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THRUST AUGMENTATION STUDY OF HIGH PERFORMANCE EJECTORS

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This technical report has been reviewed and is approved for publication.

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**Title:** Thrust Augmentation Study of High Performance Ejectors

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**Abstract:**

The use of ejectors in energy conversion processes and thrust augmentation requires that the mixed flow be produced at a high efficiency. Although many definitions of efficiency have been used to describe the efficiency of an ejector, we have used an efficiency based on thermodynamic availability which is referenced to the stagnation properties of the secondary flow (the flow being "pumped"). As is well known, a compressible-flow,
20. control-volume approach to analyzing a constant area ejector yields two solutions: one with a subsonic mixed flow and one with a supersonic mixed flow. The supersonic mixed flow produces the best efficiencies and highest total pressures. The properties of the supersonic mixed flow are of necessity related to the properties of the subsonic mixed flow by the normal shock relations. Nonetheless, in practice, the subsonic mixed flow is, in general, not achieved through a normal shock (or pseudo-normal shock) from the supersonic mixed flow solution.

A technique is developed for determining a representative value of the maximum efficiency that can be achieved with high performance ejectors when operating on the supersonic solution branch of an ejector. These efficiencies are used to calculate thrust augmentation for an ejector over a wide range of parameters including operation with a hypothetical engine.

Reasonable values of thrust augmentation can be achieved at low subsonic flight Mach numbers. However, at flight Mach numbers near one, little or no thrust augmentation was found. At supersonic flight Mach numbers, thrust augmentation was achieved.

Basic studies indicated that the effects of temperature was opposite at subsonic and supersonic flight Mach numbers. Thrust augmentation decreased with increasing temperature at subsonic Mach numbers and increased with increasing temperature at supersonic Mach numbers.

Basic concepts are presented for cross flow and counter flow momentum exchangers which may afford more efficient transfer of kinetic energy between fluid streams.
FOREWORD

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LIST OF SYMBOLS

\( \beta \)  
- bypass ratio

\( \gamma \)  
- ratio of specific heats

\( \eta \)  
- efficiency

\( \eta_c \)  
- compressor polytropic component efficiency

\( \eta_t \)  
- turbine polytropic component efficiency

\( \tau \)  
- thrust

\( \tau_m \)  
- mixed flow thrust

\( \tau_p \)  
- thrust of the primary flow

\( \phi \)  
- thrust augmentation ratio

\( A_p \)  
- primary flow area at the inlet

\( A_s \)  
- secondary flow area at the inlet

\( A_{ep} \)  
- primary flow area at the station e

\( A_{es} \)  
- secondary flow area at the station e

\( C_p \)  
- specific heat at constant pressure

\( M_\infty \)  
- flight Mach number

\( M_p \)  
- primary inlet Mach number

\( M_s \)  
- secondary inlet Mach number

\( M^* \)  
- ratio of velocity to the speed of sound at Mach one

\( M^*_p \)  
- primary \( M^* \)

\( M^*_s \)  
- secondary \( M^* \)

\( M_{ep} \)  
- primary Mach number at the station e

\( M_{es} \)  
- secondary Mach number at the station e

\( \dot{m} \)  
- mass flow rate

\( P \)  
- pressure

\( P_{amb} \)  
- ambient pressure

\( P_{comp} \)  
- compressor pressure

\( P_e \)  
- compressor pressure ratio

\( P_{plp} \)  
- primary inlet static pressure

\( P_{pls} \)  
- secondary inlet static pressure

\( P_{op} \)  
- primary total pressure

\( P_{os} \)  
- secondary total pressure

\( PR \)  
- \( P_{op}/P_{os} \)
<table>
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<td>R</td>
<td>gas constant</td>
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<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>T_{op}</td>
<td>primary total temperature</td>
</tr>
<tr>
<td>T_{os}</td>
<td>secondary total temperature</td>
</tr>
<tr>
<td>T_{R}</td>
<td>T_{op}/T_{os}</td>
</tr>
<tr>
<td>U</td>
<td>transport velocity</td>
</tr>
<tr>
<td>V</td>
<td>velocity</td>
</tr>
<tr>
<td>W</td>
<td>molecular weight</td>
</tr>
<tr>
<td></td>
<td>bar over a symbol is a ratio of primary to secondary</td>
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LIST OF SYMBOLS
FOR TABLES 1 THROUGH 5

A/AP* value of A/A* mixing tube area to primary nozzle throat area
AP/AS area ratio A_p/A_s required to match the inlet pressure
EFAVL efficiency based on availability
EFPOS efficiency based on expansion to P_os
GM gamma value of the mixture
GP primary gamma value
GS secondary gamma value
MM mixed flow Mach number
MP primary Mach number
MS secondary Mach number
PR pressure ratio
Pipzig/Pos primary nozzle pressure ratio: P_{lp}/P_{op}
Pis/Pos secondary nozzle pressure ratio: P_{ls}/P_{os}
PM/PM total pressure ratio P_m/P_os
POM/POM total pressure ratio P_{om}/P_{os}
SR entropy increase
TR temperature ratio
Tlp/Tos primary nozzle temperature ratio: T_{lp}/T_{op}
Tls/Tos secondary nozzle temperature ratio: T_{ls}/T_{os}
TM/TM temperature ratio T_m/T_{os}
VP/VS ratio of primary velocity to the secondary velocity
VM/VS ratio of the mixed velocity to the secondary velocity
WP primary molecular weight
WS secondary molecular weight
WR ratio of molecular weights
SECTION 1
INTRODUCTION

Ejectors have been used in many useful applications in industry. Ejectors have also been investigated for flight applications and their potential usefulness has been demonstrated in experimental aircraft.

It has been known for some time (see, for example, references 1 through 4) that a control volume analysis of a constant area ejector leads to a double-valued solution: one where the mixed flow is subsonic, and the other where the mixed flow is supersonic. Further, it is well known that these two solutions are related by the normal shock relations.

Recently, Alperin and Wu\textsuperscript{5} have pointed out the potential advantages of the supersonic branch for applications to thrust augmentation. Minardi et al.\textsuperscript{6}, who were studying two fluid ejectors for applications in turbines, also found better results for their applications when the supersonic branch was utilized.

Extremely high efficiencies (based on thermodynamic availability) are indicated on the supersonic solution branch. In fact, following Hoge\textsuperscript{3}, it can be shown\textsuperscript{7,8} that extremely high efficiencies can also occur on the subsonic solution branch. However, as discussed in reference 7, these extremely high efficiencies cannot be achieved in an ejector. Rather, they would require rotating machinery within the control volume to produce the extremely high efficiencies. It was shown in reference 7 that an ejector with a subsonic inlet for the secondary flow (the primary is supersonic at the inlet) operates at a single operating point when the exit flow is supersonic. This operating point can be determined by the methods described by Fabri and Siestrunk\textsuperscript{2}. Thus, a procedure is available to evaluate the performance of an ejector operating on the supersonic solution branch.

It was the purpose of the study reported herein to determine reasonable estimates of thrust augmentation that could be achieved with an ejector over a range of Mach numbers from zero to supersonic.
SECTION 2
DISCUSSION OF THE SUPersonic SOLUTION BRANCH

In reference 7 an analysis for a constant area ejector, such as shown in Figure 1, is presented. The equations resulting from the solution are presented in Appendix A. Figure 2 presents supersonic branch solutions to the equations for air driving air for a pressure ratio of six and a temperature ratio of 3.7 and for a series of bypass ratios, \( \beta \), from 0.5 to 10. The curves show efficiency, based on thermodynamic availability, versus the secondary, inlet Mach number, \( M_s \). As stated earlier there is also a subsonic solution branch (not shown on Figure 2) which is related to the supersonic solution branch by the normal shock relations: if the supersonic mixed flow experiences a normal shock, the conditions would be the same as the conditions on the subsonic solution branch at the same value of \( M_s \).

At the higher values of \( \beta \) the mixed flow becomes choked, i.e., the mixed flow Mach number is one. This choking eliminates a section of the curve as indicated on the bypass ratios of five and ten. The region over which the solutions do not exist are referred to by Hoge\(^3\) as the forbidden region. We have indicated the boundaries of the forbidden region on Figure 2.

Hoge\(^3\) also showed that the control volume equations developed for an ejector could be solved over the entire region of the \( M^*-M^* \) plane. The parameter \( M^* \) (the ratio of velocity to the speed of sound at Mach one) was chosen in place of the Mach number since a finite range of the parameter \( M^* \) covers all Mach numbers from zero to infinity. An efficiency map for a pressure ratio of six, temperature ratio of 3.7, and a bypass ratio of 10 is shown on Figure 3. On Figure 3 the forbidden region becomes egg-shaped, as shown cross-hatched. In calculating the data for Figure 2, it was assumed that the inlet static pressures of the primary, \( P_{lp} \), and secondary, \( P_{ls} \), were equal. In Figure 3 we show the curve in the \( M^*-M^* \) plane for which \( P_{lp} = P_{ls} \). After
Figure 1. Inlet Flow Pattern for an Ejector Operating with a Supersonic Mixed Flow and Having a Supersonic Primary Flow and a Subsonic Secondary Flow.
Figure 2. Supersonic Branch Solutions Showing Efficiency Versus Secondary Inlet Mach Number, $M_s$, for Various Bypass Ratios, $\beta$, at the Indicated Conditions. This is a Design Curve Because the Physical Size of the Ejector Changes as One Moves Along the Curves.
Figure 3. Supersonic Branch Efficiency Map. Also Shown is the Curve for which $P_1 = P_{1e}$. The Curve is a Design Curve because the Physical Size of the Ejector has to Change as One Moves Along the Curve.
proper adjustments for changing \( M^* \) to \( M_s \) we could take data from Figure 3 along the curve \( P_{1p} = P_{1s} \) and plot the corresponding curve for \( \beta = 10 \) on Figure 2.

The shape of the forbidden region is independent of the pressure ratio and depends only on the value of the bypass ratio and the temperature ratio. Hoge\(^3\) gives the equation for the boundaries of the forbidden region as

\[
\left( \frac{M^*+1}{M^*} \right) + \beta \left( \frac{M^*+1}{M^*_s} \right) / \sqrt{TR} = 2\sqrt{(1+\beta)(1+\beta/TR)}.
\]  \( (1) \)

It is interesting to note that for either the primary or the secondary flow that if \( M^* \) satisfies Equation (1), then so does the value of \( M^* \) equal to the reciprocal of the first value. Also, the curve for which \( P_{1p} = P_{1s} \) can be obtained from the isentropic relations which yield

\[
M^*_p = \sqrt{\frac{Y+1}{Y-1} - \frac{PR}{Y}(\frac{Y-1}{Y} - \frac{M^*_s}{Y-1} - \frac{M^*_s^2}{Y-1})}.
\]  \( (2) \)

The simultaneous solution of Equations (1) and (2) yields the pair of points where the curve \( P_{1p} = P_{1s} \) intersects the boundaries of the forbidden region. These results will be used in Section 5.

At this time, however, we wish to consider Figure 2 and 3 further. The very high values of efficiency located at the low subsonic values of \( M_s \) for all of the bypass ratios would provide excellent values of thrust augmentation if they could be achieved in an ejector. However, Figure 2 raises serious doubts that this could happen in an ejector. For example, the data of Figure 2 indicate that at a Mach number of \( M_s = 0.51 \) the efficiency at a bypass ratio of 10 is about 0.9 which is higher than the efficiency (at the same \( M_s \)) associated with any other value of \( \beta \) indicated on Figure 2. Even though the value of efficiency of 0.9 does not violate the second law (since it is less than one) it is highly unlikely that such a result could be achieved in an ejector.

It was considerations such as these that led us to consider the control volume solution in more detail in reference 7. It
was concluded that although the equality area boundary shown in Figure 1 was a sufficient condition to define the set of equations used in the analysis, it was not a necessary condition. Thus, some of the solutions generated from the equations might not be valid solutions for an ejector. On the other hand, all of the solutions valid for an ejector would be obtained from the solutions to the equations.

Thus, some other criteria would be required to sort out those solutions that are valid in an ejector. Such criteria are discussed further in the next section.
SECTION 3
INLET CONDITIONS

The curves of Figures 2 and 3 should be considered design curves since the physical size of the various ejector areas (and area ratios) would have to change to obtain the performance indicated at various points on the curves. However, once an ejector design is selected its operation will be determined by the boundary conditions imposed on the ejector.

In addition to the primary and secondary total pressures and total temperatures, the back pressure (see Figure 1) must also be known to determine the ejector operation. At any value of \( M_s \) the design is the same for a point on the supersonic or subsonic branch. Thus, if the back pressure is set at a value equal to the mixed-flow, static pressure on the subsonic branch, then the ejector would operate at that design point and \( P_{lp} \) would be equal to \( P_{ls} \). This follows since the exit flow is subsonic.

A whole series of values of \( M_s \) could be achieved in the ejector on the subsonic branch by adjusting the back pressure.

On the supersonic branch this is not the case. If the back pressure is reduced sufficiently, the ejector will make a transition to the supersonic solution branch and the operation becomes independent of further reductions in pressure. Thus, there is only one operating point that the ejector will achieve on the supersonic branch. This operating point can be determined by the methods described by Fabri and Siestrunk. We have developed equations based on their approach in reference 7 and these equations are also presented in Appendix A.

In reference 7, using the Fabri and Siestrunk approach, we calculated the data presented in Figure 4 for a temperature ratio of 3.7 and the other conditions shown on the figure. We chose various area ratios and plotted the efficiency versus the mass flow ratio achieved at various pressure ratios. Each curve
Figure 4. Efficiency Versus Mass-Flow Ratio for Different Geometry Mixing Tubes.
terminates at the point where the value of $M_s^*$ first reaches Mach one. The points are indicated by the hack marks on Figure 4 along each area ratio curve. It is clear from Figure 4 that the maximum efficiency is achieved for a given mass flow ratio with the smallest diameter tube operating at a value of $M_s^*$ nearly equal to one.

These results were achieved for an ejector that operates at a value of $M_s \leq 1$. However, from Figure 2 we see that for all of the data shown that the efficiency at $M_s = 1$, on a curve for a given bypass ratio, is higher than at any supersonic values of $M_s$. However, as shown on Figure 5, this result does not hold for all temperature ratios. At the low temperature ratios (e.g., 1.0, and 1.5) there is an increase in efficiency at higher values of $M_s$. Since most of our applications are concerned with temperature ratios higher than two, we will not consider the increased efficiencies resulting from operation at values of $M_s > 1$ at the lower temperature ratios.

In Table 1 we present a computer printout of the supersonic branch solution for a pressure ratio of six, temperature ratio of 3.7, and a bypass ratio of four. The first column contains the secondary Mach number $M_s$ which is taken as the independent variable. The second column is $P_{ls}/P_{os}$ (because of lack of space, it is labeled $P_{ls}$) and the third column is the temperature ratio $T_{ls}/T_{os}$. Each of these values is obtained from $M_s$ using isentropic relations. The exit area of the primary nozzle is chosen to match the pressure of the primary to the secondary. This sets the value of the primary Mach number, $M_p$ (located in column 10), and enables one to find $T_{lp}/T_{op}$ and $P_{lp}/P_{op}$ (located in columns 11 and 12).

Column four gives the ratio of the primary velocity to the secondary velocity $V_p/V_s$ and column five gives the ratio of the mixed velocity to the secondary velocity.

Column six gives the mixed flow Mach number. Column seven is the pressure ratio $P_m/P_{os}$ and column eight is the temperature
### TABLE 1

**SUPERSONIC BRANCH SOLUTION**

**CONSTANT MASS FLOW**
**CONSTANT AREA**
**SUPERSONIC**

**PRIMARY VAPOR AIR**  **SECONDARY GAS AIR**

<table>
<thead>
<tr>
<th>PR</th>
<th>4 000 TR</th>
<th>1 700 CP</th>
<th>1 400 GS</th>
<th>1 400 WR</th>
<th>1 00 WP</th>
<th>25 00 WP</th>
<th>25 00 WP</th>
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<tr>
<td>ET-PASS RATIO</td>
<td>4 000</td>
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<th>PIP</th>
<th>TIS</th>
<th>VPI/VS</th>
<th>VM/VS</th>
<th>MM</th>
<th>PM</th>
<th>TM</th>
<th>GR</th>
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<th>TIP/TOE</th>
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**TABLE I**

| 1.400 2 | 4.000 2 | 7.400 2 | 10.400 2 | 13.400 2 | 16.400 2 | 19.400 2 | 22.400 2 | 25.400 2 | 28.400 2 | 31.400 2 | 34.400 2 | 37.400 2 | 40.400 2 |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.4000 | 4000 | 2 | 16.400 2 | 19.400 2 | 22.400 2 | 25.400 2 | 28.400 2 | 31.400 2 | 34.400 2 | 37.400 2 | 40.400 2 | 43.400 2 | 46.400 2 | 49.400 2 | 52.400 2 | 55.400 2 |
| 0.4000 | 4000 | 2 | 16.400 2 | 19.400 2 | 22.400 2 | 25.400 2 | 28.400 2 | 31.400 2 | 34.400 2 | 37.400 2 | 40.400 2 | 43.400 2 | 46.400 2 | 49.400 2 | 52.400 2 | 55.400 2 |
| 0.4000 | 4000 | 2 | 16.400 2 | 19.400 2 | 22.400 2 | 25.400 2 | 28.400 2 | 31.400 2 | 34.400 2 | 37.400 2 | 40.400 2 | 43.400 2 | 46.400 2 | 49.400 2 | 52.400 2 | 55.400 2 |
| 0.4000 | 4000 | 2 | 16.400 2 | 19.400 2 | 22.400 2 | 25.400 2 | 28.400 2 | 31.400 2 | 34.400 2 | 37.400 2 | 40.400 2 | 43.400 2 | 46.400 2 | 49.400 2 | 52.400 2 | 55.400 2 |

12
ratio \( T_m/T_{os} \). The gamma value of the mixture is given in column nine which would vary if two different species were being used.

Column thirteen gives the area ratio \( A_p/A_s \) required to match the inlet pressures and column fourteen gives the value of \( A_p/A_s^* \). The last four columns give the two efficiencies, total pressure, and the entropy increase. The efficiency based on availability, labeled ERAVL, is plotted on Figure 2.

The Fabri conditions cannot be directly applied to the solution given in Table 1 since the geometry changes at each point. However, if we select designs (i.e., fix the geometry) from those presented in Table 1 we can then apply the inlet criteria to determine the ejector operating point for each of the selected designs. We have marked four such selections in Table 1 at values of \( M_s < 1 \).

Each of these four selections was used to generate an operating characteristic. These solutions, for the supersonic branch, are presented in Tables 2, 3, 4, and 5. The occurrence of the design points is indicated in each of the tables. These tables are basically different from Table 1 since they are for an ejector of given geometry which is presented at the top of each table. The columns referring to the primary conditions and to area ratios have been removed and replaced by the mass flow ratio of primary to secondary flow which is labeled \( M_p/M_s \) (the reciprocal of the bypass ratio).

Following the procedures that we developed in reference 7 for applying the Fabri and Siestrunck inlet conditions (the equations are given in Appendix A) we constructed the graphs presented in Figure 6 for each of the four designs. Figure 6 presents the total pressure ratio required to operate at the indicated value of \( M_s \). Thus, the intersection of each of the curves with the pressure ratio of six gives the value of \( M_s \) at which the particular ejector would operate. Only the design labeled "4" operates at the chosen design value of \( M_s \) which was \( M_s = 1 \). One can prove (see Appendix B) that the inlet conditions always allow for operation at \( M_s = 1 \) and \( P_{1p} = P_{1s} \).
### TABLE 2

**CASE 1**

**CONSTANT GEOMETRY**
**CONSTANT AREA**
**SUPERSONIC**

**PRIMARY VAPOR AIR**
**SECONDARY GAS AIR**

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**VIOlATION**

- Violation of 2nd Law requires negative temperatures.
- Violation of 2nd Law requires negative temperatures.

**IMAGINARY SOLUTION**

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- IMAGINARY SOLUTION

**WORK INPUT AND COOLING**

- Work input and cooling.
- Work input and cooling.
- Work input and cooling.
- Work input and cooling.
- Work input and cooling.
- Work input and cooling.
- Work input and cooling.
- Work input and cooling.
- Work input and cooling.

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**IMAGINARY SOLUTION**

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**EAIR/ER**

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- EAIR/ER = 1.8
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- EAIR/ER = 1.8
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- EAIR/ER = 1.8
# Table 3

## Case 2

### Constant Geometry

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### Conditions

- Violation of 1st Law Requires Negative Temperatures
- Violation of 2nd Law Requires Negative Temperatures
- Violation of 3rd Law Requires Negative Temperatures

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<th>PM/</th>
<th>TM/</th>
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</table>

**TABLE 5**

**CASE 4**

**CONSTANT GEOMETRY**

**CONSTANT AREA: SUPERSONIC**

**PRIMARY VAPOR AIR SECONDARY GAS AIR**

|------------|----------|----------|----------|----------|----------|----------|

**MF: a 1 24 100/TOP: = 499 PIP/POS: = 0 088**
Figure 6. Total Pressure Ratio Required to Operate at the Indicated Value of $M_s$ for Each of the Four Designs. The Intersection with the Pressure Ratio of Six Yields the Actual Operating Point at the Design Pressure Ratio.

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<th>$A_p/A_s$</th>
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<td>4</td>
<td>2.24</td>
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We have plotted the results of our study on Figure 7 where we show the supersonic branch solution for the bypass ratio of four where $P_{lp} = P_{ls}$ (solid line, data from Table 1) and the four operating characteristic curves (dashed lines with data from Tables 2, 3, 4 and 5) with their actual operating points indicated as determined from Figure 6. Thus, we see that an ejector cannot operate on the supersonic solution branch when $M_s < 1$ and the mass flow is given and $P_{lp} = P_{ls}$. Designs chosen from this branch actually operate at efficiencies lower than the design value and lower than the value of efficiency at $M_s = 1$.

In general, the design curves for which the bypass ratio is given and the value of $P_{lp} = P_{ls}$ are valid in the supersonic solution branch only for values of $M_s > 1$. The curves of the subsonic solution branch are valid over the entire range of $M_s$. As discussed in Appendix C an isolated point may exist on the supersonic solution branch for which $P_{lp} = P_{ls}$ and $M_s < 1$. Indications are that this point occurs in the flat region (e.g., from $M_s = 0.8$ to 1 in Table 1) or dips below the efficiency at $M_s = 1$. The value of efficiency at $M_s = 1$ has always been the highest efficiency obtainable if $M_s < 1$ for a given mass flow.

In the rest of our studies we used the value of efficiency at $M_s = 1$ as representative of the maximum efficiency achievable for the given mass flow and $M_s < 1$.

If the value of $M_s = 1$ lies within the forbidden region, we then used the value of $M_s$ (less than one) found from simultaneous solution of Equations (1) and (2) in the studies presented in Section 5.
Figure 7. Results of the Study Showing the Actual Operation of the Designs at Values of $M_s \leq 1$. Note: Although the Bypass Ratio is Four at the Design Points, it would not be Four on the Actual Operating Curve.
SECTION 4
IDEAL THRUST AUGMENTATION OF AN EJECTOR

In order to assess the potential of ejectors used for thrust augmentation we have made the following assumptions:

(1) For a given bypass ratio, the efficiency on the supersonic solution branch of an ejector at $M_s = 1$ and $P_{lp} = P_{ls}$ is representative of the maximum efficiency achievable at that bypass ratio;

(2) Inlets, nozzles, and diffusers are ideal and operate at 100 percent efficiency.

As explained in the last section, if we restrict operation of an ejector to subsonic inlet Mach numbers ($M_s < 1$) then the value of efficiency at $M_s = 1$ is a maximum (or very close to it) when the inlet restriction is considered (see Figures 4 and 7).

If supersonic secondary inlet Mach numbers are considered, then at low temperature ratios, higher efficiencies can be achieved at supersonic, secondary, inlet Mach numbers (e.g., see Figure 5).

These efficiencies, however, are not sufficiently higher to make a major change in the results.

The equations for thrust augmentation using ideal conditions were developed in reference 7 and are given in Appendix A. Figure 8 presents thrust augmentation for a pressure ratio of six, temperature ratio of 3.7, and four bypass ratios (0.5, 1, 2, and 4) as a function of flight Mach number $M_\infty$. The thrust augmentation is good at high and low values of the flight Mach number, but in the range of 0.7 to 1.1 it is low or even less than one for all four bypass ratios.

Figure 9 presents similar curves for a pressure ratio of three and a temperature ratio of two. Again the thrust
Figure 8. Thrust Augmentation as a Function of Flight Mach Number for an Ejector Operating on the Supersonic Branch at $M_s = 1$ at Various Bypass Ratios at the Indicated Conditions.
Figure 9. Thrust Augmentation as a Function of Flight Mach Number for an Ejector Operating on the Supersonic Branch at $M_s = 1$ at Various Bypass Ratios at the Indicated Conditions.
augmentation is good at low and high Mach numbers but poor in
the range of Mach numbers from 0.9 to 1.5.

Figures 10 through 14 present the effects of temperature
ratio on thrust augmentation, $\phi$, for five pressure ratios: two,
three, four, five, and six.

Figures 10 through 12 are for a bypass ratio of 0.5 and
Figures 13 and 14 are for a bypass ratio of two. On each of the
five figures the flight Mach number is the curve parameter.

Examination of the figures indicates only a weak dependence
of $\phi$ on pressure ratio. The dependence of $\phi$ on temperature is
reversed as we change from subsonic to supersonic flight Mach
number. It also becomes obvious from these figures that the
thrust augmentation in the neighborhood of Mach one is small or
less than one for all pressure ratios, temperature ratios, and
bypass ratios.

The low temperature end of the curves could be improved
slightly by operating the ejector at supersonic values of $M_s$.
It is doubtful, however, that this effect would raise the low-
temperature-ratio end of the curves above one for supersonic
flight Mach numbers, $M_\infty$.

At supersonic flight Mach numbers the thrust augmentation
is more like a ram jet effect than an ejector effect since it
depends so strongly on the energy transfer due to the high tem-
perature of the primary gas.

In the next section we will examine the thrust augmentation
of an ejector when operating with a hypothetical engine.
Figure 10. Effect of Temperature Ratio at a Pressure Ratio of Two and a Bypass Ratio of 0.5.
Figure 11. Effect of Temperature Ratio at a Pressure Ratio of Three and a Bypass Ratio of 0.5.
Figure 12. Effect of Temperature Ratio at a Pressure Ratio of Four and a Bypass Ratio of 0.5.
Figure 13. Effect of Temperature Ratio at a Pressure Ratio of Five and Bypass Ratio of Two.
BYPASS RATIO = 2
PR = 6

Figure 14. Effect of Temperature Ratio at a Pressure Ratio of Six and a Bypass Ratio of Two.
SECTION 5
AN EJECTOR ENGINE COMBINATION

The performance of an ejector when coupled to a jet engine was studied for hypothetical engines over a range of flight Mach numbers and altitudes. Hypothetical engines with pressure ratios ranging from 5 to 40 were investigated for two turbine inlet temperatures: 2500°F and 3000°F. We investigated cycles with ideal processes as well as those with nonideal processes. Figure 15 shows a sketch of a T-s diagram for an engine with losses.

The process from 1 to 2 involves ram compression which is isentropic in the ideal case, or according to MIL-E-5007D as quoted in reference 9:

\[
\frac{P_{02}}{P_{01}} = 1 \quad M_\infty = 0 \text{ to } 1.0
\]

\[
\frac{P_{02}}{P_{01}} = 1.0 - 0.076(M_\infty - 1)^{1.35} \quad M_\infty > 1.0 \text{ to } 5.
\]

The compression process from 2 to 3 was assumed to be adiabatic with a polytropic efficiency of 1 in the ideal case, and 0.85 in the nonideal case. The rpm of the engine was assumed to vary with flight Mach number so that the pressure ratio remained constant (i.e., a constant specific speed was assumed).

In the combustor, process 3 to 4, the temperature was assumed to be raised to the maximum allowable turbine inlet temperature with no loss in total pressure in the ideal case or a 5 percent loss in total pressure in the nonideal case.

In the turbine the gas expands in process 4 and 5 and supplies the work required to drive the compressor. No account was taken for work supplied to accessories and \(C_p\) and \(\gamma\) were assumed constant and equal to those of air. The polytropic efficiency was set to 1 for the ideal case and to 0.85 for the nonideal case.
Figure 15. Sketch of T-s Diagram for a Hypothetical Engine.
The process from 5 to 6 represents the adiabatic expansion to ambient pressure in the nozzle with a polytropic efficiency of 1.0 in the ideal case and 0.98 in the nonidealized case. The thrust per unit of mass flow of the engine is calculated as a function of the flight Mach number from the change in momentum.

Figures 16 and 17 show the primary engine thrust per unit of mass flow at two altitudes as a function of the flight Mach number for engines with a compressor pressure ratio, $Pe$, of 5 and 35, and turbine inlet temperatures, $T_4$, of 2500$^\circ$R and 3500$^\circ$R.

The low pressure ratio engine can, of course, achieve higher Mach numbers than the high pressure ratio engine before its thrust fall off to zero because of the high temperature limit. At each flight Mach number the pressure ratio, $PR$, can be determined ($PR=P_5/P_2$); and the temperature ratio, $TR$, can be determined ($TR=T_5/T_2$). The performance of an ejector for the values of $PR$ and $TR$ can then be calculated at $M_S=1$ and various bypass ratios. If the ejector is choked, the ejector performance is calculated on the boundary of the forbidden region for that bypass ratio.

Once the total pressure and temperature of the mixed flow is known, the thrust per unit mass of the mixed flow can readily be calculated assuming an adiabatic expansion to ambient pressure. A polytropic efficiency of 1.0 was assumed in the ideal case, and 0.98 in the nonideal case. The thrust per unit mass flow can be determined from the momentum change and the thrust augmentation, $\phi$, calculated from the following equation:

$$\phi = (1 + \beta)(\tau_m/\dot{m}_m)/(\tau_p/\dot{m}_p).$$  \hspace{1cm} (5)

The calculations were terminated when the pressure ratio, $PR$, dropped below one.

In addition to thrust augmentation of an ejector, we also calculated the performance of a mixing-fan jet engine. Using the same polytropic efficiencies as for the engine alone, we determined a common pressure at the exits of the turbine and fan which matched
Figure 16. Basic Engine Specific Thrust for a Compressor Pressure Ratio of Five at a Turbine Inlet Temperature of 2500°F (16a) and 3000°F (16b) at Sea Level and 30,000 feet.
Figure 17. Basic Engine Specific Thrust for a Compressor Pressure Ratio of 35 at a Turbine Inlet Temperature of 2500°F (17a) and 3000°F (17b) at Sea Level and 30,000 feet.
the work required by the fan (at a particular bypass ratio) and the work produced by the turbine. The two flows are then mixed at this pressure at low velocity; the common pressure is then the stagnation pressure of the mixture. Since the exit temperatures from the fan and turbine are also known, the mixed total temperature can be evaluated and the final state of the mixed gas is known. Thus, the thrust per unit mass can be calculated and the thrust augmentation determined from Equation (5).

A substantial amount of data were obtained from this study and Table 6 details the various conditions used to present Figures 18 through 29. As indicated in Table 6, Figures 18 through 21 present results for the ideal case for compressor pressure ratio of 35. As with the previous study in the neighborhood of Mach one the thrust augmentation is near one or less than one for the ejector. However, for the high pressure ratio engine it remains low even in the supersonic region, although it does slightly exceed one in that region. On the other hand, the thrust augmentation for the ejector is substantial at low subsonic Mach numbers on all the figures, even after taking account for losses.

The mixing fan shows substantial thrust augmentation throughout the entire Mach number range even after reasonable component efficiencies are assumed on the later figures; Figures 22 through 29.

If we consider the low pressure ratio engine in the non-ideal case, Figures 26 through 29, we see that the ejector again shows a thrust augmentation at the supersonic Mach numbers. Again this is the result of an action more like a ram jet since it results from the energy transfer which results from the temperature difference between the primary and secondary.

Generally mass flow entrainment in ejectors would be very large (i.e., of the order of 10 or more). If one considers a mixing fan with similar flow rates being pumped into the system, then the size of the engine will make integration with airframe a very difficult problem. On the other hand, distributing the engine air and entraining the outside air in large volumes will
not present insurmountable problems for integration. Ejectors therefore are attractive devices from the point of view of simplicity and integrability.

Curves such as the ones in Figures 18 to 29 must be viewed in the light of the comments made above. Otherwise, it might deter consideration of ejectors.


TABLE 6

THRUST AUGMENTATION DATA ARE PRESENTED FOR THE FOLLOWING COMBINATIONS OF FIGURES 18 to 29

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Figure 18: Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, Sea Level, $Pe = 35$, $n = 1$, $2500 R$.  

Sea Level, $Pe = 35$, $n = 1$, $T_{max} = 2500 R$.  

$FAN_\beta = 3.0$  

$FAN_\beta = 2.0$  

$FAN_\beta = 1.0$  

$FAN_\beta = 0.5$
Figure 19. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_s = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, Sea Level, $Pe = 35$, $\eta = 1$, 3000°F.
Figure 20. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_a = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, 30,000 feet, $Pe = 35$, $\eta = 1$, $2500^\circ R$. 
Figure 21. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_\infty = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, 30,000 feet, $Pe = 35$, $\eta = 1$, $3000^\circ R$. 

ALTITUDE = 30,000 FT
$Pe = 35$
$\eta_c = \eta_t = 1.0$
$T_{max} = 3000^\circ R$
Figure 22. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_S = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, Sea Level, $Pe = 35$, $\eta = 0.85$, $2500^\circ R$. 
Figure 23. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_S = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, Sea Level, $Pe = 35$, $\eta = 0.85$, $3000^\circ R$. 

SEA LEVEL
$Pe = 35$
$\eta_c = \eta_L = 0.85$
$T_{max} = 3000^\circ R$
Figure 24. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_s = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, 30,000 feet, $Pe = 35$, $\eta = 0.85$, $2500^\circ R$. 

**ALTITUDE = 30,000 FT**
*Pe = 35*
*\( \eta_c = \eta_t = 0.85 \)*
*\( T_{max} = 2500^\circ R \)*
Figure 25. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_S = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, 30,000 feet, $Pe = 35$, $\eta_s = 0.85$, $T_{max} = 3000^\circ R$. 
Figure 26. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_s = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, Sea Level, $Pe = 5$, $\eta = 0.85$, $2500^\circ R$. 
Figure 27. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_s = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, Sea Level, $Pe = 5$, $\eta = 0.85$, $3000^\circ R$. 
Figure 28. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at \( M_s = 1 \) (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, 30,000 feet, \( Pe = 5 \), \( \eta = 0.85 \), 2500°F.
Figure 29. Thrust Augmentation for an Ejector Operating on the Supersonic Branch at $M_\infty = 1$ (or the Boundary of the Forbidden Region) and a Mixing-Fan at Various Bypass Ratios, 30,000 feet, $Pe = 5$, $\eta = 0.85$, 3000°F.
SECTION 6
CONCLUSIONS FROM THE EJECTOR STUDY

Based on the results of the studies presented in the previous sections for constant area ejectors, we have drawn the following conclusions:

1. In general, on the supersonic branch of the design curves constructed for a given bypass ratio and $M_s = 1$, the curves cannot be realized in an ejector of values of $M_s > 1$ (see Figure 7).

2. In general, these curves, therefore, cannot be achieved at values of $M_s > 1$ on the supersonic solution branch.

3. Isolated points on the supersonic solution branch may not exceed the value of efficiency at $M_s = 1$ (see Figure 5) for an example).

4. The value of the efficiency at $M_s = 1$ on the supersonic branch is representative of the maximum efficiency if not the actual maximum efficiency (see Figures 4, 7, and 2).

5. At lower temperature ratios the efficiency on the supersonic solution branch may be higher at values of $M_s > 1$ than at $M_s = 1$ (see Figure 5). However, conclusion (4) is still valid since these higher efficiencies are probably not high enough to make a significant difference in the overall conclusions.

6. All of the thrust augmentation studies presented herein indicated little or no thrust augmentation at flight Mach numbers near one.

7. The basic study of thrust augmentation shows a reversal of the effect of temperature at subsonic and supersonic flight Mach numbers: at subsonic flight Mach numbers, increases in temperature reduce thrust augmentation while at supersonic flight Mach numbers, increases in temperature result in increases in thrust augmentation ratio (see Figures 10 through 14).
(8) Pressure ratio has only a very small effect on thrust augmentation ratio (see Figures 10 through 12 and 13 to 14).

(9) Thrust augmentation ratios at low subsonic flight Mach numbers were substantial for all cases studied.

(10) Thrust augmentation levels at supersonic flight Mach numbers were adequate for the low pressure ratio engine cycle, but less than one for the high pressure ratio cycle.

(11) As expected, mixing-fan thrust augmentation was superior to the ejector for the same bypass ratio and flight Mach numbers (see Figures 18 through 29). This superior performance is achieved at the cost of complex rotating machinery and total weight.

(12) The thrust augmentation of the mixing-fan also fell off substantially with increasing flight Mach numbers.

Since no analytical technique was found that could determine the maximum efficiency of an ejector, it was necessary to base our results on a numerical study of an ejector. The Fabri and Siestrunck inlet condition enabled us to determine a reasonable representation of the maximum efficiency that would be obtained for a wide range of parameters. Even though we know of some cases where higher efficiencies could be achieved (low temperature ratios and \( M_s > 1 \)) we do not believe that an actual ejector (with wall friction for example) could achieve efficiencies higher than those that we used. Thus, we believe that our results give reasonable estimates of the upper limits to performance that one can expect from an ejector over the range of parameters studied.

Conclusion (6) states that the thrust augmentation ratio of "constant area ejectors" is less than one at a flight Mach number around one. Therefore, this type of thrust augmenting ejector cannot be considered as a potential substitute for a turbo fan in a propulsion system for transonic aircraft. This raises two questions: (1) is thrust augmentation less than one near a Mach number of one an inherent characteristic of all types of thrust augmenting ejectors, and more broadly, of any type of momentum
exchange process not employing rotating machinery? And, (2) can ejector techniques in view of the limitation near Mach one be of any use for aircraft applications?

We feel that many useful applications of ejectors to aircraft are still possible at low flight speeds and that there are methods available to obtain thrust augmentation at Mach numbers near one. Some of the application areas are as follows:

(1) ejectors for STOL and VSTOL;
(2) boundary layer ejector systems;
(3) new approaches to momentum exchange
   - co-flowing
   - cross flow
   - counter flow.

Thrust augmenting ejectors for VSTOL and STOL have the following advantages:

(1) high thrust augmentation capability of advanced compact ejectors at take-off and low flight speeds;
(2) feasibility of a large variety of shapes for the ejector shroud (from axisymmetrical to high aspect ratio slot configurations);
(3) resulting high performance capabilities and synergistic effects of ejector and wing
   - prevention of flow separation by wing-boundary layer energization
   - strong supercirculation; very high $C_{L,\text{max}}$; improved $L/D$
   - favorable transition from "hover" to conventional flight (as flight speed increases, the vertical thrust component of the ejector decreases to a lesser degree than the aerodynamic lift increases).

In the boundary layer ejector systems the trailing edge region of the wing is configured as a very compact two-dimensional
ejector, which uses only wing boundary layer air as secondary ejector fluid medium. This results in performance characteristics and synergistic effects as follows:

(1) thrust augmentation does not drop to zero at flight Mach number around one and is extended into the supersonic flight regime;

(2) drag reduction due to restoration of the wake to free stream velocities;

(3) prevention of flow separation by boundary layer energization;

(4) improved wing characteristic (increased $C_{L,max}$ and L/D);

(5) possibility of boundary layer control by suction (if desired, the boundary layer ejector can be employed for a greatly simplified flow laminarization system).

In order to capitalize on the potential that these systems have, we recommend the following approach toward an ejector-aircraft design:

(1) Determination of major performance and design (geometry) parameters of two-dimensional ejectors under true environmental conditions, including simulated wing-boundary layer air for secondary fluid medium.

(2) Ejector-aircraft systems are major long-range projects requiring team efforts which combine expertise in ejector-engine-aircraft design.

However, before building an ejector-aircraft prototype, lessons learned from past failures of ejector-aircraft must be carefully avoided. This requires:

- a thorough comparison analysis of potential ejector wing fuselage configurations
- functional development of all critical system components and
- performance verification under true environmental conditions of these components.

In order to further our understanding of ejector processes we also recommend that studies of co-flowing ejectors be made using computational fluid dynamics (CFD). The code used by Scott and Hankey\textsuperscript{10} to study the flow in combustors would be ideal for this purpose. Fundamental understanding of the processes could be greatly enhanced by such studies. In addition, studies of ejectors not having a constant area can also be studied with this code to determine the performance of such ejectors.

In addition to the more traditional approaches just discussed, we present some entirely new concepts in the next section. These new concepts have the potential of providing improved methods of transferring availability between fluid streams without using moving parts.
Processes of transferring energy from one working medium of high energy content to one of lower energy content play a key role in aeropropulsion and other technology areas. Usually, in these processes, turbines and compressors are employed transferring the energy from one working substance to another, except in the so-called direct energy transfer processes. In these processes the energy transfer is accomplished by direct contact of the interacting media without the use of machinery.

Direct energy transfer processes have attracted a great deal of interest because of their inherent structural simplicity, low weight and cost, and high reliability; their problem, however, is to achieve high energy transfer efficiencies.

Distinction can be made between two major classes of direct energy transfer processes, namely:

- unsteady flow energy transfer processes
- steady flow energy transfer processes.

Examples for processes in which energy is transferred from one fluid medium to another one by means of unsteady flow are:

- shock tubes
- pulse jets
- dynamic pressure exchangers
- non-steady flow ejectors (non-steady flow in either primary or secondary flow or both).

Examples of steady flow direct energy transfer processes are:

- Continuous flow ejectors (for jet pumps, thrust augmentors, and other applications).

The following discussion deals with steady flow direct energy transfer processes, specifically with continuous flow
ejector-type devices for various applications in the field of thrust augmentation and pumps. Before discussing new approaches and concepts which depart from conventional ejectors, it will be most beneficial to briefly synopize the major characteristics of current ejector processes:

(a) Current ejector processes are based on momentum exchange between two mass-streams flowing in the same direction through a mixing duct (Figure 1). In the following, such processes will be referred to as "co-flowing momentum exchange processes." The momentum exchange in these processes is accomplished by mixing. The shape of the mixing duct is not limited to simple geometries, i.e., straight, constant duct cross-section. The mixing duct may be curved and its cross-section may vary with length.

(b) At the onset of mixing, the two interacting media have differences in one or more of the following fluid flow parameters:

• velocity
• total and/or static pressure
• total and/or static temperature
• physical or chemical characteristics (chemical reactions during mixing will not be considered).

The medium having at the onset of mixing the greater total pressure is called the "primary medium", the medium having the lower total pressure is called the "secondary medium."

(c) Within the mixing duct:

• the total mass flow is conserved
• in case of an adiabatic mixing process, the stagnation enthalpy is conserved
• if the internal flow does not exert a force component in the flow direction upon the inner surface of the mixing duct, then the impulse functions at mixing duct entrance and exit are equal.

(d) In "co-flow" type ejectors, mass and momentum transport are coupled by the transport velocity $U(x,y)$ as follows

- mass transport $\dot{m} = \int \rho(x,y) \cdot U(x,y) \, dA$
- momentum transport $\dot{M} = \int \rho(x,y) \cdot U^2(x,y) \, dA$

where $\rho(x,y)$ and $U(x,y)$ are, respectively, the mass density and transport velocity at a given duct cross-section as a function of position $(x,y)$.

(e) Two major categories of loss mechanisms can be identified in current ejectors:

- intrinsic losses associated with the mixing process
- parasitic drag losses.

The parasitic drag losses are due to skin friction and possible flow separation at the inner surfaces of the ejector shroud, which consists of inlet duct, mixing duct and diffuser (a diffuser is employed if necessary for matching mixing duct exit conditions with environmental conditions).

The parasitic drag is not of a fundamental nature; it can be strongly influenced by the design of the ejector shroud and method of primary injection. For example, the parasitic drag can be reduced by reducing the mixing duct length required for adequate mixing. This can be made possible, for example,
by employing a multiplicity of primary nozzles, by hypermixing nozzle shapes, or by accepting a certain degree of incomplete mixing.

The intrinsic losses associated with the mixing process are of a more fundamental nature and depend on the particular co-flow geometry chosen. For obtaining an upper bound of ejector performance and efficiency, the mixing losses can be determined analytically without taking parasitic drag losses into account. In flows where the total enthalpy is conserved, the ejector performance is completely determined by the total pressure after mixing, when mass flows and total pressures and temperatures of primary and secondary media are given.

The ratio of actual ejector performance to the performance resulting from an idealized mixing process having constant entropy for the system is most conveniently given by the ratio of availabilities, before and after mixing. We refer to this as the availability efficiency ($\eta_{av}$).

Another useful pseudo efficiency is the ratio of actual ejector exiting kinetic power to the kinetic power of the primary working medium expanding to a pressure equal to the stagnation pressure of the secondary working medium. This is called the kinetic energy efficiency ($\eta_K$).

Perhaps the most important characteristic of all co-flowing ejectors is that the differences between the flow parameters of the interacting working media (as previously defined under "b") are greatest at the beginning of the mixing process and decrease through the process of mixing. At the end of the mixing process (in the ideal case), the mixed flow has a homogeneous gas composition, temperature,
pressure, and velocity. The consequences of this type of equilibration process are, two-fold, namely:

- An intrinsic increase in system entropy (or loss in availability) will occur during the mixing process. This increase in system entropy will be substantial, if at the onset of mixing the differences between the flow parameters such as temperature, pressure, velocity of the interacting fluid media, are large.
- Only a fraction of the initial momentum of the primary working medium can be transferred to the secondary working medium, because the primary and secondary working media are brought by mixing to equal speed pressure and temperature \( (v_{m}', P_{om}', T_{om}', \text{respectively}) \).

From the above synopsis it follows that major, intrinsic performance shortcomings of current ejectors are a direct consequence of the co-flowing momentum exchange process. In the following, a discussion is given about momentum exchange processes which depart from the co-flowing type.

7.1 FUNDAMENTAL RESEARCH AND NEW CONCEPTS IN THE FIELD OF MOMENTUM EXCHANGE PROCESSES

Practically all research and development efforts on mixing phenomena and ejectors have been conducted in the area of co-flowing momentum exchange processes under shrouded and unshrouded (free jet mixing) conditions. Basic research as well as development work is needed on novel momentum exchange processes of the cross flow as well as the counter flow momentum exchange types. (These terms are chosen because of their close analogy to heat exchange processes, which are categorized according to co-flow, cross flow, and counter flow types of heat exchange; the most efficient heat transfer processes are of the counter flow type.)
First, the counter flow momentum exchange process will be described. In one-dimensional flow, only co-flow momentum exchange processes are possible. However, in axisymmetrical or two-dimensional flow configurations both a co-flow and a counter flow momentum exchange process can be realized, as illustrated in Figures 30 and 31, respectively. In Figure 31 the flow conditions are similar to those in fluid flow machinery: the mass flow per second is determined by the axial flow component, the density, and the annular area, while the angular momentum per second is determined by the tangential velocity component and the mass flow per second.

Since the axial velocity component can be small in comparison to the tangential velocity component, the primary and secondary gas flows can have either the same or opposite axial flow directions. It is important to realize the inherent functional differences resulting from these two possibilities; the latter allows for a counter flow momentum exchanger.

In the co-flowing process the greatest differences in total pressure exist at the onset of the mixing between the primary and secondary fluid media. As a consequence, this mixing process results in intrinsic losses or in an increase in total system entropy. In contrast, in the counter flow mixing process the secondary fluid medium comes first in contact with the primary medium after the energy of the primary medium is attenuated. As a consequence, the intrinsic mixing losses or system entropy increase are greatly reduced (under ideal conditions they can become zero).

In Figure 30 a cross flow concept is illustrated for a two-dimensional configuration. In the schematic shown, a rotor is used to establish vortices at a frequency corresponding to the natural Strouhal number of about 0.3 for the configuration (other configurations would not require a rotor). Eddies produced would exist for a longer duration before breaking into smaller scales. Because of the curvature of the channel, the eddies flow across
Figure 30. Schematic of a Cross Flow Momentum Exchanger Utilizing Organized Large-scale Eddies.
Figure 31. Schematic of a Counter Flow Momentum Exchanger.
the primary flow and can be accelerated to higher velocities with little exchange of thermal energy. (The primary and secondary are in general at different total temperatures.) The primary and secondary flows are separated at the exit.

Thus, we see that it is possible to describe momentum exchangers that operate on counter flow and cross flow configurations and permit, to some degree, the uncoupling of the thermal energy transport from the momentum transport. Such devices may have applications in many fields and could produce fundamental information concerning mixing phenomena. Thus, we believe that a research program should be established to investigate these concepts in view of many promising Air Force applications that can be envisioned.
REFERENCES


APPENDIX A
RELEVANT EJECTOR EQUATIONS

MIXING FORMULA

\[ \dot{C}_{p_m} \dot{m}_m = \dot{C}_{p_p} \dot{m}_p + \dot{C}_{ps} \dot{m}_s \]

\[ \dot{C}_{v_m} \dot{m}_m = \dot{C}_{v_p} \dot{m}_p + \dot{C}_{v_s} \dot{m}_s \]

\[ \dot{R}_m \dot{m}_m = \dot{R}_p \dot{m}_p + \dot{R}_s \dot{m}_s \]

\[ \bar{w} = \frac{\dot{w}_p}{\dot{w}_s} \]

\[ \bar{m} = \frac{\dot{m}_p}{\dot{m}_s} \]

\[ \beta = \frac{1}{\bar{m}} \]

\[ \gamma_m = \gamma_p \frac{\frac{\bar{m} \Gamma}{\bar{w}} + 1}{\frac{\bar{m} \Gamma}{\bar{w}} + \frac{\gamma_p}{\gamma_s} \frac{\bar{m} \Gamma}{\bar{w}}} \]

\[ \Gamma = \left[ \frac{\gamma_p}{(\gamma_p - 1)} \right] / \left[ \frac{\gamma_s}{(\gamma_s - 1)} \right] \]

\[ \frac{C_{pp}}{C_{ps}} = \frac{\Gamma}{\bar{w}} \]

\[ \frac{C_{pm}}{C_{ps}} = \frac{\Gamma \bar{m} + \bar{w}}{\bar{w} (\bar{m} + 1)} \]

\[ \frac{R_m}{R_s} = \frac{\bar{m} + \bar{w}}{\bar{w} (\bar{m} + 1)} \]

\[ \frac{C_{pp} \dot{m}_p}{C_{pm} \dot{m}_m} = \frac{\Gamma \bar{m}}{\bar{w} (\Gamma \bar{m} + \bar{w})} \]

\[ \frac{R_p \dot{m}_p}{R_m \dot{m}_m} = \frac{\bar{m}}{\bar{m} + \bar{w}} \]

\[ \frac{T_{os}}{T_{os}} = \left( 1 + \frac{\Gamma \bar{m}}{\bar{w}} \right) \frac{TR}{1 + \frac{\Gamma \bar{m}}{\bar{w}}} \]
ISENTROPIC FLOW

\[
\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \frac{M^2}{\gamma - 1}
\]

\[
\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma / \gamma - 1}
\]

\[
\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma / \gamma - 1}
\]

\[
A = \frac{1}{M} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma + 1 / 2 (\gamma - 1)}\right]
\]

\[
\frac{T_0}{T} \frac{V^2}{2h_0} = \frac{\gamma - 1}{2} M^2
\]

\[
\frac{\dot{m}}{A} = \sqrt{\frac{\gamma W}{R} \frac{P_0}{\sqrt{T_0}}} \frac{M}{(1 + \frac{\gamma - 1}{2} M^2)^{\gamma + 1 / 2 (\gamma - 1)}}
\]

EJECTOR EQUATIONS

\[
C_1 = \frac{\Gamma \dot{m} \frac{T_0}{\Gamma \dot{m} + \bar{W}}}{\Gamma \dot{m} + \bar{W}} \left(1 + \frac{\gamma_s - 1}{2} M_s^2\right)
\]

\[
C_2 = \frac{\bar{W} (\dot{m} + 1)}{\Gamma \dot{m} + \bar{W}} \frac{\gamma_s - 1}{2} M_s^2
\]

\[
C_3 = \left(\frac{\bar{P}_1 \bar{A} + 1 - \frac{2 \pi l}{P_{1S}}}{\bar{P}_1 \bar{A}} (1 + \bar{A})\right) \frac{\bar{W}}{\dot{m} + \bar{W}}
\]

\[
C_4 = \frac{\bar{m} \bar{W}}{\dot{m} + \bar{W}} \frac{\bar{V}}{\gamma_s M_s^2}
\]

\[
C_5 = \frac{\bar{W}}{\dot{m} + \bar{W}} \gamma_s M_s^2
\]

\[
C_6 + \frac{\bar{W} (\dot{m} + 1)}{\bar{W} + \dot{m}} \gamma_s M_s^2
\]
(C2 - C6) \( \bar{V}_m^2 \) + (C3 + C4 + C5) \( \bar{V}_m \) - C1 = 0

\( \bar{V}_m = \frac{V_m}{V_{ls}} \)

\[
\frac{T_m}{T_{os}} = \frac{1 + \bar{m}}{1 + \frac{m}{T_m}} \left[ \frac{1 + \frac{\bar{m}}{T_m}}{W} \right] - \left( 1 - \frac{T_{ls}}{T_{os}} \right) \bar{V}_m^2
\]

\[
\frac{P_m}{P_{os}} = \frac{\bar{m} + W}{W(1 + AR)} \cdot \frac{T_m}{T_{ls}} \cdot \frac{P_{ls}}{P_{os}} \cdot \frac{1}{V_m}
\]

\[
M_m = \frac{\bar{V}_m M_s}{\sqrt{\frac{\gamma_m}{\gamma_s} \frac{T_m}{T_{op}} \frac{W + m}{W(1 + m)}}}
\]

\[
M_m = \frac{\bar{V}_m M_s}{\sqrt{\frac{\gamma_m}{\gamma_s} \frac{T_m}{T_{ls}} \frac{W + m}{W(1 + m)}}}
\]

The remaining Equations are for a single species (e.g., air-air)
\[ \eta = \left( N + \ln \frac{P_{om}}{P_{os}} \right) / D \]

\[ D = \frac{\gamma}{\gamma - 1} \frac{m_p}{m_m} \left[ \frac{T_{op}}{T_{os}} - 1 - \ln \left( \frac{T_{op}}{T_{os}} \right) \right] + \frac{m_p}{m_m} \ln \left( \frac{P_{op}}{P_{os}} \right) \]

\[ N = \frac{\gamma}{\gamma - 1} \left[ \frac{m_p}{m_m} \left( \frac{T_{op}}{T_{os}} - 1 \right) - \ln \left( \frac{m_p}{m_m} \frac{T_{op}}{T_{os}} - \frac{m_s}{m_m} \right) \right] \]

**INLET CONDITIONS**

\[ \frac{1}{M^*_{ep}} \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M^*_{ep}^2 \right) = \frac{1}{\gamma - 1} \]  

\[ = \frac{A_p}{A^*} - \frac{A_s}{A^*} \]  

\[ M^* \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M_s^2 \right) \]  

\[ = \frac{1}{\gamma - 1} \]

\[ \frac{1 + M^*_{ep}^2}{M^*_{ep}} = \frac{1 + M^*_{p}^2}{M^*_{p}} + \frac{A_s}{A^*_p} \frac{P_{os}}{P_{op}} \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M_s^2 \right) \left( 1 - M_s^* \right)^2 \]
APPENDIX B

THE INLET FLOW CONDITIONS ALWAYS ADMIT THE SOLUTION OF $P_1 = P_{os}$ AND $M_s = 1.0$

In order to determine the value of $P_{os}/P_{op}$ at a value of $M_s^* = 1$, we can solve the second inlet condition equation given in Appendix A for $P_{os}/P_{op}$:

\[
\frac{P_{os}}{P_{op}} = \frac{A_s^*}{A_p^*} \frac{(1 + M_e^*) - (1 + M_s^*)}{(1 - M_s^*)^2 \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M_s^* \right)^{1/\gamma - 1}}.
\]

Now from the first Equation of the inlet conditions we see that if $M_s^* = 1$ then $M_e^* = M_s^*$. Thus, Equation B.1 gives a $0/0$ or indeterminate form when $M_s^* = 1$. However, L'Hospital's Rule can be used to determine the value of $M_s^* = 1$. A double application of L'Hospital's Rule is required. It results in:

\[
\frac{P_{os}}{P_{op}} = \frac{1}{2} \frac{A_s^*}{A_p^*} \frac{M_p - 1}{M_e^*} \frac{d^2 M_e^*}{d M_s^*} \bigg|_{M_s^* = 1} \quad \text{(B.2)}
\]

In order to evaluate Equation B.2 we must use the first of the inlet equations given in Appendix B:

\[
M_e^* \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M_e^* \right) - \frac{1}{\gamma - 1} = \frac{A_s}{A_p} - \frac{A_s^*}{A_p^*} M_e^* \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M_s^* \right) \left( \frac{1}{\gamma - 1} \right)
\]

(B.3)
After differentiating Equation B.3 twice we obtain:

\[
\frac{d^2 M^*_{ep}}{d M^*_{s}^2} \bigg|_{M^*_{s} = 1} = 2 \frac{A_s}{A_p} \cdot \frac{M^*_{p}^{2}}{M^*_{p} - 1} \cdot \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} \frac{M^*_{p}^{2}}{M^*_{p}} \right) \frac{\gamma}{\gamma - 1} \quad (B.4)
\]

Substitution of Equation B.4 into Equation B.2 results in

\[
\left( \frac{P_{os}}{P_{op}} \right)_{M^*_{s} = 1} = \left( \frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} \frac{M^*_{p}^{2}}{M^*_{p}} \right) \frac{\gamma}{\gamma - 1} \quad (B.5)
\]

Now

\[
\left( \frac{P_{ls}}{P_{op}} \right)_{M^*_{s} = 1} = \frac{P_{ls}}{P_{os}} \cdot \frac{P_{op}}{P_{op}} \cdot \frac{P_{os}}{P_{op}} \quad (B.6)
\]

Using Equation B.5 along with the well known isentropic relations yield

\[
\left( \frac{P_{ls}}{P_{lp}} \right)_{M^*_{s} = 1} = 1 \quad (B.7)
\]

Therefore, the inlet conditions always admit the solution where \( M^*_{s} = M^*_{s} = 1; P_{ls}/P_{op} = 1 \) and the ratio \( P_{os}/P_{op} \) is determined from Equation B.5.
APPENDIX C

AN EXAMPLE OF AN ISOLATED POINT ON THE SUPersonic BRANCH WHERE \( M_s < 1 \) AND \( P_{lp} = P_{ls} \)

As stated in the text an isolated point may exist on the supersonic branch where \( M_s < 1 \) and \( P_{lp} = P_{ls} \). The investigation of the supersonic solution branch where \( M_s < 1 \) showed that most points could not be achieved in an ejector. However, one can show, by an inverse procedure, that an isolated point may exist where \( M_s < 1 \) and \( P_{lp} = P_{ls} \).

Table C.1 presents the result of applying the Fabri and Siestrunck inlet conditions to an ejector design where \( M^*_p = 1.7 \) \( (M_p = 2.1555), A_p/A_p^* = 1.928 \) (Al in Table C.1), \( A_s/A_s^* = 2.000 \) (as in Table C.1) and TR = 1.15. Note that \( A/A_p^* = 2 + 1.928 = 3.928 \). Using the procedure described in reference 7, we found pressure ratios, \( PR \), required to achieve various values of \( M^*_s \) \( (M_s in the table). The ratio of \( P_{lp}/P_{ls} \) is also listed in Table 3.1 and we note that it achieves a value of 1 at \( M^*_s = 1 \) and about at \( M^*_s = 0.7 \) at which point the bypass ratio \( (MU in the table) is 0.25848 and \( PR = 7.40451 \).

Thus, if we run the design program with the values of \( \beta, PR, and TR \) indicated at \( M^*_s = 0.7 \) \( (M_s = 0.6668) \) we should recover this point and we know that it satisfies the Fabri and Siestrunck inlet conditions.

The computer output for this "inverse" run is shown in Table C.2. Indeed we see that at \( M_s = 0.6668, M_p = 2.155 \) is reasonable and \( A/A_p^* = 3.928 \) is also reasonable. Thus, this particular point on the curve can be achieved and the value of the efficiency is about 0.883. Note, however, that this value of efficiency is less than the value at \( M_s = 1 \), where it is 0.890. The value of the efficiency at \( M_s = 1 \) is slightly less than the values found at supersonic values of \( M_s \).
| MS | 1 1000 | M^2/P^2 | 1 94534 | AE/|AP^+ | 1 92736 | (A-AE)/|AP^+|-O 0000 | PR= 18 45680 | MU= 0 00000 | PIP/IP^2= 1 8505 |
|---|---|---|---|---|---|---|---|---|---|---|
| MS | 0 0500 | M^2/P^2 | 1 93636 | AE/|AP^+ | 1 77028 | (A-AE)/|AP^+|-O 1576 | PR= 17 64767 | MU= 0 01549 | PIP/IP^2= 1 7514 |
| MS | 0 1000 | M^2/P^2 | 1 92287 | AE/|AP^+ | 1 61318 | (A-AE)/|AP^+|-O 3142 | PR= 16 45100 | MU= 0 03348 | PIP/IP^2= 1 6591 |
| MS | 0 1500 | M^2/P^2 | 1 91072 | AE/|AP^+ | 1 45905 | (A-AE)/|AP^+|-O 4689 | PR= 15 48255 | MU= 0 05147 | PIP/IP^2= 1 5678 |
| MS | 0 2000 | M^2/P^2 | 1 89786 | AE/|AP^+ | 1 30714 | (A-AE)/|AP^+|-O 6205 | PR= 14 54310 | MU= 0 06956 | PIP/IP^2= 1 4761 |
| MS | 0 2500 | M^2/P^2 | 1 88430 | AE/|AP^+ | 1 15951 | (A-AE)/|AP^+|-O 7603 | PR= 13 63170 | MU= 0 08762 | PIP/IP^2= 1 3846 |
| MS | 0 3000 | M^2/P^2 | 1 87008 | AE/|AP^+ | 1 01649 | (A-AE)/|AP^+|-O 9114 | PR= 12 74660 | MU= 0 10554 | PIP/IP^2= 1 2927 |
| MS | 0 3500 | M^2/P^2 | 1 85524 | AE/|AP^+ | 1 87915 | (A-AE)/|AP^+|-O 1047 | PR= 11 93650 | MU= 0 12342 | PIP/IP^2= 1 2047 |
| MS | 0 4000 | M^2/P^2 | 1 83992 | AE/|AP^+ | 1 74836 | (A-AE)/|AP^+|-O 1195 | PR= 11 15240 | MU= 0 14130 | PIP/IP^2= 1 1167 |
| MS | 0 4500 | M^2/P^2 | 1 82418 | AE/|AP^+ | 1 62493 | (A-AE)/|AP^+|-O 1342 | PR= 10 41010 | MU= 0 15918 | PIP/IP^2= 1 0284 |
| MS | 0 5000 | M^2/P^2 | 1 80820 | AE/|AP^+ | 1 50563 | (A-AE)/|AP^+|-O 1462 | PR= 9 71396 | MU= 0 17706 | PIP/IP^2= 0 9404 |
| MS | 0 5500 | M^2/P^2 | 1 79219 | AE/|AP^+ | 1 38632 | (A-AE)/|AP^+|-O 1584 | PR= 9 06450 | MU= 0 19492 | PIP/IP^2= 0 8521 |
| MS | 0 6000 | M^2/P^2 | 1 77639 | AE/|AP^+ | 1 26722 | (A-AE)/|AP^+|-O 1705 | PR= 8 46391 | MU= 0 21270 | PIP/IP^2= 0 7648 |
| MS | 0 6500 | M^2/P^2 | 1 76112 | AE/|AP^+ | 1 14835 | (A-AE)/|AP^+|-O 1825 | PR= 7 91072 | MU= 0 23161 | PIP/IP^2= 0 6775 |
| MS | 0 7000 | M^2/P^2 | 1 74673 | AE/|AP^+ | 1 03099 | (A-AE)/|AP^+|-O 1948 | PR= 7 36851 | MU= 0 25048 | PIP/IP^2= 0 5903 |
| MS | 0 7500 | M^2/P^2 | 1 73366 | AE/|AP^+ | 1 02789 | (A-AE)/|AP^+|-O 2070 | PR= 7 01072 | MU= 0 26846 | PIP/IP^2= 0 5030 |
| MS | 0 8000 | M^2/P^2 | 1 72214 | AE/|AP^+ | 1 02412 | (A-AE)/|AP^+|-O 2193 | PR= 6 69451 | MU= 0 28542 | PIP/IP^2= 0 4156 |
| MS | 0 8500 | M^2/P^2 | 1 71274 | AE/|AP^+ | 1 02412 | (A-AE)/|AP^+|-O 2318 | PR= 6 43093 | MU= 0 30248 | PIP/IP^2= 0 3282 |
| MS | 0 9000 | M^2/P^2 | 1 70574 | AE/|AP^+ | 1 02412 | (A-AE)/|AP^+|-O 2443 | PR= 6 16001 | MU= 0 31944 | PIP/IP^2= 0 2408 |
| MS | 0 9500 | M^2/P^2 | 1 70044 | AE/|AP^+ | 1 02412 | (A-AE)/|AP^+|-O 2569 | PR= 5 92324 | MU= 0 33632 | PIP/IP^2= 0 1534 |
| MS | 1 0000 | M^2/P^2 | 1 69600 | AE/|AP^+ | 1 02412 | (A-AE)/|AP^+|-O 2696 | PR= 5 70664 | MU= 0 35320 | PIP/IP^2= 0 0661 |

| M^2 | 2 000 A^2 | 2 000 A^2 | 2 000 A^2 | 1 500 |
TABLE C-2

**CONSTANT MASS FLOW**

| PS | TIS/ | VP/WS | VM/WS | MM | TP | TH/ | CM | MP | TIP/TOS | PIP/POR | AP/AS | A/R | EFPQ | PSO | PMP | ETAVL | SR |
|----|------|-------|-------|----|----|-----|----|----|--------|---------|-------|-----|-----|-----|-----|-----|-------|----|
| 0 000 | VIO| VIOLATION OF 2ND LAW REQUIRES NEGATIVE TEMPERATURES | 0 000 | VIO| VIOLATION OF 2ND LAW REQUIRES NEGATIVE TEMPERATURES | 0 000 | VIO| VIOLATION OF 2ND LAW FOR ADIABATIC SYSTEMS REQUIRES WORK INPUT AND COO | 0 000 | VIO| VIOLATION OF 2ND LAW FOR ADIABATIC SYSTEMS REQUIRES WORK INPUT AND COO | 0 000 | VIO| VIOLATION OF 2ND LAW FOR ADIABATIC SYSTEMS REQUIRES WORK INPUT AND COO | 0 000 | VIO| VIOLATION OF 2ND LAW FOR ADIABATIC SYSTEMS REQUIRES WORK INPUT AND COO |
| 0 000 | 0 04 | 0 06 | 0 08 | 0 10 | 0 12 | 0 14 | 0 16 | 0 18 | 0 20 | 0 22 | 0 24 | 0 26 | 0 28 | 0 30 | 0 32 | 0 34 | 0 36 | 0 38 | 0 40 | 0 42 | 0 44 | 0 46 | 0 48 | 0 50 |
| 0 000 | 0 04 | 0 06 | 0 08 | 0 10 | 0 12 | 0 14 | 0 16 | 0 18 | 0 20 | 0 22 | 0 24 | 0 26 | 0 28 | 0 30 | 0 32 | 0 34 | 0 36 | 0 38 | 0 40 | 0 42 | 0 44 | 0 46 | 0 48 | 0 50 |
| 0 000 | 0 04 | 0 06 | 0 08 | 0 10 | 0 12 | 0 14 | 0 16 | 0 18 | 0 20 | 0 22 | 0 24 | 0 26 | 0 28 | 0 30 | 0 32 | 0 34 | 0 36 | 0 38 | 0 40 | 0 42 | 0 44 | 0 46 | 0 48 | 0 50 |
| 0 000 | 0 04 | 0 06 | 0 08 | 0 10 | 0 12 | 0 14 | 0 16 | 0 18 | 0 20 | 0 22 | 0 24 | 0 26 | 0 28 | 0 30 | 0 32 | 0 34 | 0 36 | 0 38 | 0 40 | 0 42 | 0 44 | 0 46 | 0 48 | 0 50 |
| 0 000 | 0 04 | 0 06 | 0 08 | 0 10 | 0 12 | 0 14 | 0 16 | 0 18 | 0 20 | 0 22 | 0 24 | 0 26 | 0 28 | 0 30 | 0 32 | 0 34 | 0 36 | 0 38 | 0 40 | 0 42 | 0 44 | 0 46 | 0 48 | 0 50 |

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**NORSE**

**SECONDARY VAPOR AIR**

**PR**  
**TR**  
**1**  
**GP**  
**1**  
**400**  
**GS**  
**1**  
**400**  
**WS**  
**1**  
**100**  
**2**  
**00**  
**2**  
**57**

**BY-PASS RATIO**  
**0**  
**258**