with some slight broadening (Fig. 2). The mirror was adjusted until the slit images became straight, whereupon the system was assumed to be properly adjusted.
Fig. 3.2. Experimental results, using conventional (i.e., static) holography. (a) conventional (not holographic) imaging with partially coherent light through the aberration. (b) imaging using holography and reduced coherence. Also, a flat glass plate was coupled to the emulsion surface using an index matching fluid. (c) same as b, but without the glass plate. (d) imaging using holography and complete spatial coherence.
For the real-time, or phase conjugation process, a thin film of nematic liquid crystal was used as the phase conjugation medium, in conjunction with an argon laser (488 A line) with an optimum power of 3.0 watts. The laser was divided equally into the reference and object beams. The total laser power (object, reference and reconstruction beams all together) incident on the nematic film, after passage through the optical system, was about 0.2 watts. A second beam splitter BSI was placed between the object and the mirror so as to separate the phase conjugated image from the object. The liquid crystal MBBA (p-methoxybenzylidene-p-n-butylaniline) was homeotropically aligned, with the director axis parallel to the plane defined by the two incident beam propagation vectors (reference and object) and the optical polarization of the beams was normal to the director axis. For this configuration, the nonlinearity arises from the thermal indexing effect (i.e., d n /dT, where n is the refractive index for the ordinary ray, and T the temperature. The diffraction efficiency for the MBBA film was about 0.1 percent at room temperature, but it can be increased by raising the temperature to close to the nematic + isotropic transition. Typically a diffraction (or wavefront conjugation) efficiency of about 3 percent is obtainable.

We show experimental results using two different objects. Figure 3 shows results using as an object just the laser beam, a circular distribution of light about 1 mm in diameter. The aberrated beam, the phase conjugation beam using light of reduced spatial coherence, and the phase conjugation beam using coherent light are shown in Figs. 3 a, b, and c, respectively. The noise reduction in the incoherent case is quite evident.

Figure 4 shows similar results, except that the object was a slit. Again, the phase conjugation image using light of reduced spatial coherence has resolution just about as good as for the coherent case, and the signal to noise ratio is much superior.
Fig. 33. Phase conjugation results, using a circular aperture for the object. (a) Photographic record of the object, after imaging through the aberrating medium, and using light of reduced coherence. (b) Phase conjugation imaging with light of reduced spatial coherence. (c) Same as b, but with completely coherent light.
Fig 3.4. Image of a slit (a) formed with incoherent light, (b) formed using phase conjugation with light of reduced coherence, (c) formed using phase conjugation with completely coherent light.
REFERENCES


4. Fourier Transforms with Spatially Incoherent Light

Toward the close of last year's reporting period we invented a new method for performing Fourier transform holography with monochromatic, spatially incoherent light. This idea has now been developed and explored. A complete treatment is given in the doctoral thesis of G. Collins. Here we describe the main features. The system is the first true spatially incoherent counterpart of conventional coherent holography in that it is linear in the complex amplitude of the object. As a result, all the capabilities which characterize conventional coherent holography are preserved in this system. The physical realization of the proposed technique is also quite similar in many respects to a conventional coherent holographic system. The system uses the unique properties of a grating interferometer to achieve the objective.

In the past, a number of systems were described for doing quasi-holography with incoherent light. None of these systems were true holography; none of them employed a separate reference beam, and were thus unlike conventional holography, and did not preserve the phase of the object distribution. The system we have developed is basically different; it does employ a coherent reference beam, and there are a number of interesting consequences that result.

The basic system consists of the interferometer shown in Figure 4.1. The incident illumination is from a spatially extended (incoherent) source and is quasi-monochromatic. It is assumed that the source is not so spatially broadband that the beams cannot be separated within the interferometer. This is a practical consideration only, and is otherwise unnecessary for the proposed technique. The ideal system behavior requires that the incident illumination be completely (spatially) incoherent as would be the case if the source were infinitely extended. In actual practice, the beam separability requirement can be satisfied as the idealized behavior is approximated to any degree of accuracy, by constraining the spatial bandwidth of the object amplitude
Fig. 4.1. The interferometric system for recording holograms in quasi-monochromatic, spatially incoherent illumination. G1, G2, and G3 are gratings of spatial frequency $f_g$. $t_A$ is the complex amplitude transmittance of the object.
transmittance to be suitably narrowband with respect to the spatial carrier frequency of the gratings.

The basic interferometer, composed of the gratings G1, G2, and G3, is the grating equivalent of the classical Mach-Zehnder interferometer. The grating G1 splits the incident light into two beams: an object beam, derived from a diffracted order, and a reference beam, derived from the undiffracted or zero order. The object beam propagates to the grating G2 where it is redirected to become parallel with its original direction. It intercepts the object transparency, which assumes the role played by a test section in conventional interferometry, and propagates onward. The reference beam propagates to the grating G3 where it is redirected, by diffraction, to intercept the object beam.

The basic three-grating interferometer is initially set up so that, without the object present, high contrast fringes are obtained that are localized at the z = 0 plane under broad source conditions. It is essential to the proposed technique that the interferometer be capable of forming, under conditions of broad source illumination, spatially invariant, high contrast, strongly localized interference fringes of arbitrary spatial frequency.

With the object transparency inserted into the system, each spatial frequency of the amplitude transmittance acts as a grating to diffract and thus redirect a portion of the incident light. Considered separately, each of these diffracted beams interferes with the reference beam to form a unique (constant z) plane of localized fringes. The geometry of the system is such that the distance z, at which the fringes for a given spatial frequency of the object localize, is linearly proportional to that spatial frequency. Thus, for an arbitrary object transparency, interference fringes will generally be formed throughout the beam overlap region.
The interference fringe distribution is recorded photographically on a tilted plane in the beam overlap region. Analysis shows that the interference fringes contain a term, on a spatial carrier, which is proportional to the Fourier transform of the diffracted (or Fresnel) field of the object at the recording plane. The intensity distribution at the recording plane is thus, with the exception of the Fourier transform relationship, very similar to that which would be found in the equivalent, but coherent holographic system.

The system is capable of two-dimensional imagery within the plane of the interferometer, i.e., it does not image in the dimension transverse to the plane of the interferometer. Within this plane, however, the imaging process is entirely analogous to conventional coherent holography. The system is linear in, and can reconstruct an image of, the complex amplitude associated with an object transparency. Depth or z-dimension information, is preserved in the same fashion as in conventional holography. The imaging process is not constrained to a single object plane or distance (as one might expect if the fundamental process were a spatial domain compensation technique analogous to temporal domain compensation techniques), and in fact, is fully capable of imaging two-dimensional complex amplitude distributions within the plane of the interferometer. In short, the system is capable of essentially arbitrary wavefront construction within this plane.

Experimental results have shown that this system works as theory indicates, and that it works very well indeed. Our intention is to apply it to such areas as robotic vision.

5. The Construction and Evaluation of HOES Made in Light of Reduced Coherence

5.1 Introduction
Fringes formed in light of reduced coherence, either spatial or temporal, are less noisy than fringes formed with coherent light. However, in general, fringes formed in light of reduced coherence are usually limited in various ways; the fringes may be of lower contrast, fewer in number, may exist only over a small region, or it may prove difficult or impossible to modulate the fringes, that is, form fringes of non-uniform spacing such as a zone plate pattern. Here we describe two complimentary optical systems with which we can modulate fringes to obtain zone plate patterns made using interferometry with a spatially broad, monochromatic source. Such interferometrically recorded zone plate patterns are often called holographic optical elements (HOEs) or diffractive optical elements and have been finding increasing application. Diffraction efficiency and noise level of these elements are also discussed.

5.2 System

Initial experiments of this type were discussed by Swanson, who described a method for modifying the grating interferometer so as to form zone plate structures instead of grating structures. In this method lenses are placed in each branch of the interferometer so as to image a common plane at the same magnification, but with different curvature being given to the interfering wavefronts. The interferometer configuration is shown in Fig. 1. For example lenses $L_2$ and $L_3$ may be in the telescopic (or afocal) configuration, so that $P_1$ is imaged to $P_{out}$ in such a way that a plane wave at $P_1$ becomes a plane wave at $P_{out}$. Lens $L_1$ also images plane $P_1$ onto $P_{out}$ but in such a way that the wave impinging on $P_{out}$ is a diverging spherical wave of radius $F$, the focal length of the three lenses being used. The two beams interfere under broad source monochromatic illumination, but the fringes, being the result of interference between a plane and spherical wave, form, after recording, a diffraction lens of focal length $F$. Thus to make low noise diffractive lenses, we have at $P_1$ a uniform distribution of light. No structures are placed at $P_1$, since such structures would inevitably contain noise.
Fig. 5.1
System for making HOES in extended source light.
(dirt, scratches, etc.), thus the fringe pattern is very clean when the light source is of reduced spatial coherence.

Important to the incoherent HOE construction method is the range of HOEs that can be achieved. We would like to be able to produce any focal length. This is relatively easy. More important is to achieve any range of conjugate focal planes. Noting that if we desire the object and image distances to be $d_{ob}$ and $d_{im}$, respectively, then for best results the HOE should be made with two interfering beams converging to or diverging from, points at these same distances. To achieve any possible set of $d_{ob}$ and $d_{im}$ is a more difficult problem. It is by no means a priori evident that it has a solution.

For convenience of analysis, suppose the optical systems within the two branches of the interferometer to be removed; the justification is that the interferometer itself need not be part of the analysis. All we must do to have these optical systems function in the interferometer is to impose a few constraints on them so that their presence would not disrupt the fringe forming capability of the interferometer. These two optical systems are shown in Fig. 2. The top system images plane $P_1$ to $P_{out}$, the fringe forming plane, and the lower system images $P_1'$ to $P_{out}$. $L_{sys}$ and $L_{sys}$ are imaging systems. We consider here the various forms that these systems can take, and the capabilities and limitations of these forms.

In order to obtain interference with a spatially incoherent monochromatic source, we require that at $P_{out}$ both beams have undergone the same amount of Fresnel diffraction and that the magnifications of the two beams be equal. Now, the Fresnel diffraction process occurs between the source plane and planes $P_1$ or $P_1'$. Since $P_1$ and $P_1'$ are each imaged to $P_{out}$, the Fresnel diffraction process stops at $P_1$ and $P_1'$. Thus, it is necessary that planes $P_1$ and $P_1'$ be equally distant from the source. Otherwise, the interference would be weak or absent. Since the diffraction paths stop at $P_1$ and $P_1'$ we must allow for the optical path from $P_1$ to $P_{out}$ and $P_1'$ to $P_{out}$ to be different, a condi-
Fig. 5.2
The basic concept.
tion that gives a useful flexibility in the realization of HOES with arbitrary conjugate focal planes. Indeed, it is in general necessary to image over different distances in each arm of the interferometer, that is, the distance from $P_1$ to $P_{out}$ must be different than that from $P'_1$ to $P_{out}$. Since the distances source to $P_1$ and source to $P'_1$ must be equal, it follows that the total optical paths must in general be unequal, the path difference being the difference between the $P_1 - P_{out}$ and the $P'_1 - P_{out}$ paths.

We have considered two optical systems that fulfill the above requirements. One is a modified version of a grating interferometer setup described by Swanson and the other is a modified version of the Mach-Zehnder interferometer (Fig. 3), where the usual beam splitter is replaced with a mirror. Both systems have advantages and drawbacks.

The required optical path difference can be generated in various ways. Such generation is easy in the modified Mach-Zehnder interferometer by simple positioning of the various mirrors. To generate a large path difference requires a high degree of monochromaticity, so that the beams can travel unequal path lengths and still interfere. This idea can be taken to an extreme by making the path length difference equal to twice the laser cavity length and compensating for Fresnel diffraction by imaging over the path length difference with appropriate lenses and unity magnification. This system will produce high contrast fringes in broad source monochromatic illumination. Besides the requirement of a high degree of monochromaticity, another disadvantage of this system is that the number of fringes is limited, being given by $V = \frac{1}{2\Delta\theta^2}$, where $\Delta\theta$ is the source subtense at the lens (not shown) that collimates the light incident on the interferometer. For a large $\Delta\theta$ the number of fringes may be inadequate. There is no similar constraint for the grating interferometer, which can produce a completely unlimited number of fringes regardless of source subtense.

Actual physical pathlength differences need not be introduced to compensate for differences in Fresnel diffraction. Inserting a cascade of appropriate unity
Fig. 5.3

Modified Mach Zehnder interferometer for HOE construction.
magnification telescopic imaging systems into the grating interferometer will produce
the desired Fresnel diffraction compensation. The disadvantage with this system is
that multiple lenses introduce more noise and more aberrations, both of which will
reduce fringe contrast.

A completely general mathematical analysis of the systems is somewhat tedious
and does not easily yield a simple physical viewpoint. There are basically two ways of
describing the process physically. Each branch of the optical system images the plane
$P_1$ (or $P'_1$) but with a quadratic phase factor. Alternatively we can say that each optical
system, in addition to imaging the plane $P_1$ (or $P'_1$) also images the source. The
distances $d_{ob}$, $d_{im}$ (Fig. 2) from the source image to the plane $P_{out}$ becomes the
designed conjugate focal planes of the resulting HOE. This viewpoint can lead to some
basic insights as to what range of $d_{ob}$, $d_{im}$ is possible.

For example, if the incident beam is collimated, where $l_{sys}$ and $l'_{sys}$ of Fig. 2
become imaging lenses in each branch of the interferometer for imaging $P_1$ and $P'_1$ to
$P_{out}$, then the source image is found at a distance of one focal length downstream from
the imaging lens. Thus, by appropriately adjusting the focal lengths $F_1$ and $F_2$, any set
of conjugate focal planes are possible, subject to the constraint that both source
images are formed to the left of $P_{out}$. This constraint always leads to a HOE designed
as a negative lens. This is a severe restriction, as almost all practical lenses are positive lenses. Besides, there is no justification for using an incoherent system for producing a negative HOE, since this can always be accomplished just as well and in a much simpler manner by conventional means, with light diverging from two pinholes a
distance $d_{ob}$ and $d_{im}$ from the recording plate, and with no optical elements between
either pinhole and plate, there will be no coherence induced noise.

The challenge is thus to cause one of the beams to be converging at $P_{out}$, i.e., have
either $d_{ob}$ or $d_{im}$ be a negative quantity, thus producing a positive HOE. The philosophy
of accomplishing this is the following. A lens can be considered as always preserving the sense of the image (Fig. 4). That is, two points $A$ and $B$, separated axially, are always imaged into points $A'$ and $B'$ with no axial inversion (i.e., $B$ and $B'$ both form to the right of $A$ and $A'$, respectively). Suppose we consider a dynamic situation, in which $A'$ and $B'$ are imaged continually farther from the lens, which can be accomplished by bringing the object points $A$ and $B$ continually closer to the lens. When $A$ reaches the front focal plane of the lens, $B'$ goes to infinity and further movement of $B$ towards the lens results in $B'$ coming in from minus infinity.

We thus have achieved one of the goals, we have effectively inverted $A$ and $B$, though the principal stated above has in a sense not been violated, since both $A$ and $B$ always moved in the forward axial ($z$) direction, without one overtaking the other. We see that the path is cyclic with the positive and negative ends being connected at infinity. Adding more focal power, either with shorter focal length lenses or the addition of more lenses, merely moves $A$ and $B$ (here, the source plane and the plane being imaged) to new positions along the $z$ track.

To apply this viewpoint to the HOE problem, we identify planes $A$ and $B$ with the source image and $P_{\text{out}}$, respectively, and we place a second lens in each branch of the system, which reimages both the source point and $P_{\text{out}}$. The viewpoint given above then seems to suggest that the constraint on positive focal length HOES should be no greater than that for negative HOES. For the analysis we return to Fig. 2. $P_1$ and $P_1'$ are both to be imaged at $P_{\text{out}}$ with equal magnification. The separations $P_1 - P_{\text{out}}$ (labeled $d_\alpha$) and $P_1' - P_{\text{out}}$ (labeled $d_\alpha'$) can be anything, including negative values, since it is immaterial what planes get imaged to $P_{\text{out}}$ as long as the distance $d_\alpha' - d_\alpha$ remains equal to the interferometric path difference, and indeed this path difference can be altered in any way we choose.
Imaging by lens of two axial points.
Considering Figs. 2 and 4 one can see that the addition of one more lens, which accepts both beams (Fig. 5) will produce any positive HOE desired. The conjugate focal planes are given by \( d_{ob} \) and \( d_{um} \) where,

\[
\begin{align*}
    d_{ob} &= \frac{d_1 f_3}{d_1 - f_3} \\
    d_{um} &= \frac{d_2 f_3}{d_2 - f_3}
\end{align*}
\]

Simply by varying \( f_3, d_1, \) and \( d_2 \) one can achieve any range of conjugate focal planes desired for a positive lens. The constraint for the HOE to be a positive lens are \( F_1 > f_2 \) and \( d_2 < F_3 < d_1 \). The diffraction pathlength difference for this HOE forming system is seen to be \( l_c = 4 (F_1 - F_3) \) in Fig. 6. Thus when using the Mach-Zehnder interferometer we require the coherence length to be greater than \( l_c \), the required physical path-length difference between the two arms of the interferometer (Fig. 3). Likewise \( l_c \) is the extra pathlength difference over which we must image in order to obtain equal diffraction pathlengths when using the grating interferometer of Fig. 2 integrated with the optical system of Fig. 5.

Here we have considered two systems; the grating interferometer, which is limited in fringe contrast by the number of lenses introduced into the system, and the modified Mach-Zehnder interferometer, which is limited in the number of fringes obtainable and by the coherence length of the laser being used. With these two systems almost any range of positive HOE's desired can be obtained using broad source monochromatic illumination.

5.3 Diffraction Efficiency

We desire that the diffraction efficiency of HOES made with light of reduced coherence be comparable to those made with coherent light. Thus, the fringe contrast of HOES should be high over the entire recording area and for even the highest spatial frequencies. Theoretically, such a high contrast fringe pattern will be produced, but any system defects will tend to lower fringe contrast as the source is broadened.
General configuration for H0E construction.
Various HOES were recorded with the interferometric arrangement of Fig. 1 both for coherent illumination and for illumination of reduced coherence. The HOES produced had a mean spatial frequency of 300 cycles per mm, the highest and the lowest spatial frequencies being 100 and 400 respectively. Three source sizes were used: a point source and sources of angular subtense 0.01 rad and 0.05 rad measured at the collimator.

A set of HOES was made for each of the three source sizes and the diffraction efficiency measured. The results, shown as the diffraction efficiency vs. exposure curve of Fig. 6, indicates that diffraction efficiency has decreased somewhat with increasing source size, but not to a significant degree. High diffraction efficiency (say, 60 to 90%) could be obtained by either bleaching or recording on dichromated gelatin for the optimal exposures of any of the source sizes used.

It appears that the greatest loss of diffraction efficiency arises from system imperfections, such as field curvature, which cause the surface of fringe localization to depart from the ideal planar shape, and the recording plate will therefore be in worse places, outside the surface of highest fringe contrast. Assuming that such aberrations were absent, there remain two factors that affect diffraction efficiency. First, the noise reduction is a smoothing process, wherein spatial frequency energy representing noise is converted to an ambient background. This results in a loss of fringe contrast and resulting diffraction efficiency. Since the noise, even in a rather noisy fringe pattern, has a power that is only a small fraction (a few percent) of that in the signal, the contrast loss for this effect should be similarly small. On the other hand, the presence of noise means locally that the beam ratio may be different from the nominal or average beam ratio, and the fringe contrast is thus lowered. In addition, the resulting local mean exposure may not be optimum, again providing lower diffraction efficiency.

Thus, as we broaden the source, we find factors that decrease the diffraction efficiency
Fig. 5.6
Diffraction efficiency vs. intensity transmittance. Solid curve--using coherent source. Dotted curve--using source of angular extent .005 radian. Dashed line--as before, but with angular extent .01 radian.
and others that increase it, and it appears that rigorous analysis is required to determine how these opposing factors balance.

5.4 Noise

To investigate the noise, various types of data were collected. First, HOEs were made using the grating interferometer setup of Fig. 1, with an extended source of over 0.005 rad. Second, HOEs were made the same way, but with a nonbroadened source. The results show the enormous noise reducing effect of the coherence reduction, but the comparison is not entirely realistic, since the interferometer contains noise sources, such as gratings, that would not be present in the normal HOE-forming system. Thus, to test the efficacy of the method, the comparison should be made with coherent illumination in a conventional HOE-forming system.

We have distinguished two HOE-forming systems, each rather simple. In one, interference is obtained between two divergent beams, and in the other, between a diverging and a converging beam. The former produces a HOE designed for use as a negative lens, and the latter a HOE designed for use as a positive lens. In the former case, light emanates from two pinholes, with no optical elements between pinhole and recording plate. Thus, the entire pattern should be quite free from setup noise except for back reflections from the recording plate, and experience shows that with good film backing, this can be negligible. In the second case, a lens must be placed downstream from the pinhole in one of the beams; this lens is unavoidably a noise source. Since positive lenses are far more common than negative lenses, the system with the noise-producing lens will be the usual case.

In general, noise on either of the two interfering beams will be recorded and will appear in the beam generated by the hologram. However, this noise can be minimized by proper hologram construction procedures. Suppose one of the beams has a noise

\[ n(x,y) = |n| \exp(i\phi), \]

i.e., both an amplitude and a phase component. On the basis of
conventional first order theory, both the amplitude and the phase components of the noise will be modulated onto the fringe pattern and will appear in the reconstructed beams. However, it is possible to considerably reduce, sometimes almost eliminate, the amplitude component. If diffraction efficiency is plotted as a function of exposure, the resulting curve typically shows a broad, flat maximum. Thus, if one aims to record at the center of the maximum region, but misses the proper exposure by a small amount, the diffraction efficiency is not affected.

Now, suppose one of the two beams to have an amplitude noise \( n \) of small magnitude, so that the beam has a spatial fluctuation across it of perhaps 10%. Consequently, the total exposure received by the recording plate will vary by some similar amount, or actually less, if the other beam is uniform. But, since the diffraction efficiency is insensitive to these exposure variations, the noise amplitude fluctuations \( n \) will not appear in the reconstructed beam, i.e., amplitude noise is suppressed. This suppression may be in fact only partial, since the fluctuations alter not only the exposure, but also the fringe contrast, and the lowered contrast results generally in HOES of lower diffraction efficiency.

However, the noise suppression that on balance results can be rather dramatic. For example, we recorded a fringe pattern about 1 cm$^2$ 10 separate times on different portions of a plate with different exposures. The exposed areas that had the highest diffraction efficiency were also found to diffract the most uniformly. Thus, the nonlinearities of the recording process indeed offer a significant mechanism for suppression of amplitude noise.

In the first noise experiment, the HOES were photographed through a microscope at various magnifications, results being shown in Fig. 7. Figures 7a and d show the HOE made in the grating interferometer with an extended source at two different magnifications, and Figs. 7c and d show the same, but with a point source. The noise is
Magnified image of HOE surface, using 0, +1 and -1 orders. a. extended source, low magnification. b. higher magnification. c. as in a, but using extended source. d. as in b, but with extended source.
setup noise produced by defects in the optical elements. The pictures show how effectively the higher spatial frequency noise is eliminated, whereas the very high spatial frequency noise is only partly removed by the reduction in spatial coherence. Not shown is the result for a HOE made in the conventional setup, both for the case of no lens in either beam beyond the pinholes, in which case the hologram is quite noise free, and for the case of a lens in one beam, yielding a result intermediate between the coherent and the incoherent cases of Fig. 7.

Of greater significance is the amount of noise that is to be seen in the first diffracted order. Indeed, it is conceivable, although unlikely, that none of the noise visible on the photograph of Fig. 4 is present in the first diffracted order. To examine the first diffracted order, a white light source of moderate, but not large, extent (a microscope illuminator) was used, the HOE was reimaged through a unity magnification telescopic system, and a spatial filter removed all diffractive orders except for one first order beam. The results, not shown here, are comparable to Fig. 7. It was also found that the highest diffraction efficiency exposures gave the least amplitude noise.

Since practical HOES are phase holograms, and since the amplitude-to-phase transformation amplifies the noise, we bleached some HOES and repeated the observations of the first order. At low magnification, noise is similar to that of Fig. 7 was observed.

Higher magnification revealed, however, yet another noise, much finer and basically different in appearance. This is shown in Fig. 8. Figure 8a, made with a coherent beam containing no optical elements downstream from the pinhole, revealed a noise-free field, except for a few spots and some grain noise, both produced by the film that recorded the beam. Similarly, Fig. 8b shows the result for a grating interferometer, containing various lenses and gratings, using an extended source. There is a small
trace of noise that we attribute to the HOE (the low frequency mottling, not the much finer grain noise). Figure 8 c-e show the result for the conventional case with a single lens in one beam. The noise level is considerable. In Fig. 8c, the camera is focussed on the emulsion, whereas in Fig. 8 c-e, the focus is moved slightly to one side of the emulsion, on the order of .5 mm. Again the small black spots are not relevant. The noise is enormously greater for the defocussed position. The noise thus appears to be predominately phase noise. It almost completely disappears when the emulsion was covered using a cover glass with xylene between the two glass surfaces. The noise thus is found to be a surface relief and the emulsion surface thus exhibits an orange peel effect. This seems somewhat curious, since the bleaching process (R10 bleach) produces a phase image primarily of the refracted index modulation type, and the brightness of the first diffracted order was only slightly affected by the liquid gate.

A search for the origin of the noise revealed that it arises from two causes, first, from diffraction from the aperture edge of the lens, and second, from noise on the lens. The effect could be duplicated with no lens, just a diffracting aperture (iris) in the beam, or by a large diameter lens, producing no edge diffraction but having some scattering centers (dirt, etc) on and in the glass.

We also observed the zero order and found again the same noise, but less intense. We looked for traces of such noise on the unbleached HOE's but failed to find any. Our presumption is that the noise is present on the unbleached plate, but at a level too low for observation. The bleaching process amplifies the noise, bringing it up to a visible level.

These noise measurements, by no means exhaustive, indicate that whenever scattering structures are present in one of the two beams in a HOE-making system the reduced coherence method will remove almost all the noise, except that of very low spatial frequency. However, if there are no scattering structures, the use of coherent
Fig. 5.8
Magnified image of surface of HOE, imaged in one 1st order only. a. HOE made with two diverging beams, and no optics between pinhole and recording plate. No noise visible, except for some emulsion defects on HOE surface. b. HOE made with grating interferometer, and lenses in system. Faint noise (mottling) is seen. c. HOE made in conventional system, with one lens in one beam. Some noise is seen. d. Microscope focused to one side of image plane (about .1 mm.) Noise is greater. e. Same as d, but on other side of image plane.
light will do quite well and is preferred because of the simplicity.

5.5 Concluding Comments

The results presented here, although by no means exhaustive, clearly indicate that HOES made with light of reduced coherence can have diffraction efficiency comparable to HOES made by the conventional coherent methods, and may have significantly better SNR. Finally, the reduced coherence methods are quite versatile.
References


6. Journal Articles
   a. Articles published during this period.


   b. Articles prepared during this reporting period, but not yet published.


   c. Symposium Presentations

d. Theses

Temporally and Spatially Incoherent Methods for Fourier Transforming and Optical Information Processing, G. D. Collins, April, 1983.

7. Persons Associated with this Effort

E. Leith (PI)

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G. J. Swanson (Graduate Student)

Y. S. Cheng (Graduate Student)

S. Leon (Graduate Student)

8. New Discoveries

The principal new discovery was of a way to make, by spatially incoherent light, HOES of any conjugate focal planes. This is the generalization of the rather specific method for incoherent light HOE generation we previously described.
Fig. 2.9 Tilted-Grating Interferometer.