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Title: Temperature Effects on the Evolution of Ionospheric Barium Clouds in the Presence of a Contacting Background Ionosphere

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We show that under realistic ionospheric conditions, barium ion clouds hundreds of meters in diameter can be long-lived, quasi-stable, nonbifurcating structures. These structures may resemble "tadpoles," with a dense head, steep density gradients at the front, and a long, less dense tail. We assume that these structures are the final products of the recursive bifurcation of a considerably larger barium ion cloud, i.e., striations. The realistic ionospheric conditions to which we refer consist of a barium ion cloud with ion temperatures $T_i$ of approximately $1000^\circ R$, coupled electrically to a background ionosphere of lower compressibility than itself, i.e., and F region. We show analytically that this combination of finite $T_i$ and relatively incompressible background results in an effective diffusion of barium plasma, but more importantly, of total magnetic-field-line-integrated Pedersen conductivity, $\Sigma_P$. The diffusion coefficient has a special form which allows the inner portions of the cloud to diffuse slowly, giving the cloud a long lifetime.
Finite Temperature Effects on the Evolution of Ionospheric Barium Clouds in the Presence of a Conducting Background Ionosphere

I. A High Altitude Incompressible Background Ionosphere

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and allows the outer, less dense portions of the cloud to diffuse rapidly, preventing cloud bifurcation. Numerical simulations of the full nonlinear dynamics are then used to show that this diffusion does in fact give rise to quasi-stable barium striations hundreds of meters in diameter. These findings are consistent with the linear analysis of Francis and Perkins (1975).
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FINITE TEMPERATURE EFFECTS ON THE EVOLUTION OF IONOSPHERIC BARIUM CLOUDS IN THE PRESENCE OF A CONDUCTING BACKGROUND IONOSPHERE

I. A High Altitude Incompressible Background Ionosphere

1. Introduction

The nonlinear evolution of an artificial plasma cloud (e.g., a barium ion cloud) released in the earth's ionosphere is characterized by an overall bulk \( E \times B \) motion, a one-sided steepening of plasma gradients, and by bifurcation [see Ossakow, 1979; Ossakow et al., 1982 and references therein] a process which we define here to mean the splitting of the cloud into two or more pieces. These pieces are sometimes referred to as "striations". All of the above motion and the resultant plasma structures are observed to be magnetic-field-aligned. That is, the plasma motion parallel to magnetic field lines consists primarily of diffusion and falling, while the perpendicular plasma motion is virtually identical for all plasma along a given field line. Hence one's best view of the structuring of the cloud is from a vantage point looking parallel to the magnetic field \( B \). This simplified description of barium cloud dynamics, seen along a line of sight parallel to \( B \), is depicted in Fig. 1. The sequence of drifting, steepening and bifurcating is, for sufficiently large striations, recursive: each striation becomes a new cloud which in turn forms its own striations. In the absence of some dissipative mechanism, it is believed that this process would continue indefinitely. In reality, the striation size becomes small enough to be comparable to the characteristic scale lengths of physical dissipation mechanisms, and further bifurcation ceases. A previous study by McDonald et al. [1981] attempted to quantify this concept within the context of a "one-level model", a mathematical model in which all conducting plasma was constrained to lie in a single two-dimensional plane perpendicular to the magnetic field. Implicit in

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this model is the assignment of a single Pedersen mobility and hence compressibility to both the barium and background ionospheric plasma. In this model, the leading candidate for a dissipative mechanism is the ambipolar diffusion of electrons caused by electron-ion collisions. Using this mechanism, McDonald et al. [1981] calculated final unbifurcating striation sizes (from a so-called U shaped curve) for barium clouds of ~ 20 m (perpendicular to B). However, there is experimental evidence for the existence of long-lived "blobs" of plasma several hundred meters in diameter. The apparent stability of such large plasma structures has yet to find a satisfactory explanation within the one-level model. Ossakow et al. [1981] proposed including a zero temperature second level compressible background ionosphere below the cloud (see Scannapieco et al., 1976). This would allow image striations to build up and allow the conductivity in a striation to be amplified. This in turn would allow for larger conductivity ratios than if one had just a one level cloud. In turn, this could result in kilometer size unbifurcating striations by extrapolating the U shaped curve of McDonald et al. [1981] to higher conductivity ratios.

Here we postulate that the existence of a conducting background plasma with ion Pedersen mobility different from that of the barium plasma, combined with the finite temperature of the barium plasma, may allow the existence of stable striations with scale sizes of hundreds of meters. The essence of the mechanism is a phenomenon commonly referred to as "end shorting" first investigated in the context of barium clouds by Shiau and Simon [1974], who considered a completely incompressible background ionosphere, and showed that the normal electron ambipolar diffusion rates would be replaced by some fraction of the much higher ion diffusion rates caused by collisions of ions with neutrals. Francis and Perkins [1975] generalized the work of Shiau and Simon [1974] by
considering the cases of both incompressible and compressible background ionospheres. They concurred with Shiau and Simon [1974] that incompressible backgrounds exert a stabilizing influence, but noted that compressible backgrounds are destabilizing. In contrast, Vickrey and Kelley [1982] concluded that a background (compressible) E region should exert a stabilizing influence on high latitude irregularities. We shall address the question of the correctness of the above two analyses in a later paper. For the moment we note that: 1) Francis and Perkins [1975] performed a rigorous stability analysis of the problem; Vickrey and Kelley [1982] did not; 2) Francis and Perkins [1975] self-consistently included image formation (compressibility) in both the cloud and the background; Vickrey and Kelley [1982] did not (they took a passive load model); 3) Vickrey and Kelley [1982] included recombination chemistry; Francis and Perkins [1975] did not.

Our own conclusions are based on our view of barium cloud dynamics as consisting of a two-stage feed-back loop:

1) At any given time, the distribution of plasma density will, through the effect of these distributions on magnetic-field-line integrated Pedersen and Hall conductivities, bring about the creation of polarization electric fields whose purpose it is to maintain quasi-neutrality;

2) these electric fields, through Hall and Pedersen mobilities, affect the velocity of the plasma, which in turn affects the distribution of plasma density at the next instant of time.

We emphasize that it is only through its influence on magnetic-field-line integrated Pedersen and Hall conductivities that a change in plasma distribution [e.g., diffusion] will affect the polarization fields, which are the engine of plasma structure.
In the present study we have isolated the "enhanced" difusive effects (due to finite temperature effects and a two level model, where only ion-neutral collisions are included) on barium cloud striation evolution. A major conclusion derived from the nonlinear numerical simulation results presented here is that coupling to a nearly incompressible background ionosphere, i.e., an F region, can result in a cessation of striation bifurcation in F region ionospheric barium clouds.

In addition, a simplified analysis of the equation for the time evolution of the total field line integrated Pedersen conductivity shows that coupling to a compressible background ionosphere, i.e., E region, would result in destabilization of the striations (see Eq. (4-36) and the discussion following). Both of these results are consistent with the linear analysis of Francis and Perkins [1975].

In section 2 we describe the motion of ionospheric plasma and in section 3 the mathematical model is presented. Section 4 presents a simplified two level model. Section 5 describes the diffusion characteristics for a simplified two level model with a nearly incompressible background. Numerical simulation results using an incompressible background are presented in section 6. Section 7 presents the conclusions and thoughts on future work.
2. The Motion of Ionospheric Plasma

We shall be concerned here with the motion of plasma consisting of ions and electrons in the presence of a neutral gas and magnetic field $B$, subject to an external force. We shall also be interested in the electric current $J$ arising from the differential motion of the various species comprising the plasma. In the course of deriving the equations we make the following assumptions:

1) We assume the plasma can be adequately described by the fluid approximation. This assumes that the effective collision rate of each plasma species with itself is sufficiently high to maintain near Maxwellian distribution functions on time scales short compared to the times of interest, and is well satisfied for the plasmas we treat here.

2) We assume that the electric fields $E$ are electrostatic (i.e., $\nabla \times E = 0$) and hence can be described using a scalar potential $\phi$ such that $E = -\nabla \phi$. Note that this implies $\partial B/\partial t = 0$. The validity of this assumption can be related to the fact that the Alfven velocity is much larger than any other propagation speed of interest for the plasmas we treat here. The assumption is also checked a posteriori by verifying that the calculated currents and displacement currents produce negligible time variations in $B$ which in turn produce negligible $\nabla \times E$.

3) We assume plasma quasi-neutrality; that is,

$$\sum n_i q_i = n_e e$$  \hspace{1cm} (2.1)

where $n$ is the number density, $q$ is ion species charge, $e$ is the electron charge, the subscripts $i$ and $e$ refer to ions and electrons respectively, and the sum is taken over all ion species. This assumption is a statement
that the Debye length is small compared to all length scales of interest, and again can be verified a posteriori by evaluating $\nabla \cdot \mathbf{E}$. Note that this assumption implies that $\nabla \cdot \mathbf{J} = 0$, where $\mathbf{J}$ is the electric current.

In addition to the above there are some other assumptions which, while they are not essential to the basic model, are nonetheless valid for many of the physical situations which we shall treat and impart a simplicity which we shall find convenient here.

4) We assume the electrostatic potential $\phi$ to be constant along magnetic field lines. As we shall see later, the electrical conductivity along magnetic field lines is much greater than that perpendicular to magnetic field lines, meaning that appreciable differences in potential along a field line will quickly be reduced by the resultant current. This assumption will break down for sufficiently small scale lengths perpendicular to the magnetic field, and for sufficiently large distances along the magnetic field.

5) We assume that the inertial terms in the plasma species momentum equations, i.e., the left hand side of Equation (2.3), are negligible with respect to the other terms in the equation. This assumption is justified whenever the time scales of interest are longer than the mean time between collisions for ions.

6) We neglect collisions between ions and electrons and between ions of different species. Later we shall also neglect collisions of electrons with neutrals. There are two reasons for this. First, the ion-neutral collision term $v_{in}$ can be shown to be the dominant term in the diffusion physics we consider here. (One must be careful, however, since these same assumptions will yield zero diffusion when the background conductivity is set to zero. To obtain the correct electron ambipolar rate in this limit,
the electron-ion and electron-neutral collision terms must be retained. We are not interested in this limit in this paper). Second, although exact closed form expressions for the ion and electron velocities in terms of the applied forces is possible when ion-electron collisions are retained [Fedder, 1980], the expressions are considerably more complex than those given below.

The continuity and momentum equations describing a single ion species and its associated electrons are:

\[
\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n v_\alpha) = 0
\]  \hspace{1cm} (2.2)

\[
(\frac{\partial}{\partial t} + v_\alpha \cdot \nabla) v_\alpha = \frac{q_\alpha}{m_\alpha} \left( E + \frac{v_\alpha \times B}{c} \right) - \nu_\alpha n \left( v_\alpha - U_n \right) - \frac{\nabla P_\alpha}{n_\alpha m_\alpha} + g
\]  \hspace{1cm} (2.3)

where the subscript \( \alpha \) takes on values \( i \) and \( e \) (denoting ions and electrons, respectively), \( n \) is the number density, \( v \) is the fluid velocity, \( P \) is pressure, \( E \) is the electric field, \( g \) is the gravitational acceleration, \( q \) is the charge, \( \nu_\alpha \) is the collision frequency with the neutral gas, \( U_n \) is the neutral wind velocity, \( c \) is the speed of light, and \( m \) is the particle mass. We can rewrite this equation as

\[
\frac{E_\alpha}{m_\alpha} + \frac{q_\alpha}{m_\alpha c} (v_\alpha \times B) - \nu_\alpha n v_\alpha = 0
\]  \hspace{1cm} (2.4)

where
\[ F_\alpha \equiv q_\alpha E + m_\alpha g + \nu m_\alpha U_\alpha - \gamma_\alpha / n_\alpha \]
\[ - \left( \frac{\partial}{\partial t} + \nu_\alpha \cdot \nabla \right) v_\alpha m_\alpha \]  
\[ (2.5) \]

If we place ourselves in a Cartesian coordinate system in which \( \mathbf{B} \) is aligned along the \( z \) axis, and if we treat \( F_\alpha \) as a given quantity then a componentwise evaluation of Equation (2.4) yields a set of three equations in three unknowns, the three components of \( v_\alpha \). The formal solution is

\[ v_\alpha = k_{1\alpha} F_{\alpha \perp} + k_{2\alpha} F_{\alpha \parallel} \times \hat{z} \]  
\[ (2.6) \]

\[ v_{\alpha \parallel} = k_{0\alpha} F_{\alpha \parallel} \]  
\[ (2.7) \]

where

\[ k_{1\alpha} = \frac{v_{\alpha \parallel}}{n_\alpha} \left( \frac{c}{q_\alpha B} \right) \left[ 1 - \frac{(v_{\alpha \parallel} / \Omega_\alpha)^2}{1 + (v_{\alpha \parallel} / \Omega_\alpha)^2} \right] \]  
\[ (2.8) \]

\[ k_{2\alpha} = \frac{c}{q_\alpha B} \left[ 1 - \frac{(v_{\alpha \parallel} / \Omega_\alpha)^2}{1 + (v_{\alpha \parallel} / \Omega_\alpha)^2} \right] \]  
\[ (2.9) \]

\[ k_{0\alpha} = \left( m_\alpha v_{\alpha \parallel} \right)^{-1} \]  
\[ (2.10) \]

\[ \hat{z} \equiv B / |B| \]  
\[ (2.11) \]

\[ n_\alpha \equiv \left| \frac{q_\alpha B}{m_\alpha c} \right| \]  
\[ (2.12) \]

The vector subscripts \( \perp \) and \( \parallel \) refer to the components of the vector which are perpendicular and parallel respectively to \( \hat{z} \). The quantities \( k_1, k_2, \)
and $k_0$ above are referred to as the Pedersen, Hall, and direct mobilities respectively. It should be pointed out that Equations (2.6) and (2.7) are only truly closed form expressions when the inertial terms (the last term on the right hand side of Equation (2.5)) are neglected, an assumption we have made previously. Typical ranges for collision frequencies are: $v_{in} \sim 30 \text{ sec}^{-1}$, $v_{en} \sim 800 \text{ sec}^{-1}$ at 150 km altitude; and $v_{in} \sim 10^{-1} \text{ sec}^{-1}$, $v_{en} \sim 1 \text{ sec}^{-1}$ at 500 km altitude.

As we will see later, we will use the concept of "layers" to distinguish the various ion species, so for the moment we consider only a single ion species, denoted by subscript $i$, and the associated electrons, denoted by subscript $e$. We also consider only singly charged ions so that $q_i = e$ and $q_e = -e$. Noting that $v_{en}/\Omega_e = 0$, we obtain

$$k_{1i} = \frac{v_{in}}{\Omega_i} \frac{R_i}{e|\mathbf{B}|}$$  (2.13)

$$k_{1e} = 0$$  (2.14)

$$k_{2i} = R_i \frac{c}{\Omega_i}$$  (2.15)

$$k_{2e} = -\frac{c}{\Omega_i}$$  (2.16)

where

$$R_i \equiv (1 + v_{in}^2/\Omega_i^2)^{-1}$$  (2.17)

We now define the perpendicular current
\[ \mathbf{J}_\perp = \sum \limits_\alpha n_\alpha q_\alpha \mathbf{v}_\alpha \ \ \ \ \ \ (2.18) \]

Substituting Equations (2.13) through (2.15) and (2.6) into Equation (2.18), and using the quasi-neutrality approximation

\[ n_i = n_e \approx n \ \ \ \ \ \ (2.19) \]

we obtain

\[ \mathbf{J}_\perp = v_{in} \mathbf{R}_i \frac{nc}{|B|} \mathbf{F}_i \cdot \mathbf{B} \]
\[ + \frac{nc}{B} (R_i \mathbf{F}_i \cdot \mathbf{B} + F_{e} \cdot \mathbf{B}) \times \mathbf{z} \ \ \ \ \ \ (2.20) \]

For the barium cloud problem we shall treat here, we shall only consider neutral winds, electric fields, gravity, and pressure gradients as external forces. Hence

\[ F_{i\perp} = e \mathbf{E} + m_i g_i + v_{in} m_i \mathbf{U}_n \cdot \mathbf{B} - \mathbf{P}_i/n \ \ \ \ \ \ (2.21) \]

\[ F_{e\perp} = -e \mathbf{E} + m_e g_l - \mathbf{P}_e/n \ \ \ \ \ \ (2.22) \]

Note that we have neglected the small term \( v_{en} m_e \mathbf{U}_n \cdot \mathbf{B} \) in Equation (2.22).

We obtain
\[
J_\perp = \frac{\nu_{\text{in}}}{\Omega_1} \frac{R_1}{|B|} \left( e \frac{E_\perp}{\Omega_1} + m_1 \frac{\mathbf{g}_\perp}{\epsilon} + \nu_{\text{in}} m_1 \frac{U_{\perp l}}{\Omega_1} - \nabla P_{\perp l}/n \right)
\]

\[
+ R_1 \frac{nc}{|B|} \left[ e \frac{E_\perp}{\Omega_1} (1 - R_1^{-1}) + \left( m_1 + \frac{m_e}{R_1} \right) \frac{\mathbf{g}_l}{\epsilon} + \nu_{\text{in}} m_1 U_{\perp l} \right]
\]

\[
- \nabla P_{\perp l}/n - \nabla P_{\perp e} R_1^{-1}/n \right] \times \hat{z}
\]

(2.23)

Since \(0.01 < R_1 < 1.0\) we may neglect \(m_e/R_1\) with respect to \(m_1\).

Defining the Pedersen conductivity

\[
\sigma_p = R_1 \frac{\nu_{\text{in}} nc e}{|B|}
\]

and noting that \(1 - R_1^{-1} = -\nu_{\text{in}}^2/\Omega_1^2\) we obtain

\[
J_\perp = \sigma_p \left[ e \frac{E_\perp}{\Omega_1} + \frac{m_1}{\epsilon} \frac{\mathbf{g}_l}{\epsilon} + \nu_{\text{in}} \frac{m_1}{\epsilon} \frac{U_{\perp l}}{\Omega_1} - \frac{\nabla P_{\perp l}}{ne} \right]
\]

\[
+ \frac{B}{|B|} \left( - \frac{\nu_{\text{in}}}{\Omega_1} \frac{E_\perp}{\epsilon} + \frac{\Omega_1 m_1}{\nu_{\text{in}}} \frac{\mathbf{g}_l}{\epsilon} + \frac{\Omega_1 m_1}{\epsilon} \frac{U_{\perp l}}{\Omega_1} \right)
\]

\[
- \frac{\Omega_1}{\nu_{\text{in}} ne} \left( \nabla P_{\perp l} + R_1^{-1} \nabla P_e \right) \times \hat{z}
\]

(2.25)

Our need for an expression for \(J_\perp\) stems from our need for its divergence to evaluate \(\nabla \cdot J\) (\(= 0\) by quasi-neutrality), as we shall see in the next section.
3. Mathematical Model

We shall model our physical system using a simplified model as depicted in Figure 2. The magnetic field lines are assumed to be straight, to be aligned along the z axis of our cartesian coordinate system, and to terminate in insulators at \( z = \pm \infty \). The plasma of interest is threaded by these magnetic field lines, and is divided into thin planes or "layers" of plasma perpendicular to the magnetic field. Since we have neglected collisions between different plasma species, we may use the device of layers to treat multiple ion species at a single point in space simply by allowing multiple layers to occupy the same plane in space, one for each ion species. In this way a "layer" consists only of a single ion species and its associated electrons.

Our quasi-neutrality assumption demands that

\[
\nabla \cdot \mathbf{J} = \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y + \frac{\partial}{\partial z} J_z = 0 \quad (3.1)
\]

Integrating Equation (3.1) along z and noting from Figure 2 that \( J_z \) vanishes at \( z = \pm \infty \) we obtain

\[
\int_{\infty}^{+\infty} \mathbf{v}_\perp \cdot \mathbf{J}_\perp \, dz = 0 \quad (3.2)
\]

where

\[
\mathbf{v}_\perp = \mathbf{v} \left( \frac{\partial}{\partial x} + \mathbf{v} \frac{\partial}{\partial y} \right) \quad (3.3)
\]
From our model as depicted in Figure 2 we may approximate the integral in Equation (3.2) by a discrete sum

$$\sum_{k=1}^{K} \nabla_\perp \cdot J_{-1k} \Delta z_k = 0 \quad (3.4)$$

where the subscript $k$ refers to the layer number, $K$ is the total number of layers in the system, and $\Delta z_k$ is the thickness of layer $k$ measured along the magnetic field line. Within a layer, both the ion density $n$ and the ion-neutral collision frequency $v_{in}$ are assumed to be constant along a magnetic field line. This enables us to introduce the three magnetic-field-line integrated quantities:

$$N_k \equiv n_k \Delta z_k \quad (3.5)$$

$$\Sigma_{pk} \equiv \sigma_{pk} \Delta z_k = N_k \left( \frac{v_{in}}{R_1} \right)_k \frac{ce}{|B|} \quad (3.6)$$

$$\Sigma_{hk} \equiv \left( \frac{v_{in}}{R_1} \right)_k E_{pk} \quad (3.7)$$

By our assumption of equipotential magnetic field lines and electrostatic electric fields

$$E_{-1k}(x,y) = E_o - \nabla_\perp \phi(x,y) \text{ for all } k \quad (3.8)$$

where $E_o = \frac{\hat{E}_x}{\partial x} \hat{x} + \frac{\hat{E}_y}{\partial y} \hat{y}$ is a constant, externally imposed electric field.

Then Equation (3.4) becomes
\[ V_1 \cdot \left[ \sum_{k=1}^{K} (\Sigma_{pk}) V_1^{\phi} \right] + \sum_{k=1}^{K} H_k = \sum_{k=1}^{K} V_1 \cdot J_{\text{ext}}^{\phi} \]  

(3.9)

where

\[ H_k = -\frac{3}{\partial x} \left( \Sigma_{hk} \frac{3\phi}{\partial y} \right) + \frac{3}{\partial y} \left( \Sigma_{hk} \frac{3\phi}{\partial x} \right) \]

\[ = -\frac{3\phi}{\partial y} \frac{\partial \Sigma_{hk}}{\partial x} + \frac{3\phi}{\partial x} \frac{\partial \Sigma_{hk}}{\partial y} \]  

(3.10)

\[ J_{\text{ext}}^{\phi} = \sum_{k} \left[ \frac{m_1}{e} g_1 + v_{\text{in}} \frac{e}{m_1} U_{1} - \frac{\nabla P_1}{ne} + E_0 \right] \]

\[ + \frac{B}{B_1} \left( \frac{\Omega_1 m_1}{e} g_1 + \Omega_1 \frac{m_1}{e} U_{1} - \frac{\Omega_1}{ne} v_{\text{in}} \left( \nabla P_1 + R_{1}^{-1} \nabla P_{1} \right) \right) \]

\[ - \frac{v_{\text{in}}}{\Omega_1} E_0 \times \hat{z}_k \]  

(3.11)

and the subscript \( k \) denoting layer number on terms within parenthesis operates on all terms within those parentheses.

The system of equations we must solve consists of (3.9)-(3.11) and an equation of continuity for each layer:

\[ \frac{\partial n_k}{\partial t} + \nabla \cdot (n_k v_1^{k}) = 0 \quad k=1, \ldots, K \]  

(3.12)

where \( v_1^{k} \) is given by Eq. (2.6)-(2.12).
4. Simplified Two-Level Model

We make the following further simplifications in our model:

a) We find that for ionospheric barium clouds the currents parallel to \( B \) are carried primarily by electrons, and the motion of ions parallel to \( B \) consists primarily of a slow diffusion plus a bulk falling of the cloud to lower altitudes (since \( g \cdot B \neq 0 \)). It is therefore sufficient to represent the ions as an array of two-dimensional planes or layers of plasma perpendicular to \( B \), each moving with the bulk "falling" velocity along \( B \), and hence to treat numerically only the transport of ions perpendicular to \( B \) within each layer.

b) We limit our discussion to a model consisting of only two layers or species of ions.

c) We assume that the ion neutral collision frequency \( v_{in} \) is constant within a layer.

d) We assume that the only forces acting on the plasma are an external electric field \( E_o \) and pressure gradients.

With the above assumptions we have

\[
\begin{align*}
J_{\perp k}^{\text{ext}} = & \sum_k \left( E_o - \frac{\nabla P_i}{n e} \right)_k \\
& + \frac{B}{|B|} \left[ - \frac{v_{in} E_o}{n_i} - \frac{\Omega_i}{n e v_{in}} \left( \nabla P_i + R_i^{-1} \nabla P_e \right)_k \times \hat{z} \right] \\
& \quad \text{ (4.1)}
\end{align*}
\]

If we assume \( B \) is along the positive \( z \) axis then \( B/|B| = 1 \) and we get

\[
\nabla_\perp \cdot J_{\perp k}^{\text{ext}} = \nabla_\perp \cdot \left( \sum_k \left( E_o - \frac{\nabla P_i}{n e} \right)_k \\
- \nabla_\perp \cdot \left( \frac{v_{in} E_o}{n_i} \frac{E_x}{\hat{z}} \right)_k + H_{pk} \right) \\
\quad \text{ (4.2)}
\]
where

\[ H_{pk} = -\nabla \cdot \left( \Sigma_{p} \frac{\Omega_{1}}{v_{in}} (\nabla p_{1} + R_{1}^{-1} \nabla p_{e}) \times \phi \right)_{k} \quad (4.3) \]

Now

\[ \left[ \Sigma_{p} \frac{\Omega_{1}}{v_{in}} \right]_{k} = \left[ R_{1} \frac{v_{in} \nabla c e}{|B|} \Delta z \frac{\Omega_{1}}{v_{1} ne} \right]_{k} \]

\[ = \left[ R_{1} \frac{c}{|B|} \Delta z \right]_{k} \quad (4.4) \]

which is a constant within a layer if \( v_{in} \) is constant within a layer (one of our assumptions). Then

\[ H_{pk} = -\nabla \cdot \left( \text{constant} \nabla (p_{1} + R_{1}^{-1} p_{e}) \times \phi \right) = 0 \quad (4.5) \]

(since \( \nabla \cdot (\nabla C \times \phi) = 0 \) for any scalar field \( C \)).

Our final equations to be solved are then

\[ \frac{\partial N_{k}}{\partial t} + \nabla \cdot (N_{k} \nabla) = 0 \quad k=1,2 \quad (4.6) \]

\[ \nabla \cdot \left[ \left( \Sigma_{p1} + \Sigma_{p2} \right) \nabla \phi \right] + H = E_{o} \cdot \nabla \left( \Sigma_{p1} + \Sigma_{p2} \right) \]

\[ - \nabla \cdot \left( \Sigma_{p1} \frac{\nabla p_{1}}{n_{1} e} + \Sigma_{p2} \frac{\nabla p_{2}}{n_{2} e} \right) \quad (4.7) \]
where

\[
H = - \frac{3}{\delta x} \left[ \left( \Sigma h_1 + \Sigma h_2 \right) \left( \frac{\partial \phi}{\partial y} - E_{oy} \right) \right] \\
+ \frac{3}{\delta y} \left[ \left( \Sigma h_1 + \Sigma h_2 \right) \left( \frac{\partial \phi}{\partial x} - E_{ox} \right) \right]
\]  

(4.8)

Looking at Eq. (4.7) we notice that \( \phi \) may be separated into two parts:

\[
\phi = \phi_E + \phi_P
\]  

(4.9)

\[
\nabla \cdot \left[ \left( \Sigma p_1 + \Sigma p_2 \right) \nabla \phi_E \right] + H = E_o \cdot \nabla \left[ \left( \Sigma p_1 + \Sigma p_2 \right) \right] 
\]  

(4.10)

\[
\nabla \cdot \left[ \left( \Sigma p_1 + \Sigma p_2 \right) \nabla \phi_P \right] = - \nabla \left( \Sigma p_1 \frac{\nabla p_{11}}{n_1 e} + \Sigma p_2 \frac{\nabla p_{12}}{n_2 e} \right)
\]  

(4.11)

For reasons which shall become clear as we progress, we shall regard \( \phi_E \) as that part of the potential field which tends to drive the cloud toward bifurcation. In fact, Eq. (4.10) coupled with (4.6) just form the basic inviscid two-level equations [Scannapieco et al., 1976]. We shall regard \( \phi_P \) as that part of the potential field which tends to diffuse or anti-diffuse the edges of the cloud. First let us look at solutions to (4.11). If \( \nabla P_{11} \) and \( \nabla P_{12} \) vanish at our boundaries, the unique solution consistent with zero Neumann boundary conditions (vanishing of the normal derivative of \( \phi_P \) at the boundary) gives:

\[
- \left( \Sigma p_1 + \Sigma p_2 \right) \nabla \phi_P = \Sigma p_1 \frac{\nabla P_{11}}{n_1 e} + \Sigma p_2 \frac{\nabla P_{12}}{n_2 e}
\]  

(4.12)

In general, the motion resulting from the forces proportional to \( \nabla \phi_P, \nabla P_{11} \) and \( \nabla P_{12} \) will be quite complex, not describable as the simple
superposition of a shear and a diffusion. However, in the special case of one of the layers being uniform, e.g., $N_2$ and $T_{i2}$ ($T_i$ = ion temperature) hence $\Sigma_{p2}$ and $P_{i2}$ uniform, then

$$\nabla \phi_p = \frac{-\Sigma_{p1}}{\Sigma_{p1} + \Sigma_{p2}} \frac{\nabla P_{i1}}{n_1 e}$$  \hspace{1cm} (4.13)

We shall now treat this special case. One may conveniently think of level 1 as representing the barium cloud and level 2 the uniform background ionosphere.

We now use the equation of state

$$P_1 = n_T$$

where $T_1$ is in energy units ($T_1$ is the product of Boltzmann's constant and the temperature in degrees Kelvin), and assume thermal conductivities high enough such that $T_1$ is constant and hence

$$\nabla P_1 = T_1 \nabla n$$  \hspace{1cm} (4.14)

Then

$$\nabla \phi_p = \frac{-\Sigma_{p1}}{\Sigma_{p1} + \Sigma_{p2}} \frac{T_{i1} \nabla n_1}{n_1 e}$$
The velocity in level 1 resulting from the combined action of $\nabla P_1$ and $\tau$ is

$$\nu_{1iP} = \left[ \frac{\nu_{in}}{\Omega} \frac{R_1 \nu_{1} c}{eB} \left( - e\nu_{\phi} - T_1 \frac{\nu_n}{n} \right) \right]_1 + \left[ R_1 \frac{c}{eB} \left( - e\nu_{\phi} - T_1 \frac{\nu_n}{n} \right) \right]_1 \times \hat{z}$$

(4.15)

Now

$$\left( - e\nu_{\phi} - T_1 \frac{\nu_n}{n} \right)_1$$

$$= T_{11} \frac{\nu_{n1}}{n_1} \left( \frac{\nu_{p1}}{\nu_{p1} + \nu_{p2}} - 1 \right)$$

$$= - T_{11} \frac{\nu_{n1}}{n_1} \left( \frac{\nu_{p2}}{\nu_{p1} + \nu_{p2}} \right)$$

(4.16)

Thus

$$\nu_{1iP} = \frac{-\nu_{p2}}{\nu_{p1} + \nu_{p2}} \left[ \frac{\nu_{in}}{\Omega} \frac{R_1 \nu_{1} c}{eB} \frac{T_1}{N} \nu_N \right]_1 + \left[ R_1 \frac{c}{eB} \frac{T_1}{N} \frac{\nu_N}{n} \right]_1 \times \hat{z}$$

(4.17)

Note we have used $\frac{\nu_{n}}{n} = \frac{\nu_N}{N}$.

It is convenient to divide $\nu_{1iP}$ into two parts:

$$\nu_{1iP} = \nu_{D1} + \nu_{S1}$$

with the subscripts 1 and P suppressed but understood.
That \( \mathbf{v}_{S1} \) represents shear flow at plasma gradients can be seen by noting that the velocity is always perpendicular to \( \nabla N \), and is largest where \( \frac{\mathbf{p}_2}{\mathbf{p}_1 + \mathbf{p}_2} \frac{\nabla N}{N} \) is a maximum, decaying in magnitude away from the region. The quantity \( \mathbf{v}_S \) is of interest because it is a shear flow and hence may be stabilizing in certain cases (see Perkins and Doles [1975], Huba et al. [1982]). We do not consider the effects of \( \mathbf{v}_S \) further in this paper except to note that for the special case of a uniform layer 2 being considered here, \( \nabla \cdot \mathbf{v}_{S1} = 0 \) and \( \nabla N_1 \cdot \mathbf{v}_{S1} = 0 \) which together imply \( \nabla \cdot (N_1 \mathbf{v}_{S1}) = 0 \), which is to say that \( \mathbf{v}_{S1} \) has no effect on the time evolution of \( N_1 \). Since this breaks the feedback loop, it is difficult to see how a shear stabilization mechanism could be active in this case.

A consideration of \( \mathbf{v}_D \) is really the heart of this paper. The quantity \( \mathbf{v}_D \) is the source for ion diffusion in level 1. More importantly, it is the source for the diffusion of the total integrated Pedersen and Hall conductivities if level 2 is sufficiently incompressible (i.e., if \( \mathbf{v}_{in}/\mathbf{p}_1 \) in level 2 is sufficiently small), as we will show. Let us look at the effect of \( \mathbf{v}_D \) on the ion continuity equation for level 1:

\[
\frac{\partial N_1}{\partial t} = - \nabla \cdot [N_1 \mathbf{v}_D]
\]

\[
= + \nabla \cdot [\frac{\mathbf{p}_2}{\mathbf{p}_1 + \mathbf{p}_2} \frac{\mathbf{v}_{in}}{\mathbf{p}_1} \frac{c}{e|B|} T_1 \nabla N_1]\]  

(4.20)
This is just a diffusion equation for \( N_1 \) with diffusion coefficient

\[
D_o = \frac{\Sigma p^2}{\Sigma p^1 + \Sigma p^2} \left( \frac{\nu_{\text{in}}}{\Omega_1} R_1 \frac{c}{e} \right) T_1 \]  

(4.21)

(It should be noted that this diffusion coefficient is similar to that derived by Goldman et al., 1976, and Vickrey and Kelley, 1982.)

For a barium cloud (level 1) at 180 km altitude we take \( T_1 \sim 1000^\circ \text{K} \), \( \frac{\nu_{\text{in}}}{\Omega_1} \sim 0.06 \), \( B = 0.5 \text{ g} \), and get

\[
D_o = \frac{\Sigma p^2}{\Sigma p^1 + \Sigma p^2} (100 \text{ m}^2/\text{sec}) 
\]

(4.22)

In the less dense regions of the barium cloud where \( \Sigma p^1 \) is small compared to \( \Sigma p^2 \), the ion diffusion coefficient is quite large indeed. By contrast, ambipolar diffusion rates induced by electron-ion and electron-neutral collisions yield diffusion coefficients on the order of 1 m\(^2\)/sec. Thus our neglect of these collision terms seems justified. Unfortunately, \( N_1 \) is not the relevant quantity if one is interested in the effect of \( v_D \) on barium cloud dynamics. A look at Eq. (4.10) will convince the reader that if one wants to affect the time evolution of \( \phi_E \), the quantity which determines the bifurcation process, it is necessary to change the quantities \( \Sigma_p = \Sigma p^1 + \Sigma p^2 \) and \( \Sigma_h = \Sigma h^1 + \Sigma h^2 \), the total field line integrated Pedersen and Hall conductivities. Again we assume an initially uniform distribution of \( \Sigma p^2 \) and \( N_2 \). We can only show the effects of \( v_D \) in the first instant of time here. Once images form in level 2, the problem yields only to numerical techniques. We need the velocity \( v_{D2} \) in level 2.
\[ \nu_D = (\frac{\nu_{in} R_1}{\eta_1}) \frac{c}{e} \frac{I_{p1}}{I_{p1} + I_{p2}} (T_{11} \frac{N_1}{N_1}) \]  \hspace{1cm} (4.23)

As a check, let us see the effect on \((N_1 + N_2)\)

\[
\frac{\partial N_2}{\partial t} = - \nabla \cdot \left( N_2 \nu_D \right)
\]

\[
= - \nabla \cdot \left( \frac{I_{p2} e^{-2}}{I_{p1} + I_{p2}} \frac{I_{p1}}{T_{11} \frac{N_1}{N_1}} \right) \hspace{1cm} (4.24)
\]

\[
\frac{\partial N_1}{\partial t} = - \nabla \cdot \left( N_1 \nu_D \right)
\]

\[
= + \nabla \cdot \left( \frac{I_{p2}}{I_{p1} + I_{p2}} \frac{I_{p1} e^{-2}}{T_{11} \frac{N_1}{N_1}} \right)
\]

\[
= - \frac{\partial N_2}{\partial t} \hspace{1cm} (4.25)
\]

So we see that

\[
\frac{\partial (N_1 + N_2)}{\partial t} = 0 \hspace{1cm} (4.26)
\]

which when combined with quasi-neutrality, is simply a statement that electrons can't diffuse. This is consistent with our neglect of electron-ion and electron-neutral collisions.

Now let us look at the effect of \(\nu_D\) on \(I_p = I_{p1} + I_{p2}\). A similar analysis can be performed for \(\nu_h\) with similar results. However the bifurcation effects of \(\nu_h\) are small compared to those of \(I_p\), and we shall not consider them here.
\[ \frac{\partial \Sigma_{p_1}}{\partial t} = (\frac{\nu_{in}}{\Omega_i} R_i)_1 \frac{\partial}{\partial t} \frac{\Delta N_1}{\Delta t} = \nabla \cdot (\nu_{Di} \Sigma_{p_1}) \] (4.27)

\[ \frac{\partial \Sigma_{p_2}}{\partial t} = (\frac{\nu_{in}}{\Omega_i} R_i)_2 \frac{\partial}{\partial t} \frac{\Delta N_2}{\Delta t} = \nabla \cdot (\nu_{D2} \Sigma_{p_2}) \] (4.28)

but

\[ \frac{\partial \Delta N_1}{\partial t} = - \frac{\partial \Delta N_2}{\partial t} = - \frac{|B|}{\mu_0} (\frac{\nu_{in}}{\Omega_i} R_i)_1^{-1} \frac{\partial \Sigma_{p_2}}{\partial t} \] (4.29)

\[ \frac{\partial \Sigma_{p_1}}{\partial t} = - (\frac{\nu_{in}}{\Omega_i} R_i)_1 \frac{\partial \Sigma_{p_2}}{\partial t} \] (4.30)

\[ \frac{\partial (\Sigma_{p_1} + \Sigma_{p_2})}{\partial t} = \frac{\partial \Sigma_{p_1}}{\partial t} \left[ 1 - (\frac{\nu_{in}}{\Omega_i} R_i)_2 / (\frac{\nu_{in}}{\Omega_i} R_i)_1 \right] \] (4.31)

Now

\[ \frac{\partial \Sigma_{p_1}}{\partial t} = - \nabla \cdot (\nu_{Di} \Sigma_{p_1}) \] (4.32)

We note that in Eq. (4.18) we may set \( \nu N_1 / N_1 = \nu_{p_1} / \sigma_{p_1} \) so that

\[ \frac{\partial \Sigma_{p_1}}{\partial t} = \nabla \cdot \left( \frac{\Sigma_{p_2}}{\Sigma_{p_1} + \Sigma_{p_2}} \left( \frac{\nu_{in}}{\Omega_i} R_i \frac{c}{e |B| T_i} \right)_1 \nu \Sigma_{p_1} \right) \] (4.33)

Now since

\[ \nabla \Sigma_{p_2} = 0 \] (4.34)

we have
\( \nabla (\Sigma_{p1} + \Sigma_{p2}) = \nabla \Sigma_{p1} \) \hspace{1cm} (4.35)

and

\[
\frac{\partial (\Sigma_{p1} + \Sigma_{p2})}{\partial t} = \nu_1 \cdot \left[ (1 - \left( \frac{\nu_{in}}{\Omega_1} \right)_2 / \left( \frac{\nu_{in}}{\Omega_1} \right)_1 ) \left( \frac{\Sigma_{p2}}{\Sigma_{p1} + \Sigma_{p2}} \right) \right]
\]

\[
\left( \frac{\nu_{in}}{\Omega_1} \right)_1 \frac{c}{eB} T_1 L \nabla (\Sigma_{p1} + \Sigma_{p2}) \]

(4.36)

so we see that for \( \Sigma_p \) we again have a diffusion equation, but this time our nominal 100 m\(^2\)/sec diffusion coefficient is not only multiplied by \( \Sigma_{p2}/(\Sigma_{p1} + \Sigma_{p2}) \), reducing its effectiveness in the dense portions of the cloud, but it is further multiplied by the factor \( F \) where

\[
F = 1 - \left( \frac{\nu_{in}}{\Omega_1} \right)_2 / \left( \frac{\nu_{in}}{\Omega_1} \right)_1 \]

(4.37)

Note that if \( \nu_{in} / \Omega_1 \) is the same for both layers, the diffusion is reduced to zero. Furthermore, if \( \left( \nu_{in} / \Omega_1 \right)_2 > \left( \nu_{in} / \Omega_1 \right)_1 \) (a compressible background plasma, e.g., an E region) the diffusion coefficient is negative. That is, gradients in \( \Sigma_p \) will actually be steepened, rather than smoothed. For this reason we conclude that unless some rather fast recombination chemistry is taking place in the E region, the presence of a background E region is destabilizing, in accord with the findings of Francis and Perkins [1975].

The opposite extreme of \( \left( \nu_{in} / \Omega_1 \right)_2 \ll \left( \nu_{in} / \Omega_1 \right) \) (background F region ionosphere) is more interesting since it closely approximates the conditions of most barium releases. It also yields the full nominal 100 m\(^2\)/sec diffusion coefficient, since \( F = 1.0 \), giving us some hope of
stopping the bifurcation process at scale sizes of hundreds of meters. Note that in this case the diffusion rate for $\Gamma_p$ is the same as that for $N_1$. Also note that the case of $[v_{in}R_1/N_1]_2$ small is the case where the Pedersen mobility and hence compressibility of the background is extremely low, meaning that only minimal images will be formed. It is a good approximation to take $\Gamma_p$ to be uniform for all time. One might well ask how this is possible given that the electrons can't diffuse, and that the barium ions are being allowed to diffuse in level 1. The answer is that there are (small) images: the electrons lost to a field line in level 1 are "soaked up" on the same field line in level 2. But the number of electrons $N_2$ in level 2 is extremely large. ($N_2$ must be large in order to contribute a significant Pedersen conductivity and yet have small $v_{in}R_1/N_1$ (see Eq. 2.24)). Thus the electrons that are soaked up in level 2 induce only an extremely small percentage change in $\Gamma_p$.

5. Diffusion Characteristics for a Simplified Two-level Model with a Nearly Incompressible Background Layer

In the last section we demonstrated that when the background ionosphere (layer 2) is nearly incompressible ($[v_{in}R_1/N_1]_2$ is small) then the primary effect of a finite barium ion temperature is to introduce into the continuity equation for both $N_1$ and for $\Gamma_p$ a diffusion term with diffusion coefficient $D_0$ given by Eq. (4.21). For parameters appropriate to a barium cloud at 180 km altitude we get Eq. (4.22):

$$D_0 = \frac{\Gamma_p}{\Gamma_p + \frac{\Gamma_p}{100 \text{ m}^2/\text{sec}}}$$

Since level 2 is nearly incompressible, we may regard $\Gamma_p$ as a constant.
Since it is proportional to \( N_1 \), the integrated barium cloud density, \( \Sigma_{pl} \) is larger in the center of the cloud and decays toward zero at the cloud edges. This means that the diffusion coefficient approaches 100 m\(^2\)/sec near the cloud edges, but may be considerably less than that value near the cloud center if \( \Sigma_{pl} \gg \Sigma_{p2} \).

Previous attempts to explain the persistence of scale sizes of hundreds of meters for long periods of time after barium releases have inevitably encountered the following problems: 1) The diffusion coefficients required to stop the bifurcation of plasma clouds several hundred meters in diameter were higher than one could explain theoretically; 2) assuming the existence of such large (ordinary) diffusion coefficients, their effect would not only be to stop the bifurcation of the cloud, but also to diffuse the cloud away entirely in several minutes. The form of the diffusion coefficient given by Eq. (4.22) would seem to offer us some hope of overcoming both of these problems. Its success depends on the accuracy of the following qualitative picture of barium cloud bifurcation: We postulate a barium cloud such that \( \Sigma_{pl} \geq 5 \Sigma_{p2} \) so that the diffusion well inside the cloud is considerably reduced over that at its edges. We further postulate that bifurcation is a process which, in the absence of diffusion, starts at the edges of barium clouds and works its way inward. Our diffusion then satisfies the needed criteria perfectly: 1) It is very effective at the edges of the cloud, stopping bifurcation before it starts; 2) it is very ineffective at the central core of the cloud, giving it a long lifetime. Note that if this picture is correct then the barium clouds commonly known as large \( M \) clouds where...
will have extremely long lifetimes. In fact for \( M \geq 100 \), the decay of the core is probably determined by ambipolar diffusion rates. The accuracy of this picture is best determined by numerical simulation techniques, to which we turn in the next section.

6. Numerical Simulations Using a Simplified Two-level Model with a Nearly Incompressible Background Layer

In this section we attempt to answer numerically the question posed in the last section: if we consider a barium cloud whose background conductivity is due primarily to high F region plasma (nearly incompressible plasma), is the diffusion coefficient given by Eq. (4.22) sufficient to stop the bifurcation of that cloud at a diameter of several hundred meters? In order to isolate the effects of this diffusion coefficient, we will make some further simplifications in our two-level model: 1) We assume that \( N_2 \) and hence \( \Sigma_{p2} \) remain uniform during all times of interest. This is completely consistent with our assumption of a nearly incompressible background plasma, as discussed at the end of Section 4; 2) We neglect the shear component \( v_S \) of the pressure-induced ion velocity as defined in Eq. (4.19). Again this is consistent with our assumption of a nearly incompressible background plasma which in turn implies a uniform \( \Sigma_{p2} \), as shown in the discussion following Eq. (4.19); 3) We neglect the "Hall term" \( H \) in Eq. (4.4) and hence in Eq. (4.10). This is a good approximation as long as \( v_{in}/n_i < 0.1 \); 4) We neglect Pedersen convection. That is, for all terms except those causing ion diffusion, we approximate the ion velocity with the electron velocity (see Eq. (6.4))
below). We know from our previous study [Zalesak et al., 1983] that the Pedersen mobility of the ions in response to \( (E_o - \nabla \phi_E) \) can have an effect on the "freezing" phenomenon, and it is our desire to isolate the effects of the pressure-induced diffusion. Hence we neglect ion Pedersen mobility except in the diffusion term.

The final equations to be solved numerically are then

\[
\frac{3N_1}{3t} + \nabla \cdot (N_1 \cdot \nabla) = \nabla \cdot (D \nabla N_1) \quad (6.1)
\]

\[
\frac{3N_2}{3t} = 0 
\quad (6.2)
\]

\[
\nabla \cdot ((E_{p1} + E_{p2}) \nabla \phi_E) = \frac{\varepsilon}{\varepsilon_{p1} + \varepsilon_{p2}} \nabla \cdot (E_{p1} + E_{p2}) \quad (6.3)
\]

\[
\nabla_1 = \frac{c}{B^2} (E_o - \nabla \phi_E) \times B \quad (6.4)
\]

\[
D = \frac{E_{p2}}{E_{p1} + E_{p2}} D_{10} \quad (6.5)
\]

\[
\Sigma_{pk} = \frac{N_k}{(1 + (v_{in} / v_{k1})^2)} \frac{ce}{|B|} \quad (6.6)
\]

The initial conditions for the simulations were as follows

\[
\Sigma_{p2} = 1.0 \quad (6.7)
\]

\[
\Sigma_{p1} = 4.0 \exp \left(-\frac{r^2}{r_o^2}\right) \quad (6.8)
\]

\[
r^2 = (x - x_o)^2 + (y - y_o)^2 \quad (6.9)
\]
\[ r_o = (250 \text{ m}) (1 + 0.001 \sin \theta) \]  
(6.10)

\[ \theta = \arctan \left( \frac{y-y_o}{x-x_o} \right) \]  
(6.11)

\[ B = 0.5 \text{ gauss} \]  
(6.12)

\[ cE_o/B = 100 \text{ m/sec} \]  
(6.13)

where \( x_o \) and \( y_o \) are the coordinates of the cloud center and the \( E_o \times B \) velocity \( (cE_o/B) \) is in the negative \( y \) direction.

The above equations are solved numerically on a finite difference grid in an \( x-y \) cartesian geometry perpendicular to the magnetic field, which is assumed to be aligned along the \( z \)-axis. The \( 83 \times 60 \) grid is stretched in all four directions, with the central \( 56 \times 32 \) portion of the grid, which is centered on the steepening backside of the cloud, having a grid spacing of 10 m in both directions. The grid stretching allows the boundaries to be placed 4 km away from the edges of the central uniform mesh in all directions. The potential equation (6.3) is solved using a vectorized incomplete Cholesky conjugate gradient algorithm due to Rain (1980), which is an extension of the algorithm of Kershaw (1978). The continuity equation (6.1) is integrated forward in time using the fully multidimensional flux-corrected transport algorithms of Zalesak (1979).

In Figure 3 we show isodensity contours of \( \rho_{pl} \) for the initial conditions for all of the calculations we shall show. We have performed three calculations, varying the value of \( D_{10} \) from zero up to the physically realistic value of 100 m\(^2\)/sec. The purpose of this sequence is to show first that the diffusion does have an effect on the evolution of the cloud,
and second to show that this effect is much larger than that due to any numerical diffusion which may be present in the calculations. In Figure 4 we show the results for $D_{10} = 0$. Note that even at the very early time of 14 seconds, bifurcation has already started. By 24 seconds, the main portion of the cloud has completely sheared in two. In Figure 5, we show the results for $D_{10} = 25 \text{ m}^2/\text{sec}$, one fourth of the physically realistic value. The development of the cloud has slowed considerably, showing that even at one fourth its physically realistic value, the physical diffusion is dominating the effects of whatever numerical diffusion may be present. At 14 seconds the cloud has not clearly begun to bifurcate. At 25 seconds, bifurcation has occurred, but is qualitatively and quantitatively different from that which took place when $D_{10} = 0$. In Figure 6 we show the evolution of the cloud with the full, physically realistic value for $D_{10}$ of 100 $\text{m}^2/\text{sec}$. Bifurcation has now been halted. Rather, the cloud evolves into a streamlined "bullet" or "tadpole" shape in time, and then undergoes a slow diffusive decay. Even at the latest time calculated, 75 sec, there is no hint whatsoever of imminent bifurcation. Noting that for this cloud $M = \left(\Sigma p_1 + \Sigma p_2\right)/\Sigma p_2$ is 5 in the center of the cloud, which yields an effective diffusion coefficient of 20 $\text{m}^2/\text{sec}$ there. This is still enough to result in substantial decay of the central core on time scales of minutes. Clouds with larger values of $M$ in their centers would of course be longer-lived.
7. Conclusions and Future Work

The primary conclusion of this paper can be stated simply: Under realistic ionospheric conditions (where the plasma cloud couples to an incompressible background i.e., in F region), barium striations hundreds of meters in diameter can be long-lived, quasi-stable, non-bifurcating structures. These structures may resemble "tadpoles", with a dense head, steep density gradients at the front, and a long, less dense tail. If what one means by the term "freezing" is the above phenomenon, then we have shown that freezing does indeed exist.

The next obvious step is to test some of the approximations we have made in our preliminary numerical simulation model. Pedersen convection should be put back into the model, as well as the "Hall terms" in the potential equation. Finite images should be accounted for. Also we would like to test the combined effect of the presence of both a relatively incompressible F region background and a compressible E region on the dynamics of the cloud. Recombination chemistry must be included in this E region if it is to be modeled accurately (see Vickery and Kelley, 1982).

Another of our approximations which should be studied in detail is that the barium cloud and background ionosphere are being driven by an externally imposed electric field, or equivalently, by a neutral wind which doesn't vary in altitude. Were we to address an altitude-dependent neutral wind as a driver, some of our simple results would become less so. The most important factor would be that one could no longer argue that the bifurcation of the cloud depended only on $E_{p1}$ and $E_{p2}$, since the potential equation (4.10) would then contain terms which depended explicitly on $N_1$ and $N_2$. Although this would not change the results for the incompressible background ionosphere considered here, it would appear to allow for the
possibility of this end-shorting-induced diffusion being an effective bifurcation inhibiting mechanism even when the background ionosphere is compressible. We shall consider this case in our future work in this area.

It is also clear that we must address the validity of the perfect mapping of the electrostatic potential along $B$. Given that we are now dealing with structures only a few hundred meters in diameter, with gradient scale lengths of a few tens of meters, it seems likely that we will have to deal with the question of how far the polarization field produced by the barium cloud maps along the magnetic field (Goldman et. al, 1976, Glassman and Sperling, 1983). One possible effect might be that the effective value of $M$ would be increased, since the barium cloud could no longer “see” background plasma that was far from the cloud position along $B$.

Acknowledgments

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Figure 1. Sketch of the time evolution of a typical barium cloud in a plane perpendicular to the magnetic field, subject to an upward directed neutral wind or equivalently to a rightward directed external electric field. Lines demarking the cloud denote plasma density contours, with the highest plasma density in the center of cloud.
Figure 2. Model of plasma and magnetic field geometry used in this paper. Field lines terminate on insulators at \( z = \pm \infty \). Plasma is divided into "layers" along \( z \) for mathematical and numerical treatment. Each layer consists of a single ion species and its associated electrons. Multiple collocated ion species are treated by having multiple collocated "layers".
Figure 3. Isodensity contours, with areas between alternate contours shown cross-hatched, for the barium cloud used for the three numerical simulations shown in this paper. The center of the cloud is located at the point (0,0). The boundaries in x are located at ±4.9 km. The boundaries in y are located at -4.0 km and +5.3 km respectively. The cross-hatching is done at every other grid line, so that one rectangle of cross hatching represents a 2 x 2 array of computational cells. Contours for this and all other plots in this paper are drawn at values for $\Sigma_{pl}$ of 0.31, 0.71, 1.24, 1.92, and 2.82 (logarithmic spacing for total Pedersen conductivity).
Figure 4. Time evolution of the cloud depicted in Figure 3 for the case $D_{10} = 0$. Shown are isodensity contours at a) 14 sec, b) 24 sec. Contour values are as given in Figure 3.
Figure 5. Time evolution of the cloud depicted in Figure 3 for the case $D_{lo} = 25 \text{ m}^2/\text{sec}$. Shown are isodensity contours at a) 14 sec, b) 24 sec. Contour values are as given in Figure 3.
Figure 6. Time evolution of the cloud depicted in Figure 3 for the case $D_{10} = 100 \text{ m}^2/\text{sec}$ (the physically realistic case). Shown are isodensity contours at a) 14 sec, b) 25 sec, c) 39 sec, d) 50 sec, e) 64 sec, f) 75 sec.
Figure 6, cont'd. Time evolution of the cloud depicted in Figure 3 for the case $D_{io} = 100 \, m^2/sec$ (the physically realistic case). Shown are isodensity contours at a) 14 sec, b) 25 sec, c) 39 sec, d) 50 sec, e) 64 sec, f) 75 sec.
Figure 6, cont'd. Time evolution of the cloud depicted in Figure 3 for the case $D_{10} = 100 \text{ m}^2/\text{sec}$ (the physically realistic case). Shown are isodensity contours at a) 14 sec, b) 25 sec, c) 39 sec, d) 50 sec, e) 64 sec, f) 75 sec.
References


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