DESIGN METHODOLOGY FOR REAL-TIME DISTRIBUTED SYSTEMS

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This report establishes a methodology for the structured design of complex, real-time digital systems, involving a high degree of concurrency. The design is based on the initial decomposition of the system specification into a control part and a data processing part. Formal models are developed for both parts, and a design methodology closely related to structured programming, is shown to be applicable. The proposed methodology is particularly suitable for precise and concise system require-
ments specifications as well as for the application of advanced verification methods.
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1. INTRODUCTION

This report is concerned with establishing a methodology for the design of complex real-time digital systems. These systems are dedicated to a single objective, such as flight-guidance, communication switching, patient monitoring, or industrial process-control. The overall task of the system can be decomposed into several subsidiary tasks, each of which contributes to the overall objective.

Efficient implementations exploit, as much as possible, the high degree of concurrency usually involved in such systems. Multimicrocomputer and VLSI implementations are of particular interest. Structured programming [LI-MI-WI] has become a generally accepted approach in modern software engineering. A similar approach can be applied to the design of complex, combined hardware/software systems, leading to a structured design methodology. The importance of such a design methodology has recently been emphasized, particularly in connection with the growing trend towards computer-aided design of VLSI-systems [LEW], [MEA-CON].

The major steps involved in a structured, top-down design approach are the following:

(1) system requirements specification
(2) stepwise refinement
(3) implementation
(4) verification.

In the following section we survey some of the publications dealing with the above design steps. In Sections 3-6 we develop an alternative methodology of specifying system requirements. In Section 7 we very briefly indicate the applicability of this method to the derivation of efficient and correct implementations.
2. SURVEY OF RELATED WORK

The difficulties involved in designing and maintaining complex software have led to extensive studies of suitable methodologies. In particular, the problem of software requirements specification has received considerable attention. Consequently, a variety of requirements specification languages have recently been developed. Typical examples of such languages are described in [DAV], [LEV-MUL], [ZAV]. These languages are mainly intended to facilitate the development of software, rather than hardware systems or combined hardware/software systems. They assume a well-defined, fixed architecture, for which a particular software is to be developed.

However, an essential advantage of any suitable structured system design is the integrated approach to hardware and software, enabling the designer to postpone his decision about hardware/software partitioning to a late stage in his design. Such a structured system design methodology calls for requirement specification methods applicable to both hardware systems as well as combined hardware/software systems. Of especial interest are specification methods which clearly establish feasible concurrences in the system.

Various research groups have recently devoted considerable efforts to the development of specification methods for complex, highly-concurrent systems, based on suitable modifications and extensions of the concept of Petri net. Some of these efforts are described in [VAL-COU], [MOA-DAV], [QUE], [WOJ], [YOE 82a], [YOEB-AR], [VOSS], [KYNG]. Closely related to these Petri-net oriented approaches is the Graph Model of Behavior, which forms part of the SARA design methodology being developed at UCLA [EST], [RAZ].
Recently, methods for the specification and verification of protocols have been extensively studied [SUN79], [SUN82]. Some of these methods are applicable to the more general problem of a specification methodology for digital highly-concurrent systems.

An extensive literature is presently available on the design and implementation of multi-microcomputer systems (for an annotated bibliography see [SAT]). However, most of the papers describe selected aspects of particular, experimental systems. On the other hand, valuable contributions towards a systematic design methodology are [WEI], [VAL-COU], [CAM-ROS], [EST], [KER]. Specific issues relevant to a systematic design methodology are discussed in e.g. [AND-JEN], [ADA-ROL], [LAM], [LYN-FI].
3. TOWARDS A STRUCTURED REQUIREMENTS SPECIFICATION METHODOLOGY

3.1 Main Features and Advantages of Specification Methodology

In this and the following three sections we describe a system requirements specification method which has the following features.

(a) It uses extended net concepts to provide a concise and mathematically precise model.

(b) It introduces a clear separation between control structure and data (processing) structure.

(c) It is based on a structured approach to parallel programming.

In view of the above features the specification method facilitates analysis, design, implementation, verification and testing of the overall system.

3.2 Control/Data Decomposition

The digital systems we are concerned with may be considered as consisting of two parts: a control structure and a data structure [BRU-ALT], [YOE-BRZ], [LEW], [VAL-COU]. The data structure consists of specific devices (operational units) such as adders, counters, etc. The control structure supervises the activities and sequencing of these devices.

Another essential feature of the digital systems we are interested in, is their high degree of concurrency. Furthermore, we assume the devices to operate asynchronously. The combined effect of concurrency and asynchronous operations may be utilized in order to achieve high-speed overall performance of the system.
3.3 Some Basic Control Structures

We first consider a few simple control structures, as well as methods for using them to form more complex structures. As will become evident in the sequel, our approach is strongly related to basic aspects of structured programming.

We shall use Figure 1 to explain some basic concepts, as well as to introduce our first example of a simple control structure.

![Diagram](image)

**Figure 1.** (a) Outside connections of PAR2 control structure
(b) Abbreviated notation.

All the signals indicated in Figure 1 are instantaneous; they may correspond e.g. to the rising edge (0 → 1 transition) of suitable pulse signals.

Assume the system shown in Figure 1 to be idle. Upon the arrival of the "Go" input, the control structure PAR2 becomes active,
by issuing the signals $\bar{a}$ and $\bar{b}$ either concurrently, or one after the other. These signals initiate the operation of the corresponding devices. Each device issues, upon completion of its operation, the corresponding completion signal ($a$ or $b$). The control structure PAR2 awaits the arrival of both completion signals $a$ and $b$, whereupon it produces the output $D$ ("Done") and returns to its idle state.

Thus the sequence of signals $G\bar{b}a\bar{a}b\bar{D}$ is an example of a feasible input-output sequence which takes the control structure PAR2 exactly once through the cycle of states idle-active-idle. We call any such input-output sequence a basic behavior sequence and denote by $B(CS)$ the basic behavior, i.e. the set of all basic behavior sequences, of the control structure $CS$. For the control structure PAR2 of Figure 1 we obtain:

$$B(PAR2) = \{G\bar{a}b\bar{a}bD, G\bar{a}b\bar{a}bD, G\bar{b}a\bar{a}bD, G\bar{a}b\bar{a}bD, G\bar{b}\bar{a}D\}.$$ 

Two points concerning this definition of basic behavior need clarification. Firstly, we replace the simultaneous occurrence of two or more signals by their sequential occurrences, in all possible orders. Since we assume all signals to be instantaneous, this approach is well motivated and is closely related to the "Single-Observer Principle" in [MIL], as well as the "Arbitration Condition" in [KEL74]. Secondly, we make no assumptions as to the relative speeds of the control structure and the devices. Hence, we consider e.g. the input-output sequence $G\bar{a}b\bar{b}D$ feasible. Namely, we admit the possibility that the completion signal $a$ is received before the initiation signal $b$ has been produced.

The above expression for $B(PAR2)$ can be simplified by means of the formal language operators introduced in Appendix A. Indeed,

$$B(PAR2) = G \circ (a \| b) \circ D .$$
The preceding considerations are easily extended to a control structure PAR\(k\), controlling the concurrent operation of \(k \geq 2\) devices. We denote by \(\tilde{a}_i\) the initiation signal of the \(i\)-th device, and by \(a_i\) its completion signal. Then (see Appendix A)

\[
B(\text{PAR}k) = \text{GO} \parallel \{\tilde{a}_i a_{i-1} \mid 1 \leq i \leq k\} \circ D.
\]

One easily sees, at least informally, that a PAR3 control structure can be obtained by interconnecting two PAR2 structures, as indicated in Figure 2.

\[\text{Figure 2. Two PAR2 control structures interconnected to form a PAR3 control structure.}\]

Another simple control structure is SEQ\(k\), \(k \geq 2\). SEQ\(k\) activates \(k\) devices sequentially (\(a_1\) first, \(a_k\) last). Its outside connections are the same as those of PAR\(k\), and its basic behavior is specified by

\[
B(\text{SEQ}k) = \text{GO} \parallel \big( \prod_{i=1}^{k} \tilde{a}_i a_{i-1} \big) \circ L.
\]
From a purely logic viewpoint, $\text{SEQ}_k$ can be simply realized by connecting corresponding parts, namely $G \rightarrow \tilde{a}_1, \tilde{a}_1 \rightarrow \tilde{a}_2, \ldots, \tilde{a}_{k-1} \rightarrow \tilde{a}_k, a_k \rightarrow D$. From a circuit viewpoint, however, signal regeneration might be necessary. The abbreviated notation for $\text{SEQ}_k$ is shown in Figure 3.

Generally speaking, we assume that the data structure provides status information to the control structure, by means of suitable level-type status signals.

![Diagram](image)

**Figure 3.** Abbreviated notation for $\text{SEQ}_k$.

The DEC control structure shown in Figure 4 corresponds to the if-then-else construct of conventional programs.

![Diagram](image)

**Figure 4.** (a) Outside connections of DEC control structure (b) Abbreviated notation.
In Figure 4 we denote by \( p \) an incoming (level-type) status signal. We write \( \bar{p} \) (instead of \( \neg p \) or \( \gamma_p \)) to indicate NOT-\( p \).

The basic behavior of the DEC control structure (Figure 4) is then specified by

\[
B(\text{DEC}) = \{Gp\bar{a}D, Gp\bar{b}D\}.
\]

Another control structure taken over from conventional (structured) programming is the WHILE structure shown in Figure 5.

![Figure 5. (a) WHILE control structure (b) Abbreviated notation.](image)

The basic behavior of the WHILE structure of Figure 5 is given by

\[
B(\text{WHILE}) = G(p\bar{a}a)^* \bar{p}D.
\]

Figure 6 shows an example of a parallel computation structure, which illustrates the application of a composite control structure. One easily verifies that for an integer \( y > 0 \) and an arbitrary integer \( x \) the computation structure of Figure 6 will produce the product of \( x \) and \( y \).

So far we have introduced a few basic control structures and have illustrated the possibility of composing them in order to obtain the control part of a parallel computation structure.
It is noteworthy that the simple control structures introduced so far are quite powerful, when considered as basic building blocks by means of which more complex structures can be composed. Hence these or similar building blocks may be selected as basis for a structured approach to the design of complex control structures (cf. [WEI], [BRU-ALT], [DAC-BLA]).

However, we also wish to investigate control structures which cannot be obtained by the composition of the simple control structures discussed so far. In the sequel we introduce a suitable formalism which will enable us to deal with this problem in a precise and concise way.
4. PARALLEL CONTROL GRAPHS

In this section we introduce the concept of parallel control graph (PCG), following [BOL-YOE] and [YOE-GIN].

4.1 Basic Concepts

Definition 4.1  A parallel control graph (PCG) is a finite, directed graph $G$ with the following properties:

1. Each node of $G$ is of one of the seven types shown in Figure 7.
2. Multiple edges are not admitted.
3. $G$ has exactly one START node $S$ and exactly one HALT node $H$.
4. There exists a directed path from $S$ to every other node $v$ of $G$.
5. There exists a directed path from every node $v \neq H$ of $G$ to the node $H$.

Evidently a PCG cannot have self-loops (i.e. cycles of length 1).

Examples of PCGs are shown in Figure 8.

We shall refer to nodes of type FORK, JOIN, DECIDER, and UNION as control nodes. A PCG with DECIDER and UNION nodes as only control nodes is purely sequential. Similarly, a PCG with FORK and JOIN nodes as only control nodes is purely parallel.

Definition 4.2  Let $G$ be a PCG. A marking $m$ of $G$ is a function $m : E \rightarrow \omega$, where $E$ is the edge set of $G$ and $\omega$ is the set of non-negative integers. A marked PCG is an ordered pair $(G, m)$, where $G$ is a PCG and $m$ is a marking of $G$. 
<table>
<thead>
<tr>
<th>NODE TYPE</th>
<th>INDEGREE</th>
<th>OUTDEGREE</th>
<th>GRAPHICAL REPRESENTATION</th>
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</thead>
<tbody>
<tr>
<td>START</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>HALT</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>FORK</td>
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<td>2</td>
<td></td>
</tr>
<tr>
<td>JOIN</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>DECIDER</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>UNION</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>OPERATION</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 7.** Node types of PCG.

**Figure 8.** Examples of PCGs.
Let \( e \) be an edge of the marked PCG \((G,m)\). We refer to \( m(e) \) as the number of tokens on \( e \). If \( m(e) > 0 \), we say that \( e \) is marked. In the graphical representation of marked PCGs, tokens are indicated by dots (\( \ast \)). Figure 9 shows examples of marked PCGs.

![Figure 9. Examples of marked PCGs.](image)

**Definition 4.3** Let \((G,m)\) be a marked PCG. A node of type OPERATION or DECIDER or FORK is enabled iff its inedge is marked. A JOIN node is enabled iff both its inedges are marked. A UNION node is enabled iff at least one of its inedges is marked. A node which is enabled may fire.

The firing rules, illustrated in Figure 10, are as follows:

**Definition 4.4**

(a) The firing of a FORK node decreases the marking of its inedge by 1 and increases the marking of both its outedges by 1.
<table>
<thead>
<tr>
<th>NODE TYPE</th>
<th>BEFORE FIRING</th>
<th>AFTER FIRING</th>
</tr>
</thead>
<tbody>
<tr>
<td>PORK (F)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JOIN (J)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DECIDER (D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNION (U)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OPERATION (OP)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 10. - Examples of "firings"
(b) The firing of a JOIN node decreases the markings of both its inedges by 1, and increases the marking of its outedge by 1.

(c) The firing of a DECIDER node decreases the marking of its inedge by 1, and increases the marking of either one of its outedges by 1.

(d) The firing of a UNION node decreases the marking on one of its marked inedges by 1, and increases the marking of its outedge by 1.

(e) The firing of an OPERATION node decreases the marking of its inedge by 1 and increases the marking of its outedge by 1.

For example, node J in Figure 9(a) is enabled. The firing of J yields the marked PCG of Figure 9(b).

Marked PCGs can, of course, also be defined in terms of Petri nets (cf. [YOE79]).

4.2 Well-Formed PCGs

We now define well-formed PCGs. Let m and m' be markings of the PCG G.

We write \( m \xrightarrow{v} m' \) to indicate that the marking m' is obtainable from the marking m by firing node v. We write \( m \rightarrow m' \) to state that m' is reachable from m by the successive firing of one or more nodes of G. Furthermore, we set

\[ [m] = \{m' | m \rightarrow m'\} \cup \{m\}. \]

We shall refer to [m] as the set of all markings reachable from m.

We denote by \( e_S \) the outedge of the START node S, and by \( e_H \) the inedge of the HALT node H.

**Definition 4.5** The initial marking \( m_0 \) of a PCG G is defined as follows:

\[ m_0(e_S) = 1 \text{ and } m_0(e) = 0 \text{ for every } e \neq e_S. \]
A marking \( m \) of \( G \) is final iff \( m(e_H) > 0 \). We denote by \( M_f \) the set of all final markings of \( G \).

Let \( G \) be the PCG shown in Figure 9, \( m_a \) its marking shown in Figure 9(a) and \( m_b \) the marking shown in Figure 9(b). Then \( m_a \in \{ m \}, m_b \in \{ m \}, \) and \( m_b \in M_f \).

**Definition 4.6** A PCG \( G \) is terminating iff \((\forall m \in \{ m \}) (|m \cap M_f| \neq \emptyset)\)
i.e. if \( m \) is reachable from \( m_0 \), then there exists a final marking reachable from \( m \).

By deadlock we mean a marking \( m \) such that \([m] \cap M_f = \emptyset\), i.e. no final marking is reachable from \( m \). Thus, \( G \) is terminating iff no deadlock is reachable from \( m_0 \).

One easily verifies that the PCGs of Figures 8(a), 8(b) and 9 are terminating, whereas the graph of Figure 8(c) is not terminating.

**Definition 4.7** Let \( G \) be a PCG and \( E \) its edge set. \( G \) is residue-free iff

\[
(\forall m \in \{ m \})(\forall e \in E)(e) = 1
\]
i.e. for any final marking \( m \) reachable from \( m_0 \), the marked PCG \((G,m)\) contains exactly one token (namely on \( e_H \)).

**Definition 4.8** A PCG \( G \) with edge set \( E \) is safe iff

\[
(\forall m \in \{ m \})(\forall e \in E)(e) \leq 1
\]
i.e. the number of tokens on any edge \( e \) cannot exceed 1, under any marking \( m \) reachable from \( m_0 \).

The following proposition is an immediate consequence of

**Theorem 3.1** of [YOE-GIN].

**Proposition 4.1** Every well-formed PCG is safe.
5. PARALLEL CONTROL STRUCTURES

This section is based on [BOL-YOE].

5.1 Basic Concepts

A parallel control structure (PCS) is a suitably labelled PCG [YOE79].

Definition 5.1 A parallel control structure (PCS) $\Gamma$ consists of the following:

1. A PCG $G(\Gamma)$

2. A finite alphabet $\Sigma$ of operation letters. Every OPERATION node of $G(\Gamma)$ is labelled by a letter of $\Sigma$.

3. A finite alphabet $\Pi$ of predicate letters. Every DECIDER node $D$ of $G(\Gamma)$ is labelled by a letter of $\Pi$. Furthermore, one outgoing edge of $D$ is labelled $T$ (true), and the other edge $F$ (false).

An example of a PCS is shown in Figure 11.

![Diagram of PCS](image)

$\Sigma = \{a,b,c\}$

$\Pi = \{p\}$

Figure 11. Example of PCS ($\Gamma_1$).
A PCS \( \Gamma \) is well-formed iff \( G(\Gamma) \) is well-formed.

**Definition 5.2** Let \( G \) be a PCG. A node sequence
\[
(v_1, v_2, \ldots, v_n)
\]
is a **firing sequence** of \( G \) iff there exist markings \( (m_1, m_2, \ldots, m_n) \) of \( G \) such that:
\[
m_{i-1} \xrightarrow{v_i} m_i \quad \text{for } 1 \leq i \leq n,
\]
where \( m_0 \) is the initial marking of \( G \) and \( m_n \) is final (i.e. \( m_n \in M_f \)).

**Definition 5.3** Let \( \Gamma \) be a PCS. We denote by \( \bar{\Pi} \) the set of **negated** predicate letters, i.e.
\[
\bar{\Pi} = \{ \bar{p} | p \in \Pi \}.
\]

Let \( \alpha = (v_1, v_2, \ldots, v_n) \) be a firing sequence of \( G(\Gamma) \) and \( (m_1, m_2, \ldots, m_n) \) the corresponding sequence of markings. We associate with every \( v_i \) in \( \alpha \) a symbol \( \tilde{v}_i \) in \( \tilde{\Sigma} \cup \{ \lambda \} \), where \( \tilde{\Sigma} = \Sigma \cup \Pi \cup \bar{\Pi} \) and \( \lambda \) denotes the empty sequence, in accordance with the following rules:

(a) if \( v_i \) is a FORK or a JOIN or a UNION, then \( \tilde{v}_i = \lambda \).

(b) if \( v_i \) is an OPERATION node, then \( \tilde{v}_i = \sigma \), where \( \sigma \in \Sigma \) is the label of \( v_i \) in \( \Gamma \).

(c) if \( v_i \) is a DECIDER with label \( p \in \Pi \), outedge \( e_1 \) labelled \( T \) and outedge \( e_2 \) labelled \( F \), then \( \tilde{v}_i = p \) if \( m_i(e_1) = m_i-1(e_1) + 1 \), else \( \tilde{v}_i = \bar{p} \).

We set \( \tilde{\alpha} = \tilde{v}_1 \tilde{v}_2 \ldots \tilde{v}_n \). Thus \( \tilde{\alpha} \in (\tilde{\Sigma})^* \).
Definition 5.4 Let $\Gamma$ be a PCS. With $\Gamma$ we associate the language $L(\Gamma) \subseteq (\hat{\beta})^*$ defined as follows:

$$L(\Gamma) = \{\hat{a} \mid a \text{ is a firing sequence of } G(\Gamma)\}.$$ 

For example, for the PCS $\Gamma_1$ of Figure 11 we have

$$L(\Gamma_1) = \{pab, pba, \hat{pc}\}.$$ 

If $L(\Gamma) = L(\Gamma')$, $\Gamma$ and $\Gamma'$ are said to be $L$-equivalent.

Proposition 5.1 Let $\Gamma$ be a well-formed PCS. Then $L(\Gamma)$ is regular.

Proof This follows from Proposition 4.1, stating that every well-formed PCG is safe. Thus the set of markings reachable from the initial marking is finite. Hence, there exists a finite automaton $A$ such that $L(A) = L(\Gamma)$. $\Box$

Any well-formed PCS $\Gamma$ represents a control structure $CS$ (see Section 3) in the following sense. Let $\hat{\Gamma}$ be the PCS obtained from $\Gamma$ by replacing each OPERATION node labelled $\sigma$ by a sequence of two OPERATION nodes, the first labelled $\hat{\sigma}$ and the second labelled $\sigma$. Then

$$B(CS) = G \circ L(\hat{\Gamma}) \circ D.$$ 

5.2 Composition of PCSs

Structured programs are obtained by "successive composition", using a given set of basic ("primitive") control structures [LE-MAR]. In the following definition we extend this concept of "composition" to PCGs (cf. [YOE79]).
**Definition 5.5** Let $G_1$ and $G_2$ be disjoint PCGs and $v$ an OPERATION node of $G_1$. We define the composition $G_1(v + G_2)$ to be the PCG $G$ obtained by substituting $G_2$ for $v$ in $G_1$, as indicated in Figure 12.

![Figure 12](image-url)

**Figure 12.** Illustrating the concept of composition
(a) PCG $G_1$,
(b) PCG $G_2$,
(c) Composition $G = G_1(v + G_2)$.

One easily verifies the following (see [BOL-YOE]).

**Proposition 5.2** Let $G_1$ and $G_2$ be disjoint PCGs, and $v$ an OPERATION node of $G_1$. Then their composition $G = G_1(v + G_2)$ is well-formed iff $G_1$ and $G_2$ are well-formed.

The concept of "reducibility" plays an important role in the theory of structured programming (cf. [LE-MAR]).

**Definition 5.6** Let $\Delta$ be a set of well-formed PCGs, $\Delta = \{G_1, G_2, \ldots\}$, and $\Gamma$ a PCS. $\Gamma$ is reducible with respect to $\Delta$ iff there exists a PCS $\Gamma'$, such that
(1) \( L(\Gamma') = L(\Gamma) \)

(2) \( G(\Gamma') \) can be obtained by successive compositions of PCGs in \( \Delta \).

Figure 13 shows primitive "D-structures" (D for Dijkstra, see [LE-MAR]). \( D_1 \) corresponds to SEQ2 (Section 3) in a rather evident way. Similarly, \( D_2 \) corresponds to the DEC control structure defined in Section 3.

![Figure 13. Primitive, cycle-free D-structures](image)

The following proposition is proven in [BOL-YOE].

**Proposition 5.3** Let \( \Gamma \) be a well-formed, cycle-free, purely sequential PCS. Then \( \Gamma \) is reducible w.r.t. \( \{D_0, D_1, D_2\} \), where the \( D_i \)'s are shown in Figure 13.

The reducibility of purely parallel PCSs is studied extensively in [GIN-YOE].

**Proposition 5.4** The PCS \( C_2 \) shown in Figure 14 is irreducible with respect to \( \Delta = \{H_1, H_2\} \), where \( H_1 = D_1 \) (see Figure 13) and \( H_2 \) is shown in Figure 15.
Proposition 5.5  The PCS $C_3$ shown in Figure 16 is irreducible with respect to any set $\Delta$ of purely parallel PCGs, each having less OPERATION nodes than $C_3$.

**Figure 14.** PCS $C_2$

**Figure 15.** PCG $H_2$
Figure 16. PCS C_3

One easily verifies that the preceding two propositions remain valid even if the corresponding sets Δ are replaced by larger sets Δ' = Δ ∪ Δ_s, where Δ_s is an arbitrary set of purely sequential PCGs.

The above observations clearly indicate the limitations involved in selecting the simple control structures of Section 3 as a basis for a structured approach to the design of complex control structures. Indeed, the irreducibility results derived in [GIN-YOE] and [BOL-YOE] lead to the establishment of various infinite hierarchies of bases suitable for the structured design of complex PCSs.
6. PARALLEL PROCESSING STRUCTURES

As discussed in Section 3.2, we assume that complex digital systems are composed of two parts: a control structure and a data (processing) structure. The formal concept of PCS, introduced so far, is suitable for modeling the control part of the overall digital system. In this section we define the formal concept of "Parallel Processing Structure" (PPS). Informally, a PPS is derived from a PCS by adding an "interpretation", which represents the data processing part of the system. A PPS is therefore suitable for precisely modeling the overall system. The formal definition of the PPS concept is as follows.

Definition 6.1 A parallel processing structure (PPS) consists of the following:

1. A PCS \( \Gamma \) (see Definition 5.1)
2. An interpretation \( I \) of \( \Gamma \), \( I = (D, A, C) \), where
   - \( D \) is a non-empty set of data (the domain of \( I \));
   - \( A \) is a mapping associating with every operation letter \( \sigma \) of \( \Gamma \) a binary relation \( A[\sigma] \) on \( D \), i.e. \( A[\sigma] \subseteq D \times D \), in particular, \( A[\sigma] : D \rightarrow D \) i.e. \( A[\sigma] \) is a function.
   - \( C \) is a mapping associating with every predicate letter \( p \) of \( \Gamma \) a one-place predicate \( C[p] \) on \( D \).

An example of a PPS \( (\Gamma, I) \) is shown in Figure 17. This PPS will perform (similarly to the parallel computation structure of Figure 6) the multiplication \( x \cdot y \), for an arbitrary integer \( x \) and a nonnegative integer \( y \). The product is obtained as final value of \( d_2 \), provided the initial value of \( d = (d_1, d_2) \) is set to \((0,0)\).
domain

\[ D = \omega \times Z, \]

where \( \omega \) is set of nonnegative integers
\( Z \) is set of integers.

Let \( d = (d_1, d_2) \in D \), where \( d_1 \in \omega, d_2 \in Z \);

mapping \( A \)

\[ A[a](d_1, d_2) = (d_1 + 1, d_2) \]
\[ A[b](d_1, d_2) = (d_1, d_2 \times x), \text{ for given integer } x \; ; \]

mapping \( C \)

\[ C[p](d_1, d_2) \equiv d_1 < y, \text{ for given } y \in \omega. \]

(b) Interpretation \( I \)

Figure 17. Example of a PPS \((\Gamma, I)\).
Any given PPS performs some computation. This concept will be made precise in the following definition.

**Definition 6.2** Let \((\Gamma, I)\) be a PPS. For every operation letter \(\sigma\) of \(\Gamma\), we set \(\hat{\sigma} \triangleq A[\sigma]\). For every predicate letter \(p\) of \(\Gamma\) we set

\[
\hat{p} \triangleq \{(d,d) \mid d \in D \land C[p](d)\}
\]

\[
\hat{\neg p} \triangleq \{(d,d) \mid d \in D \land \neg C[p](d)\}.
\]

Let \(w \in \tilde{\Sigma}^*\), where \(\tilde{\Sigma} \triangleq \Sigma \cup \Pi \cup \Pi^*\) (see Definition 5.3). With \(w\) we associate the binary relation \(\hat{w}\) on \(D\) as follows.

1. if \(w = \lambda\), then \(\hat{w}\) is the identity relation on \(D\), i.e.
   \[
   \hat{w} = \{(d,d) \mid d \in D\}.
   \]
2. if \(w = w_1 w_2 \ldots w_r\), where \(w_i \in \tilde{\Sigma}\) \((1 \leq i \leq r)\), then
   \[
   \hat{w} = \hat{w}_1 \circ \hat{w}_2 \circ \ldots \circ \hat{w}_r,
   \]
   where \(\circ\) denotes composition of binary relations, defined as usual.

The computation performed by the PPS \((\Gamma, I)\) is the binary relation \(C[\Gamma, I]\) on \(D\) defined by

\[
C[\Gamma, I] \triangleq \hat{\bigcup \{w \mid w \in L(\Gamma)\}}.
\]

For the PPS \((\Gamma, I)\) of Figure 17, one easily verifies that

\[(0,0)C[\Gamma, I](d_1, d_2)\] implies \(d_2 = x \times y\).
7. CONCLUSIONS

7.1 Structured Design of Concurrent Digital Systems

The theory developed in this report provides a suitable framework for a structured, top-down approach to the design of complex, highly concurrent digital systems. It is based on a distinct separation between the control part and the data (processing) part of the system. The control part is modeled by the formal concept of parallel control structure (PCS). It is shown how the well-known methodology of structured programming may be extended to the design of well-formed (particularly deadlock-free) PCSs. An important aspect of any structured approach to design is the selection of suitable, primitive building blocks. The theory of irreducible PCSs, discussed in this report, is therefore an essential contribution to the structured design methodology this report is concerned with.

The overall digital system is modeled by the concept of parallel processing structure (PPS). A PPS consists of a PCS, representing the control part, together with an interpretation, representing the data processing part of the system.

The structured, top-down design of a complex, highly concurrent digital system is best started from a high-level specification in PPS format. This specification is then transformed by stepwise refinements into a low-level description, suitable for direct implementation. Each refinement step can be verified, using well-known techniques of proving parallel programs correct (cf. [KEL76]).
7.2 Proposed Extensions of the PPS Model

The PPS model introduced in this report can easily be extended in order to provide additional modeling power. The incorporation of arbiters as additional building blocks is, no doubt, essential. In [YOE82b] methods were developed for the behavioral specification of arbiters. The formal concepts introduced in [YOE82b] can easily be combined with the PPS model developed so far.

Another important extension of the PPS model consists of the provision of relevant timing information, such as the (minimal and maximal) duration of an operation, maximal delays involved, etc. Similar timing concepts are introduced in [MER] and [MOA-DAV].

7.3 Implementation of a PPS System Description

Assume now that the structured, top-down design methodology summarized in Section 7.1 has led to a low-level PPS description of the required system.

Various techniques are available for the direct, asynchronous hardware implementation of the corresponding PCS. In particular, we refer to [DAC-BLA], [VAL-COU], and [WOJ-CAM]. The data-processing part of the required system, represented by the interpretation of the low-level PPS description, can also be implemented by a variety of techniques. A direct, register-transfer-level approach is discussed in [WOJ-CAM] and [WOJ]. For a VLSI-implementation of the system, the method of implementing a data-path chip described in [MEA-CON] becomes applicable. Alternatively, the data-processing part can be implemented by means of off-the-shelf hardware available for (loosely coupled)
multi-microcomputer systems (cf. [WLT]).

As for the direct hardware implementation of arbiters, we refer to [MUE] and [SEI].
8. REFERENCES


REFERENCES (cont'd)


REFERENCES (cont'd)


APPENDIX A

BASIC LANGUAGE CONCEPTS

In this Appendix we introduce a few, basic language concepts, used in this Report.

Let $\Sigma$ denote a finite alphabet. We denote by $\Sigma^*$ the set of all finite strings (words) of symbols from $\Sigma$, including the empty string $\lambda$.

A language $L$ over $\Sigma$ is any subset of $\Sigma^*$.

Let $L_1$ and $L_2$ be languages over $\Sigma$. Their concatenation is defined to be

$$L_1 \circ L_2 = \{xy | x \in L_1 \land y \in L_2\},$$

where $xy$ denotes the concatenation of the strings $x$ and $y$, i.e. string $x$ followed by string $y$. Usually, we write $x \circ L$ for $(x) \circ L$.

$$\prod_{i=1}^{k} L_i$$

denotes the concatenation $L_1 \circ L_2 \circ \ldots \circ L_k$.

For any language $L$ we set $L^0 = \{\lambda\}$, and $L^n = L^{n-1} \circ L$ for $n \geq 1$. Thus $L^1 = L$, $L^2 = L \circ L$, etc. Furthermore, we introduce the usual star-operation $L^* = \bigcup_{i=0}^{\infty} L^i$.

Given $x \in \Sigma^*$ and $y \in \Sigma^*$, the shuffle $x \| y$ is the language over $\Sigma$ defined recursively as follows:

1. $\lambda \| \lambda = \{\lambda\}$;
2. $\sigma \| \lambda = \lambda \| \sigma = \{\sigma\}$, for every $\sigma \in \Sigma$;
3. Let $\sigma \in \Sigma$, $\tau \in \Sigma$, $x \in \Sigma^*$, $y \in \Sigma^*$.

Then $\sigma x \| \tau y = [(\sigma) \circ (x \| y)] \cup [(\tau) \circ (\sigma x \| y)]$.

Thus, if $x = \sigma_1 \sigma_2 \ldots \sigma_n$, $y = \tau_1 \tau_2 \ldots \tau_k$, and $z \in x \| y$, then all $\sigma_i$'s and $\tau_j$'s appear in $z$ exactly once; the relative ordering
of the \(a_i\)'s in \(z\) is the same as in \(x\), and the relative ordering
of the \(\tau_j\)'s in \(z\) is the same as in \(v\).

For example, \(ab \parallel cd = \{abcd, acbd, acdb, cdab, cabd, cdab\}\).

For languages \(L_1\) and \(L_2\) over \(\Sigma\) we define their shuffle as:

\[
L_1 \parallel L_2 = \bigcup_{x \in L_1, y \in L_2} \{x \parallel y\}.
\]

Let \(\Lambda = \{L_1, L_2, \ldots, L_k\}, k \geq 1\) be a finite set of languages
over the alphabet \(\Sigma\). We define the shuffle of \(\Lambda\) (notation: \(\parallel / \Lambda\)) as:

\[
\parallel / \Lambda = L_1 \parallel L_2 \parallel \ldots \parallel L_k.
\]
APPENDIX B

SUMMARY OF RESEARCH ACTIVITIES SUPPORTED UNDER THIS GRANT

Manuscripts


Interim Scientific Reports


Conferences
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