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Memorandum 3632

TITLE: A REASSESSMENT OF THE MEASUREMENT UNCERTAINTY OF THE XT-90 TWIN T DUAL ADMITTANCE BRIDGE

AUTHORS: G J Slack and T N W Graham

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SUMMARY

This memorandum supplements and updates the original report on the Twin T Bridge which was written over thirty years ago.

An explanation of both the practical and theoretical circuit development is given leading up to an equivalent circuit and the bridge balance difference equations. A brief description of the calibration procedures used is given, together with details of the method used to measure standards at RSRE. Finally a new assessment of overall bridge measurement uncertainty has been made, and a single equation for this produced for each of the three frequencies at which standards are currently calibrated.

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A REASSESSMENT OF THE MEASUREMENT UNCERTAINTY OF THE XT-90 TWIN T DUAL ADMITTANCE BRIDGE

G J Slack and T N W Graham

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1. INTRODUCTION

The RSRE XT-90 Twin T Dual Admittance Bridge, commonly called the Woods bridge after its designer, operates over the frequency range from 5 MHz to 200 MHz and is capable of measuring each component of an unknown admittance, within the range zero to 50 pF and/or zero to 50 mS. The main purpose of this memorandum is to describe both the developments carried out and measurement methods used on the bridge at RSRE in order to update the original report [1] and produce a reassessment of the bridge measurement uncertainty. A simpler explanation of the bridge circuit design has also been included (Sections 2, 3 and 4) since the quantity and intensity of information in [1] is usually found to be rather overwhelming; it should be stressed however that this explanation only supplements and does not replace the full report.

The bridge was designed over 30 years ago, at the then Aeronautical Inspection Directorate (A.I.D.) Harefield, Middlesex and was built by KGM Electronics Ltd. Since then the bridge has been used to provide top echelon standard measurements of admittance in coaxial line and is at present located in L4 Division (r.f. and microwave standards) at RSRE. Work on two other similar bridges is also continuing at the National Physical Laboratory, Teddington [2] and the Electrical Quality-Assurance Directorate.

Since frequent reference will be made to [1] in this memorandum the designation (A.I.D.) will be used for convenience.

2. BASIC BRIDGE CIRCUIT

The basic circuit configuration of the VHF Twin T Admittance Bridge is shown in fig. 1 and can be seen to consist of two separately identifiable T-networks (hence the name Twin-T) connected in parallel between an R.F. source and detector (see also fig. 2). The two T-networks will be referred to as the A arm and B arm networks for convenience. Each of the bridge shunt elements (impedances \( Z_3 \) and \( Z_6 \)) includes a precision standard variable capacitor but the remaining four impedances (series elements) are of fixed value. Before a device under test (DUT) is connected for measurement, these standard capacitors are adjusted to obtain a null at the detector output, so ensuring zero-transmission between the source and detector and leaving the bridge in an initial referenced state. After a DUT has been connected in parallel with \( Z_6 \), the bridge is rebalanced by adjusting the standard capacitors to re-establish this zero transmission condition, so that the measured value for the DUT is calculated from the observed change in each capacitor.

The components chosen for the basic circuit are shown in fig. 3, where \( C_1 \) corresponds to \( Z_1 \) in fig. 1, \( L_a, g_a \) and \( C_3 \) (precision variable) to \( Z_3 \), \( C_2 \) to \( Z_2 \), \( C_3 \) to \( Z_4 \), \( L_b \), \( g_b \) and \( C_b \) (precision variable) to \( Z_6 \) and finally \( G \) to \( Z_5 \). The elements \( g_a \) and \( g_b \) represent shunt losses associated mainly with the inductors \( L_a \) and \( L_b \) respectively. It should be noted that although the principal measurement terminals are on the B arm, a similar set exists on the A arm mainly for the purpose of measuring unwanted circuit residuals to be discussed later.

In order to determine the admittance of the DUT from the changes in \( C_a \) and \( C_b \), and analysis of the bridge zero transmission balance conditions is required. From fig. 4 the transfer impedance \( Z_{ta} \) of the A arm, defined as the input voltage \( V_{in} \) divided by the output current with the output terminals short circuited, is given by

\[
Z_{ta} = Z_1 + Z_2 + Z_1 Z_2/Z_3
\]  

(1)
Similarly for the B arm

\[ Z_{tb} = Z_4 + Z_5 + Z_4 Z_5 / Z_6 \]  \hspace{1cm} (2)

The condition for zero transmission through the bridge circuit is then given by

\[ Z_{ta} + Z_{tb} = 0 \]  \hspace{1cm} (3)

Applying (1), (2) and (3) to the circuit of fig. 3 gives the following balance equations

\[ \frac{1}{\omega^2 L_a} = C_a + C_1 + C_2 + \frac{C_1 C_2}{C_3} \left( 1 + \frac{g_b}{g_s} \right) \]  \hspace{1cm} (4)

\[ \frac{1}{\omega^2 L_b} = C_b + C_3 \left( 1 - \frac{g_a g_s}{\omega^2 C_1 C_2} \right) \]  \hspace{1cm} (5)

where \( g_a = g_b < g_s \)

Using both (4) and (5) for the cases of initial bridge balance (i.e. measurement terminals open circuited) and then final balance (with DUT connected) gives the following expressions (the balance difference equations) for the unknown DUT admittance \( Y_x = C_x + j \omega C_x \) in terms of \( C_a \) and \( C_b \).

**DUT connected to B arm measurement terminals**

\[ C_{xb} = \frac{C_3 g_a}{C_1 C_2} (C_{a1} - C_{a2}) \]  \hspace{1cm} (6)

\[ C_{xb} = (C_{b1} - C_{b2}) \]  \hspace{1cm} (7)

where \( C_{a1} \) and \( C_{b1} \) are the initial values, and \( C_{a2} \) and \( C_{b2} \) are the final values of \( C_a \) and \( C_b \), and subscript b on both \( G_x \) and \( C_x \) relates to measurements made on the B arm.

**DUT connected to A arm measurement terminals**

\[ C_{xa} = \omega^2 \frac{R_s C_1 C_2}{C_3} (C_{b1} - C_{b2}) \]  \hspace{1cm} (8)

\[ C_{xa} = (C_{a1} - C_{a2}) \]  \hspace{1cm} (9)

where subscript a on both \( G_x \) and \( C_x \) relates to measurements made on the A arm, and \( R_s = 1/g_s \).

Both the derivation of these equations and an explanation for the particular choice of components shown in fig. 3 are given in [A.I.D.] sections 2.6 - 5.1, pages 3-9.
In order to gain a better understanding of how the bridge circuit operates it is worth considering equations (4) to (9) further. An examination of equations (6) and (7) shows that for an unknown admittance \( Y_{xb} \) connected to the B arm measurement terminals, the capacitive element of that admittance, \( C_{xb} \), causes a change only in the B arm capacitor \( C_b \) (i.e. the same arm) whereas the conductive element, \( G_{xb} \), changes only the A arm capacitor \( C_a \) (i.e. the opposite arm). A similar situation exists for measurements made on the A arm terminals. This lack of interaction between the two precision variable capacitors is an important feature of the bridge circuit and enables rapid balancing to be achieved during measurement. The simple forms of both (7) and (9) are logical since any susceptance (i.e. positive or negative capacitance) connected across the A or B arm terminals can be directly compensated for by altering \( C_a \) or \( C_b \) respectively. The situation for conductance is not easily seen, but notice that equations (6) and (8) can be derived simply from (4) and (5) in that placing a conductance across either terminal will increase \( g_a \) or \( g_b \) accordingly. Because the measured admittance of the DUT is expressed in terms of \( C_1, C_2, C_3, G_s \) and changes in \( C_a \) and \( C_b \), these are referred to as the dependent circuit elements and are required to have long term stability in order to maintain calibration. \( L_a \) and \( L_b \), which do not appear in equations (6) to (9), are called the independent elements and need only have short term stability between initial and final bridge balance. In practice, \( L_a \) and \( L_b \) take the form of a set of plug-in inductors, an appropriate pair being chosen and connected to the bridge for the particular frequency of measurement.

The balance equations (4) and (5) both have the form:

\[
\frac{1}{\omega^2L} = C
\]

which suggests a resonant type behaviour. It can be shown ([A.I.D.] section 9.2, pages 27-29) that for bridge balance at frequencies greater than approximately 50 MHz, transmission through both A and B arms is near resonant maximum even though the detector output is nulled. This explains the very high level of discrimination i.e. 2 parts in \( 10^5 \) obtained from this circuit. However for lower frequencies the term \( g_a G_s/\omega^2 C_1 C_2 \) in (5) becomes greater than unity requiring larger values of \( C_b \) to maintain balance. This causes the B arm network to depart from near resonant transmission at bridge balance and is accompanied by a significant deterioration in its discrimination. At frequencies below 5 MHz \( L_b \) is no longer required (making \( L_b = \infty \)), \( C_b \) is made very large (\( \approx 400 \) pF) with the use of external capacitors and the B arm discrimination drops to 0.05 pF or 1 part in \( 10^3 \). The A arm meanwhile is unaffected in this way and maintains its full discrimination (2 parts in \( 10^5 \)) over the whole frequency range.

The measurement range of the bridge can be calculated from the component values given in fig. 3. For the B arm terminals the capacitance range is 50 pF (equation (7)) and the conductance range is zero to 50 mS from equation (6). For the A arm terminals the capacitance range is 25 pF (equation (9)) whereas the conductance range is frequency dependent, covering zero to 40 mS at 200 MHz and yet only zero to 0.1 mS at 10 MHz (equation (8)). The B arm terminals are therefore more commonly used because of their greater measurement range although the A arm does offer significantly greater discrimination when measuring low loss devices at low frequencies (e.g. dielectric loss measurements).
3. PRACTICAL BRIDGE CIRCUIT

The practical circuit is more complex than as shown in fig. 3, firstly because of certain bridge operational requirements and secondly in order to allow for the effects of component residuals. A first stage towards describing the full circuit is shown in fig. 5 which has been drawn in keeping with [A.I.D.] but still has the same form as fig. 3. The bridge shunt elements now include inductors \( l_{a1}, l_{a2}, l_{b1}, l_{b2} \) and capacitors \( C_{at1}, C_{bt}, C_{b1t} \); the series capacitors \( C_1, C_2 \) and \( C_3 \) each have associated residual inductance and \( G_s \) is shunted by residual capacitance \( C_s \).

When performing a measurement using the B arm terminals the auxiliary variable capacitors \( C_{b1t} \) (coarse adjustment) and \( C_{b1t} \) (fine adjustment) are used to obtain initial bridge balance instead of the precision capacitor \( C_b \). This enables \( C_b \) to be set to a convenient initial reference position and allows its full capacitance change, in either a positive or negative direction to be used if required during measurement. Variable capacitors \( C_{b1t} \) and \( C_{b1t} \) are in fact used in two different circuit configurations; below 180 MHz the inductors \( l_{b1} \) and \( l_{b2} \) have negligible effect therefore placing these capacitors directly in parallel with \( L_b \) and \( C_b \). Above 180 MHz however it is difficult to make \( L_b \) small enough to suit the balance requirement of equation (5) and so the circuit comprising \( C_{b1t}, C_{b1t}, l_{b1} \) and \( l_{b2} \) is used with \( L_b \) removed altogether. This circuit is operated above series resonance in order to offer a small positive (inductive) reactance, although care must be taken to avoid actual series resonance and the resulting false balance condition. Operation of the A arm is similar to that explained in this paragraph for the B arm.

It can be seen from equations (6) and (8) that the series capacitors \( C_1, C_2 \) and \( C_3 \) appear only in the form \( C_3/C_1C_2 \). If the value of this term can be maintained relatively constant then errors in the individual capacitance values, which result from residual series inductance and can be several per cent at 200 MHz, are of no concern. By choosing \( C_1 = C_2 = \frac{1}{2} C_3 \) and arranging for the residual inductance of \( C_1 \) and \( C_2 \) (\( l_{k1} + l \) in fig. 5) to be slightly greater than that of \( C_3 \) (\( l_{k1} \)) then it can be shown ([A.I.D.] section 12.2 pages 33-34) that a compensation effect occurs and the resulting variation of \( C_3/C_1C_2 \) from its zero frequency value is less than 3 parts in \( 10^4 \) at 200 MHz.

In order to correct the errors introduced by the residual shunt capacitance \( C_s \), the bridge circuit has to be reanalysed treating the resistor \( G_s \) now as an admittance \( G_s + j\omega C_s \). The resulting new balance difference equations are given below.

**DUT connected to B arm measurement terminals**

\[
\begin{align*}
G_{xb} &= \frac{C_3G_s}{C_1C_2} \Delta C_a \\
C_{xb} &= \frac{\Delta C_b + G_{xb}}{C_s} \cdot C_s
\end{align*}
\]

where \( \Delta C_a = (C_{a1} - C_{a2}) \) and \( \Delta C_b = (C_{b1} - C_{b2}) \)
DUT connected to A arm measurement terminals

\[ G_{xa} = \frac{-\omega^2 C_1 C_2}{C_3 G_s \left( 1 + \frac{\omega^2 G_s^2}{G_s^2} \right) \Delta C_b} \]  

(13)

\[ C_{xa} = \Delta C_a - \frac{G_{xa} C_s}{G_s} \]  

(14)

Equation (11) shows that \( C_{xb} \) is unaffected by introducing \( G_s \) (i.e. equations (6) and (11) are identical) whereas for conductance measurements across the A arm (equation (13)), \( R_s \) in equation (8) has been replaced by the term \( 1/G_s \) \((1 + \omega^2 C_s^2/G_s^2)\) namely the equivalent series resistance of the admittance \( G_s + j\omega C_s \). Equations (12) and (14) show that \( C_{xb} \) and \( C_{xa} \) both now include a term in \( C_s \) similar in magnitude but of opposite sign. This feature enables \( C_s \) to be determined as part of the calibration procedure to be discussed later. More information on this practical circuit as well as a detailed description of the mechanical design of the Twin T bridge are given in [A.I.D.] sections 5.2-6.5 pages 9-17. It should be noted that each bridge terminal has a coaxial form for reasons explained in [A.I.D.] section 2.5 pages 2-3 and that on the RSRE bridge a terminal adaptor has been added (to be described later) in order to create a measurement plane in terms of the now common GR900 precision coaxial connector.

The full equivalent circuit as originally derived by Woods is shown in fig. 6 where the residual inductances associated with the series capacitors \( C_1 \), \( C_2 \) and \( C_3 \) have not been included since they do not appear in the final circuit balance difference equations; their intended effect being to maintain the term \( C_3/C_1C_2 \) virtually frequency independent as explained earlier. The circuit contains a further number of residual terms as listed below.

(i) The common junction equivalent inductances \( L_j \) and \( L_k \) (both arms)

If the junction between three or more elements has finite area then a common junction impedance exists and the elements cannot be modelled as being connected together at a single point. To allow for this effect at junctions 1 and 2 in fig. 5 the inductances \( L_j \) and \( L_k \) have been added, their corresponding values being the same for each arm as a result of bridge symmetry. As shown in fig. 6, the B arm nodes adjacent to \( L_j \) and \( L_k \) have been termed \( \bar{A} \), \( \bar{B} \) and B (expressed as 1-bar, 2-bar and 3-bar points respectively) both for future reference and to remain in keeping with [A.I.D.]. A similar form of notation is also used on the A arm.

Further information on the equivalent circuit of these junctions, leading to the choice of \( L_j \) and \( L_k \), is given in [A.I.D.] section 15.6.1-15.6.3 pages 59-60.

(ii) The common and mutual earth junction impedances

The principle of common junction impedance referred to in (i) above will also apply to the earth junction E. In fact the full equivalent circuit of the bridge is more complex than as shown in fig. 6, taking the form of fig. 7 (where the shunt elements have been diagrammatically
simplified for convenience and the bridge resistor has been redrawn to expose its residual inductance Lg). The bridge has been designed such that a mutual impedance Z_m exists between each common earth junction inductance Lg and one of the residual inductances associated with the bridge series elements C_1, C_2, C_3 and G_s. By choosing the magnitude of Z_m to cancel out each Lg element, an effective earth junction is created at a single point thus making the circuit of fig. 6 valid. The residual inductance of each series element will also be reduced by the same amount. The equivalent shunt capacitance G_s associated with G_s in fig. 5 has however been renamed C_s in fig. 6 in order to emphasize the influence of Z_m on Lg. More information on this general topic can be found in [A.I.D.] section 12.4.1 page 36.

(iii) Residuals associated with the standard capacitors C_a and C_b

Each bridge standard capacitor has associated residuals in the form of one fixed and one variable inductance and one loss resistor i.e. L_{OB}, L_{CB} and R_{OB} respectively for the B arm capacitor.

(iv) Bridge coaxial terminals

Each bridge coaxial terminal section is modelled in terms of the equivalent lumped circuit components L_t and C_t. Note that on the RSRE bridge these have to be augmented to allow for the adaptor to GR900 coaxial connector (to be described later).

(v) Terminal fringe capacitance C_f (both arms)

At initial balance each terminal is left open circuited in order to reference the bridge to a nominally zero admittance. The resulting distortion of the electric field at this open ended coaxial measurement plane can be represented by the equivalent lumped capacitance C_f.

4. FINAL BALANCE DIFFERENCE EQUATIONS AS DERIVED BY WOODS

A general analysis of the circuit in fig. 6 is required in order to derive a relationship between the unknown admittance of a DUT connected at the measurement terminals and the observed changes in C_a and C_b. This full analysis commences in [A.I.D.] sections 12.4.2 to 14.3 pages 37-49, where the balance difference conditions at the bridge 3-bar points are established, followed by [A.I.D.] sections 15.6.4 to 16.4 pages 60-64, where the necessary conditions are referred to the 1-bar points producing the 1-bar balance difference equations given below.

DUT connected to the B arm measurement terminals

$$G_{xb} = \frac{G_a G_s C_3}{c_1 c_2} \Delta \alpha - \frac{1}{w^2} - \frac{1}{w^2} \left( \frac{2 \alpha_1 - \alpha_2 + \alpha_3 - \alpha_4}{c_1 c_2} \right) - \frac{w^2 L_k (\Delta \alpha + C_3)}{2(L_j + L_k)}$$

(15)
\[
\frac{C_{xb}}{C_s} = \Delta C_b + \frac{C_{xb}}{C_s} C_{sg} + (L_j + L_k) \frac{C_{xb}^2}{C_s} + 2(L_j + L_k) \frac{R_b C_{xb}}{C_s} - 2L_k R_s C_{xb} \\
\frac{R_b C_{xb}}{C_s} - \frac{R_{oa} C_3}{R_s} - \frac{C_{a1}^2}{C_{a2}^2} + C_f
\]

(16)

where \( C_5 = 2(C_1 + C_2 + \frac{C_1 C_2}{C_3} - C_3) \) is 5 pF.

DUT connected to the A arm measurement terminals

\[
\frac{C_{xa}}{C_s} = -\frac{C_{1a} C_2}{C_3} \Delta T_a \left(1 + \frac{\omega^2 L_j}{C_3} \left(2C_{a2} C_{b1} - C_{a1} + C_{b2} + C_5\right) + \frac{L_k C_s^2}{C_3} \right) \\
+ 2(L_j + L_k) \left(\frac{C_3 G_s G_a}{C_1 C_2} + \frac{\omega^2 C_{1a} C_2 G_a}{C_3 G_a}\right) + \frac{R_{oa} C_{a1}^2}{C_{a2}^2} + C_f
\]

(17)

\[
\frac{C_{xa}}{C_s} = \frac{\Delta C_a}{C_s} - \frac{C_{xa}}{C_s} C_{sg} + \left(\frac{L_j + L_k}{C_3} \right) \frac{C_{xb}^2}{C_s} + 2(L_j + L_k) \frac{R_b C_{xa}}{C_s} - 2L_k R_b C_{xa} \\
- \frac{R_{oa} C_1 C_2}{G_s C_3} \left(\frac{C_{b2}^2}{C_{b1}^2} - \frac{C_{b1}^2}{C_{b2}^2}\right) + C_f
\]

(18)

These four equations, repeated here for convenience, correspond to |A.I.D.| equations (128), (129), (132) and (133) respectively. Note that the R.H.S. of equation (17), unlike its |A.I.D.| counterpart, has a negative sign associated with both its \( \Delta C_b \) terms; this maintains the definition \( \Delta C_b = (C_{b1} - C_{b2}) \) for measurements made on either terminal and avoids the confusion of having \( \Delta C_b = (C_{b1} - C_{b2}) \) for B arm terminal measurements, but \( \Delta C_b = (C_{b2} - C_{b1}) \) for those on the A arm, as is used in |A.I.D.|. The terms \( C_{xb} \) and \( C_{xb} \) above (or equally \( C_{xa} \) and \( C_{xa} \) for the A arm) represent changes in conductance and capacitance occurring at the 1-bar points between initial and final bridge balance. These changes then have to be transferred through the corresponding terminal section to give the final required \( C_X \) and \( C_X \) values of the unknown admittance \( Y_X \). The circuit elements \( C_{bt}, C_{bt1}, C_{b1}, C_{b2} \) and \( L_b \) (taking the B arm as an example), which are physically contained within the secondary block of the bridge, do not appear in equations (15) to (18) and are therefore independent elements. This does not apply to \( L_{cb} \) and \( L_{ob} \) because they are considered effectively part of \( C_b \).

In order for the bridge to make precision admittance measurements, values for the relevant components and residuals in fig. 6 need to be accurately determined. This forms the bridge calibration procedure which will be described in sections 7 and 8 of this report. It should be noted that for the RSRE bridge some of these procedures differ from those originally developed by Woods and have resulted in changes to the balance difference equations (15) to (18).
5. CAPACITANCE STANDARDS

Most of the Twin T bridge calibration procedures require the use of coaxial capacitance standards of the type described in [A.I.D.] section 8.1 pages 21-24. Further work at RSRE has resulted however in the following changes being made to the fundamental assessment and practical use of these standards.

(a) The zero frequency capacitance is calculated using the formula given below.

\[
C = \frac{0.24176}{\log_{10}(\frac{R}{R_1})} \text{ pF/cm at } 20^\circ \text{C, 760 mmHg (0^\circ \text{C}) and 50\% R.H.} \tag{19}
\]

where an allowance has been made for an air dielectric relative humidity of 50\% [3] which results in a capacitance value approximately 1 part in \(10^4\) higher than for the corresponding [A.I.D.] equation (9). Note that in this [A.I.D.] equation, the length L is expressed in centimetres not inches.

(b) The standard capacitors are measured at a frequency of 1 kHz using an A.F. inductive ratio capacitance bridge. This seems a more direct method than that in [A.I.D.] where the 4.5 MHz measurement is based on an A.F. calibration of \(C_a\) and \(C_b\) and assumes that at 4.5 MHz the effects of residuals in the latter will be negligible.

The capacitance of each standard has been calculated from equation (19), using mean physical dimensions each with an associated uncertainty of \(\pm 0.002 \mu\text{m}\), and is compared with the measured A.F. values in Table 1.

**TABLE 1**

CALCULATED AND MEASURED VALUES OF THE STANDARD CAPACITORS

<table>
<thead>
<tr>
<th>Capacitor</th>
<th>Calculated Capacitance (pF)</th>
<th>Measured Capacitance at 1 kHz (pF)</th>
<th>Difference (pF)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Uncertainty</td>
<td>Mean</td>
</tr>
<tr>
<td>(C_{10})</td>
<td>2.6211</td>
<td>(\pm 0.0020)</td>
<td>2.6230</td>
</tr>
<tr>
<td>(C_{11})</td>
<td>5.2088</td>
<td>(\pm 0.0037)</td>
<td>5.2108</td>
</tr>
<tr>
<td>(C_{12})</td>
<td>10.4521</td>
<td>(\pm 0.0071)</td>
<td>10.4602</td>
</tr>
<tr>
<td>(C_{13})</td>
<td>20.9072</td>
<td>(\pm 0.0140)</td>
<td>20.9238</td>
</tr>
</tbody>
</table>

Table 1 shows that the measured values have significantly smaller associated uncertainty limits and the R.F. capacitance calculations for each standard have therefore been based on these measured values. The slight discrepancy between the measured and calculated values, particularly for \(C_{12}\) and \(C_{13}\), suggests that further examination of the metrology procedure used with these capacitors is required.
(c) The capacitors are used with their outer conductors extended by at least twice their bore diameter as shown in fig. 8. This firstly prevents significant radiation of energy, which could interact with local objects such as the operator's hands and disturb the measurement, and secondly defines more precisely the fringe capacitance $C_f$, resulting in a calculated value $[4]$ of 0.398 pF.

For the sake of completeness it is worth mentioning here that [A.I.D.] equations (10) and (12) are used to calculate the change in R.F. capacitance produced at the bridge terminals on connecting a capacitance standard, and the equation at the top of [A.I.D.] page 75 is used to transform this change to the bridge 1 bar points to which the calibrations of the precision variable capacitors $C_a$ and $C_b$ are referred.

6. EQUIPMENT AND MEASUREMENT PROCEDURE USED WITH THE RSRE BRIDGE

The RSRE bridge is driven from a crystal controlled synthesised R.F. signal generator, used normally with an output power of approximately 8 mW, having a long term drift rate of $\pm$ 3 parts in $10^9$ per day and an absolute accuracy, assessed with a frequency counter referenced to an RSRE frequency standard, of better than $\pm$ 3 parts in $10^6$. Bridge balance is detected using a spectrum analyser which has a maximum sensitivity of $-117$ dBm ($= 300$ nV) and whose frequency range enables it to replace the three separate and somewhat cumbersome communications receivers used previously. Both the R.F. source and detector are connected to the bridge via semi-rigid cable and N type or TNC connectors in order to minimise R.F. leakage. To maintain system accuracy and stability the bridge is situated in a laboratory which is temperature controlled at $20^\circ C \pm 1^\circ C$ and humidity controlled to $50\% \pm 15\%$ R.H., the exact values being regularly checked by a calibrated thermometer and hygrometer respectively.

7. CALIBRATION PROCEDURES FOR THE SHUNT ELEMENTS

(i) Precision Variable Capacitors $C_a$ and $C_b$ (including $L_{oa}$, $L_{ca}$, $L_{ob}$, $L_{cb}$)

It can be seen from [A.I.D.] Table 6 page 74 that the measured values for standard capacitor $C_{a10}$ vary with initial micrometer setting, deviations from the mean value being more than 1% for $C_b$ and 0.5% for $C_a$. In an attempt to smooth out these large variations a new calibration procedure was developed. This is very comprehensive, the final calibration for each frequency being produced from parts of several separate calibration runs. A large computer program has been developed to achieve this and a laboratory report on how to form a calibration using this program is being produced for future use.

In order to give some idea of the steps involved, the main stages in calibrating one capacitor at one frequency are described below taking $C_b$ as the example.

(a) The bridge is balanced with $C_b$ set to one extreme end of its micrometer operating range.

(b) Capacitor $C_{a10}$ is connected to the bridge terminals producing a known change of capacitance at the 1 bar points.

(c) The bridge is rebalanced using $C_a$ and $C_b$, and the $C_b$ micrometer position, $\theta_p$, is noted.
(d) \(C_{s10}\) is removed and the bridge rebalanced without altering \(C_b\).

(e) Stages (b) to (d) are repeated to produce a set of contiguous points covering the whole operating range of \(C_b\).

(f) The set of points obtained so far, which represent known changes of 1 bar capacitance against micrometer position, are then referred to an absolute value by setting \(C_b = 3.5\) pF at \(a\) = 0.825" (\(C_a = 3.0\) pF at \(a\) = 0.825" for the A arm). These values are based on Woods original estimates as derived in [A.I.D.] section 7.1 pages 17-20, the difference between the R.F. and A.F. capacitance values at \(a\) or \(b\) = 0.825" being significant.

(g) The calibration is completed by using cubic spline interpolation techniques to calculate the 1 bar capacitances required at intermediate micrometer positions.

Measurements of \(C_{s10}\) made over the 0.1" to 0.7" range of both \(C_a\) and \(C_b\) using the new calibration method gave maximum variations from the mean value of less than 0.1% of 100 MHz and 0.15% at 200 MHz which are adequately small for the accuracy requirements of the bridge. It should be noted in connection with section (f) above that absolute 1-bar capacitance values, in contrast to changes in them, are not required to any great precision since they only appear in residual bridge correction terms (ie. the term \(\omega L_j\) in equation (15) of this report).

In order to assess the accuracy of the new calibration curves the standard capacitors \(C_{s11}, C_{s12}\) and \(C_{s13}\) (having nominal A.F. values of 5 pF, 10 pF and 20 pF respectively) were measured on the bridge at various initial positions on \(C_a\) and \(C_b\) and the results compared with the calculated values for the standards. Table 2 gives the assigned uncertainty figures resulting from this exercise.

**TABLE 2**

**UNCERTAINTY OF MEASURING CAPACITORS \(C_a\) AND \(C_b\)**

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Uncertainty in (C_a) and (C_b) at 1 bar points (rectangular distribution semi-range)</th>
<th>Range of (C_a) and (C_b) over which uncertainty figure is valid</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>(\pm 0.10% \pm 0.003) pF</td>
<td>0.1&quot; to 0.7&quot;</td>
</tr>
<tr>
<td>100</td>
<td>(\pm 0.15% \pm 0.003) pF</td>
<td>0.1&quot; to 0.7&quot;</td>
</tr>
<tr>
<td>200</td>
<td>(\pm 0.3% \pm 0.003) pF</td>
<td>0.1&quot; to 0.7&quot;</td>
</tr>
</tbody>
</table>
(ii) Inductance $L_t$ and Capacitance $C_t$ of the Bridge Terminals

These elements are measured using the technique described in [A.I.D.] section 18.4 pages 75-77 from which the values $L_t = 2750 \mu H$ and $C_t = 46 \mu F$, both having an associated measurement uncertainty of ±0.5%, are obtained.

(iii) Common Junction Equivalent Inductances $L_j$ and $L_k$ (both arms)

These residuals are measured using the methods described in [A.I.D.] sections 18.6 and 18.7 pages 77-83 from which the values $L_j = 324 \mu H$ and $L_k = 276 \mu H$ are obtained. The associated uncertainty limits have however been increased to ±5% for both terms based on measurements made at RSRE by R.E. Spinney.

(iv) Shunt Conductances $g_a$ and $g_b$

In the analysis of [A.I.D.] Appendix XVIII, the term $+2w^2L_jg_bC_b$ (ie. the last term of the equation above equation (A5) is neglected as being insignificant although, as pointed out in a private communication from R.E. Spinney, this is not true for a $g_b$ value in the order, as proves to be the case, of 1 mS or greater. This term, with appropriate sign, has therefore to be added to equation (15) of this report (and a similar term for the A arm in equation (17)). Also, none of the terms containing $g_a$ and $g_b$ in equations (15) to (18) have been neglected as suggested in the [A.I.D.] paragraph above equation (130).

The method developed by R.E. Spinney at RSRE to measure $g_a$ and $g_b$ is explained in Appendix 1, resulting in an associated uncertainty for both terms of ±10%. Note that the value of $g_a$ or $g_b$ chosen is that when the bridge is initially balanced ie. without any device connected to the measurement terminal.

(v) Residuals $R_{oa}$ and $R_{ob}$ of the Precision Variable Capacitors $C_a$ and $C_b$

In the method described in [A.I.D.] section 18.8 pages 83-84 the effect of the conductances $g_a$ and $g_b$ have been ignored. To allow for this, Appendix 2 describes a modification to this method developed by R.E. Spinney at RSRE which results in curves of equivalent conductance $G_{ka}$ and $G_{kb}$, corresponding to $R_{oa}$ and $R_{ob}$ respectively, being derived as a function of the $C_a$ and $C_b$ related 1 bar capacitance values. This affects equations (15) to (18) by replacing all $R_o$ terms with $G_k$ terms based on the relationship

$$G_k = \frac{w^2R_o C^2}{2}$$

The measurement uncertainty associated with both $G_{ka}$ and $G_{kb}$ is ±10%.

(vi) Bridge Terminal Adaptors

Each bridge terminal is fitted with a brass adaptor, shown in cross section in fig. 9, to enable measurements to be made using the GR900 connector. Each device is characterised electrically by calculating the transmission line impedance of sections 1 and 2 as well as the fringing and step capacitances $C_{fa}$, $C_{a1}$ and $C_{a2}$ [4] based on mechanical measurements made in the RSRE Metrology Laboratory. An allowance is also made for skin
effect in the conductors by including a reciprocal velocity factor [3] calculated from theoretical conductivity values.

The procedure used to calculate the admittance of a DUT connected at the measurement plane allowing for the adaptor is given below.

(a) The susceptance of the adaptor fringe capacitance \( C_{fa} \), which exists at initial bridge balance (ie the DUT not yet connected) is transformed through adaptor sections 1 and 2 and then the bridge terminals, each of the three stages represented using standard transmission line formulae and an allowance being made for the step capacitances \( C_{s1} \) and \( C_{s2} \).

(b) The change in the 1 bar admittance value resulting from the connection of the DUT and calculated using the 1 bar balance difference equations, is then added to the admittance from stage (a) above.

(c) The admittance from stage (b) is transformed back through the bridge terminals and adaptor sections in a reverse process to stage (a), the resulting admittance being the actual value of the DUT itself.

The use of this procedure means that the term \( C_f \) in equations (16) and (18) of this report is no longer required.

8. CALIBRATION PROCEDURES FOR THE SERIES ELEMENTS

(a) Bridge Resistor R.F. Conductance Term, Zeta

This residual is measured using the technique of [A.I.D.] section 18.9 pages 84-85, and allows for the variation of the R.F. conductance \( G_s \) of the bridge resistor in terms of the relationship.

\[
G_s = G_{ss}(1 + \text{Zeta})
\]

where \( G_{ss} \) is the D.C. conductance of the resistor. The measurement method exploits the fact that for measurements made on the B arm the \( G_s \) term appears in the numerator of the balance difference equation unlike for the A arm where it appears in the denominator, (see equations (15) and (17)). If the same conductance is measured on both arms therefore, dividing one conductance result by the other will give the term \((1 + \text{Zeta})^2\) directly.

The uncertainty limits associated with Zeta are estimated to be ± 0.001 at 100 MHz and ± 0.0015 at 200 MHz.

(b) The Bridge Resistor Shunt Capacitance \( C_{sg} \)

This component is measured using the method described in [A.I.D.] section 18.5 pages 77-78 although it has been found simpler to find the term \( C_{sg}/G_s \) as a whole and not to separately isolate \( C_{sg} \). The measurement procedure is similar to that used in section (a) above except that now the capacitance values obtained from each arm are subtracted from each other.

This estimates the term \( 2G_xC_{sg}/G_s \) from which, knowing an approximate value for \( \overline{G_x} \), \( C_{sg}/G_s \) can be found.
The uncertainty limits associated with $C_s/g_s$ are estimated to be
$\pm 0.0005$ at 15 MHz, $\pm 0.001$ at 100 MHz and $\pm 0.0015$ at 200 MHz.

9. FINAL BALANCE DIFFERENCE EQUATIONS USED WITH THE RSRE TWIN T BRIDGE

The final balance difference equations at the 1 bar point, which differ
slightly from those in equations (15) to (18) for reasons explained in section
7 of this report, are given below.

**DUT connected to the B arm measurement terminals**

$$
\overline{e}_{xb} = \frac{G_s C_3}{C_1 C_2} \delta \overline{E}_a \left[ 1 + w^2 L_j (2 \overline{c}_{b2} - \overline{c}_{a2} - C_5) - \nu^2 L_k C_5 \right] \\
-2(L_j + L_k) \left( \frac{C_3 G_s}{C_1 C_2} + \frac{w^2 C_1 C_2 g_b}{C_3 G_s} \right) + G_{kb1} - G_{kb2} - 2 \nu^2 L_j g_b \overline{D}_b 
$$

$(21)$

$$
\overline{e}_{xb} = \delta \overline{E}_b + \frac{C_{ss}}{G_s} + (L_j + L_k) \overline{E}_{xb} \overline{c}_{xb}^2 + 2(L_j + L_k) g_b \overline{E}_{xb} - 2 L_k g_b \overline{E}_{xb} \\
- \frac{G_s C_3}{w^2 C_1 C_2} \left( 1 + \frac{w^2 G_s}{C_2} \right) (G_{kal} - G_{ka2}) 
$$

$(22)$

**DUT connected to the A arm measurement terminals**

$$
\overline{e}_{xa} = -\frac{w^2 C_1 C_2}{G_s C_3} \left( \frac{1}{1 + \frac{w^2 C_{ss}}{G_s}} \right) \delta \overline{E}_a \left[ 1 + w^2 L_j (2 \overline{c}_{b2} - \overline{c}_{a2} - C_5) + w^2 L_k (C_5 - \overline{c}_b) \right] \\
+2(L_j + L_k) \left( \frac{C_3 G_s}{C_1 C_2} + \frac{w^2 C_1 C_2 g_b}{C_3 G_s} \right) + G_{kb1} - G_{ka2} - 2 \nu^2 L_j g_a \overline{D}_a 
$$

$(23)$

$$
\overline{e}_{xa} = \delta \overline{E}_a - \frac{C_{ss}}{G_s} + (L_j + L_k) \overline{E}_{xa} \overline{c}_{xa}^2 + 2(L_j + L_k) g_a \overline{E}_{xa} - 2 L_k g_a \overline{E}_{xa} \\
- \frac{C_1 C_2}{G_s C_3} (G_{kb2} - G_{kb1}) 
$$

$(24)$

When comparing equations (15) to (18) with equations (21) to (24) note that,
in the latter, the following relationship has been used.

$$
R_s = \frac{1}{G_s} \left( 1 + \frac{G_s}{C_{ss} G_s} \right)^2 \frac{2}{G_s^2} 
$$

where $G_s = G_{ss}(1 + \text{Zeta})$ and $G_{ss}$ is the D.C. conductance of $G_s$. 

14
10. ASSESSMENT OF BRIDGE MEASUREMENT UNCERTAINTY

In order to assess the overall bridge measurement uncertainty a computer program was written. This calculates the admittance of any measured D.U.T., ten times, each time increasing the value of one of the components of uncertainty, listed in Table 3, by its own associated uncertainty limit, and then returning its value to the nominal one, ready for the next calculation. The ten admittances obtained are then compared with their nominal value giving a set of percentage variations, each representing the effect of the rectangular distribution semi-range variation of the associated component. The 95% confidence limit of measurement uncertainty is then calculated [5] for both the conductive and susceptible components of the D.U.T. admittance.

TABLE 3
LIST OF COMPONENTS OF UNCERTAINTY CONTRIBUTING TOWARDS THE OVERALL BRIDGE MEASUREMENT UNCERTAINTY

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Associated limits (Semi-range of rectangular dist.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision Variable Capacitors $C_a$ and $C_b$</td>
<td>± $0.1% \pm 0.003$ pF at 15 MHz</td>
</tr>
<tr>
<td>Inductance of bridge terminals $L_t$</td>
<td>± $0.15% \pm 0.003$ pF at 100 MHz</td>
</tr>
<tr>
<td>Capacitance of bridge terminals $C_t$</td>
<td>± $0.30% \pm 0.003$ pF at 200 MHz</td>
</tr>
<tr>
<td>Common junction equivalent inductances $L_j$, $L_k$</td>
<td>± $0.5%$</td>
</tr>
<tr>
<td>Shunt conductances $g_a$, $g_b$</td>
<td>± $10.0%$</td>
</tr>
<tr>
<td>Conductances $G_{ka}$, $G_{kb}$</td>
<td>± $10.0%$</td>
</tr>
<tr>
<td>Bridge r.f. conductance term, Zeta</td>
<td>Zero at 15 MHz</td>
</tr>
<tr>
<td>$C_{sg}/C_s$</td>
<td>± $0.0015$ at 200 MHz</td>
</tr>
<tr>
<td></td>
<td>± $0.0005$ at 15 MHz</td>
</tr>
<tr>
<td></td>
<td>± $0.001$ at 100 MHz</td>
</tr>
<tr>
<td></td>
<td>± $0.0015$ at 200 MHz</td>
</tr>
</tbody>
</table>

As mentioned in the introduction, the admittance of unknowns is measured in two parts; the conductance $G$, over a range from 0 to 50 mS, and capacitance $C_s$, over a range from 0 to 50 pF.

Standards measured on the bridge over the last eleven years fall into four categories as listed below.

(a) One-port capacitors. 1 to 20 pF
(b) One-port resistors. 33 ohms to 100 ohms
(c) Two-port resistors. 20 ohms to 1000 ohms
(One port terminated in a short circuit)

(d) Two-port resistors. 0.5 ohms to 10 ohms
(One port terminated in a 50 ohm load)

The random component of measurement uncertainty is negligible with GR900 connectors fitted to the bridge providing the connector contact faces are clean and great care is taken to avoid producing mechanical vibrations during the connect/disconnect cycle (including tightening the connector to the specified torque of 1.4 Nm using a torque spanner). The effect of mechanical vibrations on the bridge is to initiate mechanical relaxation on the shafts of the shunt capacitors C_{at} (60 pf), and C_{bt} (100 pf), which is sufficient to partially unbalance the bridge. A measurement procedure to eliminate this effect has been introduced.

The overall bridge uncertainty is a function of both frequency and the measured values of G_x and C_x. The uncertainty in capacity is further complicated by cross-terms involving G_x (as can be seen from equations (22) and (24)) so that a statement of bridge uncertainty in the form of a single equation, is necessarily only approximate. However, by using the measured values of the complete range of standards measured over the past eleven years a single equation of uncertainty has been produced for each frequency which is accurate to ± 0.15% in most cases. This is shown in table 4.

**TABLE 4**

<table>
<thead>
<tr>
<th>Freq. MHz</th>
<th>Uncertainty in Conductance G_x mS. (95% CL)</th>
<th>Uncertainty in Capacitance C_x pf. (95% CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>± 0.15% ± 0.005</td>
<td>± 0.01% ± 0.001.G_x(mS) ± 0.003</td>
</tr>
<tr>
<td>100</td>
<td>± 0.25% ± 0.005</td>
<td>± 0.17% ± 0.0013.G_x(mS) ± 0.003</td>
</tr>
<tr>
<td>200</td>
<td>± 0.5% ± 0.005</td>
<td>± 0.3% ± 0.0018 G_x(mS) ± 0.003</td>
</tr>
</tbody>
</table>

In practice when standards are calibrated the individual uncertainty in G_x and C_x is calculated by a computer program from the measured values for each standard, but the complex form of the bridge balance equations together with the minimum values of systematic uncertainty components, leads to overall bridge uncertainty figures which are not only dependent on the minimum values of G_x and C_x, but also on their ratio.

A better appreciation of the significance of this effect can be obtained from Tables 5 and 6, which list the range of uncertainties associated with standards having values of G_x and C_x within the ranges shown.

For example, from Table 6, a 1 pf capacitor having negligible G_x, has an uncertainty in C_x, in the range 0.49% to 0.7% at 200 MHz, but from Table 5 a standard having G_x > 1 mS and C_x = 1 pf can have an uncertainty in C_x of 4.5%.
**TABLE 5**

TRUE BRIDGE OVERALL UNCERTAINTY FIGURES FOR ALL STANDARDS HAVING A MINIMUM CONDUCTANCE AND CAPACITY OF 1 mS AND 0.15 pF RESPECTIVELY

<table>
<thead>
<tr>
<th>CONDUCTANCE RANGE</th>
<th>UNCERTAINTY IN CONDUCTANCE % (95% CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15 MHz</td>
</tr>
<tr>
<td>1 - 2 mS</td>
<td>0.4 - 0.65</td>
</tr>
<tr>
<td>5 - 50 mS</td>
<td>0.17 - 0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CAPACITANCE RANGE</th>
<th>UNCERTAINTY IN CAPACITANCE % (95% CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15 MHz</td>
</tr>
<tr>
<td>0.15 - 1 pF</td>
<td>2 - 13</td>
</tr>
<tr>
<td>1 - 3 pF</td>
<td>0.3 - 2</td>
</tr>
<tr>
<td>&gt; 3 pF</td>
<td>0.24 - 1</td>
</tr>
</tbody>
</table>

**TABLE 6**

TRUE BRIDGE OVERALL UNCERTAINTY FIGURES FOR STANDARD CAPACITORS

<table>
<thead>
<tr>
<th>CAPACITANCE RANGE</th>
<th>UNCERTAINTY IN CAPACITY % (95% CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15 MHz</td>
</tr>
<tr>
<td>1 - 5 pF</td>
<td>0.21 - 0.45</td>
</tr>
<tr>
<td>&gt; 5 pF</td>
<td>0.16 - 0.21</td>
</tr>
</tbody>
</table>
11. CONCLUSIONS

Since the XT-90 Twin T bridge was first built, some thirty years ago, a number of modifications have been made, the most recent being those made at RSRE to the measurement and calibration procedures. In particular the introduction of computer derived calibrations, using curve fitting techniques, has given greater calibration accuracy and extended the useful operating range of the measuring capacitors.

An assessment of the bridge systematic measurement uncertainty has been performed resulting in a definitive uncertainty figure calculated in terms of the 95% confidence probability limit.

12. REFERENCES


APPENDIX 1 - METHOD FOR MEASURING THE SHUNT RESIDUALS $g_a$ and $g_b$

The initial conductances $g_a$ and $g_b$ of the Twin T bridge result mainly from losses in the top block section, the settings of the latter being determined by the initial positions of $C_a$ and $C_b$. The change of conductance with $C_a$ or $C_b$ can be measured on the bridge itself using the method described below (using the B arm as the example).

(a) Set $C_b = 0.8''$, $C_a = 0.4''$ and balance the bridge using the top block variable capacitances.

(b) Set $C_b = 0.7''$ and balance the bridge using $C_a$ and the B arm top block variable capacitances, taking care not to disturb the A arm top block controls. The change in $g_b$ is calculated from the change in $C_a$ position using the 1 bar balance difference equations.

(c) Reset $C_a = 0.4''$ and repeat stages (a) and (b) above but with the $C_b$ values reduced by 0.1''. This process is continued over the whole $C_b$ operating range producing a set of points representing changes in $g_b$ with $C_b$ position. Cubic spline techniques are then used to calculate intermediate $g_b$ values.

The total conductance associated with each arm is then measured using the resonance method described below, the B arm again being used as the example.

(a) The bridge is balanced in order to obtain the correct relationship between the top block setting and $C_b$.

(b) The input and output leads are unplugged and shorting plugs are inserted into the input and output sockets in order to electrically isolate the A and B arms from each other.

(c) If the B arm conductance is being measured, the bridge resistor is removed.

(d) The circuit of fig. 10 is set up where $C_1$ and $C_2$ are provided by specially adapted GR874 type connectors, one of which couples to the B arm bridge terminal, the other to the B arm junction block through the upper hole in the bridge front face.

(e) The attenuator is set to 13 dB and $C_b$ adjusted to give maximum response on the receiver.

(f) The attenuator setting is reduced to 10 dB and $C_b$ adjusted each side of the maximum to give the same receiver output as in stage (e).

(g) The required conductance is equal to $0.5 \times$ the total susceptance change in stage (f).

Now that $g_b$ is known for a particular setting of $C_b$, the curve of changes in $g_b$, obtained from the first experiment can be expressed in absolute terms.
THEORY OF RESONANCE METHOD

The circuit of fig. 10 is set up with $C_1 = C_2 = C$ where $C < 0.1 \text{ pF}$ in order to prevent significant loss being coupled in from either the source or the load. Also, for convenience, we define

$$X = -\frac{1}{\omega C}$$

and $Z = R + R+jX$

where $R = 50 \Omega$ is the impedance of both the source and load (attenuator).

$B$ is equal to the sum of the precision variable capacitor susceptance and the top block susceptance. From the transfer impedance of a T network.

$$V = \frac{R \cdot e}{2(R+jX) + (R+jX)^2(G+jB)}$$

Defining $D = 2(R+jX) + (R+jX)^2(G+jB)$ is the denominator of equation A1(1)

then

$$D = 2R+j2X + (R^2-X^2)G-2RXB-j(R^2-X^2)B + j2RXC$$

$$= 2R + (R^2-X^2)G-2RXB + j[B(R^2-X^2)+2RXG+2X]$$

$$\therefore D^2 = (2R+(R^2-X^2)G-2RXB)^2 + [2X+B(R^2-X^2)+2RXG]^2$$

$$= 4R^2+(R^2-X^2)G^2+4R^2X^2B^2+4R(R^2-X^2)G-8R^2XB-4RXB(R^2-X^2)G$$

$$+ 4X^2-(R^2-X^2)B^2+4R^2X^2G^2+4X(R^2-X^2)B+8RX^2G+4RXB(R^2-X^2)G$$

$$= 4(R^2+X^2)+(R^2-X^2)G^2+(R^2+X^2)B^2+2G(2R^3-2RX^2+4RX^2)+2B(-2X^3+2R^2X-4R^2X)$$

$$\therefore D^2 = (R^2-X^2)B^2-4BX(R^2+X^2)+(R^2+X^2)C^2+4GR(R^2+X^2)G+4(R^2+X^2)$$

$$= |Z|^2B^2-4BX|Z|^2+|Z|^2C^2+4GR|Z|^2+4|Z|^2$$

$|D|^2$ will be a minimum when $\frac{d|D|^2}{dB} = 0$ (ie a minimum because $|D|^2 \to$ when $B \to$)

Now $\frac{d|D|^2}{dB} = 2|Z|^4B - 4X|Z|^2$

$\therefore 2|Z|^4B = 4X|Z|^2$ for $\frac{d|D|^2}{dB} = 0$

ie when $B = \frac{2X}{|Z|^2}$

$\therefore$ Minimum value of $|D|^2 = 4X^2-8X^2+|Z|^2C^2+4GR|Z|^2+4|Z|^2$

$$= |Z|^2C^2+4GR|Z|^2+4|Z|^2-4X^2$$

A1-2
Now \( wC < 0.1 \text{ mS} \) i.e. \( |X| > 10 \text{ K} \) and \( R = 0.05 \text{ K} \).

\[ |z|^2 = X^2 \] with an error of less than 0.0025%.

\[ \text{Minimum value of } |D|^2 = |2G^2 + 4GR|^2 \]

Now \( |D|^2 \) will be equal to twice its minimum value (i.e. \( V = V_{max}/\sqrt{2} \)) when

\[ |z|^2 B^2 - 4BX|z|^2 + |z|^2 G^2 + 4GR|z|^2 = 2|z|^2 + 4GR|z|^2 \]

i.e. when \( |z|^2 B^2 - 4BX|z|^2 - |z|^2 G^2 - 4GR|z|^2 = 0 \)

i.e. \( B^2 - \frac{4BX}{|z|^2} - G^2 - \frac{4GR}{|z|^2} + \frac{4}{|z|^2} = 0 \)

\[
B = \frac{2X}{|z|^2} \pm \sqrt{\frac{4X^2}{|z|^2} + G^2 - \frac{4GR}{|z|^2}} - \frac{4}{|z|^2}
\]

\[
B = \frac{2X}{|z|^2} \pm G \sqrt{\frac{1 + 4R}{|z|^2 G}} \quad \text{since} \quad \frac{X^2}{|z|^2 G} = \frac{1}{|z|^2}
\]

\[ = \frac{2X}{|z|^2} \pm G(1 + a) \text{ where } a < \frac{0.001}{G} \]

For \( G \) of the order of 1 mS, which proves to be the case, \( a \) can be neglected.

\[ \therefore B = \frac{2X}{|z|^2} \pm G \]
APPENDIX 2 - MODIFIED METHOD FOR MEASURING RESIDUALS $R_{oa}$ and $R_{ob}$

When the conductances $g_a$ and $g_b$ are included in the method described in [A.I.D.] section 18.8 pages 83-84, equation (176) becomes (using the B arm as an example)

$$\bar{C}_x = \frac{w^2R_oC_x(C_x + C_L)}{(1 - w^2L/C_x)} - w^2R_{ob}(\bar{C}_{b1} - \bar{C}_{b2}) + 2w^2Lg_b(\bar{C}_{b1} - \bar{C}_{b2})$$

and the curve to be plotted is

$$\Delta C' = \Delta C - 2w^2Lg_b(\bar{C}_{b1} - \bar{C}_{b2})$$

against $\frac{C_b^2 - C_{b2}^2}{C_{b1} - C_{b2}}$ A2(1)

The results of this experiment showed that a constant $R_{ob}$ value exists for lower values of $C_b$ but that at the top end the losses increase more rapidly. Because [A.I.D.] equation (117) is no longer strictly valid for a variable $R_{ob}$, it was decided to derive a graph of equivalent conductance $G_{kb}$ against $C_b$ by using the relationship

$$\frac{C_x^2 - C_{b2}^2}{C_{b1} - C_{b2}} = \frac{C_x(2C_{b2} + C_x)}{C_x - C_{b1} - C_{b2}}$$

where $C_x$ is known from the bridge measurements. Then over the range where $R_{ob}$ is constant, $G_{kb} = w^2R_{ob}C_x^2$ from [A.I.D.] equation (117) and since $R_{ob}$ is now known, a parabola of $G_{kb}$ against $C_b$ can be drawn. To extend this curve, a point $(C_{b2}, G_{kb2})$ is taken at its top end, $(C_{b2} - C_{b1})$ is calculated from equation A2(2) and the conductance change between points $C_{b2}$ and $C_{b1}$ found using the original graph obtained from equation A2(1). This process is continued until the whole operating range of the capacitor $C_b$ has been covered.

The measurement is made at a frequency of 200 MHz and the value of $C_k$ at any other frequency calculated using the relationship derived below.

Assuming that $R_o$ varies with the square root of frequency then

$$R_{of} = R_{200}\sqrt{\frac{f}{200}}$$

A2(3)

where $R_{of}$ is the value of $R_o$ at frequency $f$ (MHz)

$$R_{200}$$

is the value of $R_o$ at frequency 200 MHz

Also $G_{kf} = (2\pi)^2 f^2 R_{of} C^2$ from equation (20) ....

A2(4)

where $G_{kf}$ is the value of $G_k$ at frequency $f$ (MHz).

Therefore from A2(3) and A2(4)

$$G_{kf} = (2\pi)^2 f^2 R_{200}\sqrt{\frac{f}{200}} C^2$$

A2(5)
But $G_{k200} = (2\pi)^2 200^2 R_{200} \overline{C}^2$ from A2(6) \[A2(6)\]

where $G_{k200}$ is the value of $G_k$ at frequency 200 MHz.

Therefore from A2(5) and A2(6)

$$G_{kf} = (2\pi)^2 f^2 \sqrt{\frac{f}{200}} \cdot \frac{G_{k200}}{(2\pi)^2 200^2}$$

$$= G_{k200} \left(\frac{f}{200}\right)^{\frac{5}{2}}$$ \[A2(7)\]
FIG 1 BASIC BRIDGE CIRCUIT CONFIGURATION

A arm T-network

---

B arm T-network

FIG 2 BASIC CIRCUIT AS CONNECTION OF TWO T-NETWORKS
\[ Z_{ta} = \frac{V_{IN}}{I_{OUT(S/C)}} = Z_1 \cdot Z_2 \cdot \frac{Z_1 Z_2}{Z_3} \]

**FIG 3** BASIC CIRCUIT COMPONENTS

**FIG 4** TRANSFER IMPEDANCE OF A ARM
FIG 7  EFFECT OF COMMON AND MUTUAL EARTH IMPEDANCES
FIG 8. REVISED CONFIGURATION OF STANDARD CAPACITORS
FIG. 10 CIRCUIT ARRANGEMENT FOR MEASURING SHUNT RESIDUALS

C1, C2

TWIN T BRIDGE

SCREEN

R = 50

SOURCE

VAR. ATTENUATOR RECEPTOR

G

B
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