A COMPARISON OF THREE MAGNETIC ANOMALY DETECTION (MAD) MODELS

By

Daniel Carl Schluckebier

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**A Comparison of Three Magnetic Anomaly Detection (MAD) Models**

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**Abstract:**
This thesis presents a comparison of three Magnetic Anomaly Detection (MAD) models: a cross-correlation detection model, a square law detection model, and a model referred to as the OPTEVFOR detection model. FORTRAN and BASIC programs for the three detection models are included in this thesis. The programs yield detection probabilities for straight line encounters. Magnetic signal values for the straight line encounters are an additional output. Plots of lateral range curves and magnetic signal...
Block 20 (Cont)

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A Comparison of Three Magnetic Anomaly Detection (MAD) Models

by

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ABSTRACT

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I. INTRODUCTION

This thesis presents a comparison of three Magnetic Anomaly Detection (MAD) models. The comparison is in terms of probabilities of detection that were computed using the models. Two of the models, the cross-correlation model and the square law model, have been used to model sonar detection [Ref. 1: pp. 343-357]. The third model, referred to as the OPTEVFOR model, is a slant range threshold detection model. The results of the comparisons are presented in graphical and tabular form. In addition, plots of magnetic signals for selected lateral ranges and noise levels are shown. The effects of noise, aircraft and submarine headings, submarine displacement, and vertical separation are also indicated.

The models were implemented using the FORTRAN and BASIC programs\textsuperscript{1} that are listed in Appendix A. For those interested in using the programs for other investigations, an input parameter discussion is provided in Chapter 3. To use the FORTRAN program, the user specifies the input parameters in an input file. After execution of the program, an output file is generated that contains

\textsuperscript{1}The programs are based on an unpublished BASIC program by R.N. Forrest for an H.P.- 85 microcomputer.
probabilities of detection for each of the three models. In addition, magnetic signal values and magnetic signal values plus random magnetic noise values for one of the encounters generated by the program are included in the output file. An IBM GRAFSTAT graphical package was used to produce the graphics in this thesis.

To use the BASIC program, the user must interactively enter the input parameter values for each encounter. After execution of the program, an optional hardcopy printout supplies the input parameter values and a table of detection probabilities for each of the three models (see Appendix A). Following this, lateral range curves are displayed to the user for immediate observation. A typical program run producing 21 detection probabilities for each model requires approximately 10 minutes of computing time on an Atari 800 microcomputer.
the limiting detection capability for an automatic system that does not use information about the shape of the magnetic signal.

B. OPTEVFOR MAD DETECTION MODEL

The OPTEVFOR model is described by Forrest [Ref. 3: pp. 7-8]. In characterizing the submarine magnetic signal as a simple dipole signal, the U.S. National Defense Research Committee, [Ref. 4: p. 20], reports that the magnetic signal of the submarine "varies as the inverse cube of the distance from the source". In an OPTEVFOR report [Ref. 5: p. 1, encl. 1], the results of a regression analysis on empirical peak to peak signal output against slant range between submarines and aircraft are reported. These results also suggested this inverse cube relationship for the magnetic signal. This relationship is the basis for the OPTEVFOR detection model.

The model has a deterministic mode and a stochastic mode, each of which involves the following parameters: the submarine magnetic moment (M), an Operator Recognition Factor (ORF), the average peak to peak magnetic noise (N) in the operating area, and a slant range (R). The relationship between these quantities is given by:

$$ R = \left( \frac{c M}{(ORF) N} \right)^{1/3} $$

(eqn 2.1)
The value of the constant \( c \) is 0.10 for \( M \) in oersted centimeters\(^3\), \( R \) in meters, and \( N \) in gamma.

In the deterministic mode, detection occurs if and only if the aircraft's slant range from the submarine at CPA is less than or equal to \( R \). This mode yields a rectangular ("cookie cutter") lateral range curve with the probability of detection equal to 1 for an encounter where the slant range at CPA is less than or equal to \( R \), and 0 when it is greater than \( R \).

The stochastic mode allows a more uncertain approach to detection by allowing a gradual rise in probability of detection as the slant range at CPA decreases. In this mode one sets the probability of detection at \( R \) equal to 50 percent, and the lateral range curve is given by \( P_d = \Phi(x) \); where it is understood that \( \Phi \) is the standard normal cumulative distribution function and \( x \) is determined by the following equation:

\[
\frac{R - \text{CPA}}{(AL)R} = x
\]

where \( \text{CPA} \) is the magnitude of the slant range distance at CPA, and \( R \) is the calculated range from Equation 2.1. The product \((AL)R\) represents a standard deviation. The value of \( AL \) can be considered to be determined by "the combined uncertainty and variability in the values of \( M, N, \) and ORF" [Ref. 3: p. 8]. Two values of \( AL \) (.20 and .01) are shown in Figure 2.1. If empirical data was available, the
value of AL could be chosen to provide a best fit to the observed results. Note, as AL approaches 0, the stochastic mode approximates the deterministic mode.

Figure 2.1. Lateral Range Curves for Different Values of AL.
III. INPUT PARAMETERS

The input parameters for the FORTRAN program are all contained in one input file. This allows parameter values to be easily changed without recompiling the main program or subroutines. Also, with a few changes, this program could be altered to operate in conjunction with a larger program to yield a probability of detection on an individual MAD run.

The input parameters are divided into four areas for discussion. They are: (1) sample interval, (2) earth magnetic field, (3) submarine moments, and (4) other inputs.

A. SAMPLE INTERVAL

The choice of a sample interval is discussed by Forrest [Ref. 2: pp. 27-30]. In the program, the total observation time in seconds over which the samples are taken is entered in T7. This time should be long enough to encompass a "complete signal" at the maximum expected detection slant range.

As the slant range from the submarine to the magnetometer increases, the distance over which a significant magnetic signal is present at the magnetometer also increases. Figure 3.1 graphically shows the difference
in the amount of time that a signal is present for slant ranges of 200 meters and 805 meters. In this thesis, the total time for a straight line encounter is assumed to be 20 seconds. As can be seen from Figure 3.1, a 20 second interval adequately covers the significant portion of the magnetic signal for an 805 meter slant range at CPA.

![Graphs showing magnetic signals for 200 meters and 805 meters slant range.](image)

Figure 3.1. Magnetic Signals for Slant Ranges of 200 Meters and 805 Meters.

The time between samples is set equal to the reciprocal of twice the upper bandpass filter frequency of the MAD sensor. A value of 0.9 Hz was suggested for use by Texas Instruments [Ref. 6: p. 112] as an upper bandpass filter limit in a discussion on the effects of noise on a MAD system. This value yields a time interval between samples of 0.55 seconds.

The sample interval length and the false alarm rate (the expected number of false alarms per hour) determine the
false alarm probability. The false alarm rate \((F2)\) is assigned a value of 3 based on a report by OPTEVFOR [Ref. 5: p. 2.1].

B. EARTH MAGNETIC FIELD

Input values for the earth magnetic field intensity and inclination, or dip angle, may be taken from two Defense Mapping Agency Hydrographic Center charts, [Refs. 7 and 8 respectively], or approximated by using a program. If chart values are entered, the earth field intensity must be in units of gamma and the inclination in decimal degrees. The program used to determine the intensity of the earth field and inclination is based on a simple dipole field model that is described by Forrest [Ref. 9: pp. 39-43].

Table III-1 displays the program output values of inclination in decimal degrees and earth magnetic field in gamma for selected geographic locations. In addition, corresponding values obtained from the Defense Mapping Agency Hydrographic Center Charts Number 30 and Number 39 are also displayed. The last three columns are the average slant range in meters at which a 50 percent probability of detection is obtained for the three program detection models. The program input parameters for these slant ranges were the same as the base case, except for the following differences: a sample interval time of 40 seconds, aircraft and submarine headings of 0 degrees, and a submarine
**Table III-1**

**Slant Detection Ranges for the Three Detection Models to Compare the Inclination and Earth Magnetic Field Model Values to DMARC Chart Values**

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Longitude</th>
<th>Inclination in decimal degrees</th>
<th>Earth Magnetic Field in oersted</th>
<th>Slant Detection Ranges in Meters</th>
<th>Cross-Correlation Law</th>
<th>OPTEVFOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>180</td>
<td>74</td>
<td>.63</td>
<td>650</td>
<td>422</td>
<td>282</td>
</tr>
<tr>
<td>60</td>
<td>90</td>
<td>82</td>
<td>.68</td>
<td>615</td>
<td>346</td>
<td>264</td>
</tr>
<tr>
<td>30</td>
<td>150</td>
<td>58</td>
<td>.51</td>
<td>718</td>
<td>487</td>
<td>310</td>
</tr>
<tr>
<td>30</td>
<td>60</td>
<td>59</td>
<td>.53</td>
<td>711</td>
<td>482</td>
<td>308</td>
</tr>
<tr>
<td>30</td>
<td>-60</td>
<td>31</td>
<td>.39</td>
<td>799</td>
<td>542</td>
<td>326</td>
</tr>
<tr>
<td>30</td>
<td>-150</td>
<td>45</td>
<td>.43</td>
<td>770</td>
<td>524</td>
<td>322</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
<td>-25</td>
<td>.38</td>
<td>815</td>
<td>550</td>
<td>326</td>
</tr>
<tr>
<td>-30</td>
<td>90</td>
<td>-30</td>
<td>.39</td>
<td>703</td>
<td>472</td>
<td>304</td>
</tr>
<tr>
<td>-60</td>
<td>30</td>
<td>-69</td>
<td>.60</td>
<td>671</td>
<td>444</td>
<td>292</td>
</tr>
</tbody>
</table>

*one oersted = $10^5$ gamma.
displacement of 7,000 tons. The correlation between the slant ranges, comparing the chart values and model values, was found to be 95 to 96 percent for the three models. This suggests that, even though differences exist between the chart values and model values, there is a high degree of correlation in the final output.

A limitation to the simple dipole field model is that it does not give an angle of declination (variation) with sufficient accuracy.\(^2\) As a result, all headings entered into this program must be in magnetic degrees. The Phoenix Corporation [Ref. 10: pp. 24-25] reports on geomagnetic field models that can represent the earth field "with overall accuracies better than approximately 150-200 gammas in magnitude and .2° in direction of the field." This degree of accuracy is not needed for this program, but a simplified version of one of these models that provided satisfactory angles of declination would be beneficial if the program were to be incorporated into a larger model that utilized true headings as inputs.

C. SUBMARINE MAGNETIC DIPOLE MOMENT

If a submarine's magnetic dipole moment is known for the geographical location and the submarine's magnetic heading, 

\(^2\)Private communication from R.N. Forrest, who investigated the use of the simple dipole model for this purpose.
the following values may be entered in the program: (1) $P$, its magnitude in oersted centimeters cubed, (2) $A$, its direction in decimal degrees relative to magnetic north, and (3) $B$, its depression angle from the horizontal in decimal degrees. If it is not known, these values must be calculated for a specific location and magnetic heading. A program is included in the main program that can be used to calculate these values. The program is based on a model described by Forrest [Ref. 9: pp. 35-38]. The input to the program is submarine displacement in tons. The program also contains coefficients which relate displacement to magnetic moment. The values used in the program are based on values cited by Texas Instruments [Ref. 6: p. 4].

The past history of the submarine is represented by the permanent longitudinal, transverse, and vertical moments of the submarine ($M_4$, $M_5$ and $M_6$ in oersted centimeters cubed). For the examples in this thesis, it was assumed that effective deperming had been performed and program default values of zero were used.

D. OTHER PARAMETERS

1. **Headings and Speeds**

   Since the simple dipole earth field model used by the program does not produce accurate angles of declination, magnetic headings are required. In addition, the headings
must be in decimal degrees. The input parameters for submarine speed and aircraft speed are entered in knots.

2. Noise

The magnetic noise is assumed to be such that adjacent magnetic noise samples are independent. This assumption is based in part on the filtering that is performed on the magnetic signal by the processing system in a MAD detection sensor. The standard deviation of the noise in units of gamma is the value entered into $S_1$. This value can be approximated from operational data by taking from one-fourth to one-sixth of the measured peak to peak magnetic noise. [Ref. 2: pp. 28-29]

The OPTEVFOR detection model incorporates a value of average peak to peak magnetic noise (N) in the inverse cube law calculation. In the program, the value of N is determined by multiplying the $S_1$ entry by four.

3. Operator Recognition Factor (ORF)

The ORF is the value of the ratio of magnetic signal to magnetic noise for which the average operator would detect a signal 50% of the time in the presence of background noise for a false alarm rate of 3 per hour. An ORF value of 3 was suggested for use by OPTEVFOR [Ref. 5: p. 4.12].

4. Distance Parameters

Two parameters, $R_8$ and $N_7$, are used to define the points plotted on the lateral range curves. $R_8$ is the
maximum positive value of the lateral range in meters for which a lateral range curve value is to be computed. \( N7 \) represents the number of lateral range curve values that are to be computed from the maximum lateral range to zero lateral range.

The vertical separation (\( Z \)) is the sum of the submarine depth and aircraft altitude in meters.
IV. RESULTS

Program outputs of the three models for a set of base case conditions are presented in this section. Outputs for variations from the base case are also presented. The lateral range of an encounter (the horizontal separation between the submarine and magnetometer when the magnetometer is at CPA) for a 50% probability of detection is used as a measure of comparison. Signal and signal plus "noise" traces for several cases are presented. The traces are based on the signal and noise models that are part of the cross-correlation and square law models. These idealized signal traces appear to have the characteristics of actual signal traces. This suggests that the signal and noise models might be used for training purposes.

A. BASE CASE

The base case conditions are listed in Table IV-1. The table is ordered in the same manner that the values are read into the program. An annotation of each entry is included for clarity.

Figure 4-1 presents the lateral range curves for the base case. Points on the lateral range curves are indicated by the first letter of the name of the model from which they were derived. The slight asymmetry of the cross-correlation
detection model and square law detection model curves is reflective of the shape of the signals that are 'received' in these models.

Table IV-1. Input Parameters for the Base Case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twice the upper bandpass limit in seconds</td>
<td>1.8</td>
</tr>
<tr>
<td>Sampling time interval in seconds</td>
<td>20.0</td>
</tr>
<tr>
<td>False alarms per hour</td>
<td>3.0</td>
</tr>
<tr>
<td>Enter inclination (1 = yes, 0 = no)?</td>
<td>0</td>
</tr>
<tr>
<td>Area of operation latitude in decimal degrees</td>
<td>30.0</td>
</tr>
<tr>
<td>Area of operation longitude in decimal degrees</td>
<td>60.0</td>
</tr>
<tr>
<td>Submarine magnetic heading in decimal degrees</td>
<td>45.0</td>
</tr>
<tr>
<td>Submarine speed in knots</td>
<td>10.0</td>
</tr>
<tr>
<td>Aircraft magnetic heading in decimal degrees</td>
<td>315.0</td>
</tr>
<tr>
<td>Aircraft speed in knots</td>
<td>220.0</td>
</tr>
<tr>
<td>Enter submarine moment (1 = yes, 0 = no)?</td>
<td>0</td>
</tr>
<tr>
<td>Enter earth field (1 = yes, 0 = no)?</td>
<td>0</td>
</tr>
<tr>
<td>Enter submarine perm moments (1 = yes, 0 = no)?</td>
<td>0</td>
</tr>
<tr>
<td>Submarine displacement</td>
<td>4000.0</td>
</tr>
<tr>
<td>Vertical separation in meters</td>
<td>200.0</td>
</tr>
<tr>
<td>Noise (standard deviation) in gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum lateral range in meters</td>
<td>1500.0</td>
</tr>
<tr>
<td>Divisions of lateral range</td>
<td>50.0</td>
</tr>
<tr>
<td>ORF (Operator Recognition Factor)</td>
<td>0.2</td>
</tr>
<tr>
<td>Variability factor for OPTEVFOR model</td>
<td>0025</td>
</tr>
<tr>
<td>Lateral range iteration number for the magnetic signal and signal plus noise in the output file</td>
<td>0025</td>
</tr>
</tbody>
</table>

Table IV-2 lists lateral detection ranges and corresponding slant detection ranges at CPA for a probability of detection equal to 50 percent for the cross-correlation and square law detection models. An equivalent ORF value for each model is also listed. Due to the asymmetry of the lateral range curves for the cross-correlation and square law models, the average of the two 50
Figure 4-1. Lateral Range Curves of the Cross-Correlation (C), Square Law (S), and OPTEVFOR (O) Models for the Base Case.

Table IV-2. The Lateral Detection Ranges, Slant Detection Ranges, and ORF's of the Three Models for the Base Case.

<table>
<thead>
<tr>
<th></th>
<th>Lateral Detection Range (meters)</th>
<th>Slant Detection Range (meters)</th>
<th>ORF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-Correlation</td>
<td>885</td>
<td>907</td>
<td>.21</td>
</tr>
<tr>
<td>Square Law</td>
<td>685</td>
<td>714</td>
<td>.44</td>
</tr>
<tr>
<td>OPTEVFOR</td>
<td>318</td>
<td>376</td>
<td>3</td>
</tr>
</tbody>
</table>

percent detection ranges was used as the lateral detection range. The equivalent ORF values for the cross-correlation range.
and square law detection models were calculated using the slant detection range values with the following equation, which was obtained from Equation 2.1:

\[
\text{ORF} = \frac{c M}{R^3 N}.
\]

For the base case, the magnitude of the submarine field \(M\) at the submarine is \(6.35 \times 10^8\) orested \(\text{cm}^3\), the noise \(N\) is \(.4\) gamma, and the value of the constant \(c\) is \(.1\). This suggests that, in order to detect a magnetic signal 50 percent of the time with a false alarm rate of 3 per hour, the magnetic signal to magnetic noise ratio should be \(.21\) for an ideal cross-correlation detector and \(.44\) for an ideal square law detector.

Using the ORF values, the cross-correlation and square law detection models can be used to describe the performance of an operator. To do this, a modified value of the standard deviation \((\sigma')\) of the input noise can be used. The modified value is equal to \((\text{ORF})(\sigma')/.21\) for the cross-correlation detection model and \((\text{ORF})(\sigma')/.44\) for the square law detection model. With these modifications, the two models can be used to describe the detection capability of an operator with a specified ORF. An example of a lateral range curve with the modified noise standard deviation for an ORF of \(3\) is presented in Figure 4.2 for each model. These curves are comparable to the lateral range curve for the OPTEVFOR model that is shown in Figure 4.1.
Figure 4.2. Cross-Correlation and Square Law Lateral Range Curves to Describe the Performance of an Operator with an ORF of 3.

The automatic MAD system manufactured by Canada's CAE Electronics Ltd. is expected to produce a 50 percent increase in detection slant range [Ref. 11]. Using the detection slant range for the OPTEVFOR model of 376 meters, a 50 percent improvement would yield a detection slant range of 564 meters. The ORF for a detection system with this capability would be .88. The cross-correlation and the square law detection models could be used to yield lateral range curves for a system with an ORF of .88 by using a noise standard deviation equal to .88 (σ')/.21 and .88 (σ')/.44 respectively. Figure 4.3 shows the lateral range curves of the two detection models with a 50 percent improvement in slant range detection. Note, with the modified noise standard deviations, the models are essentially equivalent for the cases considered.
Figure 4.3. The Cross-Correlation and Square Law Models to Describe LRC's for the CAE Automatic Detection System.

Figure 4.4. Magnetic Signal and Magnetic Signal Plus Magnetic Noise at a Lateral Range at CPA of 0 Meters for the Base Case.

Figures 4.4 and 4.5 present the magnetic signal and a representation of magnetic signal plus magnetic noise that would be received under the base conditions by a magnetometer with a lateral range of 0 meters and of 780
meters. The signal plus noise trace was generated from signal plus noise values obtained by adding a signal value to a gaussian noise value. The gaussian noise value was generated by multiplying the standard deviation of the input noise by a pseudo normal random number from a population with mean 0 and variance 1. The pseudo normal random numbers were generated using LLRANDOMII, a resident program at the Naval Postgraduate School computer [Ref. 12: p. 2.2].

Figure 4.5. Magnetic Signal and Magnetic Signal Plus Magnetic Noise at a Lateral Range at CPA of 780 Meters for the Base Case.

The magnitude of the magnetic signal shown in Figure 4.4 is very large in comparison to the background noise. The peak to peak signal to noise ratio is approximately 14 to 1. An operator would have little difficulty identifying the signal in this signal plus noise trace.

Conversely, the magnetic signal shown in Figure 4.5 is small compared to the background noise. The peak to peak
signal to noise ratio is .35. The probabilities of detection for the lateral range of 780 meters are: .95 for the cross-correlation detection model, .28 for the square law detection model, and 0 for the OPTEVFOR detection model. It seems apparent that an operator would have a difficult, if not impossible, time in detecting this signal at a reasonable false alarm rate.

B. DIFFERENT NOISE INPUTS

The first variation on the base case shows the effect of different noise inputs. The standard deviation (σ) of the peak to peak noise is the input parameter that is varied. Table IV-3 lists the different σ values and the corresponding lateral detection ranges.

<table>
<thead>
<tr>
<th>Standard Deviation of Noise in Gamma</th>
<th>Cross-Correlation</th>
<th>Square Law</th>
<th>OPTEVFOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>2250 (2259)*</td>
<td>1792 (1803)*</td>
<td>1000 (1020)*</td>
</tr>
<tr>
<td>.01</td>
<td>1832 (1843)</td>
<td>1446 (1460)</td>
<td>782 (807)</td>
</tr>
<tr>
<td>.05</td>
<td>1110 (1128)</td>
<td>863 (890)</td>
<td>427 (472)</td>
</tr>
<tr>
<td>.1</td>
<td>885 (907)</td>
<td>685 (714)</td>
<td>318 (375)</td>
</tr>
<tr>
<td>.5</td>
<td>512 (550)</td>
<td>382 (431)</td>
<td>90 (219)</td>
</tr>
</tbody>
</table>

*The numbers in parentheses are the slant range distances in meters. The vertical separation is 200 meters.

Figure 4.6 displays lateral range curves for the three models when the standard deviation of the noise is .01 gamma. These three curves show an increase in lateral
detection range over the base case. Note that the asymmetry of the cross-correlation and square law detection models is more apparent in Figure 4.6 than it was in Figure 4.1.

Figure 4.7 displays the magnetic signal (which is the same as the signal in Figure 4.5) and the magnetic signal plus magnetic noise at a horizontal distance of 780 meters when the magnetometer is at CPA. The signal to noise ratio is 3.5. The figure suggests that a MAD operator, in this case, should have the ability to detect a signal at 780 meters lateral range with a satisfactory false alarm rate.

![Lateral Range Curves for the Three Models with the Standard Deviation of the Noise Set to .01 Gamma.](image)
Figure 4.7. Magnetic Signal and Magnetic Signal Plus Magnetic Noise with the Standard Deviation of Noise = .01 Gamma at 780 Meters Lateral Range.

C. DIFFERENT HEADINGS

The headings of a submarine and an aircraft in an encounter have an effect on detection ranges. The effect of different headings was investigated using the square law detection model, and the results in terms of lateral detection ranges are presented in Table IV-4. This table suggests that a submarine should choose a magnetic heading of either East or West, and, for an encounter, an aircraft should also choose a magnetic heading of East or West.

Figure 4.8 shows lateral range curves for a submarine heading North and an aircraft heading East. In this case, both the cross-correlation and square law detection model lateral range curves display noticeable asymmetry. The OPTEVFOR detection model lateral range curve is symmetric.
Table IV-4. Square Law Lateral Detection Ranges for Different Submarine and Aircraft Magnetic Headings

<table>
<thead>
<tr>
<th>Aircraft Headings (magnetic)</th>
<th>Submarine Headings (magnetic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
</tr>
<tr>
<td>0</td>
<td>700</td>
</tr>
<tr>
<td>45</td>
<td>724</td>
</tr>
<tr>
<td>90</td>
<td>730</td>
</tr>
<tr>
<td>135</td>
<td>712</td>
</tr>
<tr>
<td>180</td>
<td>685</td>
</tr>
<tr>
<td>225</td>
<td>712</td>
</tr>
<tr>
<td>270</td>
<td>730</td>
</tr>
<tr>
<td>315</td>
<td>724</td>
</tr>
</tbody>
</table>

Figure 4.8. Lateral Range Curves for the Submarine Heading North and the Aircraft Heading East.
but, like the curves for the other models, it shows an increase in detection ranges over those for the base case (where the submarine is heading NE and the aircraft is heading NW).

The APAIR MOD 2.6 [Ref. 13: p. 83] simulation uses a MAD detection model that accounts for the change in a submarine's magnetic moment (which is dependent on changes in submarine heading) by using a parameter labeled DFACTR (degradation factor for heading). In the model, \( D \) (a modified slant range at CPA) determines the probability of detection. The value of \( D \) is determined using the following relation:

\[
D = DC (1 - \text{DFACTR} \times A), \quad \text{eqn. 4.1}
\]

where \( DC \) is the slant range at CPA and \( A \) is the acute angle in decimal degrees between the submarine heading and an East-West bearing. The probability of MAD detection is determined from a table of probability of detection against slant range. A uniform \((0, 1)\) random number is drawn to determine whether or not the submarine is detected. The average slant detection ranges (computed from Table IV-4, where the vertical separation is 200 meters) for submarine headings of North and East are 741 meters and 545 meters respectively. These ranges yield a value of .003 for DFACTR. The average slant detection range from Table IV-4 for a submarine heading of NE is 682 meters; however, the slant range determined by a modified slant range of 545
meters and a DFACTR = .003 is 643 meters. If sin A instead of A is used in Equation 4.1, then DFACTR is .265 and the slant detection for a submarine heading NE is 670 meters. Since this is only a single data point and there is no supporting operational data, the modification is not proposed as one that should be adopted. However, this cursory analysis does indicate a way in which the programs presented in this thesis might be used by others.

Table IV-5 lists lateral ranges for P(det) equal to 50 percent for 3 submarine/aircraft heading combinations. The cross-correlation and OPTEVFOR detection model results show the same relationship as the results of the square law detection model.

Table IV-5. Lateral Ranges for P(det) = .50 in Meters for the Three Detection Models.

<table>
<thead>
<tr>
<th></th>
<th>Submarine</th>
<th>Aircraft</th>
<th>Cross-Correlation</th>
<th>Square Law</th>
<th>OPTEVFOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45</td>
<td>0</td>
<td>885</td>
<td>685</td>
<td>318</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>90</td>
<td>934</td>
<td>730</td>
<td>358</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0</td>
<td>754</td>
<td>498</td>
<td>230</td>
</tr>
</tbody>
</table>

For the detection ranges reported by OPTEVFOR [Ref. 5: p. 5.1], the effect of different headings was averaged out. That is, measurements were taken from the 16 possible combinations of the 4 cardinal submarine and aircraft headings in equal numbers and then averaged to yield an average slant detection range. But, as shown in Tables IV-4 and IV-5, the models show significant variability in lateral
detection range for different submarine and/or aircraft headings.

Figure 4.9 is included to show the lateral range curves when the submarine is headed East and the aircraft is headed North. These lateral range curves give the minimum lateral detection ranges for the different heading combinations. Also, for the cross-correlation and square law detection models, the lateral range curves are fairly symmetric.

Figure 4.9. Lateral Range Curves for the Submarine Headed East and the Aircraft Headed North.

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D. SUBMARINE DISPLACEMENT

The submarine magnetic dipole moment program within the main program is used to calculate a submarine's induced magnetic moments. The program is based on a model described by Forrest [Ref. 9: pp. 35-38]. The model requires submarine displacement as an input. Table IV-6 displays results when the submarine displacement is doubled in each succeeding entry.

Table IV-6. Slant Detection Ranges in Meters for Different Submarine Tonnages.

<table>
<thead>
<tr>
<th>Displacement in tons</th>
<th>Signal Magnitude in oersted cm$^3$</th>
<th>Slant Detection Ranges in Meters</th>
<th>Cross-Correlation Law</th>
<th>Square OPTEVFOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>$1.59 \times 10^8$</td>
<td>590</td>
<td>463</td>
<td>236</td>
</tr>
<tr>
<td>2000</td>
<td>$3.17 \times 10^8$</td>
<td>732</td>
<td>575</td>
<td>297</td>
</tr>
<tr>
<td>4000</td>
<td>$6.35 \times 10^8$</td>
<td>907</td>
<td>714</td>
<td>376</td>
</tr>
<tr>
<td>8000</td>
<td>$1.27 \times 10^9$</td>
<td>1127</td>
<td>885</td>
<td>472</td>
</tr>
<tr>
<td>16000</td>
<td>$2.54 \times 10^9$</td>
<td>1402</td>
<td>1099</td>
<td>597</td>
</tr>
<tr>
<td>32000</td>
<td>$5.08 \times 10^9$</td>
<td>1724</td>
<td>1363</td>
<td>753</td>
</tr>
</tbody>
</table>

As can be seen from column two in Table IV-6, the dipole moment is proportional to the displacement. Since the three detection models give a slant detection range that is proportional to the cube root of the dipole moment, doubling the submarine displacement should multiply the slant detection range by $2^{1/3}$ (1.26). This is confirmed by comparing the slant detection ranges between the entries in Table IV-6. Doubling the displacement multiplies the slant detection range by 1.24 for the cross-correlation and square
law detection models and, as expected, by 1.26 for the OPTEVFOR detection model.

Table IV-7 lists the displacement in tons of selected Soviet submarines. The values were taken from Combat Fleets of the World 1982/1983 [Ref. 14: pp. 602-614]. This table is presented solely for the purpose of the information it contains. The submarine magnetic dipole moment program should not be expected to give accurate estimates of these submarine's induced magnetic moments, since the program uses a value that relates displacement to magnetic moment that is based on submarines of World War II.

### Table IV-7. Selected Soviet Submarine Displacements.

<table>
<thead>
<tr>
<th>Class</th>
<th>Displacement in Tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typhoon</td>
<td>25-30,000</td>
</tr>
<tr>
<td>Delta III</td>
<td>10,500-13,250</td>
</tr>
<tr>
<td>Yankee</td>
<td>8,000-9,600</td>
</tr>
<tr>
<td>Echo II</td>
<td>5,000-6,000</td>
</tr>
<tr>
<td>Victor I</td>
<td>4,300-5,100</td>
</tr>
<tr>
<td>Yankee</td>
<td>8,000-9,600</td>
</tr>
<tr>
<td>Echo II</td>
<td>5,000-6,000</td>
</tr>
<tr>
<td>Victor I</td>
<td>4,300-5,100</td>
</tr>
<tr>
<td>Charlie I</td>
<td>4,000-4,900</td>
</tr>
<tr>
<td>Tango</td>
<td>3,000-3,700</td>
</tr>
<tr>
<td>Foxtrot</td>
<td>1,950-2,400</td>
</tr>
<tr>
<td>Whiskey</td>
<td>1,080-1,450</td>
</tr>
</tbody>
</table>

E. VERTICAL SEPARATION

Figure 4.10 shows three lateral range curves for a vertical separation of 500 meters. The OPTEVFOR detection model lateral range curve shows only a slight detection probability even when the aircraft passes directly over the submarine. The cross-correlation and square law detection model lateral range curves show an increase in lateral
detection range over the base case. The dip in the lateral range curves, for each of these models, suggests the complex variation of the magnetic signal with lateral range.

![Graph showing lateral range curves](image)

Figure 4.10. Lateral Range Curves for a Vertical Separation of 500 Meters.

Table IV-8 lists the lateral detection ranges for different vertical separations. It should be kept in mind that these values are for a single geographic location; consequently, they may not be representative of other locations. Note that both the cross-correlation and square
law detection models lateral detection ranges increase with an increase in vertical separation until about 500 meters.

Table IV-8. Lateral Detection Ranges for Different Vertical Separations.

<table>
<thead>
<tr>
<th>Vertical Separation in meters</th>
<th>Lateral Detection Range in Meters</th>
<th>Cross-</th>
<th>Square Law</th>
<th>OPTEVFOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Correlation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>804</td>
<td>614</td>
<td>360</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>885</td>
<td>685</td>
<td>318</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>942</td>
<td>720</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>974</td>
<td>724</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>980</td>
<td>699</td>
<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>974</td>
<td>629</td>
<td></td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>936</td>
<td>262</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*No longer attains a probability of detection equal to 50 percent.

A factor related to vertical separation is the effect of ocean wave noise on a MAD system. As the altitude of a magnetometer is decreased, the magnitude of the ocean wave noise increases. Because of the rate of this increase, for a given submarine and submarine depth there is a minimum altitude at which an aircraft should prosecute a submarine using MAD. Further investigation using an ocean wave noise model might be valuable.
V. CONCLUSIONS

This thesis has presented a comparison of three MAD detection models. The cross-correlation detection model, which models an optimum detector under the conditions of the detection model, yields the maximum detection range for a set of given conditions. The square law detection model does not describe an optimum detector under the conditions of the model and yields shorter detection ranges. In the stochastic mode, with an appropriate choice for the parameter $\alpha$ that determines the standard deviation, the lateral range curves for the OPTEVFOR detection model become similar to the other two detection models. Detection ranges for the OPTEVFOR detection model depend on the choice for the Operator Recognition Factor (ORF). With a value of 3 for the ORF, it yields the shortest detection ranges. Adjusting the magnetic noise level by an amount proportional to the effective ORF, the cross-correlation and square law models can be used to describe the performance of an operator or an automatic detection system.

The magnetic signal and magnetic signal plus noise traces appear to have the characteristics of actual signal traces. This suggests that the signal and noise models, which are the basis for the cross-correlation and square law detection models, might be useful for training purposes.
Variations on a set of base case parameters were used to show relative changes in the detection models. The parameters included: magnetic noise, submarine and aircraft magnetic headings, submarine displacement, and vertical separation. Significant results were the large asymmetry of the lateral range curves under certain conditions and the variation of the magnetic signal as shown by the changes in vertical separation.

The FORTRAN and BASIC programs, along with an input parameter discussion, are included to facilitate the use of the three MAD detection models as they are implemented by the programs.
THIS IS A COMPILED PROGRAM FOR THE COMPUTATION OF PROBABILITIES
OF DETECTION FOR THE CROSS-CORRELATION, SQUARE LAW, AND OPTIMUM FOR
NO-CL L6

C INTEGER H,F,A,NN8,2,E,1,LL

C REAL G(1000),D1(500),D2(500),K(500),XO(500),F1,T1,T7,G1,H,F2;

C E1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15,E16,E17,E18,E19,E20;

C IF1,IF2,IF3,IF4,IF5,IF6,IF7,IF8,IF9,IF10,IF11,IF12,IF13,IF14,IF15,IF16,IF17,IF18,IF19,IF20;

C

10 FORMAT(F15.6)

20 FORMAT(I4)

30 FORMAT(90, 'EARTH FIELD = ', F15.7)

40 FORMAT(' MAG P=', F20.1, ', HOR ANG=', F10.6, ', VERT ANG=', F10.6)

50 FORMAT(TL, 'R RANG', TL, 'PD(CC)', TL, 'PD(ISL)', T27, 'PD(DP)', T35)

60 FORMAT('G111', 'T43', 'TNI11')

70 FORMAT('F11.411.4', 'F6.3', 'F6.3', 'F6.3', 'F10.3', 'F10.3', 'F10.3')

80 FORMAT('PH1', 'F15.6')

90 FORMAT('T5', 'F15.8')

C THIS PART OF THE PROGRAM INPUTS THE PARAMETERS FROM A MAD INPUT FILE.

C CR=57,2954,7531

C C=90.00/1F

C INPHT MAX FREQ AND INTERVAL TIME

C READD(4,10)F1

C T1=1.0/F1

C READD(4,10)T7

C G1=T7/5.0/T1

C IF1=IFX(G1)

C IF2=FLOAT(H1)

C IF3=H*IFX(2.0*(G1-H1)

C IF4=AT(SH1+1)

C IF5=MT5,500) GOTO 50C

C WRITE(9,20) GOTO 50C

C CONTINUE

C T1=T1#FLOAT(H1)

C INPUT FALSE ALARM PER HOUR

C READD(4,10)F2

C NH=FLOAT(H1)

C P1=F2#(HH-1.0)*T1/3600.0

C INPHT MAG DIP ANGLE CR COMPUTE IT

C READD(4,30)A
IF(AA.EQ.0) GOTO 510
C INPUT RAD MAG DIP ANGLE
READ (4,10) F
F=F/DR
GOTO 515
510 CONTINUE
C COMPLETE AN APPROXIMATICA OF DIP ANGLE
L1=76.5/DR
L2=100.0/DF
C INPUT AREA LAT LONG
READ (4,10)L
READ (4,10)L
L=L/DR
C=Q/DR
F=SIN(C-L2)*COS(L)
G1=COS(0-L2)*COS(L)
F1=SIN(L)
CALL RCTATE(G1,HL,J,K1)
J=J-(C50-L1)
G1=K1*SIN(J)
HL=K1*COS(J)
CALL RCTATE(G1,F,J,K1)
F=K1
R=(COS(L1)*SIN(J))
Q=(COS(L1)*COS(J))
CALL RCTATE(H1,F,J,K1)
F=ATAN2(2.0*TAN(J))
515 CONTINUE
C INPUT DIPOLE COURSE, C1 AND SPEED, V1.
READ (4,10)L
C1=C1/CR
READ (4,10)L
C INPUT SENSOR COURSE, C2 AND SPEED, V2.
READ (4,10)L
C2=C2/CR
READ (4,10)L
W1=V2*SIN(C2)-V1*SIN(C1)
W2=V2*COS(C2)-V1*COS(C1)
CALL RCTATE(W1,W2,CO,W0)
WRITE(6,8010)CO,W0
C CC IS REL COURSE, WO IS REL SPEED
D=W0*TI*4.63/9.0
C INPUT DIPOLE FERMENT CR APPROXIMATE BY COMPUTATION.
READ (4,30)AA
IF(AA.EQ.0) GOTO 520
C INPUT MAGNITUDE P, HOR ANGLE, VERT ANGLE
READ (4,10)F
READ (4,10)A
READ (4,10)E
A=A/DR
E=B/GR
GOTC 560
52C CONTINUE
C INPUT EARTH FIELD OR APPROXIMATE
READ (4,30)AA
IF(AA.EQ.0)GOTO 530
REAL (4,10)E1
GOTC 54C
53C CONTINUE
E1=70000.0/SQRT(3.0*COS(F)*COS(F)+1.0)
54C CONTINUE
WRITE (6,41)E1
N4=0.0
M4=0.0
M6=0.0
C INPUT PERMANENT MOMENTS OR APPROXIMATE
READ (4,30)AA
IF(AA.EQ.0)GOTO 550
REAL (4,10)M4
REAL (4,10)M5
REAL (4,10)M6
55C CONTINUE
K1=7.3
K2=1.6
K3=1.6
C INPUT DIPOLE DISPLACEMENT
READ (4,10)A1
M5=E1*K3*N1*SIN(F)
M2=M5+M4
M8=E1*COS(F)*N1*(K1*CO(S(C1))*COS(C1)+K2*SIN(C1)*SIN(C1))
M1=M4*SIN(C1)+M5*COS(C1)
M2=M4*COS(C1)-M5*SIN(C1)
M7=E1*(COS(F)*K1-K2)*N1*SIN(C1)*COS(C1)
M1=M1+M1
M2=M2+M2
CALL ROTATE(M1,M2,A,V)
CALL ROTATE(M3,V,B,P)
560 CONTINUE
WRITE (6,50)P,A,B
CALL I(NORM(P),V6)
V7=H*M(1.0-2.0)*9.0/H+F*V6*SQRT(2.0/9.0/H1)**3
C WRITE(6,10)V6
C WRITE(6,10) V7
C INPUT VERT SEPARATION, NOISE, MAX LAT RANGE & NUMBER
C CF INCREMENTS.
READ(4,10)Z
READ(4,10)S1
READ(4,10)RE
READ(4,10)N1
A=4*S1
READ(4,10)CF
READ(4,10)AL
READ(4,30)LI
D4=R8/K7
N8=2.0*N7
L5=R8
N9=IFIX(N8)+1
DC 570 E=1,A8
CALL SUM(L5,Z,E,C0,A,F,P,M,D3,S0,G,K1,N8,S1,LL,E,TN)
X0(EE)=L5
CALL MEC3(P,N,CRF,AL,L9,D5,E,Z)
L9=L9+D4
CALL PCE1(S0,K,V6,K1,S1,M,V7,D1,D2,E)
570 CONTINUE
C CLTPL PRIBS IC MAD CLTPTU FILE
WRITE(7,60)
DC 580 I=1,A8
WRITE(7,70)X0(I),D1(I),D2(I),D5(I),G(I),TA(I)
580 CONTINUE
STOP
END

C THAT SUBROUTINE COMPUTES THE MAGNITUDE OF THE SIGNAL FOR USE IN
THE TWO SIGNAL DETECTION THEORY MODELS. IT ALSO RETURNS THE SIGNAL
OF A SELECTED (L) LATERNAL RANGE IN THE GG VECTOR AND A TRACE OF A
SIGNAL PLUS RECOM NCISE (WITH STANDARD DEV = SIGMA) IN THE
TA VECTOR.

C SUBROUTINE SUM(X0,Z,E,C0,A,F,P,M,D3,S0,G,K1,N8,N,L,J,TN)
C
INTEGER M,1,K,L
REAL X0,Z,E,C0,A,F,P,M,D3,S0,G,K1,N8,N,L,J,TN
C
CALL ROTATE(X0,Z,E,F,C0)
BC=COS(B)*CCS(C0-A)
J0=COS(D)*CCS(B)*SIN(C0-A)-SIN(D)*SIN(B)
AC=-COS(D)*CCS(B)*SIN(C0-A)-SIN(D)*SIN(B)
B1=COS(F)*CCS(C0)
J1=CCS(D)*CCS(F)*SIN(C0)-SIN(D)*SIN(F)
N1=(SIN(D)*CCS(F)*SIN(C0))-CCS(D)*SIN(F)
K1=P/1.0+I*3
A2=2.0*B0*EI-J0*J1-K0*N1
A1=3.0 *(NC*B1+B0*N1)
A2=2.0 *N0*B1-B0-E1-J0*J1
SC=0.0
M=FLOAT(M)-1.0
ECMAT(T5,'TRACE CF I4)
IF(J.NE.L) GOTO 30
WRITE(G,20)J
30 CONTINUE
DC 100 I=1,M
II=FLOAT(II)-1.0
S=(II-M+2.0)*C3
Q=S/H0
G=(1.0/(1.0+Q*Q))**2.5
IF(J.NE.L) GOTO 90
G=1.0/(1.0+Q*Q)
IF(J.NE.P) GOTO 80
M2=M*2
M3=FIX(M*0.5)
CALL LNORM(1234567, TN, K2, 1.0)
CO 5C K=1,M2
TA(K)=TA(K)*A
50 CONTINUE
CO 6C K=1,M
TA(K+M3)=TA(K+M3)+G(K)
60 CONTINUE
50 CONTINUE
SO=SO+G*G
100 CONTINUE
RETURN
END
C THIS SUBROUTINE RETURNS THE PROB OF DETECTION FOR THE TWO SIGNAL
C DETECTION THEORY MODELS.
C
SUBROUTINE PROB1(SO,K,V6,K1,S1,M,V7,D1,D2,E)
C
INTEGER M,E
REAL SO,V6,K(1000),V1,K1,V7,D1(1000),D2(1000),V,S1
REAL K2,V6,KM,A3,L0,B3,V9
K2=SQRT(SO)
K(E)=K2
V8=V6*K1*K2/S1
L0=K1*K1*SC/(S1*S1)
M=FLOAT(M)
A3=NM+L0
\[ B^2 = 1.0 + L0/(MM+L0) \]
\[ V5 = -\text{SQRT}(2.0 \times V7/B3) + \text{SQRT}(2.0 \times A3/B3 - 1.0) \]

C
CALL IFROB(V8,Y)  \[
D1(E) = Y
\]
CALL IFROB(V9,Y)  \[
D2(E) = Y
\]
RETURN  \[
EAD
\]

C THIS SUBROUTINE RETURNS THE ANGLE AND MAGNITUDE OF A VECTOR SUM.
C
SUBROUTINE FROTATE(L,V,J,K)
REAL \( K, U, V, J \)
C
\[ K = \text{SQRT}(U^2 + V^2) \]
IF(K.EQ.0.0) GOTO 10
\[ J = \text{ATAN2}(L, V) \]
GOTC 20
10 CCNTINUE
J = 0.0
20 CCNTINUE
RETURN
EAD

C THIS SUBROUTINE RETURNS \( Z \) GIVEN \( F(Z) \) FOR THE NORMAL PROBABILITY FCN.
C
SUBROUTINE INORM1(X,Y)
REAL \( Y, G, G1, G2, H1, H2, H3, X \)
C
\[ Y = X \]
IF(X.GT.0.5) \( Y = 1.0 - Y \)
\[ Y = \text{SQRT}(\text{LOG}(1.0/Y/Y)) \]
G = 2.515517
G1 = 0.802853
G2 = 0.010328
H1 = 1.432788
H2 = 0.189269
H3 = 0.001306
\[ Y = (G + Y*(G1 + Y*G2)) / (1.0 + Y*(H1 + Y*(H2 + H3*Y))) \]
IF(X.GT.0.5) \( Y = -Y \)
RETURN
EAD

C THIS SUBROUTINE RETURNS \( F(Z) \) GIVEN \( Z \) FOR THE NORMAL PROB FCN.
C
SUBROUTINE IFROB(X,Y)


C REAL X,Y,b,Q1,Q2,Q3,Q4,Q5,PI,YY
C
Y=X
IF(X.LT.0.5) Y=-Y
M=1.0/(1.0+0.2316415*Y)
Q1=0.31938153
Q2=0.356563782
Q3=1.781477937
Q4=1.622689998
Q5=1.330274429
IF(Y.GT.15.0) GOTO 100
PI=3.141592654
YY=1/(Q1+(Q2+Q3+Q4+Q5)))/SQRT(2.0*PI)
Y=EXP(-YY**2)/YY
GOTO 110
100 CONTINUE
Y=0.0
110 CONTINUE
IF(X.GE.0.5) Y=1.0-Y
RETURN
END

C THIS SUBROUTINE RETURNS THE PROBABILITY OF DET FOR THE OPTEVFOR MODEL
C
C SUBROUTINE DCD3(M,N,CRF,AL,L9,D5,E,V)
C
C INTEGER E
REAL M,N,OFF,AL,L9,L5(500),RH,C,SIG,Z,Y,P,V
C
C C=0.10
P=1.0/3.0
RH=(C*M/(OFF*N))**P
SIG=AL**RH
P=(1.0**2+V**2)**0.5
Z=(RH-P)/SIG
CALL IFROBZ(Z,Y)
D5(E)=Y
RETURN
END
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51
10 DIM O(200), D(100), K(100), X(100)
15 DIM D5(100)
20 DEG
30 PRINT "MAX FREQ"
40 INPUT F1
50 PRINT "MAX FREQ = " F1
60 STOP
70 PRINT "INTERVAL TIME = " T
80 T=T7/2/T1
90 A=INT(T)
100 H=H+INT(2*(O-H))
110 M=M+H+1
120 IF M>200 THEN 70
130 PRINT "INT TIME = " T
140 T=T+1
150 PRINT "ADJ INT TIME = " T
160 PRINT "SAMPLE SIZE = " M
170 PRINT "P/A RATE = "
180 INPUT F2
190 PRINT "P/A RATE = " F2
200 P=P*2*(M-1)*T/T7
210 PRINT "PF = " F1
220 PRINT "INPUT DIP ANGLE (1=YES, 0=NO)" INPUT A
230 IF A=0 THEN 200
240 PRINT "DIP ANGLE PHI = " INPUT F
250 GOTO 420
260 DO 11 L1=76, L2=100
270 PRINT "LATITUDE = " L
280 INPUT L
290 PRINT "LONGITUDE = "
300 INPUT D
310 PRINT "LAT = " LPRINT "CON = " 10
320 F=SIN(O-L2)*COS(L) I=1-COS(O-L2)*COS(L) H=SIN(L)
330 U=0+H
340 OBOSUB 1990
350 J=J-(K-L)+SIN(J)*I+K+COS(J)
360 U=0+V+F
370 OBOSUB 1990
380 F=1*G-(COS(L)*SIN(J)*I+-(COS(L1)*COS(J)) I=-(COS(L1)*COS(J))
390 U=0+V+F
400 OBOSUB 1990
410 F=ATN2(O2-COS(J)/COS(I))
420 PRINT "PHI = " "1F"
430 PRINT "DIP ANGLE COURSE = " INPUT C1 PRINT "DIP ANGLE SPEED = " INPUT V1
440 PRINT "SENSORS COURSE = " INPUT C2 PRINT "SENSORS SPEED = " INPUT V2
450 PRINT "DIP ANGLE COURSE = " C1 PRINT "DIP ANGLE SPEED = " V1
460 PRINT "SENSORS COURSE = " C2 PRINT "SENSORS SPEED = " V2
470 W1=W-V*IN(C2)-V*IN(C1) W2=W*IN(C2)-V*IN(C1)
480 U=W*V+W2
490 OBOSUB 1990
500 C0=1+M
510 PRINT "REL COURSE = " C0 PRINT "REL SPEED = " V0
520 PRINT "INPUT DIP ANGLE MOMENT (1=YES, 0=NO)" INPUT A
530 IF A=0 THEN 530
540 PRINT "MAGNITUDE P = " INPUT P PRINT "HOR ANGLE = " INPUT A
550 PRINT "VERT ANGLE OMEGA = " INPUT B
560 GOTO 420
570 PRINT "EARTH FIELD (1=YES, 0=NO)" INPUT A
580 PRINT "INPUT AA
520"
1130 NEXT I
1150 PLOT XQ(i), D2(i)
1170 FOR I=1 TO N
1180 DRAWTO XQ(i), D2(i)
1190 NEXT I
1191 PLOT XQ(i), D3(i)
1194 FOR I=1 TO N
1195 DRAWTO XQ(i), D3(i)
1199 NEXT I
1200 PRINT "PD FOR X FROM "I-RA1" TO "RA
1209 GOTO 1614
1219 PRINT "FOR HARD COPY ENTER '1'"; INPUT CC
1239 IF CC=1 THEN GOTO 1099
1259 PRINT "MAX FREQ = "F1
1279 PRINT "ADJ INT TIME = "IT7
1289 PRINT "SAMPLE SIZE = "M
1299 PRINT "F/A RATE = "F2
1319 PRINT "FF = "F1
1339 PRINT "LAT = "IL
1349 PRINT "LCN = "T0
1359 PRINT "PHI = "IF
1369 PRINT "DIPOL COURSE = "IC1
1379 PRINT "DIPOL SPEED = "IV1
1389 PRINT "SENSOR COURSE = "IC2
1399 PRINT "SENSOR SPEED = "IV2
1409 PRINT "REL COURSE = "IC0
1419 PRINT "REL SPEED = "IV8
1429 PRINT "EARTH FIELD = "IE1
1439 PRINT "LONG MOMENT = "IM4
1449 PRINT "TRAN MOMENT = "IM5
1459 PRINT "VERT MOMENT = "IM6
1469 PRINT "DISPLACEMENT = "INH1
1479 PRINT "P = "IP
1489 PRINT "L = "LA
1499 PRINT "OMEGA = "I0
1509 PRINT "VERT SEPARATION = "IZ
1519 PRINT "NOISE = "IN1
1529 PRINT "MAX LATERAL RANGE = "IR8
1539 PRINT "NUMBER OF INCREMENTS = "IN7
1549 PRINT "LPRINT IPRINT LPRINT LPRINT" PD(RC) PD(RS) PD(OPT)
1559 LS=RA
1569 FOR I=0 TO N9
1579 PRINT LS="I01(I) " "I02(I) " "IP5(I)
1595 PRINT LS=LS+0A
1599 NEXT I
1609 GOTO 1099
1619 PRINT "END"
1629 END
1639 U=X0+v1*Z
1649 D=J1+0*K
1669 RH=(L1*F/(QRF+62))\$0.333
1669 SIG=AL=RH
1679 X=(RH-HB)/SIG
1689 G=SIG 1999
1699 DO=SIG 1999
1709 DS(0)=Y
1719 BS=COS(D)=COS(D)=COS(D)=SUM(D)=COS(D)-SUM(D)=SUM(D)
1729 NS=COS(D)=COS(D)=COS(D)=COS(D)-SUM(D)=SUM(D)
1739 BS=COS(F)=COS(D)=SUM(D)=SUM(D)=COS(D)-SUM(D)=SUM(D)
1749 NS=COS(F)=COS(F)=COS(F)-SUM(D)=SUM(D)
1759 K=F/10/HB*5
1769 A2=N+1-B1-J0+J1-N0+1; A1=3*(N0+1+B0+M1); A3=2+N0+M1-B0+M1-J0+J1
1779 QB=3
1779 FOR I=0 TO N-1
1789 8=(I-(M-1)/2)*03
1789 QB=3/10
1759 G=1/(1+G)*2.5
1760 G=(A2+G+1+G+G)/G
1770 G(I)=G+G+G+G+G
1780 NEXT I
1790 RETURN
1900 K=SQR(SW)
1910 K=E*K
1920 V8=V3-K=SQR(B8)/81
1930 LD=K=K1=SQR/(81+B1)*A3=M+L0*B3=1+L0/(M+L0)
1940 V3=SQR(2+V7/6)+SQR(2+V7/83-1)
1950 X=V8:GO8UB 1990
1960 D1(K)=Y
1970 X=V3:GO8UB 1990
1980 D2(K)=Y
1990 RETURN
1900 K=SQR(U+V+V+V):IF K=0 THEN J=0:RETURN
1995 UK=U/K:KV=KW/K
1997 IF (UK) 0.999999 AND (VK) 0.999999 THEN J=0:RETURN
1998 IF (UK)<=0.999999 THEN J=ATN(VK/SQR(-UK+VK+1))+30:RETURN
1999 IF (VK)=0.999999 THEN J=0:RETURN
1995 IF MM (0 THEN J=360-J
1995 RETURN
1920 Y=X:IF X)0.5 THEN Y=1-Y
1930 Y=SQR(108(1/Y)/Y)
1940 U=2.5153171*G=0.892533*G2=0.892533
1950 H1=1.122768:K2=0.189329:K3=1.3858-83
1960 Y=-(G1+G2+G3)/(1+Y*(H1+Y*(H2+H3+Y)))
1970 IF X)0.5 THEN Y=1-Y
1980 RETURN
1990 Y=X:IF X)0 THEN Y=-Y
2000 Y=1/(1-0.215419*Y)
2010 Q1=1.3138135:1*K=2=0.35626782193=1.781477931:Q4=-1.8212593:Q5=1.3827442
2020 IF Y)24.23 THEN Y=0:GO8UB 2070
2025 P=1.14159265
2030 Y=EXP(-2(Y+Y/2))\SQR(2*PI)+\(Q1+Q2+Q3+Q4+Q5))\}
2030 IF X)0 THEN Y=1-Y
2032 Y=(INT(1000+Y))/1000
2035 RETURN

55
MAX FREQ = 1.13
ADJ INT TIME = 20.55555556
SAMPLE SIZE = 37
F/A RATE = 3
PP = 0.0166666666
LAT = 39
LON = 59
PHI = 39.48076379
DIPOLE COURSE = 43
DIPOLE SPEED = 10
SENSOR COURSE = 315
SENSOR SPEED = 220
REL COURSE = 312.397439
REL SPEED = 220.227153
EARTH FIELD = 52366.5513
LNG MOMENT = 0
THN MOMENT = 0
VERT MOMENT = 0
DISPLACEMENT = 4000
P = 65.694922
w = 32.67790791
OMEGA = 27.11173173
VERT SEPARATION = 200
NOISE = 0.1
MAX LATERAL RANGE = 1500
NUMBER OF INCREMENTS = 15

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<td>10</td>
<td>Director</td>
<td>Code 331AA Center for Wargaming Newport, Rhode Island 02840</td>
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<tr>
<td>11</td>
<td>Commander</td>
<td>Naval Electronic Systems Command Attn: PME 120 2511 Jefferson Davis Highway Arlington, Virginia 20360</td>
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<td>16</td>
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<td>Naval Ocean Systems Center San Diego, California 92152</td>
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<tr>
<td>17</td>
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<td>4301 Suitland Road Washington, D.C. 20390</td>
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<tr>
<td>18</td>
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<td>Navy Research Laboratory Washington, D.C. 20375</td>
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<td>19</td>
<td>Center for Naval Analysis</td>
<td>2000 N. Beauregard St. Arlington, Virginia 22311</td>
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<td>20</td>
<td>Lcdr Dick Grahlman</td>
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21. Lt Daniel C. Schluckebier  
    Asst. CATC  
    USS Dwight D. Eisenhower (CVN-69)  
    FPO, New York  09532

22. Commander  
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    Naval Ship Research and Development Center  
    Bethesda, Maryland  20817