THESIS

APL TUTOR:
AN ON-LINE INSTRUCTIONAL FACILITY

by

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December 1983

Thesis Advisor: R. R. Read

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This thesis describes a set of APL programs which enable a student to learn A Programming Language (APL) by using it. The student needs to know only how to log on to the computer and enter a few simple commands to begin the course.

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without going through an entire lesson. Other functions conduct interactive question-and-answer drill. A MENU function lists the units for student selection of a lesson or information on a symbol. The student can also go through a sequence of lessons in a computer-driven course.

These programs were written on and for the IBM 3033 installation at Naval Postgraduate School, using APL version 4.0 for VM/CMS with IBM 3278 terminals.
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ABSTRACT

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# TABLE OF CONTENTS

I. BACKGROUND ............................................. 7

II. USING APL TUTOR ........................................ 9

III. EXTENDING APL TUTOR ................................. 12

APPENDIX A: USER'S GUIDE TO APL TUTOR ................. 14

APPENDIX B: PROGRAMMER'S GUIDE .......................... 20
  A. FLOW CHARTS ...................................... 20
  B. FUNCTIONS ....................................... 42
  C. VARIABLES ....................................... 67
     1. Global variables ............................... 67
     2. Text variables ................................ 73
  D. MAKING MODIFICATIONS ............................. 166

APPENDIX C: SAMPLE RUN ................................... 168

BIBLIOGRAPHY ............................................. 199

INITIAL DISTRIBUTION LIST .............................. 200
LIST OF FIGURES

B.1 Interrelationship of TUTOR Functions .......... 22
B.2 Procedure of START Function ................. 23
B.3 Procedure of MENU Function .................. 23
B.4 Procedure of HELP Function .................. 24
B.5 Procedure of INFO Function .................. 25
B.6 Procedure of RUN Function ................... 26
B.7 Procedure of TEACH Function ................... 27
B.8 Procedure of LESSON Function ................. 28
B.9 Procedure of SHOW Function--1 ................. 29
B.10 Procedure of SHOW Function--2 ................. 30
B.11 Procedure of SHOW Function--3 ................. 31
B.12 Procedure of ASK Function .................... 32
B.13 Procedure of SUMASK Function ................. 33
B.14 Procedure of SCORE Function ................. 34
B.15 Procedure of RUNDRILL Function ............... 35
B.16 Procedure of DRILL Function ................ 36
B.17 Procedure of PULL Function .................. 37
B.18 Procedure of TRY Function--1 ................. 38
B.19 Procedure of TRY Function--2 ................. 39
B.20 Procedure of TEST Function .................. 40
B.21 Procedure of SQUEEZE, NEXTLESSON, and
       ORDERMAT Fns ............................ 41
I. BACKGROUND

A Programming Language (APL) is an interactive, interpretive computer language designed by Kenneth E. Iverson at Harvard in 1962. Its advantages include the ease of manipulation of multi-dimensional arrays of numbers, the lack of rigorous input/output formatting, and the use of unique symbols to represent a wide variety of built-in functions. Unlike other high-level languages, there is no requirement to write and compile a strictly formatted program before execution. APL may be used in "desk calculator" mode, where a single line of input is accepted and evaluated, returning an immediate response. While it may take quite some time for the user to master APL fully, the beginner can quickly jump in and start experimenting.

The APL TUTOR was designed to enable the beginning APL student to gain some familiarity with the basic concepts of the language without going through a formal course. It consists of a set of APL programs which enable the student to enter simple commands and receive information, questions, and drill about the symbols and functions used in APL. Since at present no information on writing functions is included, it may be used as a supplement to the normal mode of teaching at NPS.

Currently the APL student's resources at NPS consist of the instructor, other students, and texts such as those listed in the bibliography. There also exists a copyrighted IBM drill function, TEACH, found in the APLCOURS workspace in the public library of most APL installations. (At NPS, it can be accessed by entering )LOAD 1 APLCOURS while in APL mode.) This function, and a sub-function, EASYDRILL, provide simple drill questions for calculator functions
selected by the student. This workspace does not, however, provide any information concerning the correct use or syntax of any functions.

The specific audience targeted is the first to third quarter NPS students in the Operations Analysis curriculum, who are currently taught APL in one-hour weekly labs as part of three probability and statistics courses. Many, if not most, of these students arrive at NPS from six weeks to six months before their first quarter. It would be to their advantage to get a 'head start' on learning APL, but there is no separate course in this language as there is in FORTRAN or PASCAL, for example.

A basic requirement for the TUTOR, then, is that it be comprehensible to someone who has never seen APL before. In fact, the "desk calculator" functions should be understandable to someone who has never even used a computer before. This means that some of the beginning lessons may seem too simplistic to the student who has some programming experience. However, the more advanced student can easily skim through this primary material and progress to the more sophisticated concepts. Intermediate mathematics including concepts of linear algebra is the only background information that is assumed.
II. USING APL TUTOR

The APL TUTOR workspace should be available in one of the public libraries at NPS. Instructions for loading and copying it are on the first page of Appendix A. If it is not available in the public libraries, it can be obtained from Professor R. R. Read upon request.

The APL TUTOR workspace contains sixteen functions which administer the course. Their interrelationship is shown in Figure B.1. In this diagram, the ovals indicate functions which are called by the student, while rectangles indicate functions which are called internally. The procedure of each function is outlined in Figures B.4 through B.21. Further information can be found in the comment lines of each program in Appendix B, part B.

The actual text of the course is contained in APL variables which are displayed by the functions SHOW and RUN. The text of each variable can be found in Appendix B, part C.2. Also in Appendix B (part C.1) are several global variables which are used in various programs. Most important of these is MAT, which provides information on each text variable, such as the symbol it pertains to, the function type (Monadic/Dyadic/Neither), the type of arguments it takes (Numeric, character(K), Either, Boolean), the rank of the arguments permitted (Scalar, Vector, matrix(X), Any), and a sequential lesson number. The other major global variable is CUES, which consists of a number of questions that can be called upon by the ASK function.

Before commencing the APL TUTOR course, the student needs to have, as a minimum, the instructions given on the first page of Appendix A. The other information the student will need is displayed by the TUTOR, and is also included in
Appendix A, which should be given to all students who will be using TUTOR. The variable HOW is displayed each and every time the workspace TUTOR is loaded by the student, and can also be called up at any time by entering HOW. The other variables in Appendix A are displayed by the START function. Most may be called up by the student at any time, as noted.

Figure B.1 shows the variety of ways the student can access and use the information in TUTOR. The beginner enters START to get the basic background information needed to complete the course. START instructs the student to enter LESSON 100 to begin the course. After completing this lesson, the student merely has to enter LESSON NEXT to receive the instructional units in the order indicated in the variable SAT.

The student who is enrolled in a course using APL may also follow the instructor's guidance concerning lessons to take. The student enters LESSON NNN, where NNN is the lesson number assigned. A more advanced student can explore or review earlier lessons by entering TEACH and then the symbol that he/she is interested in.

The more advanced student may also be interested in the HELP function which displays information without asking questions or providing a drill. The student can enter HELP, followed by the appropriate symbol, or INFO NNN, where NNN is the same as the pertinent lesson number. These lesson numbers can be found by entering MENU and browsing the list displayed. MENU also repeats the instructions for using LESSON and INFO.

Effort has been made to ensure that the programs will not terminate abnormally in response to a student input error. Every input is checked for validity before it is processed. For example, see the function DRILL, page 59, at lines 8 to 17. However, because it is not possible to check
for every possible input or combination of inputs, there may still be some way to cause an error. Instructions to the student in case of an error are included in the BASICS displayed by the START function. The student should have a hard copy of these instructions, as well.

Any input by the student at a time when no input is called for will cause an abnormal termination. This is inherent in the APL interpreter and cannot be avoided by the programmer. However, all the student needs to do in this case is restart by entering one of the elementary commands (HELP, TEACH, LESSON, etc.)

A sample run is included at Appendix C.
There are many ways in which the APL TUTOR can be expanded to improve its usefulness. Lessons could be included on the many system commands and variables which are used in APL. Auxiliary lessons on groups of functions, e.g. logical functions, and their basic concepts could provide useful additional information. Lessons could be added to describe programming techniques, and amplifying lessons on applications could be provided. Probably any of the lessons could benefit from additional questions to be included in the ASK function. Also, the ASK function itself could be modified to permit more complex questions and answers.

Another type of improvement from the pedagogical point of view would be the inclusion of facilities for measuring and recording students' progress. The student could be tested after every lesson or group of lessons and the scores recorded for the instructor's review. Or, the course could be designed to automatically review the student's weak areas and retest before going on. In any case, it might be beneficial to include lessons which review concepts covered by the course so far.

A test and evaluation of the TUTOR would be extremely beneficial in comparing it to the current teaching methods. Students could be tested for their knowledge of APL and those who had used the TUTOR compared to those who had not. Furthermore, different versions of the TUTOR could be tested against each other. Several different texts could be used, or versions with and without the drill functions.

This could lead to a still more sophisticated modification of the course, in combination with programmed student testing. After an initial unit, the testing could be
designed to reveal what type of instruction, e.g. drill or no drill, works best for that individual student. Or, the student could simply be asked his or her preferences in terms of teaching techniques. Then the course would automatically tailor later lessons to the individual.

All of these changes require someone skilled in APL to modify the current programs. These programs have been heavily documented by flow charts and comment lines (Appendix B) in order to make the task of modification easier. Also, specific instructions for certain types of modifications are included in the variable HOWMODS, Appendix B, part D. Any advanced student can personalize a copy of TUTOR with the help of these tips. For more substantial modifications such as those suggested above, the point of contact for the public version of TUTOR at NPS is Professor R. R. Read.
APPENDIX A
USER'S GUIDE TO APL TUTOR

IF YOU HAVE NEVER EVER USED THE COMPUTER BEFORE, START HERE:

1. Go to the Registration and Accounting Office inside Ingersoll room 141. Ask for a user number. You will need to tell them the password you want to use.
2. While you are there, pick up a copy of NPS Technical Note VM-01, "User's Guide to VM/CS at NPS."
3. Follow the instructions in VM-01 for logging on and formatting your disk. If your keyboard has little red symbols on it, you may continue with step 6 below. Otherwise, log off (see VM-01) and continue with step 4 when you are ready.

IF YOU HAVE FORMATTED YOUR DISK, START HERE:

4. Find a free terminal (in Ingersoll room 141 or room 369) that has red symbols on the keyboard.
5. Log on and enter your password.
6. When you see a line beginning CMS..., press the ENTER key.
7. When you see a line beginning R: ..., enter the letters APL.
8. You should see several lines appear ending with the line CLEAR WS. If you don't, stop here and get help!
9. Now look at the bottom center of your screen, below the line. If you see the letters APL, go on to the next step. If you don't, hold down the ALT key (next to the ENTER key) and press the large key at the top right of the main keyboard which has APL ON-OFF in red letters on the front. The letters APL should now appear at the bottom of your screen. This tells the terminal to use the red symbols on the front of the keys, you must hold down the ALT key instead of the SHIFT key.
10. IF THIS IS THE VERY FIRST TIME YOU ARE USING APL TUTOR, ENTER LOAD 5 TUTOR. (Remember to use the red parenthesis, third row, far right, not the black parenthesis in the top row.) OR FOLLOW THE DIRECTIONS GIVEN BY YOUR INSTRUCTOR. If you have used TUTOR before, and you followed the directions in steps 12 and 13 below, just enter LOAD TUTOR.
11. Now just follow the directions which appear on the screen. If no directions appear, and all you see is a line which says SAVED... and possibly a line which says WSIZE..., enter HOW to see the directions. To be sure that the directions appear every time, enter START, and follow steps 12 and 13 when you finish your session.
12. IF THIS IS THE FIRST TIME you have used TUTOR, enter INSID TUTOR.
13. When you are ready to quit for the day, enter )SAVE.
14. When you see the time, date, and TUTOR, enter )OFF. This will log you off the computer completely. (For more advanced students: )OFF HOLD will return you to CMS.)
HOW

YOU MAY USE THE APL TUTOR IN THREE WAYS:

(1) ENTER: HELP
    TO SELECT THE SYMBOLS THAT YOU WANT INFORMATION ABOUT,

(2) ENTER: TEACH
    TO SELECT THE SYMBOLS THAT YOU WANT INFORMATION AND DRILL ON,

(3) ENTER: MENU
    TO SEE A LIST OF SYMBOLS AND TOPICS,

IF YOU HAVE NEVER USED THE APL TUTOR BEFORE, ENTER: START

TO SEE THESE INSTRUCTIONS AGAIN AT ANY TIME, ENTER: HOW
INTRO

WELCOME TO THE APL TUTOR,

THE PURPOSE OF THIS WORKSPACE IS TO INTRODUCE YOU TO 'A PROGRAMMING LANGUAGE' BY DESCRIBING THE FUNCTIONS OF THE MANY SPECIAL APL SYMBOLS, AND BY OUTLINING THE PROCEDURES FOR DESIGNING YOUR OWN FUNCTIONS.

BACKGROUND

THE APL TUTOR ASSUMES YOU HAVE HAD LINEAR ALGEBRA AND TRIGONOMETRY, CALCULUS IS NOT NECESSARY,

IF YOU HAVE NEVER HAD ANY COMPUTER PROGRAMMING BEFORE, DON'T WORRY, YOU CAN START USING APL RIGHT AWAY, AS A SUPER-SOPHISTICATED CALCULATOR WITH MANY BUILT-IN FUNCTIONS,

IF YOU HAVE STUDIED OTHER COMPUTER PROGRAMMING LANGUAGES, RELAX, APL IS NOT LIKE ANY OF THE OTHER MAJOR HIGH-LEVEL LANGUAGES, YOU CAN FORGET ABOUT DATA TYPES, INPUT/OUTPUT FORMATTING, AND MANY OF THE OTHER TEDIOUS DETAILS OF FORTRAN, PASCAL, ETC.

AFTER YOU ARE SUFFICIENTLY FAMILIAR WITH THE CALCULATOR MODE OF APL, YOU CAN LEARN TO DEFINE YOUR OWN FUNCTIONS WITH EASE.
HERE IS SOME BASIC INFORMATION YOU WILL NEED TO KNOW IN ORDER TO UNDERSTAND THE APL TUTOR LESSONS.

**MONADIC AND DYADIC FUNCTIONS**

You are already familiar with symbols which represent arithmetic functions, such as + or - . APL uses these symbols and many others to represent a variety of functions. Many functions, such as + , require two arguments; that is, two input numbers. In APL, these are called dyadic functions, and the symbol is placed between the arguments; for example, 3+4. Other functions, such as LN (natural log), require only one argument. These monadic function symbols are placed to the left of the data which they are to operate on, for example, LN 3 in APL is 03.

**DATA TYPES**

APL distinguishes only two types of data; numeric and character. Very simply, character data are enclosed in quotes (') when entered. '2' is character data; 2 is numeric data. Some functions will operate on both types of data, some only on numeric.

**ARRAYS**

The greatest strength of APL lies in its ability to take an entire array of numbers as a single argument, thus two matrices can be added by entering simply A+B, with no subscripts, loops, etc. This makes it important to know the rank (the number of dimensions) of data in use. A single number is normally a scalar (rank 0), a series of numbers is a vector; a one-dimensional array (rank 1). Numbers can also be arranged in rows and columns, to make a matrix (rank 2). Matrices can be 'stacked' to make up the pages of a three-dimensional array, often called a book (rank 3). In fact, there is no limit to the number of dimensions in an APL array. The length of the dimensions is also effectively unlimited. Length
Refer to the number of elements in a dimension, for example, the number of rows. Length may even be 0.

In this tutorial, a scalar may be referred to as s, l, or r.
A vector may be called v, v1, or v2, while a matrix is M, M1, or M2.
An array, which may be a vector, a matrix, or an array of any higher dimension, will be labelled A, A1, or A2.

Examples
When examples are given in the text of a function description, the symbol \( \Rightarrow \) is used between the example input and the example output.
For example: \( 2+5 \Rightarrow 7 \)
This symbol can be read as 'produces' or 'returns'.

Errors
You should not receive any error messages while using APL tutor,
however, you may see a phrase like 'value error' (without quote marks) when you are expecting a normal response,
or if you accidentally hit the enter key when the computer is not expecting an input, the function may end abruptly, like this:
Menu [6]
If either of these things happen, be sure to enter: 4 (upshift +)
then restart with any command.

Halting a lesson
You may stop any lesson at any point where a question is asked
by entering: stop

To see this information again at any time, enter: basics
HOWTEACH

YOU MAY START A LESSON IN THREE WAYS:

(1) ENTER: LESSON NEXTLESSON
TO START THE LESSON FOLLOWING THE LAST ONE YOU COMPLETED.

(2) ENTER: LESSON NNN (WHERE NNN IS A 3-DIGIT NUMBER)
TO START LESSON NUMBER NNN,
TO SEE A LIST OF LESSON NUMBERS, ENTER: MENU

(3) ENTER: TEACH
TO SELECT THE SYMBOL THAT YOU WANT A LESSON ON.

TO SEE THIS INFORMATION AGAIN AT ANY TIME, ENTER: HOWTEACH
APPENDIX B
PROGRAMMER'S GUIDE

A. FLOW CHARTS

Figure B.1 shows the relationship of the various functions within the TUTOR workspace. The first function called by the new student should be START. After that, the student will usually commence a session by calling MENU, TEACH, HELP, LESSON, or INFO. These functions in turn call the other functions of TUTOR as indicated by the arrows. The procedure of these functions is outlined in figures B.2 through B.21.

In APL, there is no difference in the programming of main routines, subroutines, and functions; all are referred to as functions. However, when one function calls another function, the calling function is suspended until the called function is completed. Then the calling function resumes operation, possibly using the result of the called function. Therefore, "return to the calling function, if any" is implied at the end of every program but is not explicit. END, implying return, is shown in the diagrams only in those functions which are normally called by another function.

It should be noted that "room for expansion" has been provided in the SHOW, RUNDRILL, DRILL, and TEST functions. RUNDRILL and DRILL have the capability to display drill questions with matrix arguments. This capacity is not currently being used because the answer must be input in vector form. It was felt that this might confuse the student. However, further development and experimentation might find uses for this type of question, so the matrix form has been left available. TEST and SHOW have calling
lines for the matrix form commented out. There are also comment lines in SHOW to indicate where matrix and higher-level arrays questions would go if they are added in the future. See the section on making modifications for more information.

The four functions at the bottom of figure B.1 are not part of the TUTOR function hierarchy. NEXTLESSON is used to provide a lesson number when the student calls for LESSON NEXTLESSON. ORDERMAT is provided for the benefit of the programmer to rearrange the sequence of lessons in the course. TEST is also provided for the programmer (or student) who wishes to run only the drill section of a lesson. SQUEEZE is a utility called by several functions to delete blanks in a string of characters.
Figure B.1  Interrelationship of TUTOR Functions.
Figure B.2 Procedure of START Function.

Figure B.3 Procedure of MENU Function.
Figure B.4 Procedure of HELP Function.
Figure B.5 Procedure of INFO Function.
Figure B.6  Procedure of RUN Function.
Figure B.7 Procedure of TEACH Function.
LESSON (NNN)

NNN scalar?

Y

Find ROW Index.

No ROW?

Y

Regrets message

N

SHOW (ROW)

Ending message

Figure B.8 Procedure of LESSON Function.
Figure B.9  Procedure of SHOW Function—1.
Figure B.10  Procedure of SHGW Function—2.
Figure B.11 Procedure of SHOW Function--3.
Figure B.12  Procedure of ASK Function.
Figure B.13 Procedure of SUMASK Function.
Figure B.14 Procedure of SCORE Function.
Figure 8.15  Procedure of RUNDRILL Function.
Figure B.16  Procedure of DRILL Function.
Figure B.17 Procedure of PULL Function.
Figure B.18 Procedure of TRY Function--1.
Figure B.19  Procedure of TRY Function--2.
Figure B.20  Procedure of TEST Function.
Figure B.21 Procedure of SQUEEZE, NEXTLESSON, and ORDERMAT Fns.
B. FUNCTIONS

The following section contains the APL functions which operate the TUTOR course. The comment lines, which begin with a symbol, indicate the procedure in general, but do not describe the specific programming techniques used.
START

[1] e ensures 'how' appears when workspace is loaded
[2] dlx+ 'how'
[3] e displays information for first-time user
[4] dl+intro
[5] dl+background
[6] dl+basics
[7] dl+howteach
[8] dl+ce,'to start your first lesson in apl, enter: lesson 101'

.
MENU;ZZZ

[1] A REQUIRES GLOBAL VARIABLE MAT

[2] INFORMATION IS AVAILABLE ON THE FOLLOWING SYMBOLS/TOPICS:

[3] FORMAT AND DISPLAY MENU

[4] ZZZ+(51)+0,(17)+0

[5] (((((P[4](11)-3),72);ZZZ\MAT[;LESCOL,SYMBOL,FRAMECOL]

[6] TO SEE MENU AGAIN, ENTER: MENU',CR,' FOR INFORMATION ABOUT FUNCTION, ENTER: INFO HNN (WHERE HNN IS MENU NUMBER)

',CR,' TO GO THROUGH TUTORIAL LESSON, ENTER: LESSON HNN'
HELP;SYMBOL;ROWS;HNNHR
[1] CALLS QUEUE, MENU, SQUEEZE
[2] "ENTER THE SYMBOL(S) YOU WOULD LIKE INFORMATION ABOUT,'" 
[3] "OR, 'FOR MENU SELECTION, ENTER: MENU'
[4] DELETE BLANKS IN RESPONSE
[5] SYMBOL=SQUEEZE SYMBOL
[6] IF RESPONSE IS 'MENU', GO TO 'OTHER'; ELSE GO ON
[7] OTHER\((4=+/+SYMBOL+,'=\'MENU\')
[8] FIND INDEXES OF ROWS THAT CONTAIN THE RESPONSE IN SYMBOL COLUMN
[9] ROWS+(MAT[;SYMCOL]SYMBO\L+\{PMAT[\[\]
[10] IF NO ROWS CONTAIN RESPONSE, GO TO 'OOPS3'; ELSE GO ON
[11] OOPS3\(=ROWS
[12] START LOOP COUNTER AND SET LESSON NR TO 0
[13] HNNH=R+0
[14] INCREMENT COUNTER
[15] NEWR\[;R=R+1
[16] 0\[=R
[17] CALL RUN TO DISPLAY HELP FUNCTION FOR ROW CORRESPONDING TO COUNTER
[18] RUN ROWS[R]
[19] IF MORE ROWS REMAIN, RETURN TO NEWR; ELSE GO ON
[20] NEWR\[;R=ROWS
[21] PRINT FUNCTION ENDING MSG AND EXIT
[22] \"FOR MORE HELP, ENTER: HELP\'
[23] 0
[24] SYMBOL NOT FOUND; PRINT ERROR MSG AND EXIT
[25] OOPS3\(="SORRY, INFORMATION ABOUT THIS SYMBOL IS NOT AVAILABLE AT THIS TIME,' FOR HELP WITH ANOTHER SYMBOL, ENTER: HELP,'"
[26] 0
[27] IF RESPONSE IS 'MENU'; EXECUTE RESPONSE AND EXIT
[28] OTHER\[;SYMBO\L

45
INFO NNN;ROW

[1] a CALLS RUN; REQUIRES GLOBAL VARIABLE MAT
[2] a IF NNN IS NOT A SCALAR, GO TO 'OOPS1'; ELSE GO ON
[3] +OOPS1x1((f;NNN));0
[4] a FIND INDEXES OF ROW
[5] ROW=((1,MAT[;LESCOL]=NNN]/1(PMAT)[1]
[6] a IF NO ROWS CONTAIN RESPONSE, GO TO 'OOPS3'; ELSE GO ON
[7] +OOPS3x10=ROW
[8] a RUN HELP FUNCTION
[9] RUN ROW
[10] a DISPLAY ENDING MESSAGE
[12] +0
[13] a VECTOR ARGUMENT; PRINT ERROR MSG AND EXIT
[14] OOPS1:0;'INFO MUST BE FOLLOWED BY A SINGLE PARAMETER,';G,'TO FESTA
RT, ENTER: INFO NNN (WHERE NNN IS A THREE-DIGIT NUMBER)'
[15] +0
[16] a INFO NOT FOUND; PRINT ERROR MSG AND EXIT
[17] OOPS3:0;'SORRY, INFORMATION ABOUT THIS SYMBOL IS NOT AVAILABLE AT T
HIS TIME,';G,'FOR HELP WITH ANOTHER SYMBOL, ENTER: HELP,'
RUN ROW;HAME;RANK;FNAME;U

1. CALLED BY HELP, INFO, TRY; CALLS SQUEEZE
2. REQUIRES GLOBAL VARIABLE MAT
3. ASSIGN THE NAME OF THE HELP VARIABLE TO THE VARIABLE 'NAME'
   NAME=MAT[ROW;HAMECOL]
4. GO TO HA IF THE HELP VARIABLE 'NAME' DOES NOT EXIST; ELSE GO ON
5. HAME10=HNC NAME
6. DISPLAY THE GENERAL DESCRIPTION PORTION OF THE HELP VARIABLE
   0=(0 1)+((zNAME)[;1]='')/zNAME
7. DISPLAY SCALAR PORTION OF HELP VARIABLE
   0=(0 1)+((zNAME)[;1]='0')/zNAME
8. IF NO MORE INFO, EXIT; ELSE GO ON
9. HAMEI-1E(zNAME)[;1]='VAX'
10. ASK IF MORE DETAILED INFO DESIRED
11. O='MORE?'
12. IF RESPONSE IS NULL, EXIT; ELSE GO ON
13. O=0=U=0
14. DISPLAY VECTOR PORTION OF HELP VARIABLE
   0=(0 1)+((zNAME)[;1]='V')/zNAME
15. DISPLAY MATRIX PORTION OF HELP VARIABLE
   0=(0 1)+((zNAME)[;1]='M')/zNAME
16. DISPLAY ALL ARRAYS PORTION OF HELP VARIABLE
   0=(0 1)+((zNAME)[;1]='A')/zNAME
17. ASSIGN FULL NAME OF SYMBOL TO VARIABLE 'FNAME'; PRINT ERROR MSG
   HAME=FNAME=SQUEEZE,MAT[ROW;FNAMECOL]
18. 'SORRY, INFORMATION ABOUT 'FNAME,' IS NOT AVAILABLE AT THIS TIME.'
CALLS SCHEDULE, MENU, SQUEEZE

"ENTER THE SYMBOL(S) YOU WOULD LIKE INFORMATION ABOUT."

CLR FOR MENU SELECTION, ENTER: MENU

DELETE BLANKS IN RESPONSE

SYMBOL=SQUEEZE SYMBOL=0

IF RESPONSE IS 'MENU', GO TO 'OTHER'; ELSE GO ON

OTHER1(4=+/SYMBO,"='MENU')

FIND INDEXES OF ROWS THAT CONTAIN THE RESPONSE IN SYMBOL COLUMN

ROWS+(MAT[,SYMBO[,SYMBO]/(MAT)[1])

IF NO ROWS CONTAIN RESPONSE, GO TO 'OOPS3'; ELSE GO ON

OOPS3:10=ROWS

INITIALIZE LOOP COUNTER AND SET LESSON NR TO 0

INCREMENT COUNTER

NEWROW=NR+1

NEWROW=NR+1

CALL SHOW TO DISPLAY TEACH FUNCTION FOR ROW INDEXED BY COUNTER

SHOW ROWS[R]

IF ROWS REMAIN, RETURN TO NEWROW; ELSE GO ON

NEWROW14(ROWS

DISPLAY END OF SCHEDULE MSG AND EXIT

O=6E, 'THIS IS THE END OF SCHEDULED LESSONS', '58, 'TO SELECT MORE LESSONS, ENTER: TEACH'

O=0

SYMBOL NOT FOUND; PRINT ERROR MSG AND EXIT

OOPS3:6 'SORRY, INFORMATION ABOUT THIS SYMBOL IS NOT AVAILABLE AT THIS TIME', '58, 'FOR LESSONS ON ANOTHER SYMBOL, ENTER: TEACH'

O=0

RESPONSE IS 'MENU'; EXECUTE RESPONSE AND EXIT

OTHER:4SYMBOL
LESSON HHH;ROW

[1] CALLS SHOW
[2] IF HHH IS NOT A SCALAR, GO TO 'OOPS1'; ELSE GO ON
[3] +OOPS1{1(HHH)0}
[4] SELECT ROW FOR LESSON NUMBER HHH
[5] ROW+{1;MAT[; LesscolJ]=HHH)/1(MAT)[1]
[6] IF NO ROWS CONTAIN RESPONSE, GO TO 'OOPS3'; ELSE GO ON
[7] +OOPS3{10=ROW
[8] CALL SHOW TO DISPLAY TEACH FUNCTION FOR ROWS SELECTED
[9] SHOW ROW
[10] DISPLAY ENDING MESSAGE AND EXIT
[12] +O
[13] VECTOR ARGUMENT; PRINT ERROR MSG AND EXIT
[14] OOPS1: 'LESSON MUST BE FOLLOWED BY A SINGLE PARAMETER, 'YES,' 'NO' TO YES
TART, ENTER: LESSON HHH (WHERE HHH IS A THREE-DIGIT NUMBER)

[15] +O
[16] A LESSON NOT FOUND; PRINT ERROR MSG AND EXIT
[17] OOPS3: 'SORRY, INFORMATION ABOUT THIS SYMBOL IS NOT AVAILABLE AT THIS
TIME,' 'GO,' 'FOR HELP WITH ANOTHER SYMBOL, ENTER: HELP, '

49
SHOW ROW;NAME;G;RANK;FNAME;RT;WT;T;AV;TYPE;ARG;SS;F;

1. CALLED BY SCHEDULE, LESSON; CALLS ASK, RUNDILL, TR', SQUEEZE.
2. REQUIRES GLOBAL VARIABLE MAT
3. SET VARIABLES CORRESPONDING TO ROW
   NAME+MAT[ROW;NAMECOL]
   FNAME+SQUEEZE,MAT[ROW;FNAMECOL]
   SYM+MAT[ROW;SYMCOL]
   RANK+MAT[ROW;SYRCOL]
   TYPE+MAT[ROW;TYPECOL]
   ARG+MAT[ROW;HCOL]
4. GO TO 'HA' IF HELP VARIABLE DOES NOT EXIST; ELSE GO ON
   HA1{0=NC NAME}
5. INITIALIZE TOTAL QUESTION, WRONG ANSWER COUNTERS
   RTWT=0
6. Y+10
7.----------------------------------------------------------------
8. DISPLAY GENERAL PORTION OF HELP VARIABLE
   0+(0 1)++((NAME)[;1]=',')/%NAME
9. GENERAL QUESTIONS
   AV+1,(5)1TYPE='D'),6
   0V+(1+TYPE='D'),SQUEEZE RANK
10. ASK AV
11. CHECK NUMBER OF RIGHT ANSWERS
   WT SUMASK RT
12. IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON
   HALT=2
13.----------------------------------------------------------------
14. DISPLAY SCALAR PORTION OF HELP VARIABLE
   0+(0 1)++((NAME)[;1]='.')/%NAME
15. SCALAR QUESTIONS
   SDX TYPE='N'
16. AV+2
17. AV+'NY'[1+SYM]+X[L=#VAAA']

50
QV ASK AV

m IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON

HALT X=2

SCALAR DRILL

SD: +VX1 = (SYM(55) + x - [1]) v

+VX1 ((SYM(5) + x) v TYPE='M') v (SYM(1) v TYPE='D')

RUNDRIll 1

m IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON

HALT X=2

------------------------

DISPLAY VECTOR PORTION OF HELP VARIABLE

v:ID+(0 1) v ((NAME)[1]=v')/NAME

ASK VECTOR QUESTIONS

+VX1 TYPE='M'

AV+3

AVF [1 +SYM(55) +[0000]]

QV ASK AV

m IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON

HALT X=2

VECTOR DRILL

VD:+X[1] ((SYM(55) +x + () ) v ((SYM(1) + x) v TYPE='M') v (SYM(1) v TYPE='D')

VS:+VX1 (SYM(1) v )

1 RUNDRIll 2

m IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON

HALT X=2

VS:+VX1 (SYM(5) v ) v TYPE='M'

+X[1] (SYM(1) v ) v TYPE='M'

2 RUNDRIll 1

m IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON

HALT X=2

VV:+X[1] SYM(1)

2 RUNDRIll 2

m IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON
[66] +HALTxy=2
[67] #---------------------------------------------------------------
[68] # DISPLAY MATRIX PORTION OF HELP VARIABLE
[69] X:ID=(0 1)4((1NAME)[[1]='n')#1NAME
[70] # ASK MATRIX QUESTIONS
[71] +AX\ TYPE='H'
[72] QV=4
[73] AV=FT'[I+SYT'+-X+X][L]=#;()vW^*']
[74] QV ASK AV
[75] # IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON
[76] +HALTxy=2
[77] # MATRIX DRILL
[78] MD: #TYPE RUNDRILL 3
[79] # IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON
[80] +HALTxy=2
[81] #---------------------------------------------------------------
[82] # DISPLAY ALL ARRAYS PORTION OF HELP VARIABLE
[83] X:ID=(0 1)4((1NAME)[[1]='n')#1NAME
[84] # ASK ALL ARRAYS QUESTIONS
[85] QV=9X\[IF++(1 2)X ARG=\KE'
[86] AV=\('HT'[I+IF]),SQUEEZE ' L3'[I+F]
[87] QV ASK AV
[88] # IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON
[89] +HALTxy=2
[90] #---------------------------------------------------------------
[91] # SUMMARIZE STUDENT PERFORMANCE
[92] SUM:RT SUMASK RT
[93] # IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON
[94] +HALTxy=2
[95] #---------------------------------------------------------------
[96] # CALL TRY FOR SELECTED SYMBOLS
[97] EXP;ENDI\#(SYM' ]+X')#TYPE='D'
[98] TRY SYM
[99] # IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON

52
[100] 4HALTXY=2
[101] #--------------------------------------------
[102] # LESSON COMPLETE; IF NHH=0 (TEACH), EXIT; ELSE GO ON
[103] END:4011NHH=0
[104] # NHH#0 (LESSON); SET LASTLESSON TO THIS LESSON HR
[105] OUT;LASTLESSON+NHH
[106] # DISPLAY MSG AND EXIT
[107] 0+CE; 'THIS IS THE END OF YOUR LESSON,'
[108] 0+ 'FOR THE NEXT LESSON IN SEQUENCE, ENTER: LESSON NEXTLESSON'
[109] 40
[110] # HELP VAR NOT AVAILABLE; DISPLAY ERROR MSG AND EXIT
[111] 4H4C; 'SORRY, INFORMATION ABOUT ',FNAME,' IS NOT AVAILABLE AT THIS
[112] TIME,'
[113] 40
[114] 4HALT;4C; 'THIS LESSON HAS BEEN HALTED,'
A CALLED BY SHOW; CALLS SCORE

REQUIRES NON-LOCAL VARIABLES RT, W, Y, GLOBAL CUES

ASSIGNED THE NUMBER OF QUESTIONS TO 'R'

ASSIGN TOTAL NUMBER OF QUESTIONS TO 'RT'

RT = RT + P

IF HR OF QUESTIONS ≠ HR OF ANSWERS, GO TO 'OOPS'; ELSE GO ON

OOPS

INITIALIZE COUNTER, CHARACTER VARS

I = 0

START LOOP; INCREMENT COUNTER; INITIALIZE WRONG ANSWER COUNT

START: I = I + 1

ASSIGN QUESTION AND ANSWER PARAMETERS FOR ASK FUNCTION

A = (1,ANS)[I]

A + CUE[CUR][I]/ANS[I];

CALL 'SCORE' TO EVALUATE THE QUESTION

QUERY; A SCORE 0

IF FLAG INDICATES 'REPEAT', RETURN TO SAME QUESTION;

IF FLAG INDICATES 'STOP' RESPONSE, EXIT; ELSE GO ON

OOPS

IF QUESTIONS REMAIN, RETURN TO 'START'; ELSE EXIT

START: I = I + 1

FOR DEBUGGING; ARGUMENTS NOT SAME LENGTH; PRINT ERROR MSG AND EXIT

OOPS: [#QUEST ≠ #ANS]
A SCORE

[1] A CALLED BY ASK
[4] 0+0
[5] A ASSIGN RESPONSE TO 'INPUT' AFTER DELETING BLANKS
INPUT=SQUEEZE INPUT+0
[6] A IF RESPONSE IS NULL, ADD 1 WRONG ANS AND GO TO 'LAST'; ELSE GO ON
WT+WT+0=INPUT
[7] +LASTX10=INPUT
[8] A IF RESPONSE IS 'STOP', GO TO 'OUT'; ELSE GO ON
+OUTX4=+/+/INPUT+,='STOP'
[9] A IF RESPONSE ≠ RIGHT ANSWER, GO TO 'WRONG'; ELSE GO ON
NEXT: WROG+(INPUT)116
[10] A DISPLAY MSG
[12] A SET FLAG TO NULL; EXIT
+0;Y=10
[13] A INCREMENT WRONG ANSWER COUNTERS
[14] WROG;W+W+1
[15] WT+WT+1
[16] A GO TO LAST IF THREE WRONG ANSWERS TO THIS QUESTION; ELSE GO ON
+LASTX113
[17] A DISPLAY MSG
[18] A 'SORRY, PLEASE TRY AGAIN'
[19] A SET FLAG TO INDICATE REPEAT QUESTION; EXIT
+0;Y=11
[20] A +0
[21] A THREE WRONG ANSWERS; DISPLAY MSG
[22] LAST;A='SORRY, THE CORRECT ANSWER IS',A
[23] A SET FLAG TO INDICATE GO TO NEXT QUESTION; EXIT
+0;Y=10
[24] A SET FLAG TO INDICATE 'STOP' RESPONSE
[25] OUT;Y=2

55
[1] CALL ALL 'ASK' CALLS
[2] IF FEWER WRONG RESPONSES THAN QUESTIONS, EXIT; ELSE GO ON
[3] +0\times 1/\text{WT}\text{R}
[4] IF WRONG RESPONSES \geq QUESTIONS; DISPLAY MESSAGE AND EXIT
[5] \text{"YOU ENTERED \ '1'(WT),' WRONG ANSWERS OUT OF \ '1'(RF),' QUESTIONS,"}
[6] \text{"IF YOU WANT TO RETAKE THIS LESSON, ENTER; LESSON \ '#1',\text{\text{RAT\{PON; LESCOL\}}}
[7] \text{\text{Y+2}}
RUNDRILL H;FUNC;R;K1;K2;R;A;M;C;H

[1] A CALLED BY SHOW; CALLS DRILL, PULL
[2] A REQUIRES NON-LOCAL VARIABLES SYM, TYPE
[4] C+M=0
[5] START:

[6] A EXTEND SYM FOR REDUCTION, SCAN
[7] SYM=((SYM,'/\A')/('+F(L=?)[?S]), SYM
[8] A FORMAT FUNCTION DISPLAY
[9] R+(1,1,2)[M]
[10] FUNC+(R,3)?', 'SYM,' 

[12] M+3
[14] K1+1 PULL L

[16] A IF K1 IS EMPTY VECTOR, EXIT; ELSE GO ON
[17] 0X0=A,K1
[18] A IF TYPE IS MONADIC, GO TO MONTY; ELSE GO ON
[19] A MONTY|TYPE='M'

[20] A FORMAT DYADIC Q/A
[21] A+(+K1),FUNC,+K2
[22] A+(+K1),SYM,+K2

[23] A RUN
[24] A FORMAT MONADIC Q/A

[25] MONTY:
[26] A+FUNC,+K2
[27] A+SYM,+K2
[28] A RUN DRILL

[29] RUN;O DRILL A
[30] A DELETE EXTRA SYMBOL (IF ANY)
[31] SYM-1+SYM

[32] A IF STOP FLAG IS SET, EXIT; ELSE GO ON
[33] +0xy=2
[34] a IF WRONG ANSWERS ≤ 3, GO TO 'REV'; ELSE GO ON
[35] +REV\{W\}3
[36] a IF RIGHT ANSWERS ≥ 1, RETURN TO 'START'; ELSE EXIT
[37] +START\{C\}{1
[38] +0
[39] REV;
[40] [OCR], 'PLEASE REVIEW THE DESCRIPTION OF THIS FUNCTION OR TALK WITH
YOUR', 'INSTRUCTOR BEFORE RETURNING TO THIS UNIT.'
[41] y+2
Z+P PULL H[R;C];H;E;FPOOL

1. CALLED BY RUNDRILL
2. REQUIRES NON-LOCAL VARIABLES SYM, H
3. SET DEFAULT VALUE
4. Z=10
5. P IS 1 FOR LEFT ARG, 2 FOR RIGHT ARG
6. SET MULTIPLIER BASED ON M FROM 1 TO 3 (EXCEPT MONADIC G)
7. MULT+((10*MAX-2)*0.1)*[1+(SYM[I]'R')*\[TYPE='M']^\[\(1+3\times M\)]\[/\(\times\)]

H IS RANK OF DESIRED RANDOM ARRAY; ASSIGN ROWS AND COLUMNS TO A
ND C
9. R+1(1,1,2)[H]
9. C+(1,1+M),2[H]
10. DETERMINE LIMITS BASED ON TYPE OF SYMBOL
11. POND;POOL+10
12. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(=1\)\]'\[\(\neq\)']}/MULTXH,0,-H+1)
13. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
14. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
15. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
16. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
17. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
18. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
19. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
20. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
21. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
22. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
23. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
24. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
25. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
26. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
27. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
28. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H
29. POOL+POOL,((SYM[I]'='\[\^{\[\times\]}]L,\[\(\neq\)']}/MULTXH,0,-H

Z+R[C]POOL([(R[C])100]
A drill is called by RUNDRILL; calls SQUEEZE

Requires non-local variables C, W; global CR

START:

Y+10

DISPLAY DRILL QUESTION

ASK: [RETURN] 'WHAT IS THE RESULT OF:'

[RETURN] D

IF RESPONSE is 'STOP', GO to 'HALT'; ELSE GO ON

HALT$x_{14}=+/+/STOP'; =SB+SQUEEZE 96

IF ANSWER is NULL, GO to WR; ELSE GO ON

WR:0+PSB

CHECK FOR NUMERIC RESPONSE

CK: [RETURN] (0<=SB(1234567890,')<=0<SB(,')

IF LENGTH OF RESPONSE = LENGTH OF GIVEN ANSWER, GO to 'WR'; ELSE GO ON

WR:0+PSB

IF RESPONSE = GIVEN ANSWER, GO to 'RT'; ELSE GO ON

RT: [RETURN] ((A),=,SB)

ANSWER IS WRONG; DISPLAY ANSWER

WR:0+SORRY, CORRECT ANSWER IS:

D+10

INCREMENT WRONG ANSWER COUNTER AND RETURN TO RUNDRL

W=W+1

D

IF ANSWER is RIGHT; DISPLAY MESSAGE

RT:0+CORRECT;

INCREMENT RIGHT ANSWER COUNTER AND RETURN

C=C+1

D

ANSWER IS NOT ACCEPTABLE, PLEASE ENTER A NUMERIC ANSWER.

[RETURN] 60

ASK
[31] STOP AT STUDENT REQUEST; SET FLAG AND EXIT
[32] HALT; t = 1, y = 2
0 TRY SYM; B; T; C; TW

[1] A CALLED BY SHOW; CALLS RUN
[2] A REQUIRES NON-LOCAL VARIABLES ROW, SYM, FNAME
[3] TW=0
[4] RESTART: TW=TW+1
[5] A GO TO OUCH IF RESTART COUNTER EQUALS 3
[7] A INITIALIZE FLAGS AND WRONG ANSWER COUNTER
[8] T+C+TW=0
[10] START:
[11] 0=CF; WRITE A TRUE STATEMENT USING ', SYM', ', CF', 'FOR EXAMPLE: 4=2-

[12] A IF RESPONSE IS 'STOP', GO TO 'HALT'; ELSE GO ON
[14] A IF RESPONSE IS A NULL OR A NAME, GO TO 'WRONG'; ELSE GO ON
[15] T=WRONG: (4#0NC B)!0=FB
[16] A SET T=1 IF STATEMENT IS TRUE
[17] NEXT:T+1=FB
[18] A SET C=1 IF STATEMENT CONTAINS THE SYMBOL
[19] C+SYM=FB
[20] A GO TO RIGHT IF T AND C 20TH EQUAL 1; ELSE GO ON
[21] T=RIGHT: I=AC
[22] A ANSWER IS WRONG; INCREMENT COUNTER
[23] TW+1
[24] A GO TO REV IF WRONG ANSWER COUNTER EQUALS 3
[25] T=REV: T=3
[26] A GO TO NOSYM IF THE STATEMENT IS TRUE; ELSE GO ON
[27] T=NOSYM
[28] PRINT ERROR MSG
[29] T=Q,'SORRY, THIS STATEMENT IS NOT TRUE,'
PRINT MSG AND RETURN TO START

TRY AGAIN,

PRINT ERROR MSG AND RETURN TO START

YOUR ANSWER DOES NOT USE ',SYM', TRY AGAIN,

PRINT MSG AND EXIT

RIGHT; 'CORRECT';

0

PRINT MSG AND RESTART

DO YOU WISH TO REVIEW THE DESCRIPTION OF ',FRAME',?', ENTER Y OR N;'

IF RESPONSE IS NULL OR NOT Y, GO TO RESTART, ELSE GO ON

RESTART X 0=FBH

RESTART X 1='Y'=#B

DISPLAY HELP VARIABLE, THEN RESTART

RUN ROW

RESTART

THREE RESTARTS; DISPLAY MESSAGE AND EXIT

'PLEASE SEE YOUR INSTRUCTOR FOR ASSISTANCE,'

STOP AT STUDENT REQUEST; SET FLAG AND EXIT

HALT; 0,Y=2
a test
name: s; rank: f
name: r; t: v; a: v; type: a

[1] a used to test drill; calls run drill, try, squeeze
[2] a set variables corresponding to row
[3] row + (1, mat[, lescol]) = r/m/n (mat)[i]
[4] name + mat[row; namecol]
[5] filename + squeeze, mat[row; filenamecol]
[6] sym + mat[row; symcol]
[7] rank + mat[row; rankcol]
[8] type + mat[row; typecol]
[9] arg + mat[row; argcol]
[10] y + 10
[12] s; d; v; x; i; w; y; b
[13] + v; i; ((sym[", +x", "] type = 'm') v (sym[", +x", "] type = 'd')
[14] 1 run drill 1
[15] a if flag indicates 'stop' response, go to halt; else go on
[16] + halt xy = 2
[17] v: = v; d; i; (o = n; c; name)
[18] a vector drill
[19] v; d; x; i; (sym[", s", "] type = 'm') v (sym[", +x", "] type = 'm') v (sym[", +x", "] type = 'd')
[20] + v; s; i; (sym[", s")
[21] 1 run drill 2
[22] a if flag indicates 'stop' response, go to halt; else go on
[23] + halt xy = 2
[24] v; s; d; v; x; i; (sym[", d"] type = 'm')
[25] + x; i; (sym[", f" type = 'm')
[26] 2 run drill 1
[27] a if flag indicates 'stop' response, go to halt; else go on
[28] + halt xy = 2
[29] v; v; i; s; y; m; e; i;
[30] 2 run drill 2
[31] a if flag indicates 'stop' response, go to halt; else go on

64
[32]  +HALTXY=2
[33]  %: +MD11 (0=OHC NAME)
[34]  MD:+A1w (SYN=') & TYPE='Y'
[35]  1 RUNDFILL 3
[36]  & IF FLAG INDICATES 'STOP' RESPONSE, GO TO HALT; ELSE GO ON
[37]  +HALTXY=2
[38]  A:0+'TEST OK'
[39]  +0
[40]  HALT; OR, 'THIS TEST HAS BEEN HALTED,'
\[ ZHEXTLESSON:1;V \]

[1] #FIND INDEX OF LAST LESSON IN VECTOR OF LESSON NUMBERS

[2] \[ i+(\text{V+1}, \text{MAT}[\text{LESCOL}]) \text{LASTLESSON} \]

[3] #RETURN NEXT LESSON NUMBER IN SEQUENCE

[4] \[ Z+\text{V}[\text{I}+1] \]

\[ EOFAT;OM;ICON \]

[1] #SORTS THE VARIABLE 'MAT' IN ASCENDING ORDER OF LESSON NUMBER

[2] #CREATE VECTOR OF LESSON NUMBERS

[3] \[ \text{OMT}[$i+1], \text{MAT}[i+1] \]

[4] #FIND INDEXES OF LESSONS IN ORDER

[5] \[ \text{ICOM}+\text{COM} \]

[6] #REORDER MAT ACCORDING TO INDEXES

[7] \[ \text{MAT}+=\text{MAT}[\text{ICOM}]; \]

\[ ZHSQUEEZE STRING \]

[1] #CALLED BY HELP, RUN, TEACH, SHOW, SCORE, DRILL

[2] #DELETES BLANKS IN CHARACTER STRING ARGUMENT

[3] \[ ZH(' ', \text{STRING});/\text{STRING} \]
C. VARIABLES

1. Global variables

The following section contains all of the global variables in the TUTOR workspace except the text variables (part C) and the user’s guide variables called by the START function (appendix A). MAT is the most important variable in TUTOR, as it is called on to cross-index symbols, lesson numbers, function names, and selected function characteristics. The variables with names ending in COL index the columns of MAT for use in the functions, so that these columns can be rearranged if necessary. CUES provides questions for use in the ASK function, and all of the two-letter, underlined variables (except GR) contain sets of multiple-choice answers for use with CUES. The other important global variable is LASTLESSON which is given a value by the SHOW function to indicate the last lesson completed. All other variables used in TUTOR functions are local variables at some level of the calling hierarchy.
163 \[D \& H H A A\text{MAXI MAXIMUM}\]
164 \[D \& H H A A\text{MINI MINIMUM}\]
165 \[D \& H H A A\text{RESI RESIDUE}\]
166 \[A \& M N V GRUP GRADE\_UF\]
167 \[M N V GRAD GRADE\_DOWN\]
168 \[? M H A ROLL ROLL\]
169 \[? D N N S S DEAL DEAL\]
180 \[= D E E A A EQUA EQUAL\]
181 \[\# D E E A A NOTE NOT\_EQUAL\]
182 \[< D H H A A LESS LESS\_THAN\]
183 \[> D H H A A LTEQ LESS\_OR\_EQUAL\]
184 \[> D H H A A GTEQ GREATER\_OR\_EQUAL\]
185 \[> D H H A A GREA GREATER\_THAN\]
191 \[\& M B A NOT NOT\]
192 \[\& D B B A A AND AND\]
193 \[\& D B B A A NAND NAND\]
194 \[\| D B B A A OR OR\]
195 \[\| D B B A A NOR NOR\]
196 \[\& D E E A A MEMP MEMBER\_OF\]
201 \[\# M E A REV2 REVERSE\]
202 \[\# M E S REV1 REVERSE\]
203 \[\# D H E A A ROT2 ROTATE\]
204 \[\# D H E A A ROT1 ROTATE\]
205 \[\# M E A TRAM TRANSPOSE\_MONADIC\]
206 \[\# D H E V A TRAD TRANSPOSE\_DYADIC\]
211 \[\# D H E V A TAKE TAKE\]
212 \[\# D H E V A DROP DROP\]
213 \[\# D H E V A COM2 COMPRESS\]
214 \[\# D H E V A COM1 COMPRESS\]
215 \[\# D H E V A EXP2 EXPAND\]
216 \[\# D H E V A EXP1 EXPAND\]
219 \[\# D E E A A LAMI LAMINATE\]
231 \[\# M H X MATI MATRIX\_INVERSE\]
232 \[\# D N N X X MATD MATRIX\_DIVIDE\]
241 ODEA A OUTE OUTER_PRODUCT
242 ODEA A INHE INNER_PRODUCT
301 ODEVA INDO INDEX_OF
304 TDDWA ENCO ENCODE
305 TDDWA DECO DECODE
306 TMMV EXEC EXECUTE
307 TDDVA DYFO FORMAT-DYADIC
308 TME A FORM FORMAT-MONADIC
'HOW MANY ARGUMENTS DOES THE ',FNAME', FUNCTION TAKE?'

'DOES S1,SYM,S2 EQUAL S2,SYM,S1?'

'IF ONE ARGUMENT IS A VECTOR, THE OTHER ARGUMENT MAY BE SCALAR OR VECTOR , (T/F)'

'IF ONE ARGUMENT IS A MATRIX, THE OTHER ARGUMENT MUST BE A SCALAR',SE,' OR A MATRIX OF THE SAME SHAPE, (T/F)'

'WHAT RANK CAN THE LEFT ARGUMENT OF ',FNAME', BE?',RK

'WHAT RANK CAN THE RIGHT ARGUMENT OF ',FNAME', BE?',RK

'DO THE ARGUMENTS ALWAYS HAVE TO HAVE THE SAME RANK?'

'CAN ',FNAME', TAKE CHARACTER ARGUMENTS?'

'IF SO, WHICH ARGUMENT(S) CAN BE CHARACTER?',CR

'DOES ',FNAME', TAKE BOOLEAN (1 OR 0) ARGUMENTS?'

'IF SO, WHICH ARGUMENT(S) MUST BE BOOLEAN?',AB

ENTER R FOR RIGHT,
L FOR LEFT, OR
B FOR BOTH

ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK
2. **Text variables**

The following global variables contain the basic text of the TUTOR course. They are printed in the same order as the lessons. The header above each variable is not part of the text; it lists the lesson number, the short title of the variables, and the full name of the function. The first column control characters are omitted here.
THE SYMBOL \(-\) (UPSHIFT 2) IS USED ONLY TO ENTER NEGATIVE NUMBERS.
IT CAN BE USED ONLY IN FRONT OF SCALARS (SINGLE NUMBERS), NOT VARIABLE NAMES, ARITHMETIC EXPRESSIONS, OR ARRAYS.
TO CHANGE ARITHMETIC SIGN USE THE SYMBOL \(-\) (UPSHIFT +).
THE SYMBOL + IS USED FOR THE MONADIC CONJUGATE FUNCTION,
THIS IS AN IDENTITY FUNCTION WHICH RETURNS ITS ARGUMENT, AFTER
EVALUATION IF NECESSARY, IT TAKES NUMERIC ARGUMENTS OF ANY RANK.
EXAMPLE: +2 => 2 +^-3+2 => -1.
THE SYMBOL + IS USED FOR THE DYADIC PLUS FUNCTION.
THIS PERFORMS SIMPLE ADDITION ON NUMERIC ARGUMENTS OF ANY RANK.

FOR EXAMPLE: 2+2 = 4
SPACING IS NOT IMPORTANT.

TO ADD A SCALAR TO EACH ELEMENT OF A VECTOR, ENTER EITHER S+V OR V+S.
V1+V2 WILL ADD TWO VECTORS ELEMENT BY ELEMENT. FOR EXAMPLE:

\[ 2+(3, 4, 5) \Rightarrow (5, 6, 7) \quad (3, 4, 5)+2 \Rightarrow (5, 6, 7) \quad (3, 4, 5)+(5, 6, 7) \Rightarrow (8, 10, 12) \]
IF THE TWO VECTORS BEING ADDED ARE NOT THE SAME LENGTH,
YOU WILL GET A 'LENGTH ERROR'.

MATRIX ADDITION IS DONE AS FOLLOWS:
S+M OR M+S WILL ADD THE SCALAR S TO EACH ELEMENT OF THE MATRIX M.
FOR EXAMPLE: IF M = 1 2 THEN 3+M = 4 5 AND M+3 = 4 5
3 4 6 7 6 7
M1+M2 WILL ADD THE MATRIX M1 TO THE MATRIX M2, ELEMENT BY ELEMENT.
FOR EXAMPLE: IF M1 = 1 2 AND M2 = 3 4 THEN M1+M2 = 4 6
3 4 5 6 8 10
M1 AND M2 MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.
TRYING TO ADD A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL - (UPSHIFT 4) IS USED FOR THE MONADIC CHANGE SIGN FUNCTION.
THIS CHANGES THE ARITHMETIC SIGN OF ITS NUMERIC ARGUMENT OF ANY RANK.
FOR EXAMPLE: -(1 -2 3) = \(-1 \ -2 \ -3\).
NOTE THE DIFFERENCE BETWEEN - AND - (UPSHIFT 2), WHICH INDICATES
NEGATIVE NUMBERS.
THE SYMBOL - (UPSHIFT +) IS USED FOR THE DYADIC MINUS FUNCTION.
IT PERFORMS SIMPLE SUBTRACTION ON NUMERIC ARGUMENTS OF ANY RANK.

FOR EXAMPLE: \( 5 - 2 = 3 \) \( -10 - 2 = -3 \)

TO SUBTRACT A SCALAR FROM EACH ELEMENT OF A VECTOR, ENTER \( \text{V} - \text{S} \).
TO SUBTRACT EACH ELEMENT OF A VECTOR FROM THE SAME SCALAR, ENTER \( \text{S} - \text{V} \).
\( \text{V}_1 - \text{V}_2 \) WILL SUBTRACT EACH ELEMENT OF \( \text{V}_2 \) FROM THE CORRESPONDING ELEMENT OF \( \text{V}_1 \).
\( \text{V}_1 \) AND \( \text{V}_2 \) MUST BE THE SAME LENGTH OR YOU WILL GET A 'LENGTH ERROR'.
FOR EXAMPLE: \( (3 4 5) - 2 = 1 2 3 \) \( 2 - (3 4 5) = -1 -2 -3 \)
\( (6 7 8) - (1 2 3) = 5 5 5 \)

ENTER \( \text{M} - \text{S} \) TO SUBTRACT THE SCALAR \( \text{S} \) FROM EACH ELEMENT OF THE MATRIX \( \text{M} \),
ENTER \( \text{S} - \text{M} \) TO DO THE OPPOSITE.
FOR EXAMPLE: IF \( \text{M} = 1 2 \) THEN \( 4 - \text{M} = 3 2 \) AND \( \text{M} - 2 = -1 0 \)
\( 3 4 \) \( 1 0 \) \( 1 2 \)
\( \text{M}_1 - \text{M}_2 \) SUBTRACTS MATRIX \( \text{M}_2 \) FROM MATRIX \( \text{M}_1 \), ELEMENT BY ELEMENT.
FOR EXAMPLE: IF \( \text{M}_1 = 1 2 \) AND \( \text{M}_2 = 3 4 \) THEN \( \text{M}_2 - \text{M}_1 = 2 2 \)
\( 3 4 \) \( 5 6 \) \( 2 2 \)
IF \( \text{M}_1 \) AND \( \text{M}_2 \) ARE NOT THE SAME SHAPE, A 'LENGTH ERROR' WILL RESULT.
TRYING TO SUBTRACT A VECTOR FROM A MATRIX OR VICE-VERSA WILL PRODUCE A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL \( \times \) IS USED FOR THE MONADIC SIGNUM FUNCTION,
IT RETURNS THE ARITHMETIC SIGN OF THE ARGUMENT: 1 FOR POSITIVE NUMBERS,
\(-1\) FOR NEGATIVE NUMBERS, AND 0 FOR ZERO VALUES.

IT TAKES NUMERIC ARGUMENTS OF ANY RANK.

FOR EXAMPLE: \( x(3 - 2) \Rightarrow 1 \quad -1 \quad x(3 + 2 - 5) \Rightarrow 0 \)
THE SYMB\( \times \) IS USED FOR THE DIADIC TIMES FUNCTION.

THIS PERFORMS SIMPLE MULTIPLICATION ON NUMERIC ARGUMENTS OF ANY RANK.

FOR EXAMPLE: \( 2 \times 4 = 8 \)

ORDER AND SPACING ARE NOT IMPORTANT.

TO MULTIPLY EACH ELEMENT OF A VECTOR BY A SCALAR, ENTER \( \text{SxV OR VxS} \).

\( \text{V1xV2 RESULTS IN THE PRODUCT OF CORRESPONDING ELEMENTS OF V1 AND V2.} \)

FOR EXAMPLE: \( 3 \times (1 \ 2 \ 3) = 3 \ 6 \ 9 \) \( (1 \ 2 \ 3) \times 3 = 3 \ 6 \ 9 \)

\( (1 \ 2 \ 3) \times (2 \ 4 \ 6) = 2 \ 6 \ 18 \)

IF \( \text{V1 AND V2 ARE NOT THE SAME LENGTH, YOU WILL GET A 'LENGTH ERROR'}. \)

EITHER SxS OR SxM WILL MULTIPLY A MATRIX BY A SCALAR.

THE RESULT WILL BE A MATRIX THE SAME SHAPE AS \( \text{S} \).

FOR EXAMPLE: IF \( \text{M} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) THEN \( 3 \times \text{M} = \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} \) AND \( \text{M} \times 3 = \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} \)

\( \text{M1xM2 WILL RETURN THE PRODUCT OF CORRESPONDING ELEMENTS IN M1 AND M2.} \)

\( \text{M1 AND M2 MUST BE THE SAME SHAPE OR YOU WILL GET A 'LENGTH ERROR'}. \)

FOR EXAMPLE: IF \( \text{M1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) AND \( \text{M2} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} \) THEN \( \text{M1xM2} = \begin{pmatrix} 3 & 8 \\ 15 & 24 \end{pmatrix} \)

IT IS IMPORTANT TO NOTE THAT MULTIPLYING MATRICES WILL NOT RESULT IN 'MATRIX MULTIPLICATION' (INNER PRODUCT OR DOT PRODUCT).

FOR INFORMATION ON INNER PRODUCT, REQUEST HELP ON '.,' (DOT OR PERIOD).

MULTIPLYING A VECTOR WITH A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS,
THE SYMBOL \( \div \) (upshift +) IS USED FOR THE MONADIC RECIPROCAL FUNCTION. THIS RETURNS THE RESULT OF 1 DIVIDED BY THE ARGUMENT. ITS ARGUMENT MUST BE NUMERIC AND CAN BE OF ANY RANK, HOWEVER, IF THE ARGUMENT HAS VALUE 0, A 'DOMAIN ERROR' WILL RESULT. FOR EXAMPLE: \( \div (1 \ 2 \ 3) \Rightarrow 1 \ 0.5 \ 0.333333333 \)
THE SYMBOL \( \div \) (UPSHIFT 1) IS USED FOR THE DYADIC DIVISION FUNCTION.

IT PERFORMS SIMPLE DIVISION ON NUMERIC ARGUMENTS OF ANY RANK.

FOR EXAMPLE: \( 6 \div 2 \Rightarrow 3 \quad \text{and} \quad 10 \div 4 \Rightarrow 2.5 \)

DIVIDING BY 0 RESULTS IN A 'DOMAIN ERROR', EXCEPT THAT \( 0 \div 0 \Rightarrow 1 \).

TO DIVIDE A SCALAR BY EACH ELEMENT OF A VECTOR, ENTER S-V,

TO DIVIDE EACH ELEMENT OF A VECTOR BY THE SAME SCALAR, ENTER V+S,

\( v1 \div v2 \) DIVIDES EACH ELEMENT OF \( v1 \) BY THE CORRESPONDING ELEMENT OF \( v2 \).

\( v1 \) and \( v2 \) MUST BE THE SAME LENGTH OR YOU WILL GET A 'LENGTH ERROR'.

FOR EXAMPLE: 
\[
(3 \ 4 \ 5) \div 2 \Rightarrow 1.5 \ 2 \ 2.5 \\
3 \div (3 \ 4 \ 5) \Rightarrow 1.75 \ 0.4 \\
(3 \ 6 \ 9) \div (1 \ 2 \ 3) \Rightarrow 3 \ 3 \ 3
\]

ENTER S+M TO DIVIDE THE SCALAR S BY EACH ELEMENT OF THE MATRIX M.

M-S WILL DIVIDE EACH ELEMENT OF M BY S. BOTH OPERATIONS RESULT IN A MATRIX THE SAME SHAPE AS M.

FOR EXAMPLE: IF \( M \Rightarrow 1 \ 2 \) THEN \( 2 \div M \Rightarrow 2 \ 1 \) AND \( M \div 2 \Rightarrow 0.5 \ 1 \)

\[
\begin{bmatrix}
4 & 5 \\
0.5 & 0.4 \\
2 & 2.5
\end{bmatrix}
\]

\( M1 \div M2 \) DIVIDES MATRIX \( M1 \) BY MATRIX \( M2 \), ELEMENT BY ELEMENT.

FOR EXAMPLE: IF \( M1 \Rightarrow 2 \ 4 \) AND \( M2 \Rightarrow 1 \ 2 \) THEN \( M1 \div M2 \Rightarrow 2 \ 2 \)

\[
\begin{bmatrix}
6 & 9 \\
3 & 4 \\
2 & 2
\end{bmatrix}
\]

IF \( M1 \) AND \( M2 \) ARE NOT THE SAME SHAPE, A 'LENGTH ERROR' WILL RESULT.

TRYING TO DIVIDE A MATRIX BY A VECTOR OR VICE-VERSA WILL PRODUCE A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
PARENTHESES ARE USED TO GROUP SYMBOLS FOR CORRECT PROCESSING BY THE APL INTERPRETER.

APL DIFFERS FROM EVERY OTHER COMPUTER LANGUAGE IN THE WAY IT DETERMINES WHICH OPERATION IS PERFORMED FIRST.

IT SIMPLY READS FROM RIGHT TO LEFT—'BACKWARDS'—AND PERFORMS EACH OPERATION AS IT GOES ALONG.

FOR EXAMPLE: 3/2+1 = 9 BECAUSE 2+1 = 3 AND 3*3 = 9
1+2*3 = 7 BECAUSE 2*3 = 6 AND 1+6 = 7

PARENTHESES ARE USED TO OVERRIDE THIS RIGHT-TO-LEFT RULE.

FOR EXAMPLE: (3*2)+1 = 7 AND (1+2)*3 = 9

EXPRESSIONS WITHIN PARENTHESES ARE EVALUATED FIRST—ALSO FROM RIGHT TO LEFT.

USING THE RIGHT-TO-LEFT RULE CAN ELIMINATE PARENTHESES,

BUT USING PARENTHESES GENEROUSLY CAN ELIMINATE CONFUSION.
THE QUOTE MARK ' (UPSHIFT K) IS USED TO ENCLOSE CHARACTER DATA.
AFL ASSUMES THAT LETTERS (OR COMBINATIONS OF LETTERS AND NUMBERS)
ARE VARIABLE OR FUNCTION NAMES UNLESS THEY ARE ENCLOSED IN QUOTES.

FOR EXAMPLE: IN THIS TUTORIAL,

\[
\text{HOW} \equiv (\text{THE VARIABLE NAMED HOW})
\]

'HOW' \equiv \text{HOW}

YOU CAN ALSO USE SYMBOLS AND NUMBERS AS CHARACTER DATA,
THE SYMBOL + IS USED FOR SPECIFICATION, IT ASSIGNS THE VALUE OF THE RIGHT ARGUMENT TO THE VARIABLE NAME WHICH IS THE LEFT ARGUMENT.

THE VARIABLE NAME CAN BE ANY COMBINATION OF LETTERS AND NUMBERS (NO SYMBOLS AND NO SPACES) WHICH BEGINS WITH A LETTER. LENGTH IS EFFECTIVELY UNLIMITED.

THE RIGHT ARGUMENT CAN BE EITHER CHARACTER OR NUMERIC, OF ANY RANK, TO SPECIFY A CHARACTER VARIABLE, ENCLOSE THE RIGHT ARGUMENT IN SINGLE QUOTES ('') UNLESS IT IS ANOTHER CHARACTER VARIABLE.

NOTE THAT THE VARIABLE CAN BE USED IN THE SAME LINE AS THE VALUE IS SPECIFIED, FOR EXAMPLE:

X+1+2 RESULTS IN X HAVING VALUE 2 AND Y HAVING VALUE 5
X+Y+Z RESULTS IN X, Y, AND Z ALL HAVING VALUE 1

Y+1 2 3 RESULTS IN Y HAVING VECTOR VALUE (1 2 3)
V+APL RESULTS IN V1 BEING A CHARACTER VECTOR; APL
V2+V1 RESULTS IN V2 BEING A CHARACTER VECTOR; APL

SEE HELP ON ? (UPSHIFT F) FOR INFORMATION ON SPECIFYING MATRIX VARIABLES.
THE Symbol \( f \) IS USED FOR THE MONADIC SHAPE FUNCTION.

THIS RETURNS A DESCRIPTION OF THE SIZE OF ITS ARGUMENT, WHICH CAN

BE NUMERIC OR CHARACTER, OF ANY RANK.

THE Symbol CAN BE USED TWICE \( (f f A) \) TO RETURN THE RANK OF ITS ARGUMENT,

IF THE ARGUMENT IS A SCALAR, \( f \) RETURNS NO NUMBER (AN EMPTY VECTOR),

BECAUSE A SCALAR HAS NO DIMENSION, \( f f S = 0 \)

EXECUTING \( f V \) RETURNs ONE NUMBER WHICH REPRESENTS THE LENGTH OF

THE VECTOR'S ONE DIMENSION, \( f f V = 1 \)

THE SHAPE OF ARRAYS IS EXPRESSED AS A TWO-ELEMENT VECTOR,

THE FIRST ELEMENT EQUALS THE NUMBER OF ROWS (THE LENGTH OF THE

FIRST DIMENSION); AND THE SECOND ELEMENT EQUALS THE NUMBER OF

COLUMNS (THE LENGTH OF THE SECOND DIMENSION).

FOR EXAMPLE: IF \( A = \) 1 2 3 THEN \( f A = \) 2 3 AND \( f f A = 2 \)

\[ 4 \ 5 \ 6 \]

FOR HIGHER-LEVEL ARRAYS, \( f A \) WILL BE A VECTOR WITH AS MANY ELEMENTS AS

A HAS DIMENSIONS. THE NUMBER OF COLUMNS WILL ALWAYS BE THE LAST

ELEMENT OF \( f A \), THE NUMBER OF ROWS WILL ALWAYS BE THE NEXT-TO-LAST

ELEMENT, PRECEDED BY THE NUMBER OF 'PAGES' OR PLANES, PRECEDED BY

THE NUMBER OF 'BOOKS' OR SPACES, AND SO ON.

FOR EXAMPLE: IF \( D \) IS A 4-DIMENSIONAL ARRAY, \( f f D = 4 \).

IF \( f D = 5 \ 3 \ 4 \ 2 \) THEN \( D \) HAS 5 BOOKS, EACH WITH 3 PAGES, EACH OF

WHICH HAS 4 ROWS AND 2 COLUMNS. (THIS WOULD BE DISPLAYED AS

FIFTEEN SUCCESSIVE 4x2 MATRICES--THE FIRST THREE BEING THE FIRST

BOOK, THE SECOND THREE BEING THE SECOND BOOK, ETC.)
RESHAPE

THE SYMBOL \( \verb|^| \) IS USED FOR THE DIADIC RESHAPE FUNCTION.

THIS TAKES THE ELEMENTS OF THE RIGHT ARGUMENT AND REARRANGES THEM
ACCORDING TO THE SHAPE SPECIFIED BY THE LEFT ARGUMENT.

THE LEFT ARGUMENT MUST BE AN INTEGER SCALAR OR A VECTOR OF INTEGER
ELEMENTS,

THE RIGHT ARGUMENT CAN BE NUMERIC OR CHARACTER, OF ANY RANK.

IF THE LEFT ARGUMENT IS A SCALAR, THE RESULT WILL BE A VECTOR WITH
LENGTH EQUAL TO THE SCALAR, CONSISTING OF THE ELEMENTS OF THE RIGHT
ARGUMENT, TAKEN IN ORDER TOP LEFT TO BOTTOM RIGHT.

WHEN THERE ARE NOT ENOUGH ELEMENTS IN THE RIGHT ARGUMENT,
THE FUNCTION WILL START OVER AND TAKE THE ELEMENTS IN ORDER AGAIN,
AS MANY TIMES AS NECESSARY TO FILL THE VECTOR.

FOR EXAMPLE:
\[
\begin{align*}
4!(1 & 1 1 1 \\
4!(\text{'AB'}) & = \text{'ABAB'}
\end{align*}
\]

IF THERE ARE TOO MANY ELEMENTS ON THE RIGHT, THE EXTRAS ARE OMITTED.
FOR EXAMPLE:
\[
2!(1 2 3 4) = 1 2 3 4
\]

IF THE LEFT ARGUMENT IS A VECTOR, THE ELEMENTS INDICATE, IN ORDER,
LENGTH OF EACH DIMENSION IN THE RESULT,
FOR EXAMPLE:
\[
2 2!(1 2 3 4) \rightarrow 1 2 3 4
\]

THIS IS ONE WAY TO ENTER MATRICES OR ARRAYS OF HIGHER DIMENSION.

USING A MATRIX OR HIGHER-LEVEL ARRAY AS THE LEFT ARGUMENT OF RESHAPE
WILL RESULT IN A 'RANK ERROR',

87
Brackets

Byadic square brackets are used to index the elements of an array. They can be thought of as enclosing subscripts.

The left argument (left of the brackets) can be a numeric or character array of any rank; but not a scalar. Indexing a scalar will result in a 'rank error'.

The right argument (between the brackets) must be one or more integer scalars or vectors, separated by semicolons (;). There must be as many scalars/vectors as there are dimensions in the left argument; or a 'rank error' will result.

Any numeric expressions within the brackets will be evaluated as if the brackets and semicolons are parentheses.

Brackets can also be used following these symbols: /\4/8

For more information, see help on the appropriate symbol.

For example: if \( \mathbf{v} \) is a vector with value \( (3, 4, 5, 6) \),

\[
\mathbf{v}[1] = 3 \quad \mathbf{v}[1, 2] = 4 \quad \mathbf{v}[1+2] = 5 \quad \mathbf{v}[1]+\mathbf{v}[2] = 7
\]

'A Programming Language' \( [1, 3, 15] \) = APL.

There is only one scalar or vector within the brackets because vectors have only one dimension.

Matrices require two subscripts—two scalars or vectors, separated by a semicolon (;) within the brackets.

The first subscript refers to the row, the second to the column.

For example: if \( \mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \) then \( \mathbf{M}[1; 1] = 1 \) and \( \mathbf{M}[1; 2] = 2, 8, 4, 5, 6, 7, 8, 9 \)

To select an entire row (or rows), enter the row subscript followed by a semicolon, with no column subscript.

For example: \( \mathbf{M}[1; ] = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \)

For entire column(s), leave the row subscript blank and enter a semicolon followed by the column subscript(s),
FOR EXAMPLE: IF $\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ THEN $\mathbf{M}[1][1] = 1$

NOTICE THAT ONE ROW OR ONE COLUMN OF A MATRIX IS SIMPLY A VECTOR, NOT A $1 \times C$ OR $C \times 1$ MATRIX. A SINGLE ELEMENT OF A MATRIX IS A SCALAR, NOT A $1 \times 1$ MATRIX OR A VECTOR OF LENGTH 1.

THE USE OF BRACKETS CAN BE EXTENDED TO HIGHER DIMENSIONS BY ADDING A SEMICOLON AND A SCALAR OR VECTOR FOR EACH DIMENSION.

FOR EXAMPLE, IF $\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$ THE SECOND ROW, THIRD COLUMN, FIRST ROW (ALL COLUMNS) CAN BE SELECTED BY ENTERING $\mathbf{M}[2][3][1]$.
THE SYMBOL , IS USED FOR THE MONADIC RAVEL FUNCTION.

THIS FUNCTION CHANGES ITS ARGUMENT TO THE FORM OF A VECTOR. IT TAKES EITHER NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK.

TO CHANGE A SCALAR TO A VECTOR OF LENGTH 1, ENTER ,5.

FOR EXAMPLE: \( \text{if} \; 3 = \; 0 \) BUT \( \text{if} \; 3 \Rightarrow 1 \)

THIS CAN BE USEFUL TO ENSURE THAT A VARIABLE IS A VECTOR WHEN USING A SCALAR WOULD LEAD TO AN ERROR (E.G., INDEXING).

VECTORS REMAIN UNCHANGED BY RAVELLING.

MATRICES ARE CHANGED INTO A SINGLE LONG VECTOR, STARTING WITH THE TOP ROW AND READING ACROSS EACH ROW FROM LEFT TO RIGHT.

FOR EXAMPLE: IF \( \text{m1} = \; 1 \; 2 \) THEN \( \text{m1} \Rightarrow 1 \; 2 \; 3 \; 4 \)

3 4

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL \( \cdot \) IS USED FOR THE DYADIC Catenate FUNCTION.

THIS FUNCTION WILL COMBINE TWO SCALARS OR ARRAYS INTO A SINGLE ARRAY.

IT TAKES EITHER NUMERIC OR CHARACTER ARGUMENTS.

ENTERING \( S_1,S_2 \) WILL RESULT IN A VECTOR \( (S_1,S_2) \). FOR EXAMPLE:

\( v_{-1:2} \) WILL ASSIGN THE VECTOR VALUE \( (1,2) \) TO THE VARIABLE \( v \).

VECTORS CAN EASILY BE EXTENDED BY Catenating \( S,v \) OR \( v_3 \) OR \( v,v \).

FOR EXAMPLE: IF \( v \Rightarrow 1 \ 2 \) THEN

\[ v_{,3} \Rightarrow 1 \ 2 \ 3 \quad \text{or} \quad v_{,1} \Rightarrow 0 \ 1 \ 2 \quad \text{or} \quad v,v \Rightarrow 1 \ 2 \ 1 \ 2 \]

Catenating A SCALAR TO A MATRIX WILL RESULT IN A MATRIX WITH

ONE MORE COLUMN THAN THE MATRIX ARGUMENT. ALL ENTRIES IN THIS

COLUMN WILL HAVE THE VALUE OF THE SCALAR. FOR EXAMPLE:

IF \( h = \) \( 1 \ 2 \) THEN \( s,h \Rightarrow 5 \ 1 \ 2 \) AND \( h,s \Rightarrow 1 \ 2 \ 5 \)

\[ 3 \ 4 \quad 5 \ 3 \ 4 \quad 3 \ 4 \ 5 \]

Catenating TWO MATRICES WILL RESULT IN A MATRIX CONTAINING THE TWO

ORIGINAL MATRICES 'SIDE BY SIDE'.

BOTH MATRICES MUST HAVE THE SAME NUMBER OF ROWS OR A 'LENGTH ERROR'

WILL RESULT.

FOR EXAMPLE: IF \( h_1 = \) \( 1 \ 2 \) AND \( h_2 = \) \( 5 \ 6 \)

\[ 3 \ 4 \quad 7 \ 8 \]

THEN \( h_1,h_2 = \) \( 1 \ 2 \ 5 \ 6 \) AND \( h_2,h_1 = \) \( 5 \ 6 \ 1 \ 2 \)

\[ 3 \ 4 \ 7 \ 8 \quad 7 \ 8 \ 3 \ 4 \]

TRYING TO Catenate A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.

FOR MORE INFORMATION ON DIFFERENT WAYS TO JOIN ARRAYS, SEE

MENU FOR LAMINATE.

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THE SYMBOL \( \uparrow \) (UPSHIFT 1) IS USED FOR THE MONADIC INDEX GENERATING FUNCTION.

IT IS USED WITH A SINGLE NON-NEGATIVE INTEGER ARGUMENT (SCALAR, OR VECTOR OF LENGTH 1).

IT RETURNS A VECTOR OF INTEGERS, IN ORDER, BEGINNING WITH THE INDEX ORIGIN, AND ENDING WITH THE ARGUMENT.

FOR EXAMPLE: \( 13 \Rightarrow 123 \) \( 10 \Rightarrow \) (AN EMPTY VECTOR)

THE INDEX ORIGIN IS NORMALLY (BY DEFAULT) 1.

USING \( \uparrow \) WITH A NON-INTEGER, NEGATIVE OR ARRAY ARGUMENT WILL PRODUCE A 'DOMAIN ERROR'.
THE SYMBOL / IS USED FOLLOWING ANOTHER FUNCTION SYMBOL AND
PRECEDING A SINGLE ARGUMENT TO PRODUCE THE MIXED FUNCTION
CALLED REDUCTION.

REDUCTION MAY BE USED WITH NUMERIC ARGUMENTS OF ANY RANK.
REDUCTION MAY ALSO BE USED WITH CHARACTER ARGUMENTS IF THE FUNCTION
IT IS BEING COMBINED WITH ACCEPTS CHARACTER ARGUMENTS.

USING / WITH A SCALAR SIMPLY RETURNS THE SCALAR.

WHEN USED WITH A VECTOR, REDUCTION GIVES THE SAME EFFECT AS INSERTING
THE OTHER FUNCTION SYMBOL BETWEEN EACH VALUE OF THE VECTOR.

FOR EXAMPLE:

+/1 2 3) =: 6 because 1+2+3 =: 6

+/2 3 4) =: 24 because 2+3+4 =: 24

WHEN USING REDUCTION, ALWAYS REMEMBER APL READS RIGHT TO LEFT;

-/(1 2 3) =: 2 because 1-(2-3) =: 2

(Notice this produces the effect of summation with alternating signs.)

WHEN USED WITH A MATRIX, / WILL PERFORM AS IF EACH COLUMN IS
A UNIT, IT WILL ADD THE CORRESPONDING ELEMENTS OF EACH COLUMN,
SO THAT THE ANSWER WILL BE A VECTOR WITH AS MANY ELEMENTS AS
THERE WERE ROWS IN THE ARGUMENT.

FOR EXAMPLE; IF M =: 2 4 THEN M/ M =: 2

APL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE COLUMNS,
SINCE COLUMNS ARE THE SECOND OF TWO DIMENSIONS OF A MATRIX,
THIS MEANS THAT THE SYMBOL / WORKS 'OVER' THE LAST DIMENSION,
TO REDUCE OVER THE FIRST DIMENSION (E.G. ROWS OF A MATRIX), USE THE
SYMBOL / (ALT /), IN PLACE OF /.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS,
FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND
COLUMNS, / will reduce over the columns, \ over the pages. 

To reduce over any dimension, you may specify the dimension in 

brackets following the \ (or the /) symbol, 

For example, to plus reduce over the rows of a three-dimensional 

array called AA, enter +/[2]AA or +/[2]AA, to plus reduce 

over the pages, enter +/AA or +/[1]AA, to plus reduce over 

the columns, enter +/AA or +/[3]AA, 

The shape of the answer will always be the same as the shape of the 

argument with the dimension reduced over, 

For example: if AA = 3 4 5 

THE SYMBOL \( \_ \) IS USED FOLLOWING ANOTHER FUNCTION SYMBOL AND
PRECEDING A SINGLE ARGUMENT TO PRODUCE THE MIXED FUNCTION
CALLED REDUCTION.

REDUCTION MAY BE USED WITH NUMERIC ARGUMENTS OF ANY RANK,
REDUCTION MAY ALSO BE USED WITH CHARACTER ARGUMENTS IF THE FUNCTION
IT IS BEING COMBINED WITH ACCEPTS CHARACTER ARGUMENTS,

USING \( \_ \) WITH A SCALAR SIMPLY RETURNS THE SCALAR.

WHEN USED WITH A VECTOR, REDUCTION GIVES THE SAME EFFECT AS INSERTING
THE OTHER FUNCTION SYMBOL BETWEEN EACH VALUE OF THE VECTOR.

FOR EXAMPLE:
\[
\_ 1 2 3 = 6 \quad \text{because} \quad 1 + 2 + 3 = 6 \\
\_ 2 3 4 = 24 \quad \text{because} \quad 2(3+1) = 24
\]

WHEN USING REDUCTION, ALWAYS REMEMBER APL READS RIGHT TO LEFT;
\[
\_ 1 2 3 = 2 \quad \text{because} \quad 1-(2-3) = 2
\]
( NOTICE THIS PRODUCES THE EFFECT OF SUMMATION WITH ALTERNATING SIGNS.)

WHEN USED WITH A MATRIX, \( \_ \) WILL PERFORM AS IF EACH ROW IS
A UNIT, IT WILL ADD THE CORRESPONDING ELEMENTS OF EACH ROW,
SO THAT THE ANSWER WILL BE A VECTOR WITH AS MANY ELEMENTS AS
THERE WERE COLUMNS IN THE ARGUMENT.

FOR EXAMPLE; IF \( \_ M = 2 \) \( \_ \) THEN \( \_ + M = 4 \)

APL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE ROWS.

SINCE ROWS ARE THE FIRST OF TWO DIMENSIONS OF A MATRIX
THIS MEANS THAT THE SYMBOL \( \_ \) WORKS 'OVER' THE FIRST DIMENSION,
TO REDUCE OVER THE LAST DIMENSION (E.G., COLUMNS OF A MATRIX), USE THE
SYMBOL \( / \) IN PLACE OF \( \_ \).

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS.

FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND
COLUMNS, / WILL REDUCE OVER THE COLUMNS, \ over the pages, 
TO REDUCE OVER ANY DIMENSION, YOU MAY SPECIFY THE DIMENSION IN 
BRACKETS FOLLOWING THE \ (OR THE /) SYMBOL.
FOR EXAMPLE, TO PLUS REDUCE OVER THE ROWS OF A THREE-DIMENSIONAL 
OVER THE PAGES, ENTER ++[AA OR +/[1]AA, TO PLUS REDUCE OVER 
THE COLUMNS, ENTER +/AA OR +/[3]AA.
THE SHAPE OF THE ANSWER WILL ALWAYS BE THE SAME AS THE SHAPE OF THE 
ARGUMENT OMITTING THE DIMENSION REDUCED OVER.
FOR EXAMPLE: IF \AA = 3 4 5 
THEN + AA = 4 5 +/[2]AA = 3 5 +/AA = 3 4
THE SYMBOL ` (UPSHIFT P) IS USED FOR THE MONADIC EXPONENTIAL FUNCTION,  
IT RETURNS THE VALUE OF THE CONSTANT E TO THE POWER OF THE ARGUMENT,  
IT TAKES NUMERIC ARGUMENTS OF ANY RANK,  
FOR EXAMPLE:  \[ e^0 = 1 \]  \[ e^1 = 2.718281828 \]  
\[ e^{.5} = 1.648721271 \]  
\[ e^{(3 - 2)} = 20.08553692\]  
\[ e^{.5} \]  
\[ e^{(3 - 2)} = 20.08553692\]  
\[ e^{.5} \]  
\[ e^{(3 - 2)} = 20.08553692\]
THE SYMBOL \( ^* \) (UPSHIFT P) IS USED FOR THE DYADIC POWER FUNCTION, THIS RETURNS THE VALUE(S) OF THE LEFT ARGUMENT RAISED TO THE POWER OF RIGHT ARGUMENT, ARGUMENTS ARE NUMERIC OF ANY RANK, A NON-INTEGER RIGHT ARGUMENT CAN BE USED TO OBTAIN ROOTS.

FOR EXAMPLE: \( 2^3 = 8 \) \( 3^0 = 1 \) \( 4^*5 = 2 \)

TO RAISE EACH ELEMENT OF A VECTOR TO THE SAME POWER, ENTER \( V^*P \),

TO RAISE A SCALAR TO A SERIES OF POWERS REPRESENTED BY A VECTOR,

ENTER \( S^V \),

\( V^*V2 \) RESULTS IN THE ELEMENTS OF \( V1 \) BEING RAISED TO THE POWER OF THE CORRESPONDING ELEMENT IN \( V2 \).

\( V1 \) AND \( V2 \) MUST HAVE THE SAME NUMBER OF ELEMENTS OR A 'LENGTH ERROR' WILL RESULT.

FOR EXAMPLE: \( 3^*(1 2 3) = 3^9 27 \) \( (1 2 3)^*3 = 18 27 \)

\( (2 3 4)^*(5 4 3) = 32 81 64 \)

TO RAISE EACH ELEMENT OF THE MATRIX \( M \) TO THE SAME POWER \( S \),

ENTER \( M^*S \), TO RAISE THE SCALAR \( S \) TO THE POWER OF EACH ELEMENT IN THE MATRIX \( M \), ENTER \( S^M \),

THE RESULT WILL BE A MATRIX THE SAME SHAPE AS \( M \).

FOR EXAMPLE: IF \( M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \) THEN \( 3^M = \begin{bmatrix} 3^9 & 27 \\ 3^3 & 27 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 3 & 4 \end{bmatrix} \)

\( M^1*M2 \) RESULTS IN THE ELEMENTS OF \( M1 \) RAISED TO THE POWER OF THE CORRESPONDING ELEMENT IN \( M2 \).

FOR EXAMPLE: IF \( M1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \) AND \( M2 = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \) THEN \( M1*M2 = \begin{bmatrix} 1 & 8 \\ 3 & 4 \end{bmatrix} \)

IF \( M1 \) AND \( M2 \) ARE NOT THE SAME SHAPE, A 'LENGTH ERROR' WILL OCCUR.

IF ONE ARGUMENT IS A VECTOR WHILE THE OTHER ARGUMENT IS A MATRIX, THE RESULT WILL BE A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL \( \text{o} \) (ALT 9) IS USED FOR THE MONADIC NATURAL LOG FUNCTION.

IT RETURNS THE VALUE OF THE POWER TO WHICH THE CONSTANT \( e \) MUST
BE RAISED TO EQUAL THE ARGUMENT. IN OTHER WORDS, ENTERING
\( \text{o} \) WILL SOLVE THE APL EQUATION \( e^x = y \) FOR \( x \).

IT TAKES ONLY POSITIVE NUMERIC ARGUMENTS OF ANY RANK, USING \( \text{o} \) WITH
ZERO OR WITH NEGATIVE ARGUMENTS WILL RESULT IN A 'DOMAIN ERROR'.

FOR EXAMPLE: \( \text{o} 1 \) \( \Rightarrow \) 0 \( \quad \text{o} 2.718281828 \) \( \Rightarrow \) 1
\( \text{o} 1.648721271 \) \( \Rightarrow \) 0.5

FOR EXAMPLE: \( \text{o} (3 2) \) \( \Rightarrow \) 1.098612289 0.6931471806
THE SYMBOL \( \circ \) (ALT 8) IS USED FOR THE DYADIC LOGARITHM FUNCTION, THIS RETURNS THE LOGARITHM OF THE RIGHT ARGUMENT IN THE BASE OF THE LEFT ARGUMENT. THAT IS, IT RETURNS THE VALUE OF THE POWER TO WHICH THE LEFT ARGUMENT MUST BE RAISED TO EQUAL THE RIGHT ARGUMENT, ENTER \( \mathrm{L} \circ \mathrm{R} \) TO SOLVE THE EQUATION \( \mathrm{L} \circ \mathbf{n} = \mathrm{R} \) FOR \( \mathbf{n} \), ARGUMENTS ARE POSITIVE NUMERIC OF ANY RANK, USING \( \circ \) WITH ZERO OR WITH NEGATIVE NUMBERS WILL RESULT IN A 'DOMAIN ERROR'. ENTERING \( \mathrm{jS} \) WHERE \( S \) IS ANY NUMBER EXCEPT 1 WILL ALSO PRODUCE A 'DOMAIN ERROR'.

FOR EXAMPLE: \( 10 \circ 100 \Rightarrow 2 \) \( 2 \circ 1024 \Rightarrow 10 \) \( 9 \circ 1 \Rightarrow 0 \)

TO OBTAIN THE LOG OF EACH ELEMENT OF A VECTOR IN THE SAME BASE, ENTER \( \mathrm{PV} \circ \), TO DETERMINE THE LOG OF A SCALAR IN A VECTOR OF DIFFERENT BASES, ENTER \( \mathrm{V} \circ \mathrm{S} \), \( \mathrm{V} \circ \mathrm{V} \) RESULTS IN THE LOG OF EACH ELEMENT IN \( \mathrm{V} \) IN THE BASE OF THE CORRESPONDING ELEMENT IN \( \mathrm{V} \), \( \mathrm{V} \) AND \( \mathrm{V} \) MUST HAVE THE SAME NUMBER OF ELEMENTS OR A 'LENGTH ERROR' WILL RESULT.

FOR EXAMPLE: \( 3 \circ (3 \ 9 \ 27) \Rightarrow 1 \ 2 \ 3 \) \( (2 \ 4) \circ 16 \Rightarrow 4 \ 2 \) \( (2 \ 3 \ 5) \circ (4 \ 9 \ 25) \Rightarrow 2 \ 2 \ 2 \)

ANALOGOUS RESULTS ARE OBTAINED FOR MATRICES AND HIGHER LEVEL ARRAYS, UNLESSS ONE ARGUMENT IS A SCALAR, BOTH ARGUMENTS MUST BE THE SAME SHAPE, OTHERWISE, A 'RANK ERROR' OR 'LENGTH ERROR' WILL RESULT.
THE SYMBOL $\pi$ (UPSHIFT 0) IS USED FOR THE MONADIC PI TIMES FUNCTION.
IT RETURNS THE VALUE OF THE CONSTANT PI TIMES THE ARGUMENT.
IT TAKES NUMERIC ARGUMENTS OF ANY RANK.
FOR EXAMPLE:
\[ \pi 0 = 0 \]
\[ \pi 0.1 = 3.141592654 \]
\[ \pi 0.5 = 1.570796327 \]
\[ \pi (3 -2) = 9.424777961 -0.283185307 \]
THE SYMBOL @ (UPSHIFT @) IS USED FOR THE DYADIC GEOMETRIC FUNCTIONS.

THESE INCLUDE THE MAIN TRIGONOMETRIC FUNCTIONS, THE LEFT ARGUMENT,

WHICH MUST BE AN INTEGER FROM -7 TO 7, DETERMINES WHICH FUNCTION

IS CALLED, THE RIGHT ARGUMENT, WHICH CAN BE A NUMERIC ARRAY OF

ANY RANK, REPRESENTS THE VALUE OF AN ANGLE IN RADIANS.

THE FOLLOWING FUNCTIONS ARE REPRESENTED:

\begin{align*}
00Y &= \cos(\arcsin Y) \text{ or } \sin(\arccos Y) \text{ WHERE } |Y| < 1 \\
10Y &= \sin Y \\
20Y &= \cos Y \\
30Y &= \tan Y \\
40Y &= \cosh(\arcsinh Y) \\
50Y &= \sinh Y \\
60Y &= \cosh Y \\
70Y &= \tanh Y
\end{align*}

OTHER FUNCTIONS, SUCH AS SECANT, MUST BE COMPUTED BY FORMULA.

FOR EXAMPLE: (REMEMBER 01 EQUALS PI TIMES 1)

\begin{align*}
100 &= 0 \\
101 &= 0 \\
200 &= 1 \\
201 &= 1 \\
300.5 &= 5.734161139E15
\end{align*}

NOTICE THAT SOMETIMES A VERY SMALL NUMBER WILL APPEAR INSTEAD OF ZERO,

OR A VERY LARGE NUMBER WILL APPEAR INSTEAD OF A DOMAIN ERROR,

TECHNICALLY, THE TANGENT OF ONE-HALF PI DOES NOT EXIST, BUT:

\begin{align*}
399.5 &= \ \text{watch out for this!}
\end{align*}
THE SYMBOL ! (ALT +) IS USED FOR THE MONADIC FACTORIAL FUNCTION,
IT RETURNS THE FACTORIAL OF NON-NEGATIVE NUMERIC ARGUMENTS OF ANY RANK,
FOR EXAMPLE: 1 3 0 => 1 6 1

USING NON-INTEGER ARGUMENTS PRODUCES THE GAMMA FUNCTION OF THE
ARGUMENT+1,
FOR EXAMPLE: .5 => .8862269255 (GAMMA OF 1.5)
THE SYMBOL ! (ALT +) IS USED FOR THE DYADIC BINOMIAL FUNCTION.
IT TAKES NUMERIC ARGUMENTS OF ANY RANK.
FOR POSITIVE INTEGER ARGUMENTS, L : R WILL RETURN THE NUMBER OF WAYS OF
TAking R OBJECTS L AT A TIME.
THIS IS THE BINOMIAL COEFFICIENT OF R OVER L, COMPUTED (?R)/[[L][R-L]],
WITH NEGATIVE OR NON-INTEGER ARGUMENTS, THE BINOMIAL FUNCTION RETURNS
A VALUE BASED ON THE BETA FUNCTION.
FOR EXAMPLE: 3 : 4 => 4 BUT 4 : 3 => 0
BECAUSE THERE ARE FOUR WAYS OF TAKING FOUR OBJECTS THREE AT A TIME,
BUT THERE IS NO WAY TO TAKE THREE OBJECTS FOUR AT A TIME.
THESE ANSWERS ARE BASED ON CONVENTION:
0 : 0 => 1 0 : 42 => 1 42 : 42 => 1
WHEN A SCALAR ARGUMENT IS PAIRED WITH A VECTOR ARGUMENT, THE FUNCTION
COMBINES THE SCALAR WITH EACH ELEMENT OF THE VECTOR IN TURN,
2 : (3 4 5) => 3 6 10 (3 4 5) : 5 => 10 5 1
CORRESPONDING ELEMENTS OF TWO VECTOR ARGUMENTS ARE COMBINED.
(3 4 5) : (5 6 7) => 10 15 21
V1 AND V2 MUST HAVE THE SAME NUMBER OF ELEMENTS OR A 'LENGTH ERROR'
WILL RESULT.
WHEN A SCALAR ARGUMENT IS PAIRED WITH A MATRIX ARGUMENT, THE FUNCTION
COMBINES THE SCALAR WITH EACH ELEMENT OF THE MATRIX IN TURN,
FOR EXAMPLE: IF M = (1 2) THEN 3 : M => 0 0 AND M : 3 => 3 3
3 4 1 4 1 0
CORRESPONDING ELEMENTS OF TWO MATRIX ARGUMENTS ARE COMBINED.
FOR EXAMPLE: IF M1 = (1 2) AND M2 = (3 4) THEN M1 : M2 => 3 6
3 4 5 6 1 0 1 5
IF M1 AND M2 ARE NOT THE SAME SHAPE, A 'LENGTH ERROR' WILL OCCUR.
IF ONE ARGUMENT IS A VECTOR WHILE THE OTHER ARGUMENT IS A MATRIX,
THE RESULT WILL BE A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL | (UPSHIFT H) IS USED FOR THE MONADIC MAGNITUDE FUNCTION, IT RETURNS THE ABSOLUTE VALUE OF NUMERIC ARGUMENTS OF ANY RANK.

FOR EXAMPLE: 10 => 0 |(-3 -2 10) => 3 2 10
THE SYMBOL \ (UPSHIFT /) IS USED FOLLOWING A DYADIC FUNCTION SYMBOL, 
PRECEDING A SINGLE ARGUMENT PRODUCING A MIXED FUNCTION CALLED SCAN. 
SCAN MAY BE USED WITH NUMERIC ARGUMENTS OF ANY RANK, 
SCAN MAY ALSO BE USED WITH CHARACTER ARGUMENTS IF THE FUNCTION IT IS 
COMBINED WITH ACCEPTS CHARACTER ARGUMENTS.

USING \ WITH A SCALAR SIMPLY RETURNS THE SCALAR.

WHEN USED WITH A VECTOR, SCAN GIVES THE SAME EFFECT AS INSERTING 
THE OTHER FUNCTION SYMBOL BETWEEN EACH VALUE OF THE VECTOR, 
THEN OPERATING PROGRESSIVELY AS FOLLOWS:

THE FIRST ELEMENT OF THE ANSWER WILL BE THE FIRST ELEMENT OF THE 
ARGUMENT, THE SECOND ELEMENT WILL BE THE RESULT OF THE MAIN 
FUNCTION OPERATING ON THE FIRST TWO ELEMENTS OF THE ARGUMENT, 
THE THIRD ELEMENT OF THE ANSWER WILL BE THE RESULT OF THE MAIN 
FUNCTION OPERATING AS IN REDUCTION ON THE FIRST THREE ELEMENTS 
OF THE ARGUMENT, AND SO ON.

THE RESULT WILL ALWAYS BE THE SAME LENGTH AS THE ARGUMENT.

FOR EXAMPLE:

\(+(1 2 3)\) \(\Rightarrow\) 1 3 6 BECAUSE 1 \(\Rightarrow\) 1; 1+2 \(\Rightarrow\) 3; 1+2+3 \(\Rightarrow\) 6
\(\times(2 3 4)\) \(\Rightarrow\) 2 6 24

WHEN USING SCAN, ALWAYS REMEMBER APL READS RIGHT TO LEFT WHEN IT IS 
USING REDUCTION ON MORE THAN TWO ELEMENTS OF THE ARGUMENT.
\(-(-1 2 3)\) \(\Rightarrow\) 1 -1 2 BECAUSE 1 \(\Rightarrow\) 1; 1-2 \(\Rightarrow\) -1; 1-(2-3) \(\Rightarrow\) 2

WHEN USED WITH A MATRIX, SCAN WILL PERFORM AS IF EACH COLUMN IS 
A UNIT, THE FIRST COLUMN OF THE ANSWER IS THE FIRST COLUMN OF 
THE ARGUMENT, THE SECOND COLUMN OF THE ANSWER IS THE RESULT OF 
OPERATING ON THE FIRST TWO COLUMNS OF THE ARGUMENT, AND SO ON.

APL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE COLUMNS.
SINCE COLUMNS ARE THE SECOND OF TWO DIMENSIONS OF A MATRIX, THIS MEANS THAT THE SYMBOL \ WORKS 'OVER' THE LAST DIMENSION, TO SCAN OVER THE FIRST DIMENSION (E.G., ROWS OF A MATRIX), USE THE SYMBOL \ (ALT. ) IN PLACE OF \.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS, FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND COLUMNS, \ WILL SCAN OVER THE COLUMNS, \ OVER THE PAGES, TO SCAN OVER ANY DIMENSION, YOU MAY SPECIFY THE DIMENSION IN BRACKETS FOLLOWING THE \ (OR THE \ ) SYMBOL.


THE RESULT WILL ALWAYS BE THE SAME SHAPE AS THE ARGUMENT.
THE SYMBOL \( \langle \) (ALT \( , \)) IS USED FOLLOWING A DYADIC FUNCTION SYMBOL AND
PRECEDING A SINGLE ARGUMENT PRODUCING A MIXED FUNCTION CALLED SCAN.
SCAN MAY BE USED WITH NUMERIC ARGUMENTS OF ANY RANK,
SCAN MAY ALSO BE USED WITH CHARACTER ARGUMENTS IF THE FUNCTION IT IS
COMBINED WITH ACCEPTS CHARACTER ARGUMENTS,

USING \( \langle \) WITH A SCALAR SIMPLY RETURNS THE SCALAR,

WHEN USED WITH A VECTOR, SCAN GIVES THE SAME EFFECT AS INSERTING
THE OTHER FUNCTION SYMBOL BETWEEN EACH VALUE OF THE VECTOR,
THEN OPERATING PROGRESSIVELY AS FOLLOWS;
THE FIRST ELEMENT OF THE ANSWER WILL BE THE FIRST ELEMENT OF THE
ARGUMENT, THE SECOND ELEMENT WILL BE THE RESULT OF THE MAIN
FUNCTION OPERATING ON THE FIRST TWO ELEMENTS OF THE ARGUMENT,
THE THIRD ELEMENT OF THE ANSWER WILL BE THE RESULT OF THE MAIN
FUNCTION OPERATING AS IN REDUCTION ON THE FIRST THREE ELEMENTS
OF THE ARGUMENT, AND SO ON,
THE RESULT WILL ALWAYS BE THE SAME LENGTH AS THE ARGUMENT,

FOR EXAMPLE:
\[ +\langle 1 2 3 \rangle \Rightarrow 1 \, 3 \, 6 \text{ BECAUSE } 1 \Rightarrow 1, \, 1+2 \Rightarrow 3, \, 1+2+3 \Rightarrow 6 \]
\[ \times\langle 2 \, 3 \, 4 \rangle \Rightarrow 2 \, 6 \, 24 \]
WHEN USING SCAN, ALWAYS REMEMBER APL READS RIGHT TO LEFT WHEN IT IS
USING REDUCTION ON MORE THAN TWO ELEMENTS OF THE ARGUMENT,
\[ -\langle 1 \, 2 \, 3 \rangle \Rightarrow 1 \, -1 \, 2 \text{ BECAUSE } 1 \Rightarrow 1, \, 1-2 \Rightarrow -1, \, 1-(2-3) \Rightarrow 2 \]
WHEN USED WITH A MATRIX, SCAN WILL PERFORM AS IF EACH ROW IS
A UNIT, THE FIRST ROW OF THE ANSWER IS THE FIRST ROW OF
THE ARGUMENT, THE SECOND ROW OF THE ANSWER IS THE RESULT OF
OPERATING ON THE FIRST TWO ROWS OF THE ARGUMENT, AND SO ON,
APL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE ROWS,
SINCE ROWS ARE THE FIRST OF TWO DIMENSIONS OF A MATRIX,
THIS MEANS THAT THE SYMBOL \ works 'over' the first dimension.
TO SCAN OVER THE LAST DIMENSION (E.G., COLUMNS OF A MATRIX), USE
THE SYMBOL \ (UPSHIFT /), IN PLACE OF \.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS.
FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND
COLUMNS, \ WILL SCAN OVER THE COLUMNS, \ OVER THE PAGES.
TO SCAN OVER ANY DIMENSION, YOU MAY SPECIFY THE DIMENSION IN
BRACKETS FOLLOWING THE \ (OR THE \) SYMBOL.
FOR EXAMPLE, TO PLUS SCAN OVER THE ROWS OF A THREE-DIMENSIONAL
ARRAY CALLED AA, ENTER +[\2]AA OR +\[2]AA, TO PLUS SCAN OVER
THE PAGES, ENTER +[\]AA OR +\[1]AA, TO PLUS SCAN OVER THE COLUMNS,
ENTER +[\]AA OR +\[3]AA.
THE RESULT WILL ALWAYS BE THE SAME SHAPE AS THE ARGUMENT.
THE SYMBOL CEILING IS USED FOR THE NONADIC CEILING FUNCTION. IT RETURNS THE VALUE OF NUMERIC ARGUMENTS OF ANY RANK, 'ROUNDED UP' TO THE NEAREST INTEGER. FOR EXAMPLE: CEIL(-3.4 0 1.3 2) ==> -3 0 2 2
THE SYMBOL \(\lfloor\) (UPSHIFT D) IS USED FOR THE MONADIC FLOOR FUNCTION.
IT RETURNS THE VALUE OF NUMERIC ARGUMENTS OF ANY RANK, 'ROUNDED DOWN'
TO THE NEAREST INTEGER.
FOR EXAMPLE: \(\lfloor -3.4 \ 0 \ 1.3 \ 2 \rfloor = -4 \ 0 \ 1 \ 2\)
THE SYMBOL ⌈ (UPSHIFT S) IS USED FOR THE DYADIC MAXIMUM FUNCTION.
THIS RETURNS THE MAXIMUM OF ITS TWO ARGUMENTS, WHICH ARE NUMERIC
OF ANY RANK.

FOR EXAMPLE: 2⌈3 =) 3

TO CHECK A SCALAR AGAINST EACH ELEMENT OF A VECTOR, ENTER S⌈V OR V⌈S.
V⌈V2 WILL COMPARE TWO VECTORS ELEMENT BY ELEMENT, FOR EXAMPLE:
4⌈(3 4 5) =) 4 4 5    (3 4 5)⌈5 =) 5 5 5
(3 4 5)⌈(1 5 9) =) 3 5 9
IF THE TWO VECTORS BEING COMPARED ARE NOT THE SAME LENGTH,
A 'LENGTH ERROR' WILL RESULT.

MATRIX COMPARISON IS DONE AS FOLLOWS:
S⌈M OR M⌈S WILL CHECK THE SCALAR S AGAINST EACH ELEMENT OF
THE MATRIX M.
FOR EXAMPLE: IF M =) 1 2 THEN 3⌈M =) 3 3 AND M⌈3 =) 3 3
3 4
3 4
M⌈M2 WILL COMPARE THE MATRIXES M1 AND M2, ELEMENT BY ELEMENT.
FOR EXAMPLE: IF M1 =) 1 2 AND M2 =) 3 4 THEN M1⌈M2 =) 3 4
3 4
3 4
3 4
5 6
5 6
M1 AND M2 MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.
TRYING TO COMPARE A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL \( L \) (UPSHIFT D) IS USED FOR THE DYADIC MINIMUM FUNCTION.
THIS RETURNS THE MINIMUM OF ITS TWO ARGUMENTS, WHICH ARE NUMERIC
OF ANY RANK.

FOR EXAMPLE: \( 2L3 \Rightarrow 2 \)

TO CHECK A SCALAR AGAINST EACH ELEMENT OF A VECTOR, ENTER SLV OR VLS.
VLVS WILL COMPARE TWO VECTORS ELEMENT BY ELEMENT. FOR EXAMPLE:
\[
4L(3 4 5) \Rightarrow 3 4 4 \\
(3 4 5)LS \Rightarrow 3 4 5 \\
(3 4 5)L(1 5 9) \Rightarrow 1 4 5
\]
IF THE TWO VECTORS BEING COMPARED ARE NOT THE SAME LENGTH,
A 'LENGTH ERROR' WILL RESULT.

MATRIX COMPARISON IS DONE AS FOLLOWS:
SLM OR HLS WILL CHECK THE SCALAR S AGAINST EACH ELEMENT OF
THE MATRIX M.
FOR EXAMPLE: IF \( M = 1 2 \) THEN \( SLM \Rightarrow 1 2 \) AND \( MLS \Rightarrow 1 2 \\
3 4 \\
3 3 \\
3 3 \)
M1LM2 WILL COMPARE THE MATRICES M1 AND M2, ELEMENT BY ELEMENT.
FOR EXAMPLE: IF \( M1 = 1 2 \) AND \( M2 = 3 4 \) THEN \( M1LM2 \Rightarrow 1 2 \\
3 4 \\
5 6 \\
3 4 \)
M1 AND M2 MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.
TRYING TO COMPARE A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL | (UPSHIFT M) IS USED FOR THE DYADIC RESIDUE FUNCTION.
IT TAKES NUMERIC ARGUMENTS OF ANY RANK.
THE RESULT OF L | R IS THE REMAINDER WHEN R IS DIVIDED BY L. THIS IS
SIMILAR TO THE 'MODULO' FUNCTION IN OTHER COMPUTER LANGUAGES.

FOR EXAMPLE:
111.24 \rightarrow 0.24 \quad 101^{-4} \rightarrow 2 \quad ^{-4}110 \rightarrow ^{-2}
THE SIGN OF THE RESULT IS ALWAYS THE SAME AS THE SIGN OF THE LEFT
ARGUMENT, IF THE SIGNS OF THE ARGUMENTS DO NOT AGREE, THE RESULT
IS OBTAINED BY ADDING THE SMALLER ARGUMENT TO THE LARGER ONE
REPEATEDLY UNTIL THE ABSOLUTE VALUE OF THE RESULT IS LESS THAN
THE ABSOLUTE VALUE OF THE SMALLER ARGUMENT.

WHEN A SCALAR ARGUMENT IS PAIRED WITH A VECTOR ARGUMENT, THE FUNCTION
COMBINES THE SCALAR WITH EACH ELEMENT OF THE VECTOR IN TURN,
2|(1 2 3 4 5) \rightarrow 1 0 1 0 1 \quad (1 2 3 4 5)|3 \rightarrow 0 1 0 3 3
CORRESPONDING ELEMENTS OF TWO VECTOR ARGUMENTS ARE COMBINED.
(1 1.5 2)|1(1 2 3) \rightarrow 0 0.5 1
V_1 AND V_2 MUST HAVE THE SAME NUMBER OF ELEMENTS OR A 'LENGTH ERROR'
WILL RESULT.

WHEN A SCALAR ARGUMENT IS PAIRED WITH A MATRIX ARGUMENT, THE FUNCTION
COMBINES THE SCALAR WITH EACH ELEMENT OF THE MATRIX IN TURN.
FOR EXAMPLE: IF M \rightarrow 1 2 THEN 3|M \rightarrow 1 2 AND M|3 \rightarrow 0 1
3 4
0 1
0 3
CORRESPONDING ELEMENTS OF TWO MATRIX ARGUMENTS ARE COMBINED.
FOR EXAMPLE: IF M_1 = 1 2 AND M_2 = 3 4 THEN M_1|M_2 \rightarrow 2 2
3 4
5 6
2 2
IF M_1 AND M_2 ARE NOT THE SAME SHAPE, A 'LENGTH ERROR' WILL OCCUR.
IF ONE ARGUMENT IS A VECTOR WHILE THE OTHER ARGUMENT IS A MATRIX,
THE RESULT WILL BE A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
The symbol \( \& \) (ALT 4) is used for the monadic Grade Up function. It takes a numeric vector argument and returns a vector of the indexes of the elements in order from smallest to largest.

In other words, the first element in the response is the index of the smallest element in the input, the last element in the response is the index of the largest element in the input.

Equal values will be graded in order from left to right.

For example:

\[
\& \begin{pmatrix} 4 & 8 & 6 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 & 1 & 3 & 2 \end{pmatrix}
\]

Because the fourth element of the input (3) is the smallest, etc.

This can be used to sort a vector by combining with index brackets:

\[
\text{IF } v = \begin{pmatrix} 4 & 8 & 6 & 3 \end{pmatrix} \text{ THEN } v[\& v] \Rightarrow \begin{pmatrix} 3 & 4 & 6 & 8 \end{pmatrix}
\]

On the other hand, \( \& v \) will produce a vector of the position numbers (rank order) corresponding to the input vector.

For example:

\[
\& \begin{pmatrix} 4 & 8 & 6 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 4 & 3 & 1 \end{pmatrix}
\]

Since 4 is the second smallest element, 8 is the fourth smallest, etc.
THE SYMBOL ‹ (ALT 3) IS USED FOR THE MONADIC GRADE DOWN FUNCTION.
IT TAKES A NUMERIC VECTOR ARGUMENT AND RETURNS A VECTOR OF THE
INDEXES OF THE ELEMENTS IN ORDER FROM LARGEST TO SMALLEST.
IN OTHER WORDS, THE FIRST ELEMENT IN THE RESPONSE IS THE INDEX
OF THE LARGEST ELEMENT IN THE INPUT, THE LAST ELEMENT IN THE
RESPONSE IS THE INDEX OF THE SMALLEST ELEMENT IN THE INPUT.
EQUAL VALUES WILL BE GRADED IN ORDER FROM LEFT TO RIGHT.

FOR EXAMPLE: ‹ 4 8 6 3 = 2 3 1 4
BECAUSE THE SECOND ELEMENT OF THE INPUT (8) IS THE LARGEST, ETC.
THIS CAN BE USED TO SORT A VECTOR BY COMBINING WITH INDEX BRACKETS:
IF \( V = 4 3 6 3 \) THEN \( V[\langle V \rangle] = 8 6 4 3 \)

ON THE OTHER HAND, ‹ ‹ V WILL PRODUCE A VECTOR OF THE POSITION NUMBERS
(IN REVERSE ORDER) CORRESPONDING TO THE INPUT VECTOR.
FOR EXAMPLE: ‹ ‹ 4 8 6 3 = 3 1 2 4
SINCE 4 IS THE THIRD LARGEST ELEMENT, 8 IS THE LARGEST, ETC.
Roll

The symbol ? (upshift 0) is used for the monadic Roll function. This is a random number generator which takes positive integer arguments and returns a randomly selected integer up to the value of the argument. The lowest possible value which can be returned is normally (by default) 1.

Using ? with negative arguments or zero produces a 'Domain Error'.

For example: ?3 =) either 1, 2, or 3, with equal probability.

When used with vectors, Roll returns a vector with each number selected randomly from the range determined by the corresponding value in the argument.

For example:

?3 3 3 =) 1 1 2 or 1 3 2 or 3 3 2 or 2 1 3 etc.

?6 6 simulates the rolling of a pair of dice.

Roll can be used with matrices or higher level arrays in a similar fashion.
THE SYMBOL \( ? \) (UPSHIFT Q) IS USED FOR THE DYADIC DEAL FUNCTION.

THE RESULT OF THIS FUNCTION IS AN ARRAY OF RANDOM NUMBERS.

THE SHAPE OF THE ARRAY IS DETERMINED BY THE LEFT ARGUMENT,

THE NUMBERS ARE RANDOMLY SELECTED FROM THE POSITIVE INTEGERS UP TO THE VALUE OF THE RIGHT ARGUMENT, WITH NO REPLACEMENT,

THAT IS, A NUMBER CANNOT APPEAR TWICE IN THE RESULT,

BOTH ARGUMENTS MUST BE A POSITIVE SINGLE NUMBER (SCALAR OR VECTOR OF LENGTH 1), AND THE RIGHT MUST EQUAL OR EXCEED THE LEFT,

USING HIGHER-ORDER ARGUMENTS PRODUCES A 'RANK ERROR',

USING NEGATIVE ARGUMENTS OR ZERO PRODUCES A 'DOMAIN ERROR',

FOR EXAMPLE: \( 2?2 \Rightarrow 1 \ 2 \) OR \( 2 \ 1 \) BUT NOT \( 2 \ 2 \) OR \( 1 \ 1 \)

\( 5?5 \) SIMULATES THE DEAL OF A HAND OF CARDS.
THE SYMBOL = (UPSHIFT 5) IS USED FOR THE DYADIC EQUALS FUNCTION.
IT COMPARES NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK.
IT RETURNS 1 FOR EACH ELEMENT OF THE LEFT ARGUMENT THAT IS IDENTICAL
TO THE CORRESPONDING ELEMENT OF THE RIGHT ARGUMENT, AND 0 FOR
EACH ELEMENT THAT IS NOT.

FOR EXAMPLE: 2=2 => 1  2=4 => 0  'A'='B' => 0

TO COMPARE A SCALAR TO EACH ELEMENT OF A VECTOR,
ENTER EITHER S=V OR V=S,
2=(2 3 4) => 1 0 0  (3  4 5)=5 => 0 0 1
V1=V2 WILL COMPARE TWO VECTORS ELEMENT BY ELEMENT, FOR EXAMPLE;
'THIN':='THANK' => 1 1 0 1 1
IF THE TWO VECTORS BEING COMPARED ARE NOT THE SAME LENGTH,
YOU WILL GET A 'LENGTH ERROR'.

MATRIX COMPARISON IS DONE AS FOLLOWS;
S=M OR M=S WILL COMPARE THE SCALAR S TO EACH ELEMENT OF THE MATRIX M,
FOR EXAMPLE: IF M = 1 2 THEN 3=M => 0 0 AND M=3 => 0 0
   3 4
   1 0
M1=M2 WILL COMPARE THE MATRIX M1 TO THE MATRIX M2, ELEMENT BY ELEMENT.
FOR EXAMPLE: IF M1 = HOW AND M2 = DOH THEN M1=M2 => 0 1 0
   NOW
   WOW
M1 AND M2 MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.
COMPARING A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOl \# (UPSHIFT 8) IS USED FOR THE DYADIC NOT EQUAL FUNCTION.

THIS COMPARES NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK.

IT RETURNS 1 FOR EACH ELEMENT OF THE LEFT ARGUMENT THAT IS NOT EQUAL

TO THE CORRESPONDING ELEMENT OF THE RIGHT ARGUMENT, AND 0 FOR

EACH ELEMENT THAT IS.

FOR EXAMPLE: 2#2 = 0  2#4 = 1  'A'#'B' = 1

TO COMPARE A SCALAR TO EACH ELEMENT OF A VECTOR,

ENTER EITHER S#V OR V#S.

2#(2 3 4) = 0 1 1  (3 4 5)#5 = 1 1 0

V#V2 WILL COMPARE TWO VECTORS ELEMENT BY ELEMENT, FOR EXAMPLE;

'THINK'#'THANK' = 0 0 1 0 0

IF THE TWO VECTORS BEING COMPARED ARE NOT THE SAME LENGTH,

YOU WILL GET A 'LENGTH ERROR'.

MATRIX COMPARISON IS DONE AS FOLLOWS;

S#M OR M#S WILL COMPARE THE SCALAR S TO EACH ELEMENT OF THE MATRIX M.

FOR EXAMPLE: IF M = 1 2 THEN 3#M = 1 1 AND M#3 = 1 1

3 4

0 1 0 1

M#M2 WILL COMPARE THE MATRIX M1 TO THE MATRIX M2, ELEMENT BY ELEMENT.

FOR EXAMPLE: IF M1 = HOW AND M2 = OOH THEN M1#M2 = 1 0 1

HOW

HOW 1 0 0

M1 AND M2 MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.

COMPARING A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
THE SYMBOL ` (UPSHIFT 3) IS USED FOR THE DYADIC LESS THAN FUNCTION. THIS COMPARES NUMERIC ARGUMENTS OF ANY RANK, IT RETURNS 1 FOR EACH ELEMENT OF THE LEFT ARGUMENT THAT IS LESS THAN THE CORRESPONDING ELEMENT OF THE RIGHT ARGUMENT, AND 0 FOR EACH ELEMENT THAT IS NOT.

FOR EXAMPLE: 2(2 =) 0 2(4 =) 1

TO COMPAR E A SCALAR TO EACH ELEMENT OF A VECTOR, ENTER EITHER S(V OR V(S, ORDER IS IMPORTANT.

2(1 2 3) =) 0 0 1 (3 4 5)(5 =) 1 1 0

V1(V2 WILL COMPARE TWO VECTORS ELEMENT BY ELEMENT, FOR EXAMPLE:

(1 2 3)(1 3 5) =) 0 1 1

IF THE TWO VECTORS BEING COMPARED ARE NOT THE SAME LENGTH, YOU WILL GET A 'LENGTH ERROR'.

MATRIX COMPARISON IS DONE AS FOLLOWS:

S(M OR M(S WILL COMPARE THE SCALAR S TO EACH ELEMENT OF THE MATRIX M.

FOR EXAMPLE: IF M =) 1 2 THEN (M =) 0 0 AND M(3 =) 1 1

3 4 0 1 0 0

M1(M2 WILL COMPARE THE MATRIX M1 TO THE MATRIX M2, ELEMENT BY ELEMENT, FOR EXAMPLE: IF M1 =) 1 4 AND M2 =) 2 6 THEN M1(M2 =) 1 1

4 7 5 3 1 0

M1 AND M2 MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT, COMPARING A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
LESS_OR_EQUAL

\[ \leq \]

The symbol \( \leq \) (upshift 4) is used for the dyadic less than or equal to function. This compares numeric arguments of any rank. It returns 1 for each element of the left argument that is less than or equal to the corresponding element of the right argument, and 0 for each element that is not.

For example: \( 2 \leq 2 \Rightarrow 1 \)
\( 2 \leq 4 \Rightarrow 1 \)
\( 2 \leq 2 \Rightarrow 0 \)

To compare a scalar to each element of a vector,

\( \text{enter either } \lt V \text{ or } V \lt \), order is important.

\( 2 \lt 1 \lt 3 \) \( \Rightarrow 0 \ 1 \ 1 \)
\( 3 \lt 4 \lt 5 \) \( \Rightarrow 1 \ 1 \ 1 \)

\( V \lt V \) will compare two vectors element by element. For example:

\( (1 \ 2 \ 3) \lt (1 \ 3 \ 5) \Rightarrow 1 \ 1 \ 1 \)

If the two vectors being compared are not the same length, you will get a 'length error'.

Matrix comparison is done as follows:

\( \leq M \) or \( M \leq \) will compare the scalar \( S \) to each element of the matrix \( M \).

For example: if \( M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \), then \( 3 \leq M = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \) and \( M \leq 3 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \).

\( M \leq M \) will compare the matrix \( M_1 \) to the matrix \( M_2 \), element by element.

For example: if \( M_1 = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix} \) and \( M_2 = \begin{pmatrix} 2 & 6 \\ 5 & 3 \end{pmatrix} \), then \( M_1 \leq M_2 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \).

\( M_1 \) and \( M_2 \) must be the same shape or a 'length error' will result, comparing a vector to a matrix will result in a 'rank error'.

These principles can be extended to higher level arrays.
THE SYMBOH \( \geq \) (UPSHIFT 6) IS USED FOR THE DYADIC GREATER THAN OR EQUAL TO FUNCTION. THIS COMPARES NUMERIC ARGUMENTS OF ANY RANK.

IT RETURNS 1 FOR EACH ELEMENT OF THE LEFT ARGUMENT THAT IS GREATER THAN OR EQUAL TO THE CORRESPONDING ELEMENT OF THE RIGHT ARGUMENT, AND 0 FOR EACH ELEMENT THAT IS NOT.

FOR EXAMPLE: \( 2 \leq 2 \Rightarrow 1 \)  \( 2 \leq 4 \Rightarrow 0 \)  \( 2 \leq 2 \Rightarrow 1 \)

TO COMPARE A SCALAR TO EACH ELEMENT OF A VECTOR,

ENTER EITHER \( \mathbf{v} \mathbf{S} \) OR \( \mathbf{v} \mathbf{S} \mathbf{S} \), ORDER IS IMPORTANT.

\( 2 \leq (1 \ 2 \ 3) \Rightarrow 1 \ 1 \ 0 \)  \( (3 \ 4 \ 5) \leq 5 \Rightarrow 0 \ 0 \ 1 \)

\( \mathbf{v} \leq \mathbf{v} \) WILL COMPARE TWO VECTORS ELEMENT BY ELEMENT. FOR EXAMPLE:

\( (1 \ 2 \ 3) \leq (1 \ 3 \ 5) \Rightarrow 1 \ 0 \ 0 \)

IF THE TWO VECTORS BEING COMPARED ARE NOT THE SAME LENGTH,

YOU WILL GET A 'LENGTH ERROR'.

MATRIX COMPARISON IS DONE AS FOLLOWS:

\( \mathbf{s} \mathbf{M} \) OR \( \mathbf{M} \mathbf{s} \) WILL COMPARE THE SCALAR \( \mathbf{s} \) TO EACH ELEMENT OF THE MATRIX \( \mathbf{M} \).

FOR EXAMPLE: IF \( \mathbf{M} = (1 \ 2) \) THEN \( 3 \leq \mathbf{M} \Rightarrow 1 \ 1 \) AND \( \mathbf{M} \leq 3 \Rightarrow 0 \ 0 \)

\( \mathbf{M} \leq \mathbf{M} \) WILL COMPARE THE MATRIX \( \mathbf{M} \) TO THE MATRIX \( \mathbf{M} \), ELEMENT BY ELEMENT.

FOR EXAMPLE: IF \( \mathbf{M} = (1 \ 4) \) AND \( \mathbf{M} = (2 \ 6) \) THEN \( \mathbf{M} \leq \mathbf{M} \Rightarrow 0 \ 0 \)

\( \mathbf{M} \) AND \( \mathbf{M} \) MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.

COMPARING A VECTOR TO A MATRIX WILL RESULT IN A 'RANK ERROR'.

THESE PRINCIPLES CAN BE EXTENDED TO HIGHER LEVEL ARRAYS.
The symbol \( \uparrow \) (upshift 7) is used for the dyadic greater than function. This compares numeric arguments of any rank. It returns 1 for each element of the left argument that is greater than the corresponding element of the right argument, and 0 for each element that is not.

For example:

\[
2 \uparrow 2 = 0 \quad 4 \uparrow 2 = 1
\]

To compare a scalar to each element of a vector, enter either \( S \uparrow V \) or \( V \uparrow S \). Order is important.

\[
2 \uparrow (1, 2, 3) = 1 \quad (3, 4, 5) \uparrow S = 0 \quad 0 \quad 0
\]

\( V_1 \uparrow V_2 \) will compare two vectors element by element. For example:

\[
(1, 2, 3) \uparrow (1, 3, 5) = 0 \quad 0 \quad 0
\]

If the two vectors being compared are not the same length, you will get a 'length error'.

Matrix comparison is done as follows:

\( S \uparrow M \) or \( M \uparrow S \) will compare the scalar \( S \) to each element of the matrix \( M \). For example: if \( M = 1 \quad 2 \) then \( 3 \uparrow M = 1 \quad 1 \) and \( M \uparrow 3 = 0 \quad 0 \)

\[
\begin{array}{ccc}
3 & 4 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{array}
\]

\( M_1 \uparrow M_2 \) will compare the matrix \( M_1 \) to the matrix \( M_2 \), element by element. For example: if \( M_1 = 1 \quad 4 \) and \( M_2 = 2 \quad 6 \) then \( M_1 \uparrow M_2 = 0 \quad 0 \)

\[
\begin{array}{ccc}
4 & 7 & 1 \\
3 & 1 & 1
\end{array}
\]

\( M_1 \) and \( M_2 \) must be the same shape or a 'length error' will result. Comparing a vector to a matrix will result in a 'rank error'.

These principles can be extended to higher level arrays.
THE SYMBOL \( \uparrow \) (UPSHIFT T) IS USED FOR THE MONADIC NOT FUNCTION.

IT TAKES AN ARGUMENT OF ANY RANK WHICH CONSISTS ONLY OF 1'S AND 0'S,

AND RETURNS THE LOGICAL INVERSE, THAT IS, 1'S ARE CHANGED TO 0'S

AND 0'S ARE CHANGED TO 1'S.

FOR EXAMPLE: \( \downarrow 1001 = \) 0110

127
AND

THE SYMBOL A (UPSHIFT 0) IS USED FOR THE DYADIC AND FUNCTION.
IT TAKES ARGUMENTS OF ANY RANK WHICH CONSIST ONLY OF 1'S AND 0'S.
THIS FUNCTION RETURNS 1 WHERE BOTH ARGUMENTS ARE 1, AND 0 OTHERWISE.
THE ORDER OF THE ARGUMENTS IS NOT IMPORTANT, BUT ENTERING AN ARGUMENT
WHICH IS NOT 1 OR 0 WILL PRODUCE A 'DOMAIN ERROR'.

FOR EXAMPLE: 1^A => 1 1^0 => 0 0^0 => 0

TO COMPARE A SCALAR TO EACH ELEMENT OF AN ARRAY,
ENTER EITHER SSA OR AAS, FOR EXAMPLE, IF V => 2 -3 0
1^(V) => 1 0 0 (V=2) ^ V[1]=2 => 1 0 0
A^AA^2 WILL COMPARE TWO ARRAYS ELEMENT BY ELEMENT, FOR EXAMPLE:
(V)^-3) ^ (V(2) => 0 0 1
IF M1 => 1 0 AND M2 => 1 1 THEN M1^M2 => 1 0
1 0 0 0 0

A^1 AND A^2 MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.
COMPARING TWO ARRAYS OF DIFFERENT RANK, FOR EXAMPLE A VECTOR AND A
MATRIX, WILL RESULT IN A 'RANK ERROR'.

128
THE SYMBOL \( \& \) (ALT 0) IS USED FOR THE DYADIC NOT AND FUNCTION.

IT TAKES ARGUMENTS OF ANY RANK WHICH CONSIST ONLY OF 1'S AND 0'S.

THIS FUNCTION RETURNS 1 WHERE ONE OR NEITHER OF THE ARGUMENTS ARE 1;

AND 0 OTHERWISE (WHERE BOTH ARE 1).

THE ORDER OF THE ARGUMENTS IS NOT IMPORTANT, BUT ENTERING AN ARGUMENT

WHICH IS NOT 1 OR 0 WILL PRODUCE A 'DOMAIN ERROR'.

FOR EXAMPLE: \( \#1 = 0 \quad \#0 \Rightarrow 1 \quad 0\#0 \Rightarrow 1 \)

TO COMPARE A SCALAR TO EACH ELEMENT OF AN ARRAY,

ENTER EITHER \$A$ OR \$A\$, FOR EXAMPLE, IF \( V = \) 2 -3 0

\( \#(AV) = 0 \quad 1 \quad 1 \) \( (V=2) \quad A[V]=2 \Rightarrow 0 \quad 1 \quad 1 \)

\( \#A1\#A2 \) WILL COMPARE TWO ARRAYS ELEMENT BY ELEMENT, FOR EXAMPLE:

\( (V-3) \quad A (V=2) = 1 \quad 1 \quad 0 \)

IF \( M1 = 1 \quad 0 \) AND \( M2 = 1 \quad 1 \) THEN \( M1\#M2 = 0 \quad 1 \)

\( 1 \quad 0 \quad 0 \quad 1 \quad 1 \)

\( A1 \) AND \( A2 \) MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT.

COMPARING TWO ARRAYS OF DIFFERENT RANK, FOR EXAMPLE A VECTOR AND A

MATRIX, WILL RESULT IN A 'RANK ERROR'.

129
THE SYMBOL $\vee$ (upshift 9) is used for the dyadic or function.

It takes arguments of any rank which consist only of 1's and 0's.

This function returns 1 where one or both arguments are 1,
and 0 otherwise (where both are 0).

The order of the arguments is not important, but entering an argument
which is not 1 or 0 will produce a 'domain error'.

For example: $1 \vee 1 \Rightarrow 1 \quad 1 \vee 0 \Rightarrow 1 \quad 0 \vee 0 \Rightarrow 0$

To compare a scalar to each element of an array,

enter either $s \vee a$ or $a \vee s$, for example, if $v = [2, 3, 0]$

$1 \vee (v) = [1, 1, 1] \quad (v=2) \vee (v[1]=3) = [1, 0, 0]$

$a_1 \vee a_2$ will compare two arrays element by element, for example:

$(v>3) \vee (v<2) = [1, 1, 1]$

if $m_1 = [1, 0]$ and $m_2 = [1, 1]$ then $m_1 \vee m_2 = [1, 1, 0, 0, 1, 0]$

$a_1$ and $a_2$ must be the same shape or a 'length error' will result,

comparing two arrays of different rank, for example a vector and a

matrix, will result in a 'rank error'.

130
THE SYMBOL \( \lor \) (ALT 9) IS USED FOR THE DYADIC NOR FUNCTION.

IT TAKES ARGUMENTS OF ANY RANK WHICH CONSIST ONLY OF 1'S AND 0'S.
THIS FUNCTION RETURNS 1 WHERE NEITHER OF THE ARGUMENTS ARE 1,
AND 0 OTHERWISE.
THE ORDER OF THE ARGUMENTS IS NOT IMPORTANT, BUT ENTERING AN ARGUMENT
WHICH IS NOT 1 OR 0 WILL PRODUCE A 'DOMAIN ERROR'.

FOR EXAMPLE: \( 1 \lor 1 = 0 \) \( 1 \lor 0 = 0 \) \( 0 \lor 0 = 1 \)

TO COMPARE A SCALAR TO EACH ELEMENT OF AN ARRAY,
ENTER EITHER \( s \lor a \) OR \( a \lor s \). FOR EXAMPLE, IF \( v = \begin{pmatrix} 2 & 3 & 0 \end{pmatrix} \)
\( 1 \lor (v) = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \) \( (v)_2 \lor (v[1]) = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \)
\( a1 \lor a2 \) WILL COMPARE TWO ARRAYS ELEMENT BY ELEMENT. FOR EXAMPLE:
\( (v)_3 \lor (v)_2 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \)

IF \( m1 = \begin{pmatrix} 1 & 0 \end{pmatrix} \) AND \( m2 = \begin{pmatrix} 1 & 1 \end{pmatrix} \) THEN \( m1 \lor m2 = \begin{pmatrix} 0 & 0 \end{pmatrix} \)

\( 1 & 0 \)
\( 0 & 0 \)
\( 0 & 1 \)

\( A1 \) AND \( A2 \) MUST BE THE SAME SHAPE OR A 'LENGTH ERROR' WILL RESULT,
COMPARING TWO ARRAYS OF DIFFERENT RANK, FOR EXAMPLE A VECTOR AND A
MATRIX, WILL RESULT IN A 'RANK ERROR'.

131
THE SYMBOL \( \& \) (UPSHIFT E) IS USED FOR THE DIADIC MEMBER OF FUNCTION.

THIS COMPARES NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK.

IT RETURNS 1 FOR EACH ELEMENT OF THE LEFT ARGUMENT THAT IS FOUND
ANYWHERE IN THE RIGHT ARGUMENT, AND 0 FOR EVERY ELEMENT
THAT IS NOT.

WITH TWO SCALAR ARGUMENTS, \( \& \) IS EQUIVALENT TO = .

FOR EXAMPLE: \( 2 \& 2 = 1 \) \( 2 \& 4 = 0 \) \( 'A' \& 'B' = 0 \)

TO DETERMINE IF A SCALAR IS A MEMBER OF AN ARRAY, ENTER \( S \& A \).

\( 2 \& (2, 3, 4) = 1 \) \( 'A' \& 'ABC' = 1 \)

ENTERING \( S \& S \) WILL PRODUCE THE SAME RESULT AS \( A \& S \).

\( A \& A \) WILL TAKE EACH ELEMENT OF THE LEFT ARRAY AND DETERMINE IF
IT IS CONTAINED IN THE RIGHT ARRAY.

FOR EXAMPLE: 'WHICH 1' \( \& '0123456789' = 0 0 0 0 0 0 0 1 \)

IF \( A_1 = 1 \ 22 \ 14 \) AND \( A_2 = 1 \ 2 \ 3 \ 38 \ 5 \ 2 \)

THEN \( A_1 \& A_2 = 1 \ 0 \ 0 \) AND \( A_2 \& A_1 = 1 \ 1 \ 0 \ 0 \ 0 \ 1 \)

THE RESULT OF \( A_1 \& A_2 \) HAS THE SAME SHAPE AS \( A_1 \).
THE SYMBOL \$ (ALT 5) IS USED FOR THE MONADIC REVERSE FUNCTION.
IT TAKES NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK,
IT RETURNS ALL THE ELEMENTS OF THE ARGUMENT IN THE SAME SHAPE,
BUT IN A DIFFERENT ORDER, AS FOLLOWS.

USING \$ WITH A SCALAR SIMPLY RETURNS THE SCALAR.

USING \$ WITH A VECTOR RETURNS THE VECTOR IN REVERSE ORDER.
FOR EXAMPLE: \$2 3 5 =) 5 3 2
\$ 'STOP' =) POTS

WHEN USED WITH A MATRIX, \$ TAKES THE COLUMNS AS UNITS AND REVERSES
THEM. THE FIRST COLUMN BECOMES THE LAST COLUMN, AND SO ON.
FOR EXAMPLE, IF M =) 1 2 3 THEN \$M =) 3 2 1
4 5 6
7 8 9
AFL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE COLUMNS,
SINCE COLUMNS ARE THE SECOND OF TWO DIMENSIONS OF A MATRIX,
THIS MEANS THAT THE SYMBOL \$ WORKS 'OVER' THE LAST DIMENSION.
TO REVERSE OVER THE FIRST DIMENSION (E.G., ROWS OF A MATRIX), USE THE
SYMBOL \$ (ALT 7), IN PLACE OF \$.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS,
FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND
COLUMNS, \$ WILL REVERSE OVER THE COLUMNS, \$ OVER THE PAGES,
TO REVERSE OVER ANY DIMENSION, YOU MAY SPECIFY THE DIMENSION IN
BRACKETS FOLLOWING THE \$ (OR THE \$) SYMBOL.
FOR EXAMPLE, TO REVERSE OVER THE ROWS OF A THREE-DIMENSIONAL ARRAY
CALLED AA, ENTER \$[2]AA OR \$[2]AA. TO REVERSE OVER THE PAGES,
Enter \$AA OR \$[1]AA. TO REVERSE OVER THE COLUMNS, ENTER \$AA
OR \$[3]AA.
THE SYMBOL @ (ALT 7) IS USED FOR THE MONADIC REVERSE FUNCTION.

IT TAKES NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK,
IT RETURNS ALL THE ELEMENTS OF THE ARGUMENT IN THE SAME SHAPE,
BUT IN A DIFFERENT ORDER, AS FOLLOWS,

USING @ WITH A SCALAR SIMPLY RETURNS THE SCALAR.

USING @ WITH A VECTOR RETURNS THE VECTOR IN REVERSE ORDER.

FOR EXAMPLE:  \[ 2 3 5 \] \rightarrow \[ 5 3 2 \]
\[ 'STOP' \] \rightarrow \[ RTS \]

WHEN USED WITH A MATRIX, @ TAKES THE ROWS AS UNITS AND REVERSES THEM, THE FIRST ROW BECOMES THE LAST ROW, AND SO ON.

FOR EXAMPLE, IF \[ w \rightarrow 1 2 3 \] THEN \[ w \rightarrow 7 8 9 \]
\[ 4 5 6 \] \[ 4 5 6 \]
\[ 7 8 9 \] \[ 1 2 3 \]

APL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE ROWS.

SINCE ROWS ARE THE FIRST OF TWO DIMENSIONS OF A MATRIX,

THIS MEANS THAT THE SYMBOL @ WORKS 'OVER' THE FIRST DIMENSION,

TO REVERSE OVER THE LAST DIMENSION (E.G., COLUMNS OF A MATRIX),

USE THE SYMBOL $ (ALT 5), IN PLACE OF @.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS,

FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND COLUMNS, @ WILL REVERSE OVER THE COLUMNS, $ OVER THE PAGES,

TO REVERSE OVER ANY DIMENSION, YOU MAY SPECIFY THE DIMENSION IN BRACKETS FOLLOWING THE @ (OR THE $) SYMBOL,

FOR EXAMPLE, TO REVERSE OVER THE ROWS OF A THREE-DIMENSIONAL ARRAY CALLED AA, ENTER @[2]AA OR $[2]AA, TO REVERSE OVER THE PAGES, ENTER @AA OR @[1]AA, TO REVERSE OVER THE COLUMNS, ENTER @AA OR @[3]AA.

134
THE SYMBOL $ (ALT 5) IS USED FOR THE DYADIC ROTATE FUNCTION.

THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.

THE LEFT ARGUMENT MUST BE AN INTEGER SCALAR OR ARRAY.

IT RETURNS ALL THE ELEMENTS OF THE RIGHT ARGUMENT IN THE SAME SHAPE,

BUT IN A DIFFERENT ORDER, AS FOLLOWS.

USING $ WITH TWO SCALAR ARGUMENTS SIMPLY RETURNS THE RIGHT ARGUMENT.

A VECTOR RIGHT ARGUMENT REQUIRES A SCALAR LEFT ARGUMENT,

EACH ELEMENT OF THE RIGHT ARGUMENT IS MOVED TO THE LEFT BY THE NUMBER

OF PLACES INDICATED IN THE LEFT ARGUMENT,

FOR EXAMPLE; 2 3 5 $) 5 2 3 3$'ROTATE' $) 2 EROR

WHEN USED WITH A MATRIX, $ TAKES EITHER A SCALAR LEFT ARGUMENT OR A

VECTOR WITH AS MANY ELEMENTS AS THERE ARE ROWS IN THE MATRIX.

$M TAKES THE COLUMNS AS UNITS AND MOVES EACH COLUMN TO THE LEFT BY

THE NUMBER OF SPACES INDICATED BY THE SCALAR.

FOR EXAMPLE; IF M = 1 2 3 THEN 2$M $) 3 1 2

4 5 6 6 4 5

7 8 9 9 7 3

$M TAKES EACH ROW INDIVIDUALLY AND MOVES THE ELEMENTS TO THE LEFT BY

THE NUMBER OF SPACES INDICATED BY THE CORRESPONDING ELEMENT OF THE

LEFT ARGUMENT. FOR EXAMPLE;

2 3 1$M $) 3 1 2 THE FIRST ROW IS MOVED 2 SPACES LEFT

4 5 6 THE SECOND ROW IS ROTATED 3 SPACES

8 9 7 THE THIRD ROW IS ROTATED ONCE

EACH ELEMENT IS IN THE SAME ROW BUT A DIFFERENT COLUMN, SO APL

USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE COLUMNS.

SINCE COLUMNS ARE THE SECOND OF TWO DIMENSIONS OF A MATRIX,

THIS MEANS THAT THE SYMBOL $ WORKS 'OVER' THE LAST DIMENSION,

TO ROTATE OVER THE FIRST DIMENSION (E.G., ROWS OF A MATRIX), USE THE
SYMBOL $\theta$ (ALT 7), IN PLACE OF $\phi$.

This is important when working with higher-dimensional arrays.

For example, in a three-dimensional array of pages, rows, and columns, $\theta$ will rotate over the columns, $\phi$ over the pages.

To rotate over any dimension, you may specify the dimension in brackets following the $\phi$ (or the $\theta$) symbol.

The left argument must have the same shape as the right argument.

Omitting the dimension being rotated over, or it may be a scalar.

For example, to rotate over the rows of a three-dimensional array called $\mathbf{AA}$ where $\mathbf{AA} = 2 \ 3 \ 4$, enter either $\mathbf{A}[2]\mathbf{AA}$ or $\mathbf{A}[2]\mathbf{AA}$ where $\mathbf{m} = 2 \ 4$. To rotate over the pages, enter $\mathbf{A}[\mathbf{m}]\mathbf{AA}$ or $\mathbf{A}[\mathbf{m}]\mathbf{AA}$ or $\mathbf{A}[\mathbf{m}]\mathbf{AA}$ where $\mathbf{m} = 3 \ 4$. To rotate over the columns, enter $\mathbf{A}[\mathbf{m}]\mathbf{AA}$ or $\mathbf{A}[\mathbf{m}]\mathbf{AA}$ or $\mathbf{A}[\mathbf{m}]\mathbf{AA}$ where $\mathbf{m} = 2 \ 3$. 

136
THE SYMBOL ⊗ (ALT 7) IS USED FOR THE DYADIC ROTATE FUNCTION.

THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.

THE LEFT ARGUMENT MUST BE AN INTEGER SCALAR OR ARRAY.

IT RETURNS ALL THE ELEMENTS OF THE RIGHT ARGUMENT IN THE SAME SHAPE,

BUT IN A DIFFERENT ORDER, AS FOLLOWS,

USING ⊗ WITH TWO SCALAR ARGUMENTS SIMPLY RETURNS THE RIGHT ARGUMENT,

A VECTOR RIGHT ARGUMENT REQUIRES A SCALAR LEFT ARGUMENT,

EACH ELEMENT OF THE RIGHT ARGUMENT IS MOVED TO THE LEFT BY THE NUMBER

OF PLACES INDICATED IN THE LEFT ARGUMENT,

FOR EXAMPLE: 2×35 ⊗> 523 3×'ROTATE' ⊗> 3EROT

WHEN USED WITH A MATRIX, ⊗ TAKES EITHER A SCALAR LEFT ARGUMENT OR A

VECTOR WITH AS MANY ELEMENTS AS THERE ARE COLUMNS IN THE MATRIX;

⊗M TAKES THE ROWS AS UNITS AND MOVES EACH ROW UP BY THE NUMBER

SPACES INDICATED BY THE SCALAR,

FOR EXAMPLE: IF M =

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 \\
\end{bmatrix}
\]

THEN 2⊗M ⊗> 7 8 9

4 5 6

1 2 3

7 8 9

4 5 6

⊗M TAKES EACH COLUMN INDIVIDUALLY AND MOVES THE ELEMENTS UP BY

THE NUMBER OF SPACES INDICATED BY THE CORRESPONDING ELEMENT OF THE

LEFT ARGUMENT. FOR EXAMPLE:

\[
\begin{bmatrix}
2 & 3 & 1 \\
\end{bmatrix}
\]

⊗M =

\[
\begin{bmatrix}
7 & 2 & 6 \\
1 & 5 & 9 \\
4 & 8 & 3 \\
\end{bmatrix}
\]

THE FIRST COLUMN IS MOVED 2 SPACES UP

THE SECOND COLUMN IS ROTATED 3 SPACES

THE THIRD COLUMN IS ROTATED ONCE

EACH ELEMENT IS IN THE SAME COLUMN BUT A DIFFERENT ROW, SO APL

USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE ROWS,

SINCE ROWS ARE THE FIRST OF TWO DIMENSIONS OF A MATRIX,

THIS MEANS THAT THE SYMBOL ⊗ WORKS 'OVER' THE FIRST DIMENSION.

TO ROTATE OVER THE LAST DIMENSION (E.G., COLUMNS OF A MATRIX), USE THE
Symbol $\phi$ (ALT 5), in place of $\theta$.

This is important when working with higher-dimensional arrays. For example, in a three-dimensional array of pages, rows, and columns, $\phi$ will rotate over the columns, $\varphi$ over the pages. To rotate over any dimension, you may specify the dimension in brackets following the $\phi$ (or the $\varphi$) symbol.

The left argument must have the same shape as the right argument omitting the dimension being rotated over, or it may be a scalar.

For example, to rotate over the rows of a three-dimensional array called AA where $\text{pAA} = 2 \ 3 \ 4$, enter either $\phi[2]\text{AA}$ or $\varphi[2]\text{AA}$ where $\text{pM} = 2 \ 4$. To rotate over the pages, enter $\phi[0]\text{AA}$ or $\varphi[0]\text{AA}$ or $\phi[1]\text{AA}$ or $\varphi[1]\text{AA}$ where $\text{pM} = 3 \ 4$. To rotate over the columns, enter $\phi[3]\text{AA}$ or $\varphi[3]\text{AA}$ or $\phi[3]\text{AA}$ or $\varphi[3]\text{AA}$ where $\text{pM} = 2 \ 3$. 

138
THE SYMBOL $\&$ (ALT 6) IS USED FOR THE MONADIC TRANSPOSE FUNCTION.

IT TAKES NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK,

IT RETURNS ALL THE ELEMENTS OF THE ARGUMENT IN A DIFFERENT SHAPE

AND ORDER, AS FOLLOWS,

USING $\&$ WITH A SCALAR OR VECTOR SIMPLY RETURNS THE ARGUMENT.

WHEN USED WITH A MATRIX, $\&$ PERFORMS MATRIX TRANSPOSITION:

THE ROWS BECOME COLUMNS AND THE COLUMNS BECOME ROWS.

FOR EXAMPLE, IF $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ THEN $\& M = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

WHEN USED WITH HIGHER-DIMENSIONAL ARRAYS, TRANSPOSE REVERSES THE
ORDER OF THE DIMENSIONS.

FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND
COLUMNS, THE PAGES WILL BECOME ROWS AND THE ROWS PAGES,

THAT IS, THE ELEMENT IN THE FIRST PAGE, SECOND ROW, THIRD COLUMN

WILL BE TRANSPOSED TO THE THIRD PAGE, SECOND ROW, FIRST COLUMN.

IF $A[1;2;3;4] = 7$ THEN $(A[4;3;2;1]) = 7$

THE SHAPE OF THE RESULT WILL ALWAYS BE THE REVERSE OF THE SHAPE OF


139
THE SYMBOL $\otimes$ (ALT $\otimes$) IS USED FOR THE DYADIC TRANSPOSE FUNCTION.

THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.

THE LEFT ARGUMENT MUST BE A POSITIVE INTEGER SCALAR OR VECTOR.

IT RETURNS ALL THE ELEMENTS OF THE RIGHT ARGUMENT IN A DIFFERENT
SHAPE AND ORDER, AS FOLLOWS,

USING $\otimes$ WITH A SCALAR RIGHT ARGUMENT RETURNS A 'LENGTH ERROR' UNLESS
THE LEFT ARGUMENT IS AN EMPTY VECTOR, THEN IT RETURNS THE SCALAR.

$\text{INV}$ RETURNS THE VECTOR $V$. ANY OTHER SCALAR LEFT ARGUMENT USED WITH
A VECTOR RIGHT ARGUMENT PRODUCES A 'DOMAIN ERROR', ANY OTHER
RANK LEFT ARGUMENT RESULTS IN A 'LENGTH ERROR'.

WITH A MATRIX RIGHT ARGUMENT, THE LEFT ARGUMENT MUST BE A VECTOR;
EITHER (1 2) OR (2 1). (2 1)$\times M$ PRODUCES THE TRANSPOSE OF $M$;
THE COLUMNS (THE SECOND DIMENSION) BECOME THE ROW (THE FIRST
DIMENSION), (1 2)$\otimes M \Rightarrow M$

WITH A RIGHT ARGUMENT OF RANK $R$, THE LEFT ARGUMENT MUST CONTAIN ALL
THE INTEGERS FROM 1 TO $R$, IN ANY ORDER.
TRANSPOSE WILL TAKE THE DIMENSIONS OF THE RIGHT ARGUMENT AND PUT THEM
IN THE ORDER SPECIFIED BY THE LEFT ARGUMENT.
FOR EXAMPLE, IF $A = 2 3 4$ THEN (3 1 2)$\otimes A$ WILL PUT THE THIRD
DIMENSION FIRST (COLUMNS BECOME PAGES), THE FIRST DIMENSION SECOND
(PAGES BECOME ROWS), AND THE SECOND DIMENSION THIRD (ROWS BECOME
COLUMNS), SO $(3 1 2)\otimes A \Rightarrow 4 2 3$.

IN OTHER WORDS, THE ELEMENT IN THE SECOND COLUMN, THIRD ROW, FIRST
PAGE WILL BE TRANSPOSED TO THE SECOND PAGE, THIRD COLUMN, FIRST

140
TAKE

THE SYMBOL \( \uparrow \) (UPSHIFT Y) IS USED FOR THE DYADIC TAKE FUNCTION.

THE LEFT ARGUMENT MUST BE AN INTEGER SCALAR OR VECTOR.

THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.

TAKE RETURNS SELECTED ELEMENTS OF THE RIGHT ARGUMENT AS FOLLOWS:

WHEN THE RIGHT ARGUMENT IS A SCALAR, THE LEFT ARGUMENT CAN BE A SCALAR OR A VECTOR OF ANY LENGTH.

THE RESULT OF \( \text{L} \times \text{R} \) HAS A SHAPE EXACTLY EQUAL TO \( \text{L} \). \( \text{R} \) WILL BE THE FIRST ELEMENT IF \( \text{L} \) IS POSITIVE, AND THE LAST ELEMENT IF \( \text{L} \) IS NEGATIVE. THE OTHER ELEMENTS WILL BE 0 IF \( \text{R} \) IS NUMERIC, BLANK IF \( \text{R} \) IS CHARACTER.

FOR EXAMPLE:

\[
\begin{align*}
\text{1} \uparrow \text{4} & \rightarrow 4 \\
-\text{1} \uparrow \text{4} & \rightarrow 1 \\
\text{2} \uparrow \text{A} & \rightarrow \text{A} \\
\text{2} \uparrow \text{A} & \rightarrow 2 \\
\text{-4} \uparrow \text{2} & \rightarrow 0 \ 0 \ 0 \ 2 \\
\text{f}(\text{3} \ 4) \uparrow \text{1} & \rightarrow 3 \ 4
\end{align*}
\]

IF THE RIGHT ARGUMENT IS A VECTOR, THE LEFT ARGUMENT MUST BE A SCALAR OR A 'RANK ERROR' WILL RESULT.

\( \text{S} \uparrow \text{V} \) WILL RETURN THE FIRST \( \text{S} \) ELEMENTS OF \( \text{V} \), IF \( \text{S} \) IS POSITIVE, OR THE LAST \( \text{S} \) ELEMENTS OF \( \text{V} \), IF \( \text{S} \) IS NEGATIVE.

IF \( \text{S} \) EXCEEDS THE LENGTH OF \( \text{V} \), THE RESULT WILL BE PADDED WITH 0'S, OR BLANKS, ON THE RIGHT IF \( \text{S} \) IS POSITIVE, ON THE LEFT IF \( \text{S} \) IS NEGATIVE.

FOR EXAMPLE:

\[
\begin{align*}
\text{3} \uparrow \text{10} & \rightarrow 1 \ 2 \ 3 \\
\text{3} \uparrow \text{ALPHABET} & \rightarrow \text{BET} \\
\text{5} \uparrow \text{ABC} & \rightarrow \text{ABC} \\
\text{5} \uparrow \text{ABC} & \rightarrow 5 \\
\text{-4} \uparrow 23 & \rightarrow 0 \ 0 \ 23
\end{align*}
\]

WHEN THE RIGHT ARGUMENT IS A MATRIX OR HIGHER LEVEL ARRAY, THE LEFT ARGUMENT MUST BE A VECTOR WITH AS MANY ELEMENTS AS THERE ARE DIMENSIONS IN THE RIGHT ARGUMENT, OR A 'LENGTH ERROR' RESULTS.

FOR EXAMPLE:

\[
\begin{align*}
\text{M} & \rightarrow 9 \ 8 \ 7 \\
\text{-1} \uparrow \text{M} & \rightarrow 6 \ 5 \ 2 \ 4 \uparrow \text{M} & \rightarrow 9 \ 8 \ 7 \ 0 \\
& \rightarrow 6 \ 5 \ 4 \ 0
\end{align*}
\]

THE FIRST ELEMENT OF THE VECTOR DETERMINES WHICH ROWS ARE TAKEN, AND THE SECOND ELEMENT DETERMINES WHICH COLUMNS ARE TAKEN.

THE RESULT WILL BE PADDED WITH 0'S OR BLANKS AS ABOVE.
FOR HIGHER LEVEL ARRAYS, EACH ELEMENT OF THE LEFT ARGUMENT WILL TAKE ELEMENTS FROM THE CORRESPONDING DIMENSION.

FOR EXAMPLE, IF PA =: 6 5 4; (2 -3 1)\$4 WILL RETURN THE FIRST COLUMN OF THE LAST THREE ROWS OF THE FIRST TWO PAGES.

THE RESULT OF "TAKE" WILL ALWAYS HAVE A SHAPE EXACTLY EQUAL TO THE ABSOLUTE VALUE OF THE LEFT ARGUMENT.

IF THERE IS A ZERO (0) ANYWHERE IN THE LEFT ARGUMENT, THE ANSWER WILL BE AN EMPTY ARRAY WITH THE APPROPRIATE SHAPE.

FOR EXAMPLE: 2 -3 1\$A =: 2 3 1; 7 0 7\$A =: 7 0 7
THE SYMBOL ↓ (UPSHIFT U) IS USED FOR THE DYADIC DROP FUNCTION.
THE LEFT ARGUMENT MUST BE AN INTEGER SCALAR OR VECTOR,
THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.
DROP RETURNS SELECTED ELEMENTS OF THE RIGHT ARGUMENT AS FOLLOWS:

WHEN THE RIGHT ARGUMENT IS A SCALAR, THE LEFT ARGUMENT CAN BE A SCALAR
OR A VECTOR OF ANY LENGTH,
IF ALL ELEMENTS OF THE LEFT ARGUMENT EQUAL 0, THE RESULT OF L↓S WILL
BE AN ARRAY CONTAINING THE RIGHT ARGUMENT; OTHERWISE IT WILL BE AN
EMPTY ARRAY. THIS ARRAY HAS AS MANY DIMENSIONS AS THE LEFT ARGUMENT
HAS ELEMENTS AND EACH DIMENSION IS OF LENGTH 1.
FOR EXAMPLE: 1↓4 =) (EMPTY) 2↓4 =) 1 0↓'A' =) A
2↓4↓2 =) (EMPTY) 3↓(2↓4)↓2 =) 1 1 1 0 0 0↓3 =) 3

IF THE RIGHT ARGUMENT IS A VECTOR, THE LEFT ARGUMENT MUST BE A SCALAR
OR A 'LENGTH ERROR' WILL RESULT,
$↓V$ WILL RETURN V OMITTING THE FIRST S ELEMENTS, IF S IS POSITIVE,
OR THE LAST S ELEMENTS, IF S IS NEGATIVE.
IF S EXCEEDS THE LENGTH OF V, THE RESULT WILL BE AN EMPTY VECTOR.
FOR EXAMPLE: 6↓110 =) 7 8 9 10 3↓'ALPHABET' =) ALPHA
5↓'ABC' =) (EMPTY) 4↓'ABC' =) 0 0↓13 =) 1 3

WHEN THE RIGHT ARGUMENT IS A MATRIX OR HIGHER LEVEL ARRAY, THE LEFT
ARGUMENT MUST BE A VECTOR WITH AS MANY ELEMENTS AS THERE ARE
DIMENSIONS IN THE RIGHT ARGUMENT, OR A 'LENGTH ERROR' RESULTS.
FOR EXAMPLE: IF M =) 9 8 7 6 5 4 0↓M =) 6 5 0 2↓M =) 7
THE FIRST ELEMENT OF THE VECTOR DETERMINES WHICH ROWS ARE Dropped,
AND THE SECOND ELEMENT DETERMINES WHICH COLUMNS ARE DROPPED.

FOR HIGHER LEVEL ARRAYS, EACH ELEMENT OF THE LEFT ARGUMENT WILL
DROP ELEMENTS FROM THE CORRESPONDING DIMENSION.

For example, if \( A = \begin{bmatrix} 6 & 5 & 4 \end{bmatrix} \) \( (2 \ 3 \ 1)A \) will return the last three columns of the first two rows of the last four pages.

The result of drop will always have a shape exactly equal to the shape of the right argument minus the absolute value of the left argument, except as follows:

If any element of the left argument exceeds the length of the corresponding dimension in the right argument, the result will be an empty array of the appropriate shape.

In the above example:

\[
(2 \ 3 \ 1)A = \begin{bmatrix} 4 & 2 & 3 \\
3 & 1 & 0 \end{bmatrix} \\
3 & 4 & 5A = \text{(empty)}
\]
THE SYMBOL / IS USED FOR THE DYADIC COMPRESS FUNCTION.

THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.

THE LEFT ARGUMENT MUST BE 1, 0, OR A VECTOR OF 1'S AND 0'S.

COMPRESS RETURNS SELECTED ELEMENTS OF THE RIGHT ARGUMENT, AS FOLLOWS:

1/A WILL RETURN THE RIGHT ARGUMENT, WHILE 0/A RETURNS AN EMPTY ARRAY.

V1/V2 WILL RETURN THE ELEMENTS OF V2 THAT CORRESPOND TO 1'S IN V1.

FOR EXAMPLE: 1 0 1/1 2 3 =) 1 3 1 1 0/‘HALT’ =) HAT

TWO VECTOR ARGUMENTS MUST HAVE THE SAME NUMBER OF ELEMENTS OR A

'LENGTH ERROR' WILL RESULT.

V/M TAKES THE COLUMNS AS UNITS AND RETURNS THE COLUMNS IN M WHICH

CORRESPOND TO THE 1'S IN V. FOR EXAMPLE:

IF M =) 1 2 3 THEN 1 0 1/M =) 1 3

4 5 6 4 6

7 8 9 7 9

AFL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE COLUMNS.

SINCE COLUMNS ARE THE SECOND OF TWO DIMENSIONS OF A MATRIX,

THIS MEANS THAT THE SYMBOL / WORKS 'OVER' THE LAST DIMENSION.

TO COMPRESS OVER THE FIRST DIMENSION (E.G., ROWS OF A MATRIX), USE THE

SYMBOL \ (ALT /), IN PLACE OF /.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS.

FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND

COLUMNS, / WILL COMPRESS OVER THE COLUMNS, \ OVER THE PAGES.

TO COMPRESS OVER ANY DIMENSION, YOU MAY SPECIFY THE DIMENSION IN

BRACKETS FOLLOWING THE / (OR THE \) SYMBOL.

THE LEFT ARGUMENT MUST BE A SCALAR, OR A VECTOR WITH THE SAME

NUMBER OF ELEMENTS AS THE DIMENSION BEING COMPRESSED OVER.

FOR EXAMPLE, TO COMPRESS OVER THE ROWS OF A THREE-DIMENSIONAL ARRAY
THE SYMBOL \( \land \) (ALT /) IS USED FOR THE DYADIC COMPRESS FUNCTION.

THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.

THE LEFT ARGUMENT MUST BE 1\( \times \)0, OR A VECTOR OF 1's AND 0's.

COMPRESS RETURNS SELECTED ELEMENTS OF THE RIGHT ARGUMENT, AS FOLLOWS;

\( 1 \land a \) will return the right argument, while \( 0 \land a \) returns an EMPTİ ARRAY.

\( v \land v2 \) will return the elements of v2 that correspond to 1's in v1.

For example: \( 1\,0\,1\,1\,2\,3 \land 1\,3\) 1 1 0 /'HALT' \( =\) 1 1 1

Two vector arguments must have the same number of elements or a

'LENGTH ERROR' will result.

\( v \land m \) takes the rows as units and returns the rows in m which

correspond to the 1's in v. For example:

If \( m = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \) then \( 1\,0\,1\,1\,2\,3 \land m = \begin{pmatrix} 1 & 2 \end{pmatrix} \)

\( \begin{pmatrix} 4 & 5 & 6 \end{pmatrix} \)

\( \begin{pmatrix} 7 & 8 & 9 \end{pmatrix} \)

APL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE ROWS.

Since rows are the first of two dimensions of a matrix,

this means that the symbol / works 'over' the first dimension,

To compress over the last dimension (E.G, columns of a matrix),

use the symbol / in place of \( \land \).

This is important when working with higher-dimensional arrays.

For example, in a three-dimensional array of pages, rows, and

columns, / will compress over the columns, \( \land \) over the pages,

to compress over any dimension, you may specify the dimension in

brackets following the \( \land \) (or the \( / \)) symbol.

The left argument must be a scalar, or a vector with the same

number of elements as the dimension being compressed over.

For example, to compress over the rows of a three-dimensional array
THE SYMBOLO (UPSHIFT /) IS USED FOR THE DYADIC EXPAND FUNCTION.
THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.
THE LEFT ARGUMENT MUST BE A VECTOR OF 1'S AND 0'S.
IS NUMERIC, OR BLANKS IF IT IS CHARACTER, AS FOLLOWS;
EXPAND RETURNS THE RIGHT ARGUMENT, ADDING 0'S IF THE RIGHT ARGUMENT
IS NUMERIC, OR BLANKS IF IT IS CHARACTER, AS FOLLOWS;

\$V/X WILL RETURN A VECTOR CONSISTING OF THE SCALAR REPEATED AS MANY
TIMES AS THERE ARE 1'S IN V.
FOR EXAMPLE: 1 0 1 \$3 => 3 3
IF THERE ARE NO 1'S IN V, \$V/X RETURNS AN EMPTY VECTOR.

\$/V2 WILL RETURN THE ELEMENTS OF V2, WITH 0'S OR BLANKS INSERTED
TO CORRESPOND TO THE 0'S IN V1, FOR EXAMPLE;
1 1 0 1 1 2 3 => 1 2 0 3 1 0 1 \$AFL' => A F L
THE LEFT ARGUMENT MUST CONTAIN AS MANY 1'S AS THERE ARE ELEMENTS
IN THE RIGHT ARGUMENT OR A 'LENGTH ERROR' WILL RESULT.

\$V/M TAKES THE COLUMNS AS UNITS AND RETURNS THE A MATRIX WITH COLUMNS
OF 0'S OR BLANKS INSERTED TO CORRESPOND TO THE 1'S IN V.
IF M = 1 2 3 THEN 1 0 1 0 1M = 1 0 2 0 3
4 5 6
7 8 9
AFL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE COLUMNS,
SINCE COLUMNS ARE THE SECOND OF TWO DIMENSIONS OF A MATRIX,
THIS MEANS THAT THE SYMBOL \ WORKS 'OVER' THE LAST DIMENSION,
TO EXPAND OVER THE FIRST DIMENSION (E.G. ROWS OF A MATRIX), USE THE
SYMBOL \ (ALT ,); IN PLACE OF \.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS,
FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND
COLUMN, \ will expand over the columns, \ over the pages.

To expand over any dimension, you may specify the dimension in brackets following the \ (or the \) symbol.

For example, to expand over the rows of a three-dimensional array called AA where \[x = 2 3 4\], enter \(V\[2\]AA or \(V\[2\]AA\)

where \(+/V = 3\), to expand over the pages, enter \(V\[4\]AA or \(V\[1\]AA\)

where \(+/V = 2\), to expand over the columns, enter \(V\[\]AA or \(V\[3\]AA where \(+/V = 4\)
THE SYMBOL \ (ALT \ ) IS USED FOR THE DYADIC EXPAND FUNCTION.
THE RIGHT ARGUMENT CAN BE CHARACTER OR NUMERIC, OF ANY RANK.
THE LEFT ARGUMENT MUST BE A VECTOR OF 1'S AND 0'S.
EXPAND RETURNS THE RIGHT ARGUMENT, ADDING 0'S IF THE RIGHT ARGUMENT
IS NUMERIC, OR BLANKS IF IT IS CHARACTER, AS FOLLOWS:

V\S WILL RETURN A VECTOR CONSISTING OF THE SCALAR REPEATED AS MANY
TIMES AS THERE ARE 1'S IN V.
FOR EXAMPLE: 1 0 1 3 =⇒ 3 3
IF THERE ARE NO 1'S IN V, V\S RETURNS AN EMPTY VECTOR.

V\V2 WILL RETURN THE ELEMENTS OF V2, WITH 0'S OR BLANKS INSERTED
TO CORRESPOND TO THE 0'S IN V1. FOR EXAMPLE:
1 1 0 1 1 2 3 =⇒ 1 2 0 3 1 0 1 1 'AFL' =⇒ 'AFL'
THE LEFT ARGUMENT MUST CONTAIN AS MANY 1'S AS THERE ARE ELEMENTS
IN THE RIGHT ARGUMENT OR A 'LENGTH ERROR' WILL RESULT.

V\M TAKES THE ROWS AS UNITS AND RETURNS THE MATRIX WITH ROWS
OF 0'S OR BLANKS INSERTED TO CORRESPOND TO THE 1'S IN V.
IF M =⇒ 1 2 3 THEN 1 0 1 \ M =⇒ 1 2 3
    0 0 0
    4 5 6
    4 5 6

APL USERS REFER TO THIS AS WORKING 'ACROSS' OR 'OVER' THE ROWS,
SINCE ROWS ARE THE FIRST OF TWO DIMENSIONS OF A MATRIX.
THIS MEANS THAT THE SYMBOL \ WORKS 'OVER' THE FIRST DIMENSION.
TO EXPAND OVER THE LAST DIMENSION (E.G., COLUMNS OF A MATRIX), USE
THE SYMBOL \ (UPSHIFT /), IN PLACE OF \.

THIS IS IMPORTANT WHEN WORKING WITH HIGHER-DIMENSIONAL ARRAYS,
FOR EXAMPLE, IN A THREE-DIMENSIONAL ARRAY OF PAGES, ROWS, AND
COLUMNS, \ WILL EXPAND OVER THE COLUMNS, \ OVER THE PAGES.
To expand over any dimension, you may specify the dimension in brackets following the / (or the \) symbol.

The left argument must be a vector with the same number of 1's as there are elements in the dimension being expanded over.

For example, to expand over the rows of a three-dimensional array called AA where fAA = [2 3 4], enter V\[2]AA or V\[2]AA

Where +/v = 3, to expand over the pages, enter V\AA or V\[1]AA

Where +/v = 2, to expand over the columns, enter V\AA or V\[3]AA where +/v = 4
THE SYMBOL \( \cdot \) FOLLOWED BY AN NUMBER IN BRACKETS IS USED FOR THE
DYADIC LAMINATE FUNCTION, AN EXTENSION OF THE CATENATE FUNCTION.
THIS FUNCTION WILL COMBINE TWO CHARACTER OR NUMERIC ARGUMENTS
OF ANY RANK (EXCEPT TWO SCALARS) INTO A SINGLE ARRAY.

VECTORS CAN BE EXTENDED BY LAMINATING \( \mathbf{v}_1[1] \mathbf{v}_2 \) OR \( \mathbf{v}_1[1,2] \mathbf{v}_2 \).
THE NUMBER IN THE BRACKETS MUST BE 1 BECAUSE VECTOR HAVE ONLY
ONE DIMENSION, THIS WORKS EXACTLY LIKE CATENATE.

NOW FOR SOMETHING REALLY TRICKY;
TO MAKE A TWO-ROW MATRIX OUT OF TWO VECTORS OF THE SAME LENGTH,
ENTER \( \mathbf{v}_1[.5] \mathbf{v}_2 \), ANY INDEX BETWEEN 0 AND 1 WILL DO.
THIS CREATES A NEW DIMENSION PRECEDING THE FIRST DIMENSION OF
THE ARGUMENTS, THE OTHER DIMENSION STAYS THE SAME LENGTH.
TO MAKE A TWO-COLUMN MATRIX, ENTER \( \mathbf{v}_1[1,5] \mathbf{v}_2 \) (ANY INDEX FROM 1 TO 2).
NOW THE NEW DIMENSION COMES AFTER THE FIRST, WHICH IS THE SAME AS
THE LENGTH OF THE ARGUMENTS,

LAMINATING A SCALAR TO A MATRIX WILL RESULT IN A MATRIX WITH ONE
MORE ROW (IF THE INDEX NUMBER IN BRACKETS IS 1) OR ONE MORE
COLUMN (IF THE INDEX NUMBER IS 2) THAN THE ORIGINAL MATRIX.
ALL ENTRIES IN THIS ROW OR COLUMN WILL HAVE THE VALUE OF THE SCALAR.
IF \( M = 1 2 \) THEN \( 5, [1] M = 5 5 \) AND \( M, [2] 5 = 1 2 5 \)
\( 3 4 \) \( 1 2 \)
\( 3 4 \)

LAMINATING TWO MATRICES WILL RESULT IN A MATRIX CONTAINING THE TWO
ORIGINAL MATRICES SIDE BY SIDE (INDEX 2) OR VERTICALLY (INDEX 1).
FOR EXAMPLE; IF \( M_1 = 1 2 \) AND \( M_2 = 5 6 \)
\( 3 4 \) \( 7 8 \)
THEN \( M_1, [2] M_2 = 1 2 5 6 \) AND \( M_2, [1] M_1 = 1 2 \)
153
3 4 7 8
5 6
7 8

To stack the matrices by creating a third dimension, enter
M1,[[.5]]M2 (any index between 0 and 1). You can also create
a third dimension by M1,[[1.5]]M2 or M1,[[2.5]]M2. Try it.

Variations on this theme are almost endless—experiment;
in every case, the number in the brackets refers to the dimension
where the 'seam' is—the dimension that increases in length.
If this is a new dimension, it will always be length 2 in the result,
the other dimensions must be the same length in both arguments
(unless one is a scalar) or a 'length error' will result.
THE SYMBOL \( \text{MATRI} (\text{ALT } x) \) IS USED FOR THE MONADIC MATRIX INVERSE FUNCTION. IT TAKES A NUMERIC SCALAR, VECTOR, OR MATRIX ARGUMENT, AND RETURNS THE LEFT INVERSE. IN TERMS OF MATRIX ALGEBRA, THIS IS THE MATRIX WHICH PRODUCES THE IDENTITY WHEN LEFT-MULTIPLIED WITH THE ORIGINAL.

WHEN USED WITH SCALARS, \( \text{MATRI} \) IS EQUIVALENT TO \( \div \).

\( \text{MATRI} \) WILL TREAT VECTORS AS 1xL MATRICES, \( \text{MATRI} \) WILL RETURN THE VECTOR \( \text{V} \):

SO THAT \( \text{V} \cdot \text{MATRI} \text{V} = I \)

SQUARE MATRICES WILL BE INVERTED NORMALLY.

FOR EXAMPLE: IF \( M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) THEN \( \text{MATRI}M = \begin{pmatrix} 2 & 1 \\ 1.5 & -0.5 \end{pmatrix} \)

SOME NON-SQUARE MATRICES MAY ALSO BE INVERTED.

FOR EXAMPLE: IF \( M_2 = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \) THEN \( \text{MATRI}M_2 = \begin{pmatrix} -0.9444 & -0.1111 & 0.7222 \\ 0.4444 & 0.1111 & -0.2222 \\ 1.1111 & -0.3333 & 0.6666 \end{pmatrix} \)

IF \( M \) IS NOT INVERTIBLE, A 'DOMAIN ERROR' WILL RESULT.
THE SYMBOL $\cdot$ (ALT $\times$) IS USED FOR THE DYADIC MATRIX DIVIDE FUNCTION. IT TAKES ARITHMETIC ARGUMENTS WITH RANK $\leq 3$. THE TWO ARGUMENTS MUST HAVE THE SAME NUMBER OF ROWS.

IN TERMS OF MATRIX ALGEBRA, IT LEFT MULTIPLIES THE LEFT ARGUMENT BY THE LEFT INVERSE OF THE RIGHT ARGUMENT.

$\cdot$ Produces the solution to the set of linear equations $AX = B$.

When used with two scalars, $\cdot$ is equivalent to $\div$.

When used with vectors, $\cdot$ operates as if the vectors are $1\times C$ or $R \times 1$ matrices, whichever is appropriate.
THE SYMBOLS . (UPSHIFT J FOLLOWED BY A PERIOD) ARE PLACED PRECEDING ANY DYADIC FUNCTION SYMBOL TO PRODUCE THE OUTER PRODUCT FUNCTION. THE ARGUMENTS MAY BE NUMERIC OR CHARACTER, IF THE DYADIC SYMBOL ACCEPTS CHARACTER ARGUMENTS, THEY MAY BE OF ANY RANK AND DO NOT HAVE TO AGREE IN SHAPE.

THE OUTER PRODUCT RESULTS IN THE DYADIC SYMBOL OPERATING ON EACH ELEMENT OF THE LEFT ARGUMENT COMBINED WITH EVERY ELEMENT OF THE RIGHT ARGUMENT.

**Example:**

\[
\begin{align*}
1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 \implies & \quad 3 & \quad 4 & \quad 5 \\
& & 6 & \quad 8 & \quad 10
\end{align*}
\]

\[
\text{\textasciitilde Stop} \quad . &= \quad \text{\textasciitilde Post} \implies \\
0 & \quad 0 & \quad 1 & \quad 0 \\
0 & \quad 0 & \quad 0 & \quad 1 \\
0 & \quad 1 & \quad 0 & \quad 0 \\
1 & \quad 0 & \quad 0 & \quad 0
\]

OUTER PRODUCT (ALSO CALLED 'JOT DOT' BECAUSE OF THE SYMBOLS) CAN BE EXTENDED TO ARRAYS OF ANY DIMENSION OR SIZE, THE SHAPE OF THE RESULT WILL ALWAYS BE EQUAL TO THE SHAPE OF THE LEFT ARGUMENT CATENATED TO THE SHAPE OF THE RIGHT ARGUMENT.

**Example:**

IF \( a_1 = \) \( 3 \), \( 4 \) AND \( a_2 = \) \( 6 \), \( 5 \) THEN \( a_1 \cdot a_2 = \) \( 3 \), \( 4 \), \( 6 \), \( 5 \)

157
THE SYMBOL (PERIOD OR 'DOT') MAY BE PLACED BETWEEN ANY TWO DIADIC FUNCTION SYMBOLS TO CREATE THE INNER PRODUCT FUNCTION.

INNER PRODUCT IS MOST COMMONLY USED WITH + AND X TO FORM THE CONVENTIONAL DOT PRODUCT OR MATRIX PRODUCT OF MATRIX ALGEBRA.
FOR EXAMPLE: IF V₁ = [1 2 3] AND V₂ = [4 5 6]
THEN V₁ +,X V₂ = [3 13]

INNER PRODUCT MAY BE EXTENDED TO HIGHER LEVEL ARRAYS AS WELL AS MATRICES. THE SHAPE OF THE RESULT WILL BE THE SAME AS THE SHAPE OF THE LEFT ARGUMENT CATEHESATED TO THE SHAPE OF THE RIGHT ARGUMENT, OMITTING THE MATCHING DIMENSIONS (LAST ON THE LEFT, FIRST ON THE RIGHT.
FOR EXAMPLE: IF F₁ = [1 2 3] AND F₂ = [4 5]
THEN F₁ +,X F₂ = [6 15 8]

158

IF THE RIGHT ARGUMENT DOES NOT OCCUR IN THE LEFT ARGUMENT, THE RESULT WILL BE ONE PLUS THE LENGTH OF THE LEFT ARGUMENT.

FOR EXAMPLE:

\[
\begin{align*}
4 & 3 & 2 & 1 & 3 \rightarrow 2 \text{ BECAUSE } 3 \text{ IS IN THE SECOND POSITION} \\
'HELLO' & \uparrow 'P' \rightarrow 6 \text{ BECAUSE 'P' IS NOT FOUND}
\end{align*}
\]

VĄ CHECKS EACH ELEMENT OF THE RIGHT ARGUMENT INDIVIDUALLY.

FOR EXAMPLE:

\[
\begin{align*}
3 & 5 & 7 & 9 & 1 & 2 & 3 & 4 & 5 \rightarrow 5 & 5 & 1 & 5 & 2
\end{align*}
\]

IF YOU ASSIGN \( \text{ALPHABET} = 'ABCDEFGHIJKLMNOPQRSTUVWXYZ' \) THEN \( \text{ALPHABET} \uparrow 'A P L' \rightarrow 1 & 16 & 12 \)
THE SYMBOL \( \uparrow \) (UPSHIFT \( \uparrow \)) IS USED FOR THE DYADIC ENCODE FUNCTION.

IT TAKES NUMERIC ARGUMENTS OF ANY RANK AND RETURNS THE REPRESENTATION
OF THE RIGHT ARGUMENT IN THE NUMBER SYSTEM SPECIFIED BY THE LEFT
ARGUMENT.

THE ENTRIES IN THE LEFT ARGUMENT DEFINE THE NUMBER OF VALUES THAT
CAN BE PLACED IN THAT POSITION.

FOR EXAMPLE, IN THE BINARY SYSTEM, EACH POSITION CAN TAKE ONE OF TWO
VALUES, 0 OR 1. THEREFORE, TO TRANSLATE 133 INTO BINARY, ENTER:

\[
2 
2 
2 
2 
2 
2 
2 
\uparrow 
1 
3 
3 \Rightarrow 1 
0 
0 
0 
0 
1 
0 
1
\]

THE RESULT WILL ALWAYS BE THE SAME SHAPE AS THE LEFT ARGUMENT, SO BE
SURE THAT THE LEFT ARGUMENT IS LARGE ENOUGH TO BE CORRECT.

FOR EXAMPLE:

\[
2 
2 
2 
\uparrow 
1 
3 
3 \Rightarrow 1 
0 
1
\]

ENCODE CAN ALSO BE USED TO TRANSLATE TIMES, GIVEN TIME IN SECONDS,
ENTER (2000 365 24 60 60) \( \uparrow \) TIME TO RECEIVE A VECTOR OF YEARS
(0 TO 1999), DAYS (0 TO 364), HOURS (0 TO 23), MINUTES (0 TO 60),
AND SECONDS (-TO 60).

FOR EXAMPLE:

\[
2 
0 
0 
0 
3 
6 
5 
4
\Rightarrow 0 
5 
6 
5 
1 
4 
1
\]
THE SYMBOL \( \uparrow \) (upshift by) is used for the dyadic DECODE function. It takes numeric arguments of any rank and returns the decimal value of the right argument which is written in the number system specified by the left argument.

The entries in the left argument define the values of each position in the number system.

For example, in the binary system, each position can take one of two values, 0 or 1. Therefore, to translate 1000101 from binary:

\[ 2 \uparrow 1000101 \Rightarrow 133 \]

Unless one argument is a scalar, the arguments must be the same shape or a 'length error' will be produced.

DECODE can also be used to translate times, given time in years, (0 to 1999), days (0 to 364), hours (0 to 23), minutes (0 to 60), and seconds (0 to 60), enter 2000 365 24 60 60 \( \uparrow \) TIME to find the time in seconds.

For example: 2000 365 24 60 60 \( \uparrow \) 0 5 6 51 41 \( \Rightarrow \) 456701
THE SYMBOL $\&$ (ALT [] ) IS USED FOR THE MONADIC EXECUTE FUNCTION.
IT TAKES A VECTOR CHARACTER ARGUMENT ONLY, AND TREATS IT AS
AN AFL EXPRESSION.

FOR EXAMPLE: $\&'1 + 2'$ =) 3

IF $v =) 'HELP'$, $v$ WILL RUN THE HELP FUNCTION
HOWEVER, $v$ WILL NOT EXECUTE EXPRESSIONS BEGINNING WITH ) OR $.,
THE SYMBOL (ALT [J] is used for the dyadic format function. The right argument must be numeric, of any rank, and the left argument must be a vector of integers. This function converts the right argument to character data in a tabular format specified by the left argument, as follows:

If the left argument is a scalar, the right argument will be written with that number of decimal places.

For example: 2*14 = 14.00 (remember this is how character data is now character data),
3*15 28.35 0.1475 = 15.000 28.350 .148

Notice that the columns are of equal width, separated by one space.

If the left argument is a vector of length two, each column in the right argument will take up the number of spaces specified by the first element of the left argument, and each entry will have the number of decimal places specified by the second entry in the left argument. For example:

If M = 1.2 3 then 5*M = 1.2 3.1
   -4 5.55  -4.0 5.5

The column element must be large enough to write the longest entry, including one space each for a negative sign or decimal point. If it is not, a ‘length error’ will result.

Remember to allow for a blank between columns: 3 1*1 2 = 1.02.0

The left argument may also be a vector of length equal to exactly twice the number of columns in the right argument. In this case, the entries in the left argument are paired and each pair governs the format of the corresponding column, as above.

6 2 6 1 10 0 + 113.524 1 300.24 = 113.52 1.0 300

One final twist: if the left argument is a negative scalar, the
RESULT WILL BE WRITTEN IN EXPONENTIAL NOTATION, WITH THE NUMBER OF DECIMAL PLACES IN THE MANTISSA EQUAL TO ONE LESS THAN THE ABSOLUTE VALUE OF THE LEFT ARGUMENT.

FOR EXAMPLE: 
-1+3 200 => 3E00 2E02

-3+4.6167 10.56 400 => 4.62E00 1.06E01 4.00E02
THE SYMBOL + (ALT J) IS USED FOR THE MONADIC FORMAT FUNCTION.
IT TAKES A NUMERIC OR CHARACTER ARGUMENT OF ANY RANK AND CONVERTS
IT TO CHARACTER DATA.

THIS IS USEFUL IN CATENATING NUMERIC VARIABLES TO CHARACTER DATA.
FOR EXAMPLE: IF S = 3 AND V = 'GO TO LINE'

V,S WILL PRODUCE A 'DOMAIN ERROR' BUT V,IS = 'GO TO LINE 3'
D. MAKING MODIFICATIONS

The variable HOWMODS is included in the TUTOR workspace to guide the advanced student, programmer, or instructor who wishes to expand, modify, or customize the TUTOR functions and variables. Some suggestions for improvement are discussed in chapter 3. Any ideas for modification of TUTOR are welcomed—please pass them along to Professor R. R. Read or the author of this thesis, in care of the Operations Analysis curriculum office.
THE FOLLOWING ARE SOME TIPS ON MODIFYING THE TUTOR WORKSPACE:

TO MODIFY TEXT OF DESCRIPTIVE VARIABLES:
XEDIT THE VARIABLE (CHECK MAT FOR SHORT NAME)
BE SURE TO INCLUDE THE APPROPRIATE FIRST COLUMN CONTROL CHARACTERS:
* FOR GENERAL DESCRIPTION
v FOR SCALAR DESCRIPTION AND EXAMPLES
< FOR VECTOR DESCRIPTION AND EXAMPLES
n FOR MATRIX DESCRIPTION AND EXAMPLES
x FOR HIGHER-LEVEL ARRAYS DESCRIPTION AND EXAMPLES

TO CHANGE THE ORDER OF LESSONS:
XEDIT MAT TO ASSIGN NEW LESSON NUMBERS,
THEN, TO REORDER MAT ACCORDING TO LESSON NUMBERS, ENTER: ORDERMAT

TO WRITE NEW LESSONS:
WRITE A NEW TEXT VARIABLE, USING CONTROL CHARACTERS AS ABOVE,
XEDIT MAT TO ADD A NEW ROW, BE SURE TO INCLUDE:
COL 1-3: LESSON NUMBER
COL 28-31: NAME OF THE TEXT VARIABLE (LIMIT 4 CHARACTERS)
COL 35-51: FULL NAME OF THE LESSON (LIMIT 23 CHARACTERS)
THE OTHER COLUMNS MAY BE LEFT BLANK AS THEY PERTAIN TO FUNCTION SYMBOLS ONLY.

TO CHANGE OR WRITE NEW QUESTIONS (NOT DRILL):
XEDIT CUES TO CHANGE OR ADD A QUESTION, BE SURE TO ENCLOSE THE TEXT OF THE QUESTION, BUT NOT VARIABLES, IN QUOTE MARKS.
A CARRIAGE RETURN MAY BE INCLUDED BY USING THE VARIABLE CR.
ANSWER GROUPS MAY BE SEPARATE VARIABLES SUCH AS AR, RK.
XEDIT SHOW TO INCLUDE NEW QUESTIONS IN ANY QUESTION VECTOR QV,
BE SURE TO INCLUDE THE CORRESPONDING ANSWER IN THE VECTOR AV.
APPENDIX C
SAMPLE RUN

This appendix contains a printout of a session with the APL tutor. Student response to questions is preceded by a dot or period. (This is a function of the terminal and not part of the input.) Other student input is generally indented eight spaces and preceded by several blank lines. At least one example of each of the various ways to use the TUTOR is included. Also the various types of student response to questions and drills are demonstrated.
YOU MAY USE THE APL TUTOR IN THREE WAYS:

(1) ENTER: HELP
    TO SELECT THE SYMBOLS THAT YOU WANT INFORMATION ABOUT.

(2) ENTER: TEACH
    TO SELECT THE SYMBOLS THAT YOU WANT INFORMATION AND DRILL ON.

(3) ENTER: MENU
    TO SEE A LIST OF SYMBOLS AND TOPICS.

IF YOU HAVE NEVER USED THE APL TUTOR BEFORE, ENTER: START

TO SEE THESE INSTRUCTIONS AGAIN AT ANY TIME, ENTER: HOW

START

WELCOME TO THE APL TUTOR,

THE PURPOSE OF THIS WORKSPACE IS TO INTRODUCE YOU TO
'A PROGRAMMING LANGUAGE' BY DESCRIBING THE FUNCTIONS OF THE
MANY SPECIAL APL SYMBOLS, AND BY OUTLINING THE PROCEDURES FOR
DESIGNING YOUR OWN FUNCTIONS.

THE APL TUTOR ASSUMES YOU HAVE HAD LINEAR ALGEBRA AND TRIGONOMETRY.
CALCULUS IS NOT NECESSARY.
IF YOU HAVE NEVER HAD ANY COMPUTER PROGRAMMING BEFORE, DON'T WORRY,
YOU CAN START USING APL RIGHT AWAY, AS A SUPER-SOPHISTICATED
CALCULATOR WITH MANY BUILT-IN FUNCTIONS.
IF YOU HAVE STUDIED OTHER COMPUTER PROGRAMMING LANGUAGES, RELAX.
APL IS NOT LIKE ANY OF THE OTHER MAJOR HIGH-LEVEL LANGUAGES, YOU CAN FORGET ABOUT DATA TYPES, INPUT/OUTPUT FORMATTING, AND MANY OF THE OTHER TEDIous DETAILS OF FORTRAN, PASCAL, ETC. AFTER YOU ARE SUFFICIENTLY FAMILIAR WITH THE CALCULATOR MODE OF APL, YOU CAN LEARN TO DEFINE YOUR OWN FUNCTIONS WITH EASE.

HERE IS SOME BASIC INFORMATION YOU WILL NEED TO KNOW IN ORDER TO UNDERSTAND THE APL TUTOR LESSONS.

MONADIC AND DYADIC FUNCTIONS
YOU ARE ALREADY FAMILIAR WITH SYMBOLS WHICH REPRESENT ARITHMETIC FUNCTIONS, SUCH AS + OR -. APL USES THESE SYMBOLS AND MANY OTHERS TO REPRESENT A VARIETY OF FUNCTIONS. MANY FUNCTIONS, SUCH AS +, REQUIRE TWO ARGUMENTS, THAT IS, TWO INPUT NUMBERS, IN APL, THESE ARE CALLED DYADIC FUNCTIONS, AND THE SYMBOL IS PLACED BETWEEN THE ARGUMENTS; FOR EXAMPLE, 3+4.
OTHER FUNCTIONS, SUCH AS LN (NATURAL LOG), REQUIRE ONLY ONE ARGUMENT. THESE MONADIC FUNCTION SYMBOLS ARE PLACED TO THE LEFT OF THE DATA WHICH THEY ARE TO OPERATE ON. FOR EXAMPLE, LN 3 IN APL IS #3.

DATA TYPES
APL DISTINGUISHES ONLY TWO TYPES OF DATA: NUMERIC AND CHARACTER,
VERY SIMPLY, CHARACTER DATA ARE ENCLOSED IN QUOTES (') WHEN ENTERED.
'2' IS CHARACTER DATA; 2 IS NUMERIC DATA. SOME FUNCTIONS WILL OPERATE ON BOTH TYPES OF DATA, SOME ONLY ON NUMERIC.

ARRAYS
THE GREATEST STRENGTH OF APL LIES IN ITS ABILITY TO TAKE AN ENTIRE ARRAY OF NUMBERS AS A SINGLE ARGUMENT, THUS TWO MATRICES CAN BE ADDED BY ENTERING SIMPLY A+B, WITH NO SUBSCRIPTS, LOOPS, ETC.
THIS MAKES IT IMPORTANT TO KNOW THE RANK (THE NUMBER OF DIMENSIONS) OF DATA IN USE. A SINGLE NUMBER IS NORMALLY A SCALAR (RANK 0), A SERIES OF NUMBERS IS A VECTOR, A ONE-DIMENSIONAL ARRAY (RANK 1).
NUMBERS CAN ALSO BE ARRANGED IN ROWS AND COLUMNS, TO MAKE A MATRIX (RANK 2). MATRICES CAN BE 'STACKED' TO MAKE UP THE PAGES OF A THREE-DIMENSIONAL ARRAY, OFTEN CALLED A BOOK (RANK 3). IN FACT, THERE IS NO LIMIT TO THE NUMBER OF DIMENSIONS IN AN APL ARRAY. THE LENGTH OF THE DIMENSIONS IS ALSO EFFECTIVELY UNLIMITED. LENGTH REFERS TO THE NUMBER OF ELEMENTS IN A DIMENSION, FOR EXAMPLE, THE NUMBER OF ROWS. LENGTH MAY EVEN BE 0.

IN THIS TUTORIAL, A SCALAR MAY BE REFERRED TO AS S, L, OR R.

A VECTOR MAY BE CALLED V, V₁, OR V₂, WHILE A MATRIX IS M, M₁, OR M₂. AN ARRAY, WHICH MAY BE A VECTOR, A MATRIX, OR AN ARRAY OF ANY HIGHER DIMENSION, WILL BE LABELLED A, A₁, OR A₂.

EXAMPLES

WHEN EXAMPLES ARE GIVEN IN THE TEXT OF A FUNCTION DESCRIPTION, THE SYMBOL =) IS USED BETWEEN THE EXAMPLE INPUT AND THE EXAMPLE OUTPUT. FOR EXAMPLE: 2+5 =) 7

THIS SYMBOL CAN BE READ AS 'PRODUCES' OR 'RETURNS'.

ERRORS

YOU SHOULD NOT RECEIVE ANY ERROR MESSAGES WHILE USING APL TUTOR.

HOWEVER, YOU MAY SEE A PHRASE LIKE 'VALUE ERROR' (WITHOUT QUOTE MARKS) WHEN YOU ARE EXPECTING A NORMAL RESPONSE,

OR IF YOU ACCIDENTALLY HIT THE ENTER KEY WHEN THE COMPUTER IS NOT EXPECTING AN INPUT, THE FUNCTION MAY END ABRUPTLY, LIKE THIS:

MENU [6]

IF EITHER OF THESE THINGS HAPPEN, BE SURE TO ENTER: + (UPSHIFT +)

THEN RESTART WITH ANY COMMAND.

HALTING A LESSON

YOU MAY STOP ANY LESSON AT ANY POINT WHERE A QUESTION IS ASKED BY ENTERING: STOP

TO SEE THIS INFORMATION AGAIN AT ANY TIME, ENTER: BASICS

171
YOU MAY START A LESSON IN THREE WAYS:

(1) ENTER: LESSON NEXTLESSON
       TO START THE LESSON FOLLOWING THE LAST ONE YOU COMPLETED.

(2) ENTER: LESSON NNN (WHERE NNN IS A 3-DIGIT NUMBER)
       TO START LESSON NUMBER NNN.
       TO SEE A LIST OF LESSON NUMBERS, ENTER: MENU

(3) ENTER: TEACH
       TO SELECT THE SYMBOL THAT YOU WANT A LESSON ON.

TO SEE THIS INFORMATION AGAIN AT ANY TIME, ENTER: HOWTEACH

TO START YOUR FIRST LESSON IN APL, ENTER: LESSON 101

LESSON 101

................................. .................................

THE SYMBOL -(UPSHIFT 2) IS USED ONLY TO ENTER NEGATIVE NUMBERS.
IT CAN BE USED ONLY IN FRONT OF SCALARS (SINGLE NUMBERS),
NOT VARIABLE NAMES, ARITHMETIC EXPRESSIONS, OR ARRAYS.
TO CHANGE ARITHMETIC SIGN USE THE SYMBOL -(UPSHIFT +).

HOW MANY ARGUMENTS DOES THE NEGATIVE_NUMBER FUNCTION TAKE?
1
RIGHT

WHAT RANK CAN THE RIGHT ARGUMENT OF NEGATIVE_NUMBER BE?
ENTER S FOR SCALAR
   V FOR VECTOR

172
FOR MATRIX
A FOR ANY RANK

CAN NEGATIVE_NUMBER TAKE CHARACTER ARGUMENTS?

RIGHT

THIS IS THE END OF YOUR LESSON.
FOR THE NEXT LESSON IN SEQUENCE, ENTER: LESSON NEXTLESSON
TO START ANOTHER LESSON, ENTER: LESSON NNN

LESSON NEXTLESSON

THE SYMBOL + IS USED FOR THE MONADIC CONJUGATE FUNCTION,
THIS IS AN IDENTITY FUNCTION WHICH RETURNS ITS ARGUMENT, AFTER
EVALUATION IF NECESSARY, IT TAKES NUMERIC ARGUMENTS OF ANY RANK.

HOW MANY ARGUMENTS DOES THE CONJUGATE FUNCTION TAKE?

STOP

THIS LESSON HAS BEEN HALTED.
TO START ANOTHER LESSON, ENTER: LESSON NNN

173
THE SYMBOL ÷ (UPSHIFT X) IS USED FOR THE DYADIC DIVISION FUNCTION.
IT PERFORMS SIMPLE DIVISION ON NUMERIC ARGUMENTS OF ANY RANK.

HOW MANY ARGUMENTS DOES THE DIVIDE FUNCTION TAKE?
TWO
SORRY, PLEASE TRY AGAIN

HOW MANY ARGUMENTS DOES THE DIVIDE FUNCTION TAKE?
TWO

WHAT RANK CAN THE LEFT ARGUMENT OF DIVIDE BE?
ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK
S
SORRY, PLEASE TRY AGAIN

WHAT RANK CAN THE LEFT ARGUMENT OF DIVIDE BE?
ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK
A

WHAT RANK CAN THE RIGHT ARGUMENT OF DIVIDE BE?
ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK

174
SORRY, PLEASE TRY AGAIN

WHAT RANK CAN THE RIGHT ARGUMENT OF DIVIDE BE?
ENTER S FOR SCALAR
   V FOR VECTOR
   X FOR MATRIX
   A FOR ANY RANK

SORRY, PLEASE TRY AGAIN

WHAT RANK CAN THE RIGHT ARGUMENT OF DIVIDE BE?
ENTER S FOR SCALAR
   V FOR VECTOR
   X FOR MATRIX
   A FOR ANY RANK

SORRY, THE CORRECT ANSWER IS A
YOU ENTERED 5 WRONG ANSWERS OUT OF 3 QUESTIONS.
IF YOU WANT TO RETAKE THIS LESSON, ENTER: LESSON 107

THIS LESSON HAS BEEN HALTED,
TO START ANOTHER LESSON, ENTER: LESSON XXXX
HELP

ENTER THE SYMBOL(S) YOU WOULD LIKE INFORMATION ABOUT.
OR... FOR MENU SELECTION, ENTER:  MENU

INFORMATION IS AVAILABLE ON THE FOLLOWING SYMBOLS/TOPICS:

101 - NEGATIVE_NUMBER 102 - CONJUGATE 103 - PLUS
104 - CHANGE_SIGH 105 - MINUS 106 - SIGN
107 - TIMES 108 - RECIPROCAL 109 - DIVIDE
121 - PARENS 122 - QUOTE 123 - SPECIFICATION
124 - SHAPE 125 - RESHAPE 126 - BRACKETS
127 - RAVEL 128 - CATEGANATE 129 - INDEX_GENERATOR
131 - REDUCTION 132 - REDUCTION 141 - EXPONENTIAL
142 - POWER 143 - NATURAL_LOG 144 - LOGARITHM
145 - FI_TIMES 146 - GEOMETRIC 147 - FACTORIAL
148 - BINOMIAL 149 - MAGNITUDE 151 - SCAN
152 - SCAN 161 - CEILING 162 - FLOOR
163 - MAXIMUM 164 - MINIMUM 165 - RESIDUE
166 - GRADE_UP 167 - GRADE_DOWN 168 - FLOOR
169 - DEAL 180 - EQUAL 181 - NOT_EQUAL
182 - LESS_THAN 183 - LESS_OR_EQUAL 184 - GREATER_OR_EQUAL
185 - GREATER_THAN 191 - NOT 192 - AND
193 - NAND 194 - OR 195 - NOR
196 - MEMBER_OF 201 - REVERSE 202 - REVERSE
203 - ROTATE 204 - ROTATE 205 - TRANSPOSE_MONADIC
206 - TRANSPOSE-DYADIC 211 - TAKE 212 - DROP
213 - COMPRESS 214 - COMPRESS 215 - EXPAND
216 - EXPAND 219 - LAMINATE 231 - MATRIX_INVERSE
232 - MATRIX_DIVIDE 241 - OUTER_PRODUCT 242 - INNER_PRODUCT
301 - INDEX_OF 304 - ENCODE 305 - DECODE
306 - EXECUTE 307 - FORMAT-DYADIC 308 - FORMAT-MONADIC

TO SEE MENU AGAIN, ENTER: MENU

FOR INFORMATION ABOUT FUNCTION, ENTER: INFO NNN (WHERE NNN IS MENU NUMBER)

TO GO THROUGH TUTORIAL LESSON, ENTER: LESSON NNN
LESSON 107

......................... I .................................
THE SYMBOL \times \text{ IS USED FOR THE DYADIC TIMES FUNCTION.}
THIS PERFORMS SIMPLE MULTIPLICATION ON NUMERIC ARGUMENTS OF ANY RANK.

HOW MANY ARGUMENTS DOES THE TIMES FUNCTION TAKE?

\text{2}

\text{RIGHT}

WHAT RANK CAN THE LEFT ARGUMENT OF TIMES BE?

ENTER \text{S} FOR SCALAR
\quad \text{V} FOR VECTOR
\quad \text{X} FOR MATRIX
\quad \text{A} FOR ANY RANK

\text{ANY}

\text{RIGHT}

\text{FOR EXAMPLE: } 2\times4 \Rightarrow 8 \quad \text{ORDER AND SPACING ARE NOT IMPORTANT.}

DOES 51\times52 \text{ EQUAL } 52\times51?

\text{Y}

\text{RIGHT}

\text{WHAT IS THE RESULT OF:}

\text{20 \times 80}
\quad \text{160}
SORRY, CORRECT ANSWER IS:

-1600

WHAT IS THE RESULT OF:

80 \times -30

=-2400

CORRECT:

TO MULTIPLY EACH ELEMENT OF A VECTOR BY A SCALAR, ENTER S \times V OR V \times S.

V_1 \times V_2 RESULTS IN THE PRODUCT OF CORRESPONDING ELEMENTS OF V_1 AND V_2.

FOR EXAMPLE: 

3 \times (1 \ 2 \ 3) = \begin{pmatrix} 3 \ 6 \ 9 \end{pmatrix} 

(1 \ 2 \ 3) \times 3 = \begin{pmatrix} 3 \ 6 \ 9 \end{pmatrix} 

(1 \ 2 \ 3) \times (2 \ 4 \ 6) = \begin{pmatrix} 2 \ 3 \ 18 \end{pmatrix}

IF V_1 AND V_2 ARE NOT THE SAME LENGTH, YOU WILL GET A 'LENGTH ERROR'.

IF ONE ARGUMENT IS A VECTOR, THE OTHER ARGUMENT MAY BE SCALAR OR VECTOR.

(T/F)

.F

RIGHT

WHAT IS THE RESULT OF:

-0.6 \times -0.5 \ 0.9

0.3 \times -0.54

SORRY, CORRECT ANSWER IS:

0.3 \times -0.54

WHAT IS THE RESULT OF:

0.1 \times 0.2 \ 0.7

0.2 \ 0.7

SORRY, CORRECT ANSWER IS:

0.02 \ 0.07

WHAT IS THE RESULT OF:

-0.2 \times -0.1 \ 0.6

-0.02 \ -0.12

SORRY, CORRECT ANSWER IS:

0.02 \ -0.12

PLEASE REVIEW THE DESCRIPTION OF THIS FUNCTION OR TALK WITH YOUR
INSTRUCTOR BEFORE RETURNING TO THIS UNIT.

THIS LESSON HAS BEEN HALTED.
TO START ANOTHER LESSON, ENTER: LESSON XXX

LESSON 124

------------------------------- / -------------------------------
THE SYMBOL $p$ IS USED FOR THE MONADIC SHAPE FUNCTION,
THIS RETURNS A DESCRIPTION OF THE SIZE OF ITS ARGUMENT, WHICH CAN
BE NUMERIC OR CHARACTER, OF ANY RANK,
THE SYMBOL CAN BE USED TWICE ($pp$) TO RETURN THE RANK OF ITS ARGUMENT.

HOW MANY ARGUMENTS DOES THE SHAPE FUNCTION TAKE?
1
RIGHT

WHAT RANK CAN THE RIGHT ARGUMENT OF SHAPE BE?
ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK
A
RIGHT

IF THE ARGUMENT IS A SCALAR, $p$ RETURNS NO NUMBER (AN EMPTY VECTOR),
BECAUSE A SCALAR HAS NO DIMENSION, $pp = 0$
ENTERING PV RETURNS ONE NUMBER WHICH REPRESENTS THE LENGTH OF THE VECTOR'S ONE DIMENSION, \( f_f' = 1 \)

WHAT IS THE RESULT OF:
\( f \) 10 20 90 -40
\( .P_4 \)
SORRY, THIS ANSWER NOT ACCEPTABLE, PLEASE ENTER A NUMERIC ANSWER.

WHAT IS THE RESULT OF:
\( f \) 10 20 90 -40
\( .P_4 \)
CORRECT;

WHAT IS THE RESULT OF:
\( f \) 2 0 3
\( .P_3 \)
CORRECT;

THE SHAPE OF MATRICES IS EXPRESSED AS A TWO-ELEMENT VECTOR,

FOR EXAMPLE: IF \( M = ) 1 2 3 \) THEN \( f_M = 2 3 \) AND \( f_fM = 2 4 5 6 \)

FOR HIGHER-LEVEL ARRAYS, \( f_A \) WILL BE A VECTOR WITH AS MANY ELEMENTS AS \( A \) HAS DIMENSIONS, THE NUMBER OF COLUMNS WILL ALWAYS BE THE LAST ELEMENT OF \( f_A \), THE NUMBER OF ROWS WILL ALWAYS BE THE NEXT-TO-LAST ELEMENT, PRECEDED BY THE NUMBER OF 'PAGES' OR PLANES, PRECEDED BY THE NUMBER OF 'BOOKS' OR SPACES, AND SO ON.

FOR EXAMPLE: IF \( D \) IS A 4-DIMENSIONAL ARRAY, \( f_D = 4 \), IF \( f_D = 5 3 4 2 \) THEN \( D \) HAS 5 BOOKS, EACH WITH 3 PAGES, EACH OF WHICH HAS 4 ROWS AND 2 COLUMNS, (THIS WOULD BE DISPLAYED AS FIFTEEN SUCCESSIVE 4X2 MATRICES--THE FIRST THREE BEING THE FIRST
CAN SHAPE TAKE CHARACTER ARGUMENTS?

RIGHT

IF SO, WHICH ARGUMENT(S) CAN BE CHARACTER?
ENTER R FOR RIGHT,
L FOR LEFT, OR
B FOR BOTH

RIGHT

THIS IS THE END OF YOUR LESSON,
FOR THE NEXT LESSON IN SEQUENCE, ENTER: LESSON NEXT
TO START ANOTHER LESSON, ENTER: LESSON

LESSON NEXT

THE SYMBOL \( \texttt{p} \) IS USED FOR THE DYADIC RESHAPE FUNCTION,

THIS TAKES THE ELEMENTS OF THE RIGHT ARGUMENT AND REARRANGES THEM
ACCORDING TO THE SHAPE SPECIFIED BY THE LEFT ARGUMENT.

THE LEFT ARGUMENT MUST BE AN INTEGER SCALAR OR A VECTOR OF INTEGER
ELEMENTS,

THE RIGHT ARGUMENT CAN BE NUMERIC OR CHARACTER, OF ANY RANK.

HOW MANY ARGUMENTS DOES THE RESHAPE FUNCTION TAKE?

2

RIGHT
WHAT RANK CAN THE LEFT ARGUMENT OF RESHAPE BE?
ENTER S FOR SCALAR
   v FOR VECTOR
   X FOR MATRIX
   A FOR ANY RANK
, v
RIGHT

WHAT RANK CAN THE RIGHT ARGUMENT OF RESHAPE BE?
ENTER S FOR SCALAR:
   v FOR VECTOR
   X FOR MATRIX
   A FOR ANY RANK
, a
RIGHT

IF THE LEFT ARGUMENT IS A SCALAR, THE RESULT WILL BE A VECTOR WITH
   LENGTH EQUAL TO THE SCALAR, CONSISTING OF THE ELEMENTS OF THE RIGHT
   ARGUMENT, TAKEN IN ORDER TOP LEFT TO BOTTOM RIGHT.
   WHEN THERE ARE NOT ENOUGH ELEMENTS IN THE RIGHT ARGUMENT,
   THE FUNCTION WILL START OVER AND TAKE THE ELEMENTS IN ORDER AGAIN,
   AS MANY TIMES AS NECESSARY TO FILL THE VECTOR.
   FOR EXAMPLE:  4f1 = 1 1 1 1
                  4f('AB') = ABAB
   IF THERE ARE TOO MANY ELEMENTS ON THE RIGHT, THE EXTRAS ARE OMITTED.
   FOR EXAMPLE:  2f(1 2 3 4) = 1 2

DOES S1fS2 EQUAL S2fS1?
   , n
RIGHT

WHAT IS THE RESULT OF:
   2 f 0.6
SORRY, CORRECT ANSWER IS:
0.6 0.6
WHAT IS THE RESULT OF:
2 p 0.1
..1 .1 .1
SORRY, CORRECT ANSWER IS:
0.1 0.1
WHAT IS THE RESULT OF:
3 p 0.6
..6 .6 .6
CORRECT;

IF THE LEFT ARGUMENT IS A VECTOR, THE ELEMENTS INDICATE, IN ORDER,
LENGTH OF EACH DIMENSION IN THE RESULT.
FOR EXAMPLE: 2 2p1 2 3 4 => 1 2
            3 4
THIS IS ONE WAY TO ENTER MATRICES OR ARRAYS OF HIGHER DIMENSION.

IF ONE ARGUMENT IS A VECTOR, THE OTHER ARGUMENT MAY BE SCALAR OR VECTOR,
(T/F)
.T
RIGHT

WHAT IS THE RESULT OF:
3 p 0.1 4
..1 4 .1
CORRECT;

USING A MATRIX OR HIGHER-LEVEL ARRAY AS THE LEFT ARGUMENT OF RESHAPE
WILL RESULT IN A 'RANK ERROR'.

IF ONE ARGUMENT IS A MATRIX, THE OTHER ARGUMENT MUST BE A SCALAR
OR A MATRIX OF THE SAME SHAPE, (T/F)
CAN RESHAPE TAKE CHARACTER ARGUMENTS?

Y

RIGHT

IF SO, WHICH ARGUMENT(S) CAN BE CHARACTER?
ENTER R FOR RIGHT,
    L FOR LEFT, OR
    B FOR BOTH

RIGHT

THIS IS THE END OF YOUR LESSON,
FOR THE NEXT LESSON IN SEQUENCE, ENTER: LESSON NEXTLESSON
TO START ANOTHER LESSON, ENTER: LESSON MMM
TEACH

ENTER THE SYMBOL(S) YOU WOULD LIKE INFORMATION ABOUT,
OR FOR MENU SELECTION, ENTER: MENU

+ \\

+-----------+-----------+
THE SYMBOL + IS USED FOR THE NOMADIC CONJUGATE FUNCTION.
THIS IS AN IDENTITY FUNCTION WHICH RETURNS ITS ARGUMENT, AFTER
EVALUATION IF NECESSARY. IT TAKES NUMERIC ARGUMENTS OF ANY RANK.

HOW MANY ARGUMENTS DOES THE CONJUGATE FUNCTION TAKE?

1

RIGHT

WHAT RANK CAN THE RIGHT ARGUMENT OF CONJUGATE BE?
ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK

A

RIGHT

EXAMPLE: \(+2 \Rightarrow 2 \) \(+(-3+2) \Rightarrow -1\).

CAN CONJUGATE TAKE CHARACTER ARGUMENTS?

N

RIGHT

+-----------+-----------+
THE Symbol + IS USED FOR THE DYADIC PLUS FUNCTION.
THIS PERFORMS SIMPLE ADDITION ON NUMERIC ARGUMENTS OF ANY RANK.

HOW MANY ARGUMENTS DOES THE PLUS FUNCTION TAKE?

2
RIGHT

WHAT RANK CAN THE LEFT ARGUMENT OF PLUS BE?

ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK
A
RIGHT

WHAT RANK CAN THE RIGHT ARGUMENT OF PLUS BE?

ENTER S FOR SCALAR
V FOR VECTOR
X FOR MATRIX
A FOR ANY RANK
A
RIGHT

FOR EXAMPLE: 2+2 = 4 SPACING IS NOT IMPORTANT.

DOES S1+S2 EQUAL S2+S1?
Y
RIGHT

WHAT IS THE RESULT OF:
10 + 10
+
SORRY, THIS ANSWER NOT ACCEPTABLE, PLEASE ENTER A NUMERIC ANSWER.

WHAT IS THE RESULT OF:
10 + 10
.20
CORRECT;

TO ADD A SCALAR TO EACH ELEMENT OF A VECTOR, ENTER EITHER +V OR *V.
V1+V2 WILL ADD TWO VECTORS ELEMENT BY ELEMENT. FOR EXAMPLE:

2+(3 4 5) = 5 6 7  (3 4 5)+2 = 5 6 7
(3 4 5)+(5 6 7) = 8 10 12

IF THE TWO VECTORS BEING ADDED ARE NOT THE SAME LENGTH,
YOU WILL GET A 'LENGTH ERROR'.

IF ONE ARGUMENT IS A VECTOR, THE OTHER ARGUMENT MAY BE SCALAR OR VECTOR.

(T/F)
T
RIGHT

WHAT IS THE RESULT OF:
0.2 + 0.9 0.3
2 + .9 .3
SORRY, THIS ANSWER NOT ACCEPTABLE, PLEASE ENTER A NUMERIC ANSWER,

WHAT IS THE RESULT OF:
0.2 + 0.9 0.3
1.1 .5
CORRECT;

WHAT IS THE RESULT OF:
-5 4 3 + -5
,STOP

THIS LESSON HAS BEEN HALTED.

THIS IS THE END OF SCHEDULED LESSONS,
TO SELECT MORE LESSONS, ENTER:  TEACH

187
ENTER THE SYMBOL(S) YOU WOULD LIKE INFORMATION ABOUT.
FOR MENU SELECTION, ENTER: MENU

INFORMATION IS AVAILABLE ON THE FOLLOWING SYMBOLS/TOPICS:

101 - NEGATIVE_NUMBER
102 + CONJUGATE
104 - CHANGE_SIGN
105 - MINUS
107 x TIMES
108 - RECIPROCAL
121 ( PARENS
124 | SHAPE
127 , RAVEL
131 / REDUCTION
132 # REDUCTION
142 & POWER
145 Ø PI_TIMES
148 : BINOMIAL
152 % SCAN
153 T MAXIMUM
156 & GRADE_UP
159 ? DEAL
162 ( LESS_THAN
163 | MINIMUM
166 & GRADE_DOWN
169 ? EQUAL
182 < LESS_THAN
183 | LESS_OR_EQUAL
193 * HAND
196 e MEMBER OF
203 @ ROTATE
206 & TRANSPOSE-DYADIC
213 / COMPRESS
216 # EXPAND
232 B MATRIX_DIVIDE
301 \ INDEX_OF
306 & EXECUTE
211 + TAKE
214 / COMPRESS
215 \ EXPAND
219 : LAMINATE
241 x OUTER_PRODUCT
242 : INNER_PRODUCT
304 + ENCODE
307 + FORMAT-DYADIC
308 + FORMAT-MONADIC

TO SEE MENU AGAIN, ENTER: MENU
FOR INFORMATION ABOUT FUNCTION, ENTER: INFO HHH (WHERE HHH IS MENU NUMBER)
TO GO THROUGH TUTORIAL LESSON, ENTER: LESSON HHH
THE SYMBOL - (UPSHIFT +) IS USED FOR THE DYADIC MINUS FUNCTION. IT PERFORMS SIMPLE SUBTRACTION ON NUMERIC ARGUMENTS OF ANY RANK.

HOW MANY ARGUMENTS DOES THE MINUS FUNCTION TAKE?
- 2

RIGHT

WHAT RANK CAN THE LEFT ARGUMENT OF MINUS BE?
ENTER S FOR SCALAR
    V FOR VECTOR
    X FOR MATRIX
    A FOR ANY RANK
- A

RIGHT

WHAT RANK CAN THE RIGHT ARGUMENT OF MINUS BE?
ENTER S FOR SCALAR
    V FOR VECTOR
    X FOR MATRIX
    A FOR ANY RANK
- A

RIGHT

FOR EXAMPLE:  S-2 =: 3  -10-2 =: -8

DOES S1-S2 EQUAL S2-S1?
- N

RIGHT

WHAT IS THE RESULT OF:
- 0.1 - -0.3 ..4
CORRECT:

TO SUBTRACT A SCALAR FROM EACH ELEMENT OF A VECTOR, ENTER V-S.
TO SUBTRACT EACH ELEMENT OF A VECTOR FROM THE SAME SCALAR, ENTER S-V.
V1-V2 WILL SUBTRACT EACH ELEMENT OF V2 FROM THE CORRESPONDING ELEMENT OF V1.
V1 AND V2 MUST BE THE SAME LENGTH OR YOU WILL GET A 'LENGTH ERROR'.
FOR EXAMPLE: (3 4 5)-2 = 1 2 3
     2-(3 4 5) =) "1 "2 "3
     (6 7 8)-(1 2 3) =: 5 5 5

IF ONE ARGUMENT IS A VECTOR, THE OTHER ARGUMENT MAY BE SCALAR OR VECTOR.

(T/F)

RIGHT

WHAT IS THE RESULT OF:
0.9 - 0.6 0.5
..3 .4
CORRECT;

WHAT IS THE RESULT OF:
1 2 3 - "5
..4 -3 "2
SORRY, CORRECT ANSWER IS:
6 7 8
WHAT IS THE RESULT OF:
"0.9 0.4 - "0.4
..5 .8
CORRECT;

WHAT IS THE RESULT OF:
"0.7 0.8 - "0.8 0.7
..1 1.5
SORRY, CORRECT ANSWER IS:
0.1 0.1

What is the result of:

\[ 80 \ -60 \ 50 \ -30 \ -90 \ 30 \ -10 \ -50 \]

Correct:

Enter M-S to subtract the scalar S from each element of the matrix M.
Enter S-M to do the opposite.

For example: if \( M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) then \( 4-M = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix} \) and \( M-2 = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix} \).

\( M_1-M_2 \) subtracts matrix \( M_2 \) from matrix \( M_1 \), element by element.

For example: if \( M_1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) and \( M_2 = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} \) then \( M_2-M_1 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \).

If \( M_1 \) and \( M_2 \) are not the same shape, a 'length error' will result.

Trying to subtract a vector from a matrix or vice-versa will produce a 'rank error'.

If one argument is a matrix, the other argument must be a scalar or a matrix of the same shape. (T/F)

T

These principles can be extended to higher level arrays.

Can minus take character arguments?

N

Right

Write a true statement using -,

For example: \( 4=2+2 \)

\( 2-2=0 \)

Sorry, this statement is not true. Try again.
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963 A
WRITE A TRUE STATEMENT USING -. 
FOR EXAMPLE: 4=2+2
.4=2+2
YOUR ANSWER DOES NOT USE -, TRY AGAIN.

WRITE A TRUE STATEMENT USING -, 
FOR EXAMPLE: 4=2+2
.(2-2)=4

DO YOU WISH TO REVIEW THE DESCRIPTION OF MINUS?
ENTER Y OR N,
-N

WRITE A TRUE STATEMENT USING -, 
FOR EXAMPLE: 4=2+2
.0=2-2
CORRECT!

THIS IS THE END OF YOUR LESSON,
FOR THE NEXT LESSON IN SEQUENCE, ENTER: LESSON NEXTLESSON
TO START ANOTHER LESSON, ENTER: LESSON NNN
HELP

ENTER THE SYMBOL(S) YOU WOULD LIKE INFORMATION ABOUT,

OR... FOR MENU SELECTION, ENTER: MENU

HELP

********************************************************* 
THE SYMBOL \ (UPSHIFT I) IS USED FOR THE MONADIC INDEX GENERATING 
FUNCTION,
IT IS USED WITH A SINGLE NON-NEGATIVE INTEGER ARGUMENT (SCALAR, OR 
VECTOR OF LENGTH 1),
IT RETURNS A VECTOR OF INTEGERS, IN ORDER, BEGINNING WITH THE INDEX 
ORIGIN, AND ENDING WITH THE ARGUMENT,
FOR EXAMPLE: \ 13 \ 123 \ 10 \ (AN EMPTY VECTOR)
THE INDEX ORIGIN IS NORMALLY (BY DEFAULT) 1,
USING \ WITH A NON-INTEGER, NEGATIVE OR ARRAY ARGUMENT WILL PRODUCE A 
'DOMAIN ERROR'.

********************************************************* 
THE SYMBOL \ (UPSHIFT I) IS USED FOR THE DYADIC INDEX OF FUNCTION,
THE LEFT ARGUMENT MUST BE A VECTOR AND THE RIGHT ARGUMENT CAN BE 
ANY RANK, INDEX OF CAN BE USED WITH CHARACTER OR NUMERIC DATA,
IT RETURNS THE POSITION IN THE LEFT ARGUMENT OF THE ELEMENTS OF THE 
RIGHT ARGUMENT, THE SHAPE OF THE RESULT IS THE SAME AS THE 
SHAPE OF THE RIGHT ARGUMENT,
IF THE RIGHT ARGUMENT DOES NOT OCCUR IN THE LEFT ARGUMENT, THE RESULT 
WILL BE ONE PLUS THE LENGTH OF THE LEFT ARGUMENT,

FOR EXAMPLE: 4 3 2 1 3 \ 2 BECAUSE 3 IS IN THE SECOND POSITION 
'H E L L O' \ 'P' \ 6 BECAUSE P IS NOT FOUND

MORE?

193
VIA CHECKS EACH ELEMENT OF THE RIGHT ARGUMENT INDIVIDUALLY.

FOR EXAMPLE: 3 5 7 9 \{ 1 2 3 4 5 \} =) 5 5 1 5 2

IF YOU ASSIGN ALPHABET \{ ABCDEFGHIJKLMNOPQRSTUVWXYZ \}

THEN ALPHABET \{ APL \} =) 1 16 12

FOR MORE HELP, ENTER: HELP
**MENU**

**INFORMATION IS AVAILABLE ON THE FOLLOWING SYMBOLS/TOPICS:**

<table>
<thead>
<tr>
<th>Symbol/Topic</th>
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</tr>
</thead>
<tbody>
<tr>
<td>101 - NEGATIVE_NUMBER</td>
<td>102 + CONJUGATE</td>
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</tr>
<tr>
<td>104 - CHANGE_SIGN</td>
<td>105 - MINUS</td>
<td>106 x SIGNUM</td>
</tr>
<tr>
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<td>108 + RECIPROCAL</td>
<td>109 + DIVIDE</td>
</tr>
<tr>
<td>121 ( PARENS</td>
<td>122 ' QUOTE</td>
<td>123 + SPECIFICATION</td>
</tr>
<tr>
<td>124 / SHAPE</td>
<td>125 $ RESHAPE</td>
<td>126 [ BRACKETS</td>
</tr>
<tr>
<td>127 , RAVAL</td>
<td>128 ; CATEHATE</td>
<td>129 \ INDEX_GENERATOR</td>
</tr>
<tr>
<td>131 / REDUCTION</td>
<td>132 \ REDUCTION</td>
<td>141 ^ EXPONENTIAL</td>
</tr>
<tr>
<td>142 % POWER</td>
<td>143 @ NATURAL_LOG</td>
<td>144 @ LOGARITHM</td>
</tr>
<tr>
<td>145 * PI_TIMES</td>
<td>146 # GEOMETRIC</td>
<td>147 : FACTORIAL</td>
</tr>
<tr>
<td>148 : BINOMIAL</td>
<td>149</td>
<td>MAGNITUDE</td>
</tr>
<tr>
<td>152 \ SCAN</td>
<td>161 \ CEILING</td>
<td>162 ] FLOOR</td>
</tr>
<tr>
<td>163 ] MAXIMUM</td>
<td>164 L MINIMUM</td>
<td>165 L RESIDUE</td>
</tr>
<tr>
<td>166 \ GRADE_UP</td>
<td>167 \ GRADE_DOWN</td>
<td>168 ? ROLL</td>
</tr>
<tr>
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<td>184 \ GREATER_OR_EQUAL</td>
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<td>191 \ HOT</td>
<td>192 ^ AND</td>
</tr>
<tr>
<td>193 ^ HAND</td>
<td>194 \ OR</td>
<td>195 ^ NOR</td>
</tr>
<tr>
<td>196 \ MEMBER_OF</td>
<td>201 @ REVERSE</td>
<td>202 @ REVERSE</td>
</tr>
<tr>
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</tr>
<tr>
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**TO SEE MENU AGAIN, ENTER:**

**FOR INFORMATION ABOUT FUNCTION, ENTER:**

```
INFO NNN (WHERE NNN IS MENU N UNDER)
```

**TO GO THROUGH TUTORIAL LESSON, ENTER:**

```
LESSON NNN
```
INFO 180

THE SYMBOL = (UPSHIFT 5) IS USED FOR THE DYADIC EQUALS FUNCTION.
THIS COMPARES NUMERIC OR CHARACTER ARGUMENTS OF ANY RANK.
IT RETURNS 1 FOR EACH ELEMENT OF THE LEFT ARGUMENT THAT IS IDENTICAL
TO THE CORRESPONDING ELEMENT OF THE RIGHT ARGUMENT, AND 0 FOR
EACH ELEMENT THAT IS NOT.

FOR EXAMPLE: 2=2 \Rightarrow 1 \quad 2=4 \Rightarrow 0 \quad 'A'='B' \Rightarrow 0

MORE?

FOR MORE INFORMATION, ENTER: INFO HNN
TEST 211

WHAT IS THE RESULT OF:
-3 + -30 40 60 -90
40 60 -90
CORRECT:
TEST OK

TEST 203

WHAT IS THE RESULT OF:
-10 60 70 -20 -40
SORRY, CORRECT ANSWER IS:
-40 60 70 -20
WHAT IS THE RESULT OF:
20 0 -60 -40 -10
-40 -10 0 -60
CORRECT:
TEST OK

TEST 152

WHAT IS THE RESULT OF:
14 7 -6 -1
7 7 7
CORRECT:
TEST OK
TEST 166

WHAT IS THE RESULT OF:
\[ 4 \times -0.3 \times -0.4 \]
2 1
CORRECT:

WHAT IS THE RESULT OF:
\[ 3 \times 5 \times -6 \]
STOP

THIS TEST HAS BEEN HALTED.

STOP

IF YOU WANT TO END THIS SESSION, ENTER: )SAVE
WHEN YOU SEE THE TIME, DATE, AND 'TUTOR' MESSAGE, ENTER: )OFF

11:42:19 12/14/83 TUTOR

)OFF

CONNECT= 01:10:27 VIRTCPU= 000:08.88 TOTCPU= 000:11.11
CONNECT= $5.87 TOTCPU= $1.85 SIO= $0.41 TOTAL= $8.13
MULT BY SHIFT FACTOR: =1(DAY), =0.6(EVE), =0.3(NIGHTS)
LOGOFF AT 11:42:42 PST WEDNESDAY 12/14/83

VM/370 ONLINE
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<thead>
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