FINAL REPORT ON LINE OF LIGHT AID TO NAVIGATION

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and
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Avery Point Groton, Connecticut 06340

FINAL REPORT

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OFFICE OF RESEARCH AND DEVELOPMENT
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SAMUEL F. POWEL, III
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The Coast Guard Research and Development Center evaluated the feasibility of using collimated, horizontal beams of light to mark shipping channels. The original concept called for the beams to be directed, at some fixed height above the water, down the channel centerline from an appropriate transmission point. Mariners would endeavor to safely transit the channel by steering to keep the beams overhead.

Several source types were considered; the large beam divergence of incandescent sources made them unsuitable for ranges in excess of several hundred yards. Only lasers were found to offer the required intensity and beam divergence characteristics for a workable system. However, eye safety considerations limited the allowable output power density to unacceptable levels. At the allowable irradiance levels, light scattering calculations predicted poor beam visibility, except under ideal viewing conditions. Additionally, the high-powered lasers required would be expensive, unreliable, and difficult to maintain. A recommendation was made to remove the "line of light" concept from consideration due to negative safety, cost, performance, maintenance, and reliability aspects.
## METRIC CONVERSION FACTORS

### Approximate Conversions to Metric Measures

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<tbody>
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<td>Fahrenheit</td>
<td>5/9 (after subtracting temperature)</td>
<td>Celsius</td>
<td>°C</td>
</tr>
<tr>
<td>°C</td>
<td>Celsius</td>
<td>°F (then add 32)</td>
<td>Fahrenheit</td>
<td>°F</td>
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</tbody>
</table>

Note: 1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures. Price $2.25.

BD Catalog No. C13.10.286.
1.0 INTRODUCTION

The previous interim report established a maximum permissible exposure (MPE) for a continuous wave (CW) laser and determined that only lasers were suitable candidates for use in the "line of light" concept. Additionally, a brief outline of the intended approach to determine beam visibility was given. This final report expands upon the previous eye safety analysis and includes repetitively-pulsed lasers. Pulsed beams were not found to offer significant advantages over CW beams, and in most cases result in a lower maximum permissible exposure. A complete analysis of scattering and beam visibility shows that the restrictive maximum permissible exposure level limits use of the system to those locations with low background illumination levels. Such a system would be characterized by high costs and questionable reliability, and could not be easily maintained by Coast Guard personnel. These findings do not support implementation of the "line of light" concept at the present time.

During the course of this investigation, a completely different procedure for using a laser line of light was analyzed. The Appendix contains the complete text of this analysis.

2.0 EYE SAFETY ANALYSIS FOR LASER SOURCES

Calculations given in the previous interim report showed that only lasers could meet the stringent beam divergence and intensity requirements of the horizontal "line of light" concept. The "line of light" scheme depends upon scattered light to enable an off-axis observer to detect the beam against background lighting of the nighttime sky and horizon. The scattered laser radiation is not expected to pose a problem; however, intrabeam (on-axis) viewing of the laser source may subject observers to potentially hazardous levels of radiation. The beam should never be observed on-axis, but an inadvertent observer on the bridge of a supertanker or onboard a low-flying plane might view the source in this manner. Misalignment of hardware could direct the hazardous beam down toward the water surface, imperiling other observers. It is assumed that Coast Guard personnel would take adequate precautions during maintenance or that the device would be equipped with an automatic shut-off feature. However, for the sake of inadvertent observers, a maximum permissible exposure (MPE) (laser power density) was established.

The type of laser first considered was a continuous wave (CW) type operating in the visible spectrum. For visible wavelengths, absorption by the aqueous and vitreous humor of the eye is minimal, and radiation incident on the cornea is focused onto the retina. For continuous wave radiation, the damage is primarily caused by local heating of the retinal tissue, with damage generally being irreversible. The danger of permanent injury increases with absolute irradiance level. The worst case occurs when the eye is "relaxed" (focused at infinity); the eye then focuses the laser radiation onto a small section of the retina, producing very high irradiance levels. If the observer is using magnifying optics (binoculars, alidade, etc.), the irradiance increases approximately as the square of the magnification (7-power optics produce a 50-fold increase).
Fortunately, bright lights in the visible spectrum cause a natural aversion response. The U.S. Army Environmental Hygiene Agency uses an exposure duration of 0.25 seconds for this situation. Several sources 1,2,3,4 agree that for an exposure duration of 0.25 seconds, the eye irradiance should not exceed 2.5 milliwatts per square centimeter (2.5 mW/cm²). In a practical system, the raw laser output beam would be optically expanded to produce a "line of light" several inches in diameter, but system parameters must be adjusted so that the final output beam does not exceed 2.5 mW/cm². (NOTE: This is safe only for observers using unaided vision. Binoculars or other magnifying optics would produce dangerous levels of eye irradiance.)

Preliminary scattering calculations predicted poor beam visibility with a beam limited to 2.5 mW/cm². Except for very low background illumination levels, the scattered laser light would not be sufficient for beam detection at other than short ranges. Coast Guard Headquarters personnel suggested pulsing the laser to maintain the required average irradiance, yet allow the individual pulses to have irradiance levels higher than 2.5 mW/cm². Very short pulses can cause damage to tissue from acoustic shock waves; however, the primary damage mechanism is still thermal, and the effects of individual pulses are additive. Prior to calculating maximum permissible exposure (MPE) levels for pulsed lasers, it should be determined whether or not a pulsed signal offers any detection advantages.

An intermittent visual stimulus, under certain conditions, may be perceived by the eye as a steady signal. This results from the persistence of the visual image for a brief period after the stimulus is removed. The frequency at which an intermittent signal begins to appear continuous is known as the critical fusion frequency (CFF). Above this frequency a relationship known as Talbot's Law holds: a flashing light which is on P percent of the time has the same apparent brightness as another light which is continuously on, yet is only P percent as intense. 5 It is now possible to eliminate from consideration laser pulse rates higher than the critical fusion frequency; these pulse rates would not offer increased signal conspicuity over the continuous wave laser, given equal average output irradiance. CFF is chiefly dependent upon the intensity of the stimulus; however, beyond 60 Hz, fusion occurs regardless of intensity. For "line of light" luminance levels, the CFF will be approximately 40 Hz. As previously mentioned, the intermittent nature of the stimulus will become apparent at frequencies less than the CFF. Studies have shown evidence of a resonance in the visual response system at 8 to 10 Hz. A light flashing at this frequency will appear somewhat brighter than a light flashing at a lower or higher frequency. 6,7 A "line of light" system pulsing at 10 Hz would have a slight conspicuity advantage over a continuous wave system.

Consider the following two cases:

1) A CW laser producing an output irradiance of 2.5 mW/cm².

2) A repetitively-pulsed laser with a 50 percent duty cycle, an average output irradiance of 2.5 mW/cm², and an individual pulse irradiance of 5.0 mW/cm².
The pulsed laser will operate at 10 Hz to afford it a slight edge in conspicuity. Both lasers will operate in the green region (Argon-ion), and the exposure period will be 0.25 seconds, equal to the natural aversion response time. The MPE of the Case 2 laser is determined by a two-step procedure, considering both individual pulse irradiance and average irradiance.

Step 1: Individual Pulse Limitation\(^4\). The total "on time" during a 0.25 second period is:

\[
\text{Time} = \text{Pulse Width} \times \text{Frequency} \times \text{Exposure Period} = (0.05s) (10Hz) (0.25s) = 0.125s.
\]

From the ANSI publication for safe use of lasers, the maximum exposure allowed in 0.125 seconds is \(4 \times 10^{-4} \text{ J/cm}^2\). This applies to the train of pulses. The maximum exposure per pulse equals \(\frac{(4 \times 10^{-4} \text{ J/cm}^2)/(2.5 \text{ pulses})}{1.6 \times 10^{-4} \text{ J/cm}^2}\). (As a comparison, the maximum permissible exposure for a single pulse not in a train is \(2 \times 10^{-4} \text{ J/cm}^2\). For the purpose of MPE determinations, multiple pulses are considered to be in a train if they fall within the same 0.25 second interval.)

Step 2: Average Power Limitation\(^4\). For a 0.25 second period, the maximum allowed exposure is \(6.3 \times 10^{-4} \text{ J/cm}^2\). This corresponds to an MPE of \(2.5 \times 10^{-3} \text{ W/cm}^2\), the same as for the CW laser. However, the limiting exposure is defined in Step 1, where \(1.6 \times 10^{-4} \text{ J/cm}^2\) corresponds to \(1.6 \times 10^{-3} \text{ W/cm}^2\).

The higher power laser in Case 2 is unsafe, although it produces the same average power as the laser in Case 1. Generally, given the same total energy (Joules), it is safer to radiate it continuously than in discrete pulses. A duty cycle less than 50% alleviates the safety problem, but the lower average irradiance affects beam visibility negatively. For the reasons given above, pulsing is not considered a solution to the problem of poor beam visibility.

Although the optics can be designed to give a uniform output irradiance level of \(2.5 \text{ mW/cm}^2\) in a collimated beam, unwanted concentration of the beam can result from two effects. Relative mechanical motion between optical components can cause the beam to focus, or atmospheric turbulence can result in momentary increases of the irradiance level. Concerning the focusing effect, for example, if the projection optics have an 80-inch focal length, an axial shift of the optics by 0.008 inch can cause the beam to focus at a one mile range. Smaller shifts will lead to focusing at greater ranges. If uncompensated, a temperature change in the structure of the order of 50°C could result in such focusing effects. The hazard and liability involved would likely necessitate adding a safety monitor to the device to shut down the laser in case of inadvertent focusing. The second effect, focusing by random turbulence, can be serious, though the probability of an eye being at the exact spot at the exact time is very unlikely.
3.0 SYSTEM PERFORMANCE

Expected system performance is analyzed in Appendix A, "Geometry and Photometry of the "Line of Light" Beacon". From the results given in Section III.2 of the Appendix, it is concluded that a suitable laser beam cannot be seen reliably at twilight at scattering angles (angles off the bow) of tens of degrees. It can likely be seen in full darkness, depending upon the local value of sky luminance. Since measured values of sky luminance in the vicinity of ship channels were not available, the dark night case was not treated.

From the results given in Section III.3 of the Appendix, it is concluded that the laser line or lines of light can be seen on the horizon at twilight and in greater darkness. Also it is shown that the slope of the luminous line at the horizon indicates the lateral position of the observer with respect to the channel center line. For the purpose of determining lateral position, the luminous line must appear as a "line source", with sufficient angular subtense to allow estimation of its slope. Obviously, a point source would not allow slope determination. It is shown in the Appendix that, at twilight, the required length of luminous line can be seen, so that a lateral position determination can be made with sufficient accuracy at ranges up to roughly one visibility distance (visibility distance = range of visibility). At the maximum ranges some spreading of the beam will occur due to multiple scattering of the light, but this is not expected to reduce the usability of the technique seriously.

4.0 COST, RELIABILITY, AND MAINTENANCE CONSIDERATIONS

If the "line of light" project is to be continued, despite its unsuitability for the majority of range applications, several other factors should be considered. The hardware will be expensive, and prototype units may suffer from reliability and maintenance problems.

4.1 Cost

The two lasers considered were an Argon-ion gas laser and a frequency-doubled, Neodymium-YAG solid-state laser. It was essential to choose a laser with its output close to the eye's maximum sensitivity, which occurs at 555 nanometers. No other suitable lasers produce green light. The cost estimates for the two laser types and associated equipment are given below:

<table>
<thead>
<tr>
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<th>Argon-ion Laser</th>
<th>Nd-YAG Laser</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser</td>
<td>11.4K</td>
<td>22.0K</td>
</tr>
<tr>
<td>Beam expander/adjusting bracket</td>
<td>18.0</td>
<td>18.0</td>
</tr>
<tr>
<td>Sealed enclosure</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Optical grade window</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Littrow prism</td>
<td>0.4</td>
<td>---</td>
</tr>
<tr>
<td>Power supply</td>
<td>+ 5.0</td>
<td>included with laser</td>
</tr>
<tr>
<td></td>
<td><strong>$35.4K</strong></td>
<td><strong>$40.6K</strong></td>
</tr>
</tbody>
</table>
The estimates given are for basic systems intended only for experimental use. Both lasers are water-cooled and require 70°F water at 2 to 4 gallons per minute. Field sites would likely require a closed cooling system with a heat exchanger. The Argon-ion laser would require factory installation of a new plasma tube every 4000-5000 hours. The Nd-YAG laser would require a flash lamp change every 500-1000 hours and a filter change every six months. Without considering spares or routine maintenance, the units would each require a cooling system at approximately $5K. Complete spare systems would be necessary—maintenance could seldom be performed on site.

In the consideration of costs peculiar to the “line of light” system, attention should be given to the stability requirements of the tower or other means of mounting the apparatus. In the conventional beacon system, the beam widths (a degree or so) merely require that the projectors be oriented in angle within approximately 5 mrad to provide adequate visibility of the projector over the channel width. However, if a “line of light” is used for indicating the channel center, it would be necessary that its angular orientation be true within an angle of the order of w/20R, where w is the channel width and R is the range to the projector. (This assumes an angular resolution of 1/20th of the channel width.) For example, for a channel width of 500 feet and ranges of 5 and 10 n mi, the azimuthal orientation accuracy requirements would be 0.8 and 0.4 mrad, respectively. It would be difficult, if not impossible, to meet this orientation requirement with a (relatively flexible) high tower.

4.2 Reliability

Both lasers are laboratory instruments and are not designed for the harsh marine environment. A sealed enclosure would be needed to protect the system, both to keep the output optics clean and dry, and to keep the system within its operating temperature range. One method of protecting the system from moisture intrusion is to pressurize the enclosure lightly with dry nitrogen. A less effective method is to place bags of dessicant inside the enclosure. In extreme climates, an auxiliary means of controlling system temperature will become necessary.

A lifetime of two thousand hours is guaranteed for most lasers such as the two being considered, with 4000-5000 hours lifetime probable. These lifetimes assume laboratory conditions. Routine maintenance is limited to that mentioned in Section 5.3. Cooling system failure and moisture intrusion would lead to system shutdown and deterioration of beam quality, respectively. A significant amount of condensation on the output beam optics would render the beam useless. Alignment of the laser and beam expander and alignment of the system with respect to its support structure are critical. Under field conditions this alignment may be difficult to maintain.

5
Special requirements of the lasers follow:

<table>
<thead>
<tr>
<th></th>
<th>Nd-YAG</th>
<th>Argon-ion</th>
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</thead>
<tbody>
<tr>
<td>Cooling Water</td>
<td>2 gpm/70°F/40 psi</td>
<td>4 gpm/70°F/30 psi</td>
</tr>
<tr>
<td>Input Power</td>
<td>220 VAC/10/50A</td>
<td>20 VAC/30/50A</td>
</tr>
</tbody>
</table>

The Argon-ion and Nd-YAG laser/power supply systems weigh 136 pounds and 300 pounds, respectively.

4.3 Maintenance

Due to the sealed enclosure and the sensitivity of laser components, little maintenance could be performed on site. Major repairs, such as plasma tube replacement (Argon-ion), can only be done at the factory. Flash lamp and filter changes (Nd-YAG) are minor operations and could be performed by trained Coast Guard personnel. Spare parts packages are normally available for the power supplies, circulating pumps, and heat exchangers. However, most maintenance operations would necessitate removing the entire system and transporting it to a shore repair station.

5.0 CONCLUSIONS AND RECOMMENDATIONS

Since the "line of light" scheme depends upon scattered light for beam detection at off-axis angles, very high power is required at the output of the system. This drives up the cost of the laser and conflicts with safety requirements. If the beam power density is restricted to levels which are safe for on-axis observers, off-axis detection of the beam is possible only under conditions of extremely low background illumination levels. Most coastal harbors do not have the required low levels of background illumination. The previous project, conducted ten years ago on the St. Mary's River in Michigan, was done under the necessary low level background lighting conditions. That project demonstrated the concept, but did not consider varying background illumination levels.

Furthermore, if a stable, high tower is required for mounting the projector, this would be unusually expensive, and the system itself would be expensive and difficult to implement and maintain. The service power and cooling water requirements make it impractical for many sites. Once installed, reliability would be questionable and maintenance inconvenient. The system has too many disadvantages to justify the cost. Because of these reasons and the safety considerations, it is recommended that the "line of light" concept not be considered for Coast Guard implementation at this time; the range of applications is too narrow. If technological progress significantly improves the reliability of lasers and brings down production costs, the "line of light" concept might prove cost-effective in future limited applications. The "line of light" concept will be reassessed in the outyears of the recently initiated Signal Effectiveness (2704) Project, and continued then, if warranted.
REFERENCES


APPENDIX A

GEOMETRY AND PHOTOMETRY OF THE "LINE OF LIGHT" BEACON

Physics Branch Technical Memorandum by F. S. REPLOGLE, Jr.
I. INTRODUCTION

The "line of light" concept as described to the author utilizes the visual location of a narrow laser beam to indicate the lateral position of a ship in a channel. The beam is projected from one end of the channel over its center line at a height above the highest ship's bridge. If the ship is centered in the channel, the beam is directly overhead; whereas if the ship is off the center of the channel, the elevation angle to the beam may be used to indicate the direction and distance by which it is off center.

This memorandum proceeds to set down the observation geometry and visibility (photometry) of single and triple beams which could be used for this mode of ship guidance.

II. GEOMETRY OF SINGLE AND TRIPLE LINE OF LIGHT SYSTEMS

If the apparent height of the beam is to be used as an indication of the distance by which the ship is off the channel center, the beam height must be constant, and thus its direction must be horizontal. On long ranges the constant height requirement is, of course, limited by the curvature of the earth. The height change $\Delta H$ resulting from this curvature is given by

$$\Delta H = \frac{d^2}{2R_e},$$

where $d$ is lateral distance from the projector and $R_e$ is the effective earth radius, which, because of the refractive gradient in the air, is $\frac{4}{3}$ times the geometric radius. With appropriate number substitution

$$\Delta H (\text{ft}) \approx 0.65 \ d^2 \ (\text{n mi}).$$

For example, if the beam is directed horizontally over the mid-point of a five or a ten n mi range, the beam height changes between the centers and the ends of the ranges would amount to 4 and 16 feet, respectively.

If we assume for simplicity that the earth is flat, we may describe the geometry of an observer looking at the scattering from a single horizontal beam as is sketched in Figure 1.a. Here scattering from a narrow (laser) beam projected in the $-z$ direction is sensed by an observer at point $O(x_o, h_o, 0)$. The distances $\Delta h$ and $\Delta x$ are the differences in the heights and lateral positions of the beam and the observer. Then the magnitude of the angle of scattering* from the beam at point $P$ is given by

$$\theta = \tan^{-1} \left[ \sqrt{ \frac{(\Delta x)^2 + (\Delta h)^2}{R^2}} \right]$$

Figure 1.b is equivalent to Figure 1.a., but with a shift of coordinate origins to illustrate more clearly the angles $\theta_{az}$ and $\theta_{el}$ subtended by $\Delta x$ and $\Delta h$. Here it is seen that

$$\tan \theta_{az} = \frac{\Delta x}{\sqrt{R^2 + (\Delta h)^2}}$$

and

$$\tan \theta_{el} = \frac{\Delta h}{R}.$$
At all ranges the angle $\psi$ subtended by $\Delta x$ in a plane $R = \text{constant}$ is given implicitly by

$$\tan \psi = \frac{\Delta x}{\Delta h}.$$  \hspace{1cm} 4.)

By substituting the values of $\Delta x$ and $\Delta h$ from Equations 3.)

$$\tan \psi = \sqrt{1 + \left(\frac{\Delta h}{R}\right)^2} \cdot \tan \theta_{az} / \tan \theta_{el}$$ \hspace{1cm} 5.)

If the points of observation of the line are at great ranges from the observer, the locus of these points forms a line as shown in the diagram of Figure 2. Here we have shown three points and the small azimuth and elevation angles corresponding to them. The angle $\psi$ by which the locus line is tipped from the vertical is

$$\tan^{-1}\left(\frac{\theta_{az}}{\theta_{el}}\right).$$ \hspace{1cm} 6a.)

If the ranges at which the line is observed are greater than $10 \Delta x$ and $10 \Delta h$,

$$\psi \propto \tan^{-1}\left(\frac{\Delta x}{\Delta h}\right)$$ \hspace{1cm} 6.b)

within an error of less than one percent.

Thus near the horizon, the horizontal component of the inclination angle of the line is proportional to the distance of the observer (pilot) from the center of the channel. For example, if $h = 50$ feet and the channel width is 500 feet, the tipping angle of the line at the horizon is $79^\circ$ when the ship is at the edge of the channel. If the point of observation lies at intermediate ranges ($\theta_{az}$ greater than a few degrees) no simple proportionality between beam elevation and ship location is evident. If the azimuth angle is $90^\circ$, the elevation angle is, of course, $\tan^{-1}\left(\Delta h/\Delta x\right)$.

To illustrate the appearance of a narrow beam proceeding from the horizon we include the photographic recordings of Figure 3 taken from Curcio and Drummeter. The searchlight beam is originally 1/2 degree wide (on the horizon) and is spread at great ranges by aerosol scattering. From the geometry of the eighth frame we conclude that the beam remained relatively narrow for a distance of five n mi.

In lieu of using a single horizontal beam projector, one may use a triple projector array for deducing distance off the channel center. Figure 4 shows the geometry which would locate the projectors in such a way that pairs of beams will be superposed when the observer is located in the plane passing through the higher beam and a channel edge. To achieve this relationship, projectors 1, 2, and 3, placed on a central tower, must be located in a triangular array with sides proportional to the large scale triangle shown in the figure. To satisfy this proportionally, it is required that

$$\frac{2 \delta x}{\delta h} = \frac{w}{h^2 + \delta h}$$ \hspace{1cm} 7.)

A-2
The tipping angles \( \psi \) for an observer at \((x_0, h_0)\) are

\[
\psi_1 \approx -\tan^{-1}\left[ \frac{x_0 - \delta x}{h_1 - h_0} \right] \tag{8.a}
\]

\[
\psi_2 \approx -\tan^{-1}\left[ \frac{x_2}{h_2 + h_0} \right] \tag{8.b}
\]

\[
\psi_3 \approx -\tan^{-1}\left[ \frac{x_3 + \delta x}{h_3 - h_0} \right] \tag{8.c}
\]

When the observer is located on the large triangle of Figure 4 at \(\left( \frac{w}{2}, \frac{1}{2} h_0 \right)\), two lines are coincident. The tipping angles for the three beams are

\[
\psi_1 = \psi_2 = \tan^{-1}\left( \frac{w}{2(h_0 + \delta h)} \right) \tag{9.a}
\]

and

\[
\psi_3 = \tan^{-1}\left( \frac{w}{2(h_0 + \delta h)} \cdot \left( 1 + \frac{2\delta h}{h_1 - h_0} \right) \right) \tag{9.b}
\]

Then the magnitude of the difference angle \( \Delta \psi = \psi_3 - \psi_2 \) is approximately

\[
\Delta \psi \approx \frac{w \delta h}{h_1(h_1 - h_0)} \tag{10.}
\]

III. EYE RESOLUTION AND PHOTOMETRIC CONSIDERATIONS FOR SINGLE AND TRIPLE LASER BEAMS

III.1 General Visibility Equation

The visibility of a horizontal laser beam depends upon the beam power \( P \), the beam's apparent size, the angle by which light is scattered from the beam (\( \theta \) of Figures 1 and 6), the background luminance \( L \), and the resolving power of the eye at the luminance level. Expressed mathematically -- for a beam to be visible, we require that

\[
P \cdot C_{w1} \cdot \frac{W_{\text{res}}}{W_b} \cdot B \cdot \Delta R \cdot \gamma(\theta)/A \geq C(\text{size, shape, } L, \text{ variability in } L) \cdot L \tag{11.}
\]

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where \( W_{\text{res}}/W_b \) is the fraction of the beam width resolvable by eye.

- If the beam width is unresolved, \( W_{\text{res}}/W_b = 1.0 \).

\[ P \] is the total power in the beam (watts),

\[ C_{\text{wl}} \] is the luminance ratio (lumens/watt) = 680 for green light,

\[ \beta_s \] is the scattering coefficient (ft\(^{-1}\))

\[ \Delta R \] is the length of the beam at a range \( R \) which can be resolved by the eye (ft)

\[ A \] is the area of sky at a range \( R \) which can be resolved by the eye (ft\(^2\))

\( \mathcal{H}(\theta) \) is the fractional scattering per unit path length at an angle \( \theta \) normalized to 1.0 for total (4\(\pi\) steradian) scattering (ster \(\text{rad}^{-1}\))

\[ C \] is the contrast threshold \((\Delta L/L)\) required to permit the observer to detect the beam and measure its location.

We may partially evaluate the variables of Equation 11. by noting that when the range from the projector is at least equal to several channel widths, the total optical path length of the light reaching the observer is approximately equal to the range \( R_p \) of the projector (see Figure 6). Under these circumstances the power \( P \) reaching the eye is attenuated (assuming single scattering) as given by Beer's Law.

\[ P = P_0 \exp(-\beta R_p), \]

where \( P_0 \) is the power transmitted and \( \beta \) is the attenuation coefficient. For visible light, the scattering coefficient is very nearly equal to the attenuation coefficient. Also in accordance with Coast Guard practice, we may set the attenuation coefficient equal to 3 divided by the visibility range \( R_{\text{vis}} \). To evaluate the fraction \( W_{\text{res}}/W_b \), if \( W_{\text{res}} < W_b \) we let

\[ W_{\text{res}} = r_d \cdot \Delta \theta_{\text{res}}/\sin \theta, \]

where \( r_d \) is the diagonal distance shown in Figure 6, and \( \Delta \theta_{\text{res}} \) is the angular resolution capability of the eye at the line and background luminance levels.

Otherwise we let

\[ W_{\text{res}}/W_b = 1.0. \]

To evaluate the fraction \( \Delta R/A \), we let

\[ \Delta R = (R^2 + r_d^2) \cdot \Delta \theta_{\text{res}}/r_d \]
and

\[ A = (R^2 \times r_d^2) \cdot (\Delta \theta_{res})^2, \]

giving

\[ \Delta R/A = 1/(r_d \cdot \Delta \theta_{res}). \]

For data on the normalized scattering function \( \mathcal{J}(\theta) \) for a coastal-marine atmosphere we utilize the curves found in Figure 4 and 5 of Deirmendjian\(^2\), showing scattering as a function of angle for wavelengths of 700 and 450 nm. Since the variation of the scattering function is small over the visible spectrum, for scattering at a wavelength of 550 nm we utilize a weighted mean of the values shown. The following brief table lists significant cases.

<table>
<thead>
<tr>
<th>ANGLE (DEG)</th>
<th>FRACTIONAL SCATTERING (ster-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 10</td>
<td>8.0 \times 10^{-4.6} \theta (rad)</td>
</tr>
<tr>
<td>30</td>
<td>0.225</td>
</tr>
<tr>
<td>45</td>
<td>0.078</td>
</tr>
<tr>
<td>60</td>
<td>0.033</td>
</tr>
<tr>
<td>90</td>
<td>0.0086</td>
</tr>
</tbody>
</table>

To evaluate Equation 11.) numerically as far as possible, we let the power \( P_0 \) be transmitted in a 10-inch diameter collimated beam at the maximum allowable density. Then

\[ P_0 = 0.0025 \text{ w/cm}^2 \times \pi/4 \times 625 \text{ cm}^2 \times 680 \text{ l/w} = 835 \text{ lum}. \]

We choose for the background luminosity 1.0 cand/m\(^2\) = 0.093 cand/ft\(^2\), a value corresponding to low twilight. At this illumination level the resolution of the eye is of the order of two arc minutes or 5.8 \times 10^{-4} \text{ rad}.

Substituting these values, values of the fractions \( W_{resW/P} \) and \( \Delta R/A \), and Equation 12.) in Equation 11.) we obtain the general projector range relationships which must be satisfied for the laser line of light to be visible during low twilight*.

For the larger (tens of degrees) scattering angles \( \theta \) where the (10-inch diameter) beam is resolved

\[ \frac{\exp (-3 R_d/R_{vis}) \cdot \mathcal{J}(\theta)}{R_{vis} (n mi) \cdot \sin \theta} \geq 0.186C \]  

For the smaller (degrees) scattering angles where the beam is unresolved,

* Obviously, if this criterion is met, the luminous line will be highly visible in darker surroundings.
\[ \frac{\exp (-3 R_p/R_{vis}) \cdot \theta' (\theta)}{R_{vis} (n \text{ mi}) \cdot r_d (\text{ft})} \geq 0.00013 C \quad (13.\text{b}) \]

III.2 RANGES ACHIEVABLE WITH WIDE ANGLE (NEAR) VIEWING OF THE BEAM

Although contrast threshold data have not been taken for resolved lines at the low background level of 0.3 ft lambert, data taken by Lamar, et alia\textsuperscript{3} at luminance levels of 17.5 ft lamberts on rectangular luminous areas and data taken by Blackwell\textsuperscript{4} on circular luminous areas indicate that a contrast threshold of approximately 0.04 will prevail in the wide angle viewing case. Substituting this value, values of \( \theta \), and visibility ranges of 5 and 15 n mi in Equation 13.a) gives the following set of projector ranges.

<table>
<thead>
<tr>
<th>Scattering Angle (deg)</th>
<th>Projector Ranges (n mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_{vis}=5 )</td>
</tr>
<tr>
<td>30</td>
<td>0 to 4.2</td>
</tr>
<tr>
<td>45</td>
<td>0 to 1.8</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>

III.3 RANGES ACHIEVABLE WITH NARROW ANGLE (HORIZON) VIEWING OF THE BEAM

The resolving power of the eye at nighttime places limitations on the accuracy with which the tipping angles of the lines at the horizon may be sensed. To assess this, we assume that the lines describing a minimum tipping angle depend upon three points, as shown in Figure 5.a. To deduce this minimum tipping angle, we first note that the minimum angular resolution between two lines (Figure 5.b.) has a well known value as a function of the background luminance. For the maximum background luminance level determining the minimum angular separation between two lines which may be sensed, we use the (low) twilight value of 1.0 cd-m\(^{-2}\). At this illumination level the resolution of the eye is of the order of two arc minutes or 5.8 \( \times \) 10\(^{-4}\) rad. Then from Figure 5

\[ \Delta \psi_{\text{min}} = \Delta \theta_{\text{az}} / \Delta \theta_{\text{el}} = \Delta \theta_{\text{res}} / \Delta \theta_{\text{el}} \]

If, for example, we require that \( \Delta \psi_{\text{min}} \leq 40^\circ = 0.070 \text{ rad} \),

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\[ \Delta \theta_{e1} \geq 5.8 \times 10^{-4}/0.07 = 0.0083 \text{ rad} = 0.48 \text{ deg.} \quad 14. \]

Thus we will impose the photometric requirement that the length of line visible at the horizon be at least 0.5 deg (the angular subtense of the moon).

Figure 6 illustrates the practical geometry of a luminous line segment observed near the horizon proceeding from a projector at a range \( R_p \) and subtending a small angle (properly 0.5 degrees = 0.00087 rad) at the observer's eye. In this geometry the maximum scattering angle (from the nearest point of the 0.5 degree segment) for an observer located at \( (\Delta x, h_0, \theta) \) is

\[ \theta_{\text{max}} \approx 0.0087 + \frac{r_d}{R_p} \quad 15. \]

where the diagonal distance, \( r_d = \sqrt{\Delta x^2 + \Delta h^2} \)

Assuming that at small scattering angles \( \theta \), the thickness of the luminous line cannot be resolved, we employ Equation 13.a) to solve for the ranges at which a sufficiently accurate determination of the tipping angle can be made. Since the eye resolution which we previously cited was made with a high contrast line array (grating), we will let \( C = 1.0 \). Then, using the small angle form of \( \bar{\theta}(\theta_{\text{max}}) \), letting \( r_d = \sqrt{(250^2 + 50^2)} = 255 \text{ ft,} \) and substituting 1.0 for \( C \) in Equation 13.b) gives the requirement

\[ \exp[-3R_p/R_{\text{vis}} -0.45/R_p (\text{n mi})]/R_{\text{vis}} (\text{n mi}) \geq 0.0045. \quad 16. \]

Values of \( R_p \) satisfying this criterion are given in the following table for visibility ranges of 2, 5, 10, and 20 n mi.

<table>
<thead>
<tr>
<th>Visibility Range (n mi)</th>
<th>Usable Projector Range (n mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0 to 3.1</td>
</tr>
<tr>
<td>5</td>
<td>0 to 6.2</td>
</tr>
<tr>
<td>10</td>
<td>0 to 10.2</td>
</tr>
<tr>
<td>20</td>
<td>0 to 15.8</td>
</tr>
</tbody>
</table>

IV. CONCLUSIONS

From the results given in III.2 we conclude that a suitable laser beam cannot be seen reliably at twilight at scattering angles of tens of degrees. It can likely be seen in full darkness, depending upon the local value of sky luminance. Since measured values of sky luminance in the vicinity of ship channels were not available to us, we have not treated the dark night case.
From the results given in III.3 we conclude that the laser line or lines of light can be seen on the horizon at twilight and in greater darkness. Also a sufficient length of line can be seen that the observer is able to judge the rotation of the line with sufficient accuracy to indicate his deviation from the center line of the channel (beam direction). This will be possible at ranges up to roughly one visibility distance. At the maximum ranges some spreading of the beam will occur due to multiple scattering of the light, but this is not expected to reduce the usability of the technique seriously. Obviously, since the beam is observed near the horizon, the projector must not be located near strong lights.

APPENDIX REFERENCES AND BIBLIOGRAPHY


Figure 1.a

GEOMETRY OF SINGLE LASER BEAM AND OBSERVER
Figure 2

APPEARANCE OF A LINE AT GREAT RANGES
APPEARANCE OF A SEARCHLIGHT BEAM
DIRECTED OVER CHESAPEAKE BAY TOWARD THE OBSERVER

The range of the searchlight is 24.6 nmi. Elev is elevation of the searchlight beam. Time is camera exposure time. The vertical field of the pictures is about 40 degrees. The camera time exposure records portions of images too faint to be seen by eye.
Figure 4.
RELATIONSHIP OF THREE PROJECTOR ARRAY TO CHANNEL EDGES

Figure 5.
SCHEMATIC ILLUSTRATION OF MINIMUM RESOLVABLE LINE TIPPING ANGLE
Figure 6

SINGLE SCATTERING GEOMETRY WITH THE PROJECTOR AT A FINITE RANGE