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Title: Research in Stochastic Processes

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Research was conducted and directed in the area of stochastic processes by three of the principal investigators (Cambanis, Kallianpur, and Leadbetter) and their associates, and in estimation in statistical models by R.J. Carroll and co-workers. A summary of the main lines of activity in each area follows for each of the four principal investigators. More detailed descriptions of the work of all participants is given in the main body of the report.

STOCHASTIC PROCESSES - The research effort in stochastic processes was a major part of a substantial research activity organized as the Center for Stochastic Processes within the Statistics Department, involving permanent faculty, visitors and students.
ITEM #19, CONTINUED:

This organization has provided the framework for significant interaction between the participants — permanent and visiting. In addition, the research program has been enhanced by a regular seminar series (listed by speakers later in the report) which has provided an excellent vehicle for exchange of current research ideas.

The primary means for dissemination of results is by means of the Center's Technical Report series, containing current research work prior to formal journal submission. To date, 23 technical reports leading (or expected to lead) to published papers have been produced by the participants, involving research results in a wide area of stochastic process theory. The main ideas of research activity for each principal investigator and co-workers are as follows: (1) S. Cambanis - Asymptotic optimal quantizers, complex symmetric stable variables and processes, prediction and representation of stable processes, nonparametric special density estimation for stable processes, delayed delta and pulse code modulation. (2) G. Kallianpur - Feynman integrals, stochastic nonlinear filtering, stationary random fields, stochastic differential equations and diffusion approximation models for neuron activity, white noise and generalized Brownian functionals, stochastic Radon transform, splicing of measures. (3) M.R. Leadbetter - Extreme values of stationary stochastic sequences and processes, dependence structure of stochastic processes, extremes of non-stationary normal sequences, estimation of point process intensities.

ROBUST ESTIMATION IN LINEAR MODELS - R.J. Carroll - Transformations and regressions: tests for regression parameters in power transformation models, power transformations and prediction; heteroscedastic linear models: robust estimators for random coefficient regression models, adapting for heteroscedasticity, maximum likelihood and generalized least squares, bounded influence methods; linear and binary regression with errors-in-variables: robustness in the functional errors-in-variables regression model, comparison between estimators, binary models.
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MATTHEW J. REINK
Chief, Technical Information Division
SUMMARY OF RESEARCH ACTIVITY

Research was conducted and directed in the area of stochastic processes by three of the Principal Investigators (Cabanis, Kallianpur, Leadbetter) and their associates, and in estimation in statistical models by R.J. Carroll and co-workers. A summary of the main lines of activity in each area follows for each of the four Principal Investigators. More detailed descriptions of the work of all participants is given in the main body of the report.

STOCHASTIC PROCESSES

The research effort in stochastic processes was a major part of a substantial research activity organized as the Center for Stochastic Processes within the Statistics Department, involving permanent faculty, visitors and students.

This organization has provided the framework for significant interaction between the participants—permanent and visiting. In addition the research program has been enhanced by a regular seminar series (listed by speakers later in the report) which has provided an excellent vehicle for exchange of current research ideas.

The primary means for dissemination of results is by means of the Center's Technical Report Series, containing current research work prior to formal journal submission. To date 23 technical reports leading (or expected to lead) to published papers have been produced by the participants, involving research results in a wide area of stochastic process theory. The main areas of research activity for each Principal Investigator and co-workers are as follows.
S. Cambanis: Asymptotic optimal quantizers, complex symmetric stable variables and processes, prediction and representation of stable processes, nonparametric spectral density estimation for stable processes, delayed delta and pulse code modulation.


ROBUST ESTIMATION IN LINEAR MODELS

RESEARCH IN STOCHASTIC PROCESSES
The work briefly described here was developed in connection with problems arising from and related to the statistical communication theory and the analysis of stochastic signal and systems. Item 1 is the completion, and recent extension, of joint work with my student Neil L. Gerr, whose Ph.D. dissertation in its final form is described in item 5. Item 3 is the completion of joint work with Dr. Reza Soltani, a junior visitor supported by this grant. Item 4 describes continuing joint work with Elias Masry of the University of California at San Diego. Further work in progress will be described at the end of the current funding period.

1. **A simple class of asymptotically optimal quantizers** [1]

   A simple class of quantizers is introduced which are asymptotically optimal, as the number of quantization levels increases to infinity, with respect to rth mean distortion measure. These asymptotically optimal quantizers are very easy to compute. Their performance is evaluated for several distributions and compares very favorably with the performance of the optimal quantizers in all cases for which the latter have been computed. Also their asymptotic robustness is studied under location, scale and shape mismatch for several families of distributions.

2. **Complex symmetric stable variables and processes** [2]

   In order to make efficient use of spectral methods in the analysis of problems involving stationary stable processes, it is necessary to extend to complex stable variables and processes the structure and tools which have been
developed for the real case in [3,4,5] and this is done in this article. The concepts, tools and properties considered include the covariation, linear regression, moments and the stochastic integral. The stochastic integral is considered in the most general case using the concept of covariation. This approach is simple and adequate for most linear problems, but it does not provide an expression for the characteristic function of the integral. In the important special case of integration with respect to a process with independent stable increments, the characteristic function of the integral in the complex case is obtained using Hosoya's [6] approach, which is refined and completed here.


For stable processes which are Fourier transforms of processes with independent increments we obtain a Wold decomposition, we characterize their regularity and singularity, and, in the discrete-parameter case, we derive their linear predictors. In sharp contrast with the Gaussian case, regular stable processes which are Fourier transforms of processes with independent increments are not moving averages of stable motion.


It has been shown in [5] and [7] that in problems of linear prediction and filtering, when the signal and noise are stable processes with spectral representation, the "spectral density" of these stationary stable processes plays a role analogous to the role the usual spectral density plays for second order stationary processes, hence the need to develop consistent estimates of the spectral density from long records of a sample function of such a stationary
stable process. Both weakly and strongly consistent estimates are obtained, along with rates of convergence.

5. **Exact analysis of delayed delta modulator and an adaptive differential pulse-code modulator** [9]

Delayed Delta Modulation (DDM) uses a second feedback loop in addition to the standard DM loop. While the standard loop compares the current predictive estimate of the input to the current sample, the new loop compares it to the upcoming sample so as to detect and anticipate slope overloading. Since this future sample must be available before the present output is determined and the estimate updated, delay is introduced at the encoding.

The performance of DDM with perfect integration and step-function reconstruction is analyzed for each of three inputs. In every case, stochastic stability of the system is established. For a discrete time i.i.d. input, the (limiting) joint distribution of input and output is derived, and the (asymptotic) mean square sample joint error MSE(SP) is computed when the input is Gaussian. For a Wiener input, the joint distribution of the sample point and predictive errors is derived, and MSE(SP) and the time-averaged MSE (MSE(TA)) are computed. For a stationary, first-order Gauss-Markov input, the joint distribution of input and output is derived, and the MSE(SP) and MSE(TA) are computed. Graphs of the MSE's illustrate the improvement attainable by using DDM instead of DM. With optimal setting of parameters, MSE(SP) (MSE(TA)) is reduced about 15% (35%).

An Adaptive Matched Differential Pulse-Code Modulator (AMDPCM) is analyzed. The adaptation of the symmetric uniform quantizer parameter \( \lambda_n \) is performed by fixed multipliers assigned to the quantizer output levels. The input is stationary first-order Gauss-Markov. The correlation of the samples is used as the leakage parameter in the matched integrator, with the predictive...
reconstruction similarly matched.

We examine the stochastic stability of this system when the range of $\zeta_n$ is unconstrained. For a 4-level quantizer and multipliers ($\gamma^{-1}, \gamma$) we derive the limiting joint distribution of the predictive error and $\zeta_n$, and compute and plot as functions of $\gamma \in (1,2)$, $\text{MSE(SP)}$, $\text{MSE(TA)}$, and the asymptotic mean and variance of $\Delta_n$. We find that the asymptotic performance of AMDPCM does not depend on the choice of $\Delta_0$, that the increase in MSE incurred by using $A(M)DPCM$ instead of $(M)DPCM$ with $\Delta_{opt}$ is small, with $\text{MSE}(A(M)DPCM) \cdot \min_{\Delta_{opt}}$, $\text{MSE}((M)DPCM)$ as $\gamma \downarrow 1$, and that the signal-to-noise ratio of AMDPCM does not depend on the input power.

References


Research was carried out in the following areas:

1. **Feynman Integrals**
   Continuation and extension of earlier joint work with my former student C. Bromley and now partly in collaboration with Professor D. Kannan, visitor at the Center from the University of Georgia. Problems arising out of this work will be investigated jointly with Professor G. Johnson of the University of Nebraska at Lincoln.

2. **Stochastic Nonlinear Filtering Theory**
   This has been my major area of interest for many years. A completely new approach to the subject is being developed which has already yielded surprising and gratifying new results. The work which is still continuing, is jointly with Dr. R.L. Karandikar, visitor at the Center from the Indian Statistical Institute, Calcutta.

3. **Stationary Random Fields**
   Collaborative work (part of which was reported last year) with Professor V. Mandrekar, visitor at the Center from Michigan State University.

4. **Stochastic Differential Equations and Diffusion Approximation Models for the Activity of Neurons**
   This research is the outcome of discussions with Dr. M. Habib, Department of Biostatistics, and Dr. T. McKenna, formerly of the Department of Physiology, University of North Carolina at Chapel Hill. The later work is jointly with Professor R. Wolpert, visitor at the Center from Duke University.
5. **White Noise and Generalized Brownian Functionals**

This is an area of stochastic analysis of which Professor T. Hida of Nagoya University, Japan, has been one of the founders. During a brief visit to the Center, we collaborated on some aspects of the subject related to finite dimensional approximation.

6. **Stochastic Radon Transforms**

Professor D. Kolzów (visitor at the Center from the University of Erlangen-Nürnberg) and I have discussed this problem with a view to future collaboration. The work of Professor S. Takenaka, reported separately, seems to be closely related.

7. **Splicing of Measures**

This is a problem in measure-theoretic probability theory which was solved in joint work with Dr. D. Ramachandran, visitor at the Center and now at the University of Georgia.

Items 4 and 5 are new directions of research started within the last year. Items 1, 2 and 3 are a continuation of previous work.

Three of my Ph.D. students, Hans Hucke, Mauro Marques and Víctor Perez-Abreu will be working on problems arising from Items 2 and 4.

A brief summary of work done under each item is given below.

[1] Feynman integrals [1,2,3,4]

Following the analytic continuation approach involving several complex variables which was developed in [1,2] a sequential definition was given in the abstract Wiener space context. A Cameron–Martin formula was derived for a class of functionals which includes and is larger than the class of Fourier transforms of bounded complex measures on Hilbert space. The formula holds both for the
analytic Feynman and the sequential Feynman integral. This result is related to some recent work of Elworthy and Truman [3].

We are currently investigating the relationship between the sequential definition and the definitions given by Cameron and Storvick and by Truman. Further details are given in Kannan's report. A paper now in preparation gives a comprehensive survey and comparison of the various methods now available for the definition and evaluation of Feynman integrals [4]. Applications to problems of Quantum Mechanics, relationships to other definitions (such as via the Trotter-Kato product formula) will be taken up in our later work.

[2] Nonlinear filtering: A finitely additive white noise approach [5,6,7,8]

The stochastic integrals of Ito and Stratonovich, stochastic differential equations and, more generally, stochastic calculus have been used with spectacular success in the development of Filtering and Control theory [5]. It is only in recent years that a re-examination of the theory has begun from the point of view of applications. In a series of papers which are the forerunners of our own work Balakrishnan has questioned the adequacy of the existing theory (see [6]) and advocated the use of finitely additive white noise theory. His reasons are based on practical considerations, viz., that the Wiener process as a model for observation noise leads to results which cannot be implemented. In [7] and [8] we have constructed a complete theory of nonlinear filtering for the important case when signal and observation noise are independent. The starting point is that of Balakrishnan and I.E. Segal. We supply a suitable definition of conditional expectation in the finitely additive set up.

White noise versions of the Kallianpur-Striebel formula, the Zakai equation, the Kunita equation and the Fujisaki-Kallianpur-Kunita (FKK) equation are obtained. When the observation process is finite dimensional, a partial
differential equation for the unnormalized density (in the finitely additive context) is derived and the existence and uniqueness of its solution is established.

The white noise approach has the following advantages:

(a) The theoretical framework within which filtering is performed is a Hilbert space $H_T$ of (relatively smooth) observation paths of Wiener measure zero but which represent the actual observations.

(b) It leads to a robust procedure when the observations are restricted to $H_T$.

(c) The robust solutions obtained by Davis and others using the conventional Ito calculus can be approximated by the solutions in (a). (See [7]).

In the case the observation process takes values in an infinite-dimensional Hilbert space $K$, there is no conditional density. The measure-valued equations of Zakai, Kunita and FKK types are studied directly and existence and uniqueness of their solution for each path $y$ in $L^2([0,T];K)$ is established [8]. (It should be noted that these equations are not stochastic equations of Ito type but "ordinary" equations in which the observed $y$ appears as a parameter). Robustness is also shown within $L^2([0,T];K)$.

[3] Second order stationary random fields [9,10]

The first part of the work which was reported last year, was concerned with the "time domain" analysis of discrete two parameter second order random fields (s.o.r.f) [9]. A definition of pure nondeterminism was given which led to a decomposition of the Hilbert space of the s.o.r.f. and to a corresponding four-fold Wold decomposition of the s.o.r.f. itself. Also, it was shown that there were three distinct kinds of innovation spaces for this problem.

The second part of the work, carried out in the current year, generalizes the four-fold Wold and Halmos decompositions to continuous 2-parameter
These results are consequences of a generalization obtained by us of J.L.B. Cooper's work to the case of two commuting continuous semigroups of isometries acting on a separable Hilbert space and satisfying certain conditions.

A new type of Karhunen representation is derived and the Cramér-Hida theory of multiplicity is extended: It is shown that associated with a continuous, stationary, purely nondeterministic s.o.r.f. is a uniquely determined triplet of (possibly infinite) numbers called the multiplicities of the s.o.r.f. Of these, two are directional multiplicities and the third, 2-dimensional. Furthermore, they are identified as the dimensions of certain subspaces of the Hilbert space of the s.o.r.f.


There is an extensive literature on stochastic models in neurophysiological problems. The work most closely related to our interest in neuronal behavior are the papers of Tuckwell and his co-workers and those of Ricciardi and his colleagues (See [12] for references). Our concern has been to construct a precise theory which reconciles the use of the discontinuous and the continuous models considered in the literature.

(i) A diffusion approximation to a discontinuous stochastic model for neural response (the Tuckwell-Cope model) was established using the functional central limit theorem of Liptser and Shiryaev [12]. Under certain basic assumptions necessary and sufficient conditions were obtained for the weak convergence of the sequence of probability measures (on Skorokhod space) corresponding to the Tuckwell-Cope model to the measure of an Ornstein-Uhlenbeck type (OU) process. A central problem of interest in the study of neuronal activity is the distribution of interspike intervals, i.e. intervals between
consecutive (random) "firings" of the neuron. The problem is equivalent to finding the distribution of the first passage times for Markov processes if the activity of the membrane potential is modeled as a Markov process. The problem is a very difficult one and solutions in closed form are hard to get even for the most frequently studied case of the OU process. However, at least the following qualitative result has been proved: If $\tau^n_C$ is the first passage time corresponding to the $n$th Tuckwell-Cope model (for a constant threshold $c$), then $\tau^n_C - \tau_C$ in distribution, $\tau_C$ being the first passage time of the OU process diffusion approximation.

(ii) (with R. Wolpert, [13]). In (i) a single neuron is considered. A more realistic model is to study the activity of a large assemblage of neurons, in other words, to take the spatial extent of neurons also into account. Mathematically, the problem calls for more sophisticated techniques. The approach adopted here leads to infinite dimensional stochastic differential equations. See the report on Wolpert's work for details.


Details are given in Hida's report.

[6] **Stochastic Radon transforms**

Details are given in Kolzow's report.

[7] **Splicing of measures** [16,17]

Given two probabilities $\xi$ and $\eta$ on $(X,A)$ and $(X,B)$, a probability $\gamma$ on $(X,A \times B)$ is called a splicing of $\xi$ and $\eta$ if $\gamma(A \times B) = \xi(A) \eta(B)$ for all $A \in A$ and $B \in B$. We use a result of Marczewski to give an elementary proof of Stroock's result [17] on the existence of a splicing. A new proof of Marczewski's result is also given together with comments on the splicing problem for compact measures [16].
References


Professor Leadbetter's primary research effort under the contract involved extremal theory for stochastic sequences and processes. Two main directions have been investigated: (a) the role played by the "local dependence" structure of a stochastic sequence in affecting the distribution of its maxima, and (b) the effect of non-stationarity in extremal results. The results of (a) have been discussed as a technical report and those of (b) are in a technical report under current preparation.

In addition M.R. Leadbetter worked with Diane Wold on the estimation of intensity functions of point processes, with results also appearing as a technical report. The following abstracts summarize the results obtained in these three papers.

1. *Extremes and local dependence in stationary sequences* [1]

Extensions of classical extreme value theory to apply to stationary sequences generally make use of two types of dependence restriction:

(a) a weak "mixing condition" restricting long range dependence,

(b) a local condition restricting the "clustering of high level exceedances

The purpose of this paper is to investigate extremal properties when the local condition (b) is omitted. It is found that, under general conditions, the type of the limiting distribution for maxima is unaltered. The precise modifications and the degree of clustering of high level exceedances are found to be largely described by a parameter here called the "extremal index" of the sequence.
2. **Extremes of non-stationary normal sequences** [2]

It has been shown in recent years that classical extreme value theory extends to apply to stationary sequences under appropriate restrictions on the amount of dependence involved. In particular the theory applies to stationary normal sequences under a simple condition concerning the rate of decay of the correlation sequence. In this paper similar results are obtained for non-stationary normal sequences with a wide variety of possible forms for the mean and correlation structure. In particular the work includes stationary cases with added trends and seasonal components.

3. **On estimation of point process intensities** [3]

Smoothed estimations are developed for the intensity function (i.e. density for the expectation measure) of a point process. The main results concern mean square and almost sure pointwise consistency and asymptotic distributional properties of the estimator, emphasizing the features which differ from those in other forms of function estimation. The results are illustrated in the particular case of renewal processes.

**References**


Dr. Daley conducted research in two areas under support from the contract. First he considered limit laws for the maximum of weighted independent and identically distributed random variables as part of the general research effort in extremal theory being conducted under the contract. In addition he investigated the problem of obtaining useful lower bounds for the mean waiting time in a certain class of queueing problems. Abstracts of papers under preparation in each area are as follows.

1. **Limit laws for the maximum of weighted i.i.d. random variables** [1]

Define $M_n = \max(X_0, X_1, \ldots, X_{n-1})$ for a sequence $(X_n)$ of i.i.d. r.v.s. with d.f. $F$. Gnedenko (1943) exhibited the class $G$ of all possible non-degenerate limit laws for $M_n$ and discussed domains of attraction of $F$ for various elements of $G$. Prompted by the study of limit laws as $a \to 1$ of the r.v. $Y(a) = \sup_n (a^n X_n)$, we also sought limit laws as $b \to 0$ of the r.v. $Z(b) = \sup_n (X_n - nb)$ and, quite generally, of

$$M = M(w,v) = \sup_n (w_n(a)X_n - v_n(b))$$

for sequences of weights $(w_n(a))$ and translates $(v_n(b))$.

First, we show that $M$ is finite with probability zero or one, and identify conditions under which $M$ is finite with probability one, namely, in the non-trivial case that $F(x) < 1$ for all finite positive $x$, that

$$\sum_{n=0}^{\infty} (1-F((x+v_n)/w_n)) < \infty$$

for some finite $x$. We then show that a limit law for $M$ belongs to $G$ if the r.v.'s

$$M_k(w,v) = \sup_n (w_n X_n - v_n)$$

have a limit law in $G$, which is little more than a restatement of Gnedenko's definition of $G$ via a functional equation for types of d.f.'s. As corollaries, limit laws for $Y(a)$ and $Z(b)$ belong to $G$, but we also show that not every $F$
yielding $M_n$ converging to some element of $G$ yields $Y$ or $Z$ converging to an element of $G$ (nor even the same element). Further examples are given of limit laws for $M$ that are outside $G$.

2. **The tight lower bound on the mean waiting time in a class of GI/G/l queues** [2]

A representation, derived recently by Teunis Ott, for the mean waiting time $EW$ in a stationary stable GI/G/l queueing system, is established by an alternative argument. This formula is used to show that a known lower bound for $EW$ in systems with given first two moments for the inter-arrival and service time distributions is the best possible lower bound, being attained (in the limit) by a certain family of D/G/l systems, namely, service times concentrated on the lattice ($nET: n=0,1,...$) where ET is the mean inter-arrival time.

References


Dr. de Haan investigated the extent to which the term "max stable" used to describe a random variable may be extended to apply to both random sequences and continuous time processes. Representations and other properties were obtained and reported in a technical report, whose abstract follows.

A spectral representation for max-stable processes [1]

The elements of an arbitrary max-stable sequence are exhibited as functionals of a 2-dimensional Poisson point process. The result is extended to a continuous time max-stable process that is continuous in probability. We define an analogue of a stochastic integral appropriate for this context.

References

Dr. Tiago de Oliveira conducted research in the area of extreme values for bivariate sequences of random variables. Specifically pairs \((X_n, Y_n)\) were considered, where \(X_n\) and \(Y_n\) are (usually) dependent random variables but the pairs are independent for different values of \(n\). This work thus provides one step in extending the contract work on univariate theory of extremes of processes to apply to multivariate cases. The research undertaken was described in a technical report as summarized in the following abstract.

**Bivariate extremes: Models and statistical decision** [1]

After obtaining the asymptotic distribution of bivariate maxima, a direct characterization of the asymptotic distribution is given; the 5 known models are described through their dependence functions and some properties obtained. Known statistical decision results for the models are described.

**References**

Hida worked on the following topics during his stay at the Center.

1. **Delta function of Brownian motion**

   Following the idea explained in [1] it is possible to make such a functional as
   \[
   \delta_t = \delta(B(t)-y), \quad \delta \text{ the delta function, } B(t) \text{ a Brownian motion,}
   \]
   to be a generalized Brownian functional. Detailed properties of it were obtained jointly with H.H. Kuo. An alternative method of defining the Brownian local time is given by using the functional \( \delta_t \). By replacing \( B(t) \) with a linear functional of the \( \dot{B}(t) \), it was proved that the functional plays a similar role to the ordinary delta function.

2. **Infinite dimensional Gaussian kernel**

   It was rigorously proved that a functional formally expressed in the form
   \[
   \zeta = \exp[-1/2\int_0^T \dot{B}(t)^2 dt], \quad T > 0,
   \]
   can be shown to be a generalized Brownian functional. The renormalized one may be viewed as an infinite dimensional analogue of the Gaussian kernel. In fact, if we use the theory of Fourier transform introduced to the space of generalized Brownian functionals by Kuo, we can give plausible interpretations to the fact that the functional \( \zeta \) looks like a Gaussian kernel.

3. **Finite dimensional approximation to white noise**

   This work has been done and is still being done jointly by G. Kallianpur and Hida. Starting from a formal expression
   \[
   \dot{B}(t) = \sum_{j=1}^J \sum_{i=1}^I X_{ij} e_j(t),
   \]
where \( \{e_j\} \) is a complete orthonormal system of \( L^2([0,1]) \) and \( \{X_j\} \) is a system of independent identically distribute Gaussian random variables. The nth approximation is given by

\[
B^n_n(t) = \sum_{j=1}^{n} X_j e_j(n).
\]

With this approximation to \( \dot{B}(t) \) we are able to give an approximation to the differential operator \( \partial/\partial \dot{B}(t) \), the exact meaning of which was not quite well visualized. By having such an approximation we can compare the operator \( \partial/\partial \dot{B}(t) \) to finite dimensional differential operators.

A unified method of having renormalization of formal Brownian functionals has so far been given by using the T-transformation

\[
(T\phi)(x) = \int_{S^*} \exp[i <x, \xi>] \phi(x) d\nu(x), \quad \phi \in L^2(S^*, \nu).
\]

However this approximation will give another, more reasonable interpretation to renormalization.

4. Conformal group

The infinite dimensional rotation group (see [1]) leads us to discuss, as it were, infinite dimensional harmonic analysis, which will be an important part of our analysis of generalized Brownian functionals. For the one-dimensional time-parameter case one can find an interesting subgroup of the rotation group which is isomorphic to the projective special linear group. As a generalization to higher-dimensional case, Hida was able to introduce a subgroup, which is isomorphic to the conformal group arising from quantum field theory. The group involves six one-parameter subgroups that are given below:

[n-dimensional time parameter case]

1) time shifts: \( S^i_t \), \( i = 1, 2, \ldots, n \),

2) isotropic dilation of time,
3) n-dimensional rotations, i.e. the group SO(n),
4) special conformal transformations given by
   \[ R S_t R, R: \text{reflection with respect to the unit sphere,} \]
   \[ i = 1, 2, \ldots, n. \]

This approach is a development of the joint work with Mr. S.S. Lee and Mr. L.K. Lee.

References

1. **Feynman Integrals [1,2,3,4,5]**

   Further development of Kallianpur and Bromley's work has led to new research in the following directions.

   (a) Definition of sequential Feynman integrals (SFI) in an abstract Wiener space set up. Concentrating attention on two physically interesting classes of functionals, it is shown that the SFI includes and extends the Fresnel integral of Albeverio and Höegh-Krohn as well as the sequential path integrals of Cameron-Storvick and of Truman [6].

   (b) The Cameron–Martin formula for SFI obtained by Kallianpur is improved by showing that it holds when the increasing projection family in Kallianpur's proof is replaced by an arbitrary sequence of finite-dimensional projections converging strongly to the identity. As a consequence, the generality of the result is considerably enhanced.

   (c) Extension of SFI to cases that involve indefinite bilinear forms.

   (d) Applications of the above work to the study of the Schroedinger equation and of the stationary phase method is planned for future work.

References


1. **Ramsey Theory**

Kölzow gave a comprehensive series of lectures on the "Applications of Ramsey Theorems to Analysis and Probability." These lectures, which included some new research, presented a unified method of treating problems in analysis, probability theory, using Ramsey theory techniques. Some are listed below:

(i) Sucheston's and Lorentz's extension of a theorem of Menshov and Visser on mixing sequence of events. Kölzow gave a simpler proof of this result. In addition, a new existence theorem for subsequences which are stable of any order. (ii) The infinite Ramsey theorem and a theorem of Erdös-Rado. (iii) Schrader's extension of Helly's selection theorem.

2. **The Radon Transform**

Research on the Radon transform centered around the following problems:

(a) Extension of the Radon transform to measures on a separable Hilbert space with respect to a given Gaussian measure.

(b) Proof (using Radon transforms) of the Wold-Cramér theorem on the unique determination of a finite measure on $\mathbb{R}^n$ by its values on half spaces. Also, a derivation of an inversion formula.

(c) A "folklore" Wold-Cramér theorem for separable Banach spaces and the corresponding reconstruction problems were discussed.

3. **Stochastic Radon Transform**

In cooperation with G. Kallianpur work was initiated for developing the concept of a stochastic transform. In particular, the work of Čenocv and others on an integral geometric approach to Lévy's Brownian motion was studied.
References


1. Generalized Brownian Functionals and Donsker's Delta Function [1]

In addition to giving a seminar in the Center, Kuo worked in a series of private seminars with Hida and Kallianpur in which he expounded his approach to generalized Brownian functionals. He has obtained several new results including a rigorous treatment of Donsker's delta-functional. Details of collaboration with Hida have been given above in Hida's report. The possibility of using the white noise approach to the Malliavin Calculus was discussed in these seminars and we hope that it will lead to collaborative work on this subject.

References

1. Continuous two-parameter second order stationary random fields [1]
   Details are given in Kallianpur's report.

2. Markov property for Gaussian ultraprocesses [2]
   In this joint work with A.R. Soltani the authors introduce Gaussian
   processes taking values in ultradistributions. They obtain a general theorem
   giving necessary and sufficient conditions for germ field Markov property in
   terms of the structure of the reproducing kernel Hilbert space. The results of
   Kusuoka (in the Gaussian case), Kallianpur-Mandrekar, Molchan and others are
   obtained as a consequence.

   Work on this subject which was begun while Mandrekar was at the Center was
   completed during his stay in Strasbourg. The Lecture Notes of a course given at
   Strasbourg contains a general survey of the problems together with some new
   work.

References

1. G. Kallianpur and V. Mandrekar, Commuting semigroups of isometries and Kar-
   hunen representation of stationary random fields, Center for Stochas-
   Proc. of the IFIP-ISI Conference on the Theory and Application of
   Random Fields, G. Kallianpur, ed., (to be published in the Springer
   Lecture Notes Series).

2. V. Mandrekar and A.R. Soltani, Markov property for Gaussian ultraprocesses,

3. V. Mandrekar, Central limit problem and invariance principles on Banach
   spaces, Lecture Notes, Institut de Recherche Mathematique Avancee,
   Universite Louis Pasteur, Strasbourg (1982).
1. **Decomposability of p-cylindrical martingales** [1]

A class of p-cylindrical martingales in locally convex spaces is studied. We obtain a general form of convergent p-cylindrical martingales in barrelled spaces. Using the locally convex space technique, new results are deduced even in Banach spaces. It is proved that for $p \geq 1$ the adjoint to $p$-absolutely summing operator is $p$-decomposing for any p-cylindrical martingale.

This study is motivated by the remark of Metivier and Pellaumail ([2], Chapter 6) that it is possible to develop the theory of stochastic integration with respect to 2-cylindrical martingales in Banach spaces, cf. also [3]. The important examples are cylindrical Brownian motion and white noise in time and in space. Such processes have been discussed in connection with quantum field theory, partial differential equations involving random terms and filtering theory, cf. for example [4] and references therein.

2. **Prediction of processes stationary in norm** [5]

The classical $L^2$ Wiener-Kolmogorov prediction theory has been extended to the following two cases: (1) $L^0$; the concept of prediction for strictly stationary sequences of random variables has been introduced by K. Urbanik (1964) [6]. (2) $L^p$, $1 \leq p \leq 2$; S. Cambanis, G. Miller and R. Soltani (1981-82) [7,8] have developed the linear prediction theory for $p$-th order and stable processes. It is very desirable to have analogues of the classical theory for certain stochastic processes which are nearly, but not exactly, Gaussian or second order.

In the search for the greatest common denominator of these two cases, the notion of a process stationary in norm emerges. The starting points for
the present investigation are two questions arising immediately. First, does one get the most general "reasonable" linear prediction theory by amalgamating these two known techniques? The answer is yes and the Wold decomposition as well as some characterizations of completely non-deterministic processes are obtained. Secondly, linear predictors are limits, and the question arises in which sense do they converge. The present investigation concentrates on convergence in $L^p$-norm, $0 \leq p < \infty$, though many of the methods developed apply as well to convergence in the topology of certain Orlicz and Lorentz spaces.

References


5. A. Weron, Prediction of processes stationary in norm, manuscript under preparation.


CLYDE D. HARDIN, JR.

General (asymmetric) stable variables and processes [1]

Previous research in the field of stable processes has dealt almost exclusively with symmetric stable processes. This research deals with those stable variables and processes where the symmetry requirement has been dropped. Such "skewed" processes are clearly of wider applicability.

Specifically, we determine the form of all strictly stable independent increments processes and develop a Wiener-type stochastic integral with respect to these processes. We prove a generalization of the spectral representation theorem for symmetric stable processes to general stable processes: it says, loosely, that all stable processes are stochastic integrals with respect to a stable process with stationary, independent increments and "maximum skewness."

With the aid of the representation, we solve some regression problems. For example, we show that the regression of one stable variable upon another is not always linear, in sharp contrast with the symmetric case. We determine necessary and sufficient conditions for its linearity and determine the regression function when it is not linear.

Also, some decompositions of general stable distributions are given and some moment inequalities are proved.

References

Stochastic integrals for Gaussian processes:

the differential formula [1]

Stochastic integrals for Gaussian processes were developed in [2] as a natural extension of Ito's integral for a Wiener process. The corresponding stochastic calculus is developed further in this paper. By exploiting the tensor product structure of the nonlinear space of a Gaussian process, a stochastic differential formula is obtained analogous to the celebrated Ito formula. An application of the differential formula to inequalities for the multivariate normal distribution is also given.

References


JÜRGEN HÜSLER

Dr. Hüsler's work under the contract involved investigation of extremes of non-stationary stochastic sequences. In particular general dependence conditions were obtained under which certain distributional extremal results for stationary sequences still hold for non-stationary cases. This research was reported in a technical report described in the following abstract.

**Extreme values of non-stationary sequences and the extremal index [1]**

The conditions used to generalize the extreme value theory for stationary random sequences to non-stationary sequences are studied with respect to their necessity. We find that the extremal index, defined in the stationary case, plays a similar role in the non-stationary case. The details show that this index describes not only the behavior of exceedances above a high level constant boundary, but also above a non-constant high level boundary.

**References**

1. **Nonlinear Filtering Theory** [1,2]

   Details are given in Kallianpur's report.

2. **Other Related Work in Progress**

   (i) Nonlinear filtering problems for two-parameter processes. It appears that the white noise approach is a natural and, perhaps, a simpler technique than 2-parameter martingale theory. The latter has been used in work of E. Wong (and M. Zakai), by H. Korezlioglu and others in multiparameter filtering problems. (ii) Likelihood rations for 2-parameter processes, again using the white noise model. Details will be given in a later report.

**References**


Dr. McCormick has conducted research in the area of extremes of stationary Gaussian (normal) sequences. Work completed and reported as a technical report concerns the limiting distribution of the size of the jumps in the sequence of partial maxima. Current work involves probability-one results and in particular strong laws for extremes of Gaussian sequences. An abstract for the reported work and summary of work in progress are as follows.

1. **A conditioned limit law result for jumps in the sequence of partial maxima of a stationary Gaussian sequence** [1]

   Conditional on a jump occurring, the limiting distribution for the size of the jump in the partial maxima sequence for a class of stationary Gaussian sequences is derived. It is shown that the limiting distribution is exponential with mean \((1-a)^{1/2}\) where \(a\) equals the atom at zero of the spectral distribution function associated with the correlation function of the sequence. This result is generalized to include the entire jump sequence subsequent to a jump conditioned to occur.

2. **A strong law result for extreme values from Gaussian sequences** [2]

   In a recent paper V. Hebbar [3] showed the following result. If \((X_n, n \geq 1)\) is a stationary sequence of standard normal variables having correlation function \(r_n\) and if \(M_n = \max(X_1, X_2, \ldots, X_n)\), \(S_n = \text{second max} (X_1, X_2, \ldots, X_n)\), \(a_n = (\log \log n)^2 \log n^{-1/2}\) and \(b_n = (2 \log n)^{1/2} - (\log \log n + \log 4) (8 \log n)^{-1/2}\) then under the assumption that
\( r_n (\log n)^{2+a} = 0(1) \) as \( n \to \infty \)

for some \( a > 0 \) we have that the set of almost sure limit points of

\[
\left\{ \left( \frac{M_n - b_n}{a_n}, \frac{S_n - b_n}{a_n} \right), \quad n \geq 1 \right\}
\]

coincides with the set

\[ A = \{(x,y): \ 0 \leq y \leq x \text{ and } x + y \leq 1\} \]

My work improves Hebbar's result by relaxing the mixing conditions. It is shown that the above result remains true under the assumption \( r_n \log n = 0(1) \).

References


1. **Large Deviation Theory**

I investigated, jointly with Professor G. Kallianpur, possible extensions of large deviation theory (see [1] and references therein). For the case of Markov processes we attempted to derive the results of Donsker and Varadhan using multinomial type approximations. Some of the results obtained by us were found to be contained in an article of L.B. Boza and our work was continued in other directions. We worked on extending to more general cases the Bahadur approach for the i.i.d. case in the derivation of the point entropy function.

2. **Disintegration Problems**

We worked on unifying known theorems on disintegration of probability spaces and some related questions. We pursued the invariant measure problem connected with recent attempts on descriptive characterization of dynamical systems.

3. **Stochastic Filtering Theory**

In order to pursue some research problems in this area, we undertook a planned study of stochastic filtering theory as developed in the recent book of Professor Kallianpur [2] and related monographs. I gave lectures on the Ito stochastic integral and linear stochastic differential equations to graduate students who wanted to pursue this topic.
References


Weighted quantile sampling in estimating the integral
of a stochastic process [1]

The integral of a stochastic process over an interval is estimated by a weighted linear combination of observations of the process at n points. The estimate is obtained by first dividing the interval into m subintervals whose endpoints constitute a regular sequence of points corresponding to a given density. Then n' sample points are chosen from each subinterval as certain fixed quantiles of the uniform density over that subinterval. (Here n' m = n and n' does not depend on n). The weights are chosen to depend only on the quantile corresponding to each sample point within a subinterval.

Asymptotic expressions for the mean square error are obtained for midpoint sampling (n'=1) introduced by Tubilla [2] and for more general sampling schemes. It is seen that the rate of convergence of the mean square error to zero is $n^{-2 \min(\ell, n')^{-2}}$, where the process has at least $\ell$ continuous quadratic mean derivatives. The rate obtained in the literature for a sequence of asymptotically optimal estimators is $n^{-2\ell-2}$. In certain cases, e.g., when $\ell = 1$, $n'=1$, and the quantile chosen is the median, the type of estimator considered here is shown to be asymptotically optimal.

References


A.R. SOLTANI

Dr. Soltani completed his joint work with Professors Mandrekar and Cambanis described in items 1 and 2 and pursued his own work on the extrapolation of random fields described in item 3.

1. Markov property for Gaussian ultraprocesses [1]

We introduce Gaussian processes taking values of ultradistributions. We obtain a general theorem giving necessary and sufficient conditions for germ field Markov property in terms of the structure of the reproducing kernel Hilbert space. As a consequence, we obtain results of Kusuoka (in Gaussian case), Kallianpur-Mandrekar, Molchan, Rozanov, Okabe-Kotani, Kotani and Pitt. The approach also explains the role of conditions put by the latter three authors in the stationary case.


For stable processes which are Fourier transforms of processes with independent increments we obtain a Wold decomposition, we characterize their regularity and singularity, and, in the discrete-parameter case, we derive their linear predictors. In sharp contrast with the Gaussian case, regular stable processes which are Fourier transforms of processes with independent increments are not moving averages of stable motion.

3. Extrapolation and moving average representation for stationary random fields and Beurling's theorem [3]

Strong regularity for stationary random fields is discussed. An extension of the classical Beurling theorem to functions of several variables is given.
Necessary and sufficient conditions for the moving average representation of stationary random fields are obtained. A recipe formula for the best linear extrapolator is also given.

References


Efficiency and optimality of factorially balanced designs
for correlated errors [1]

Under the assumptions of a linear model with fixed effects and uncorrelated, homoscedastic errors, several classes of efficient, balanced designs for factorial experiments were constructed in [2]. In this project, the author has first studied the variations in the performance of the balanced factorial experiments constructed in [2], under the assumption that the errors are no longer homoscedastic and uncorrelated but are correlated according to the nearest neighbor covariance model. Experimental arrangements which can be derived from already constructed balanced designs so that these will attain high efficiency and/or optimality (D-, A- or E-optimal or weakly universally optimal) under the nearest neighbor correlation model, are under investigation.

References


1. Two parameter filtering problems using Radon transforms [1]

Takenaka's projected research (which is still in progress) is to use white noise and Radon transforms to solve certain 2-parameter filtering problems of practical importance — e.g. image detection in x-ray pictures. The white noise appears in the representation of Lévy's Brownian motion. A decomposition of white noise in spherical harmonics has been obtained in the course of the work. The techniques involved are related to the work of Čencov on Lévy's Brownian motion and Helgason's work on the Radon transform.

References

1. **Stochastic differential equations in infinite dimensional spaces**

This research is an outgrowth of the work on stochastic models for the activity of neurons. The problem described in Kallianpur's report (para. 4(ii)) leads naturally to problems involving stochastic differential equations (SDE's) for processes taking values in infinite dimensional Hilbert spaces or in $S(\mathbb{R}^d)$:

(i) A natural derivation of the SDE for infinite dimensional Ornstein-Uhlenbeck type process is given based on the following simple considerations [4].

(a) The process is Gaussian and (with suitable initial value) absolutely continuous with respect to $S(\mathbb{R}^d)$ -valued Wiener process;

(b) There exists a non-anticipative representation (in the sense of Kallianpur and Oodaira) for any process satisfying (a);

(c) The process is Markov;

(ii) A model which studies activity of neurons which are spatially extended (e.g. on dendrites) leads to an $S'$-valued, time-discontinuous process satisfying an SDE which can be called an infinite dimensional analogue of the Tuckwell-Cope model. The main result obtained pertains to weak convergence of the corresponding measures $\mathbb{P}^n$ defined on $D([0,\infty),S')$ to an Ornstein-Uhlenbeck type process satisfying a suitable SDE. Further generalizations are being investigated [5]. The work has obvious connections with the work (on neuronal activity) of J. Walsh, with the Holley-Stroock paper on infinite particle systems and with the string model of Funaki and of Miyahara.
References


Hazareesingh's main research work (which is being carried out partly for a Doctoral thesis at the University of Georgia under the direction of Professor Kannan) is in the theory of product integration in Banach algebras with unity, in particular, the connection between the integration theory of Masani and that of Lee. The following questions are being explored: (i) the product integral representation of the generalized Fredholm determinant, (ii) path integration using Masani's integral. The possibility of extending Hida's work on generalized Brownian functionals is also being considered.
RESEARCH IN ESTIMATION IN STATISTICAL MODELS
During the past year I have continued my research on robustness and the linear model. Research has been focused on the following areas: robust and efficient estimation in transformation models; robust estimation in models with nonconstant variances (heteroscedasticity); linear and nonlinear models in which some of the predictors are measured with error; applications of stochastic approximation to help find optima in complex simulation models. My Ph.D. student, Paul Gallo, completed his research and graduated with his Ph.D. degree in August. I now have two additional students, David Giltinan and Len Stefanski, who should complete their Ph.D. research by August, 1983. Gallo worked in the linear errors-in-variables models, Giltinan is working on robustness in heteroscedastic linear models and Stefanski is focusing on binary regression models, discussing robustness and errors-in-variables.

1. Transformations and Regressions

Note: The first and second articles in this subheading are new. The third and fourth are completion (new theory, examples of Monte Carlo) of work listed last year as "in preparation." The fourth was inadvertently omitted last year.

[1] Tests for regression parameters in power transformation models

We study tests of hypotheses for regression parameters in the power transformation model. In this model, a usual test consists of estimating the correct scale and then performing the usual linear model F-test in this estima-
ted scale. We explore situations in which this test has the correct level asymptotically as well as comparable power to Wald's test or the likelihood ratio test. In particular, the correct level is attained for simple linear regression, randomized analysis of covariance and some simple factorial designs. In most multiple regression models, the usual test has the wrong level; the exceptions depend on various forms of orthogonality.

[2] **Power transformations when fitting theoretical models to data** (with D. Ruppert)

We investigate power transformations in nonlinear regression problems when there is a physical model for the response but little understanding of the underlying structure. In such circumstances and unlike the ordinary power transformation model, both the response and the model must be transformed simultaneously and in the same way. We show by an asymptotic theory and a small Monte Carlo study that for estimating the model parameters there is little cost for not knowing the correct transform a priori; this is in dramatic contrast to the results for the usual case that only the response is transformed. Examples are included; in particular, we consider in detail the spawner-recruit relationship for Atlantic menhaden, as well as an example from chemistry.

[3] **Prediction and power transformations when the choice of power is restricted to a finite set.**

We study the family of power transformations proposed by Box and Cox (1964) when the choice of the power parameter is restricted to a finite set $R$. The behavior of the Box-Cox procedure is as anticipated in two extreme cases: when the true parameter $\theta$ is an element of $R$ and when $\theta$ is "far" from $R$. We study the case in which $\theta$ is "close" to $R$, finding that the resulting methods can be very different from unrestricted maximum likelihood and
that inferences may depend on the design, the values of the regression parameters, and the distance of \( \theta \) to \( \theta_0 \). The paper focuses on prediction and is thus a companion to [4] by Carroll and Ruppert. We find in particular that data transformation can be very costly in the sense that prediction estimates are often much more variable than is generally recognized.


The power transformation family is often used for transforming to a normal linear model. The variance of the regression parameter estimators can be much larger when the transformation parameter is unknown and must be estimated, compared to when the transformation parameter is known. We consider prediction of future untransformed observations when the data can be transformed to a linear model. When the transformation must be estimated, the prediction error is not much larger on average than when the parameter is known. However, the accuracy of prediction at individual design points can be greatly affected by data based transformation.

2. Heteroscedastic Linear Models

Note: The first article in this subheading is a completion and revision of work listed last year as in "preparation." The second and third are comprehensive revisions of earlier work, especially through the computations. The fourth is still in preparation.

[5] Robust estimators for random coefficient regression models

(with D. Ruppert).

Random coefficient regression models have received considerable attention, especially from econometricians. Previous work has assumed that the
coefficients have normal distributions. The variances of the coefficients have, in previous papers, been estimated by maximum likelihood or by least squares methodology applied to the squared residuals from a preliminary (unweighted) fit. In this paper we propose several robust estimators for random coefficient models. We compare them by Monte Carlo with estimators based on least squares applied to the squared residuals. The robust estimators are best overall, even at the normal model.


In a heteroscedastic linear model, it is known that if the variances are a parametric function of the design, then one can construct an estimate of the regression parameter which is asymptotically equivalent to the weighted least squares estimate with known variances. We show that the same is true when the only thing known about the variances is that they are an unknown but smooth function of the design or the mean response. Some preliminary Monte Carlo is very encouraging.


We consider a linear model with normally distributed but heteroscedastic errors. When the error variances are functionally related to the regression parameter, one can use either maximum likelihood or generalized least squares to estimate the regression parameter. We show that likelihood is more sensitive to small misspecifications in the functional relationship between the error variances and the regression parameter. Monte Carlo work demonstrates that in small samples, a proper robust generalized least squares as proposed in [13] by Carroll and Ruppert is far superior to maximum likelihood.
Bounded influence methods for heteroscedastic regression models (with D. Ruppert and D. Giltinan).

In [13], Carroll and Ruppert introduced a class of distribution robust estimators for the regression model with non-constant variance. This paper introduces methods for bounding the influence of extraordinary design as well as outlying responses. (Note: this manuscript is still in preparation).

3. Linear and Binary Regression Models with Errors-in-Variables
Note: The first two items in this section were listed last year as "in preparation." This third is new.

Some aspects of robustness in the functional errors-in-variables regression model (with Paul Gallo).

This paper considers regression models in which some of the predictor variables are measured with error. We present a class of distribution robust estimators for the regression coefficient and prove consistency and asymptotic normality. A Monte Carlo study is also included, showing that our methods can be very successful.

Comparisons between some estimators in functional errors-in-variables regression models (with Paul Gallo).

We study the functional errors-in-variables regression model. In the case of no equation error (all randomness due to measurement errors), the maximum likelihood estimator (MLE) computed assuming normality is asymptotically better than the usual moments estimator, (MME), even if the errors are not normally distributed. Our Monte Carlo study confirms this result, but shows that there are good reasons to favor the moments estimator for samples of the size ordinarily encountered in practice.
For certain statistical problems such as randomized two group analysis of covariance, the least squares estimate is shown to be better than the aforementioned errors-in-variables methods for estimating certain important contrasts. We also consider the robust methods we introduced in [9], showing them to be preferable to both the MLE and the MME in many circumstances.


We consider in detail probit and logistic regression models when some of the predictors are measured with error. For normal measurement errors, the functional and structural maximum likelihood estimates (MLE) are considered; in the functional case the MLE is not generally consistent. By an example and a simulation, we show that if the measurement error is large, the usual estimate of the probability of the event in question can be substantially in error, especially for events of high probability.

4. Other Topics


In [10], Carroll, et al. considered binary regression (e.g., logistic and probit regression) when some of the predictors are measured with error. If the predictors are treated as non-random (the functional model), they showed that the MLE is inconsistent in general for the binary regression parameter. We consider the case that independent replications of the predictor variables are made. We obtain sharp results on the size of m(N) relative to N necessary for consistency and asymptotic normality. (Note: this manuscript is still in preparation).
Monte-Carlo optimization by stochastic approximation, with application to harvesting of Atlantic menhaden (with D. Ruppert, R.L. Reish and R.B. Deriso).

In a recent study of the Atlantic menhaden, a commercially important fish in the herring family (Clupeidae), we made extended use of stochastic approximation. This paper is intended to introduce stochastic approximation to those statisticians unfamiliar with the area. A stochastic simulation model of the menhaden population is used as an example, but the paper is not addressed to only those working in fisheries. In this model, two variables are used to define the harvesting policy. For any values of these variables, the model will produce a random catch, and for a specified utility function the objective is to find the values of the variables which maximize the expected utility of the catch. Therefore, this is a classical response surface problem. However, nonsequential response surface methods would be extremely expensive to apply here. We used stochastic approximation to estimate the policy maximizing the expected utility of the catch.
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R.J. Carroll, Two examples of robust transformations when there are no outliers, *Appl. Statist.* 31, (1982), 149-152.


ACCEPTED FOR PUBLICATION (after November 1, 1982)


R.J. Carroll, R.N. Holt and V. Scarpello, Towards understanding the contents of the "Black Box" for predicting complex decision making outcomes, Decision Sciences, (1982), to appear.


5. "Markov property for Gaussian ultraproceses." V. Mandrekar and A. Soltani, Jan. 82.


10. "Complex symmetric stable variables and processes." S. Cambanis, June 82.


17. "A conditioned limit law result for jumps in the sequence of partial maxima of a stationary Gaussian process." William P. McCormick, August 82.

18. "On the diffusion approximation to a discontinuous model for a single neuron." G. Kallianpur, Aug. 82.


22. "Exact analysis of a delayed delta modulator and an adaptive differential pulse-code modulator." N.L. Gerr, Nov. 82.


TECHNICAL REPORTS IN PREPARATION


Carroll, R.J. and D. Ruppert: Power transformations when fitting theoretical models to data, Jan. 1982, #1383.


IN PREPARATION

Carroll, R.J. and Abbott, R.D. Interpreting multiple logistic regression coefficients in prospective observational studies.

Carroll, R.J., Giltinan, D. and Ruppert, D. Bounded influence methods for heteroscedastic regression models.

Carroll, R.J. and Lombard, F. A note on estimating the binomial N.

Carroll, R.J., Stefanski, L. and Ruppert, D. Consistency and asymptotic normality for binary errors-in-variables models.

OTHER TECHNICAL REPORTS

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June 16  Bivariate extremes: Models and statistical decision, J. de Oliveira, University of Lisbon.

July 29  Consistent estimates of parameters in continuous time stochastic processes, A. Baqchi, University of California at Los Angeles and Technical University of Twente, Holland.

Aug. 6   A general principle for limit theorems in finitely additive probability, R.L. Karandikar, Indian Statistical Institute and University of North Carolina.

Aug. 12  General boundary problems for linear differential operators, Yu. A. Rozanov, Steklov Mathematical Institute and Moscow University.

Aug. 16  Donsker's delta functional, H.H. Kuo, Louisiana State University.


Aug. 20  Some aspects of statistical inference in stochastic processes, G. Roussas, University of Patras, Greece.

Aug. 27  Brownian functionals, T. Hida, Nagoya University and University of North Carolina.

Sept. 1   Generalized Brownian functionals and applications, T. Hida, Nagoya University and University of North Carolina.

Sept. 10  Infinite dimensional Ornstein-Uhlenbeck process and string model, Y. Miyahara, Carleton University and Nagoya University.

Sept. 22  On extreme values of non-stationary sequences, J. Husler, University of Bern and University of North Carolina.

Sept. 29  Dilation theory methods in stochastic processes, A. Weron, Wroclaw Technical University and University of North Carolina.

Oct. 6   Multiparameter Brownian motion, S. Takenaka, Nagoya University and University of North Carolina.

Oct. 11  Weak compactness problems, N. Dinculeanu, University of Florida.

Oct. 13  Sequential urn problems imbedded in birth processes, J. Husler, University of Bern and University of North Carolina.

Oct. 20  Exact and limiting distribution of sustained maxima, W.P. McCormick, University of Georgia and University of North Carolina.

Oct. 27  Similarities and contrasts between stable and Gaussian processes, S. Cambanis, University of North Carolina.
LIST OF PROFESSIONAL PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT

1. Faculty Investigators:
   S. Cambanis
   R.J. Carroll
   G. Kallianpur
   M.R. Leadbetter

2. Visitors
   Senior:
   D. Daley (Jan. 81)
   L. de Haan (May-July 82)
   J. Tiago de Oliveira (June 82)
   T. Hida (Aug. 82)
   D. Kannan (May 82-present)
   D. Kölzow (Oct.-Dec. 82)
   H.H. Kuo (Aug. 82)
   V. Mandrekar (Nov. 81-Feb. 82)
   A. Weron (Sept. 82-present)

   Junior:
   C. Hardin (Sept. 82-present)
   S. Huang (June-Aug. 82)
   J. Hüsler (Sept. 82)
   R.L. Karandikar (June 82-present)
   W.P. McCormick (June 82-present)
   D. Ramachandran (Nov. 81-May 82)
   C. Schoenfelder (June-Sept. 82)
   A.R. Soltani (Nov. 81-June 82)
   C. Suen (Oct. 82)
   S. Takenaka (Sept. 82-present)
   R. Wolpert (June 82-present)

3. Graduate Students:
   D. Giltinan
   N.L. Gerr
   L. Hazareesingh
   L. Stefanski
R.J. Carroll presented invited lectures at Texas A & M University, Southern Methodist University, Virginia Technical University, Johns Hopkins University, the National Institutes of Health (National Cancer Institute) and the University of South Africa.

S. Cambanis presented invited talks at the meetings of the Institute of Mathematical Statistics in Tallahassee and of the American Mathematical Society in Washington. He also gave talks at the Conference on Information Sciences and Systems at Princeton, the International Symposium on Information Theory in Les Arcs, France and the Conference on Stochastic Processes and Their Applications in Clermont-Ferrand, France.

N.L. Gerr gave a talk at the Conference on Information Sciences and Systems at Princeton.

M.R. Leadbetter gave invited lectures at the University of Aarhus, Denmark; Aalborg University, Denmark; University of Bern, Switzerland; Chalmers University, Göteborg, Sweden and the University of Tennessee and was one of the guest lecturers (giving two talks) at the (Dec. 1981) meeting of statisticians held annually in Holland. He also completed a book (with G. Lindgren and H. Rootzén) on extreme values, to be published by Springer-Verlag in early 1983.

M.R. Leadbetter was elected to the Council of the Institute of Mathematical Statistics and began service on that body at the annual meeting in August 1982.
G. Kallianpur presented the following:

1. Invited talk at the ISI Golden Jubilee Conference on Statistics held in December 1981 at the Indian Statistical Institute (ISI), Calcutta. [On stationary random fields].

2. Invited talk at the IFIP-ISI Conference on the Theory and Application of Random Fields held at the Bangalore Campus of the ISI, January 1982. [On Feynman integrals].

3. Invited address on Nonlinear Filtering Theory at the Workshop in Filtering and Control Theory at Bonn, West Germany (June 1982).

4. Invited lecture at the Conference on Stochastic Processes of the Bernoulli Society at the University of Clermont-Ferrand, France (July 1982). [On Feynman integrals].


6. Three lectures on Stochastic Filtering Theory given at the invitation of the joint seminar of Carleton University (Ottawa), University of Ottawa and the Research Institute of the University of Montreal. (Lectures given at Montreal, April 1982).

7. Invited talk given at the session on "The Legacy of Norbert Wiener" at the Meetings of the American Mathematical Society at the University of Maryland, College Park, Maryland (October 1982).

8. The Layman Lectures in the Mathematical Sciences given at the University of Nebraska, Lincoln, Nebraska (November 1982). [Feynman integrals and modern developments in Filtering Theory].

9. Participated by invitation, in the conference on Random Fields, Quantum Field Theory and Differential Geometry sponsored by the American Mathematical Society and organized by A. Jaffe and E. Dynkin. The conference was held at the University of New Hampshire in July 1982.
(10) Together with Professor A.V. Balakrishnan of UCLA, organized the IFIP-ISI conference on the Theory and Application of Random Fields held in Bangalore, India (January 1982).
