We derive a general, nonlocal dispersion equation which describes the long wavelength limit of several interchange instabilities (viz., Rayleigh-Taylor, $E \times B$ gradient drift, and current convective). Analytical expressions for the growth rates of these modes are presented, both for the collisional and collisionless regimes. The scaling of these growth rates with wavenumber ($k$) and the Pedersen conductivity ratio ($M$) are explicitly stated.
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LONG WAVELENGTH LIMIT OF INTERCHANGE INSTABILITIES

I. INTRODUCTION

A considerable amount of DNA-sponsored research has been directed at understanding interchange instability processes in the ionosphere. This is an important topic since these instabilities can generate large-scale, long-lasting plasma structure in the ionosphere which is deleterious to radar and communications systems. The immediate concern is to understand structuring mechanisms associated with a HANE in order to obtain a predictive capability which could aid the operations of these systems in a nuclear environment. To achieve this goal, research has also been carried out to understand ionospheric structuring in a non-nuclear environment. This work involves the study of natural structuring processes which occur during equatorial spread F and in the high latitude ionosphere, as well as structuring processes associated with artificial plasma cloud releases. The underlying mechanism believed responsible for structure in all of these situations is an interchange instability, i.e., the interchange of high and low density plasma in the presence of an external force. The instabilities investigated to date are (1) the Rayleigh-Taylor instability (driven by a gravitational force), (2) the $E \times B$ gradient drift instability (driven by an ambient electric field or neutral wind), and (3) the current convective instability (driven by a current parallel to the magnetic field). The extensive research of these instabilities has entailed both theoretical and computational analyses. The theoretical studies have focussed on the fundamental linear and nonlinear behavior of the instabilities, while the computational studies have primarily dealt with the gross evolution of the plasma in the nonlinear regime. For reviews of this work see Ossakow (1979), Fejer and Kelley (1980), and Ossakow et al. (1982).

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A key feature of the structuring of ionospheric plasmas in both nuclear and non-nuclear environments is that large-scale structure develops. It has been observed experimentally and in computer simulations that the wavelengths of the irregularities that develop (i.e., striations) are generally comparable to the scale length of the density gradient in the plasma. However, the majority of theoretical analyses performed thus far have considered irregularity wavelengths much smaller than the density gradient scale length, i.e., local theory has been assumed which implies $kL \gg 1$ where $k$ is the wavenumber and $L$ is the scale length of the density gradient. Recently, the opposite limit has been considered by Huba and Zalesak (1983) in a study of the $E \times B$ instability and by Huba (1983) in a study of the current convective instability. The crux of these studies is that a nonlocal mode equation is derived and solved analytically for each instability in the long wavelength limit, i.e., $kL << 1$. Thus, they provide insight into the linear behavior of each instability in the important "large-scale" regime.

The purpose of this paper is to derive a single, general, nonlocal dispersion equation which describes the relevant interchange instabilities: Rayleigh-Taylor, $E \times B$ gradient drift, and current convective. The important equilibrium quantities considered in the derivation are gravity, an ambient electric field, a current parallel to the background magnetic field, and, of course, a density gradient. Simple analytic expressions of the growth rate for each instability in the long wavelength limit are presented in both the collisional ($v_{in} \gg \omega$) and collisionless ($\omega \gg v_{in}$) regimes, where $\omega$ is the wave frequency and $v_{in}$ is the ion-neutral collision frequency.
The organization of the paper is as follows. In the next section we derive the general nonlocal mode equation. In Section III we derive the nonlocal dispersion equation. In Section IV we present simple analytical expressions for the growth rates of the various interchange instabilities in the long wavelength limit. In the final section we present our conclusions.

II. DERIVATION OF MODE EQUATION

The plasma configuration and slab geometry used in the analysis are shown in Fig. 1a. The ambient magnetic field is in the $z$ direction ($\mathbf{B} = B_0 \hat{e}_z$), the ambient electric field is in the $y$ direction ($\mathbf{E} = E_0 \hat{e}_y$), the ambient current is in the $z$ direction ($\mathbf{J} = J_0 \hat{e}_z$), gravity is in the $x$ direction ($\mathbf{g} = g \hat{e}_x$) and the density is taken to be inhomogeneous in the $x$ direction ($\rho = n_0(x)$). A collisional plasma is assumed such that $v_{ei}/\Omega_e \ll 1$, $v_{en}/\Omega_e \ll 1$, $v_{ie}/\Omega_i \ll 1$, and $v_{in}/\Omega_i \ll 1$ where $\Omega_\alpha = eB_0/m_\alpha c$ is the cyclotron frequency of species $\alpha$, $v_{ei}$ refers to electron-ion collisions, $v_{en}$ to electron-neutral collisions, $v_{ie}$ to ion-electron collisions, and $v_{in}$ to ion-neutral collisions. Furthermore, we assume that $v_{en}/\Omega_e \ll v_{in}/\Omega_i$ in our analysis. Perturbed quantities are assumed to vary as $\delta p = \delta p(x) \exp \left[ i(k_y y + k_z z - \omega t) \right]$ and it is assumed that $\omega/\Omega_i \ll 1$, $k_0_i \ll 1$, and $k_z \ll k_y$, where $\rho_i$ is the mean ion Larmor radius. That is, we consider low frequency, long wavelength, field-aligned perturbations.

We neglect temperature effects. Finally, we consider only electrostatic oscillations and assume quasi-neutrality ($n_e = n_i$).
The fundamental equations used in the analysis are continuity, momentum transfer, and charge conservation, in the neutral frame of reference:

$$\frac{\partial n_\alpha}{\partial t} + \mathbf{v}_\alpha \cdot (n \mathbf{v}_\alpha) = 0 \quad (1)$$

$$0 = -\frac{e}{m_e} \left( \mathbf{E} + \frac{1}{c} \mathbf{v}_e \times \mathbf{B} \right) - \mathbf{v}_{en} \mathbf{v}_e - \mathbf{v}_{ei} (\mathbf{v}_e - \mathbf{v}_i) \quad (2)$$

$$\frac{\partial \mathbf{v}_i}{\partial t} = \frac{e}{m_i} \left( \mathbf{E} + \frac{1}{c} \mathbf{v}_i \times \mathbf{B} \right) - \mathbf{v}_{in} \mathbf{v}_i - \mathbf{v}_{ie} (\mathbf{v}_i - \mathbf{v}_e) + \mathbf{g} \quad (3)$$

$$\nabla \cdot \mathbf{J} = \nabla \cdot \left[ n_\alpha (\mathbf{v}_i - \mathbf{v}_e) \right] = 0 \quad (4)$$

where $\alpha$ denotes species (e: electrons, i: ions) and other variables have their usual meaning. Note that we have neglected electron inertia effects in Eq. (2) but have retained ion inertia effects in Eq. (3). However, we have assumed $\partial / \partial t \gg \mathbf{v}_i \cdot \nabla$ in Eq. (3) which is justified for the instabilities under consideration. The equilibrium drifts are given by

$$\mathbf{v}_{e0} = 0 \quad (5)$$

$$\mathbf{v}_{i0} = \frac{\mathbf{v}_{in}}{q_i} \frac{e}{m_i} \mathbf{e}_x + \left( -\frac{e}{q_i} + \frac{\mathbf{v}_{in} c \mathbf{E}_0}{\beta_0} \right) \mathbf{e}_y + \mathbf{v}_d \mathbf{e}_z \quad (6)$$

where we have taken $\mathbf{v}_x = \mathbf{v}_x - c \mathbf{E}_0 / \beta$ and have chosen the ions to carry the parallel current ($\mathbf{J} = e \mathbf{v}_d \mathbf{e}_z$).
We now linearize Eqs. (1)-(3) and take \( n = n_0 + \delta n \), \( v_\alpha = v_\alpha^0 + \delta v_\alpha \), and \( \phi = - \phi_0 \) where \( \phi \) is the perturbed electrostatic potential. Using Eqs. (2) and (3) we find that

\[
\delta v_e = - \frac{c}{B} \nabla \phi \times x e_z + \frac{\omega}{\nu_e i} \frac{c}{B} \nabla \phi \times e_z
\]

(7)

\[
\delta v_i = - \frac{c}{B} \nabla \phi \times e_z + i \frac{\omega}{\nu_i} \nabla \phi
\]

(8)

which can be written as

\[
\delta v_e = - i \frac{c}{B} k_y \phi \hat{e}_x + \frac{c}{B} \phi^\prime \hat{e}_y + i \frac{\omega}{\nu_e i} k_z \phi \hat{e}_z
\]

(9)

\[
\delta v_i = \frac{c}{B} \left[ -ik_y \phi + i \frac{\omega}{\nu_i} \phi^\prime \right] \hat{e}_x + \frac{c}{B} \left[ -k_y \frac{\omega}{\nu_i} \phi + \phi^\prime \right] \hat{e}_y
\]

(10)

where \( \tilde{\omega} = \omega + i \nu_{in} \) and the prime indicates a derivative with respect to \( x \). We have neglected collisional effects on the electron motion perpendicular to \( B \) and on the ion motion parallel to \( B \). This is justified since we have taken \( \nu_{en}/\nu_e < \tilde{\omega}/\nu_i \) which is appropriate for ionospheric conditions in the F region.

We now substitute Eqs. (9) and (10) into Eq. (4) and obtain

\[
(n_0 + \omega - n_0 k_y^2 \left( 1 + \frac{k_z^2 \nu_e}{\nu_i} \right)) \phi
\]

\[
+ \frac{c}{\omega} \frac{\nu_i}{\nu_e} \left[ k_y \left( -\frac{\omega}{\nu_i} + \frac{\nu_{in}}{\nu_i} \frac{c z_{0}}{B_0} \right) + k_z v_d \right] \delta n = 0
\]

(11)
We relate \( \delta n \) and \( \phi \) using the electron continuity equation and find that (from Eqs. (1) and (9))

\[
\delta n = \frac{c}{B} \frac{k_y}{\omega} \left[ n_0 \phi^* - (n_0 \phi)^* + i n_0 \frac{k_z \Omega_e}{k_y v_{ei} k_z} \right] \tag{12}
\]

Finally, substituting Eq. (12) into Eq. (11) we arrive at the mode equation which describes the interchange instabilities relevant to ionospheric structuring

\[
(n_0 \phi^*)^* - n_0 k_y^2 (1 + i \frac{k_z^2 \Omega_e}{k_y^2 v_{ei} \omega}) \phi + \frac{\Omega_i}{\omega} \left( k_y \frac{\varepsilon_y}{\lambda_i} - k_y \frac{\varepsilon_x}{n_i} \frac{c E_0}{B_0} - k_z \frac{v_d}{n_0} \right) = 0 \tag{13}
\]

The first term in Eq. (13) relates to the mode structure in the \( x \) direction, the second term relates to the mode structure in the \( y \) and \( z \) directions, and the final term contains the driving mechanisms for the various instabilities (viz., the Rayleigh-Taylor instability (= \( n_0^2 \)), the \( \mathbb{E} \times \mathbb{B} \) gradient drift instability (= \( E_0 n_0^2 \)), and the current convective instability (= \( v_d n_0^2 \))).

III. ANALYSIS OF MODE EQUATION

As noted in the introduction, the bulk of linear analyses of the interchange instabilities under study have made use of the local approximation so that only short wavelength modes are considered. That is, it is assumed that \( k_y^2 l^2 \gg k_x^2 L^2 \gg 1 \) where \( L = (\partial n_0 / \partial x)^{-1} \) is the density gradient scale length and \( k_x = \partial / \partial x \). With this assumption one can neglect the first term on the LHS of Eq. (13) and obtain a relatively simple
dispersion equation

\[ \omega \tilde{\Gamma} = \left( k_y \frac{8}{\Omega_L} - k_y \frac{\nu_{in}}{\Omega_L} \frac{cP_0}{\varepsilon_0} - k_z \nu_d \right) \Omega_L \left( \frac{1}{k_y \tilde{r}} \frac{n_0}{n_0} \right) \]  

(14)

where \( \Gamma = (1 + i(k_z^2/k_y^2)(\Omega_L/\omega)(\nu_e/\nu_{ei})^{1/2} \). Equation (14) has been thoroughly analyzed in the literature for each of the instabilities under consideration (see Ossakov et al. (1982) and references therein). However, we are considering the long wavelength limit \( (kL \ll 1) \) and Eq. (13) must be solved in order to obtain the nonlocal dispersion equation.

For simplicity, we consider a density profile with a single discontinuity at \( x = 0 \) (see Fig. 1b) given by

\[
n_0(x) = \begin{cases} 
n_1 & x > 0 \\
n_2 & x < 0 
\end{cases}
\]

(15)

For \( x \neq 0 \), \( n_0' = 0 \) and Eq. (13) reduces to

\[
\phi'' - k_y^2 \Gamma^2 \phi = 0
\]

(16)

where

\[
\Gamma^2 = 1 + i \left( \frac{k_y^2 \Omega_L \nu_e}{k_z^2 \nu_e \nu_{ei}} \right)
\]

(17)

the solution to Eq. (16) is taken to be

\[
\phi(x) = \phi_1 e^{k_y r x} + \phi_2 e^{-k_y r x}
\]

(18)
Since the modes are assumed to be bounded as \( x \to \pm \infty \) we note that

\[
\phi(x) = \begin{cases} 
-k_y \Gamma x & x > 0 \\
-k_y \Gamma x & x < 0 
\end{cases}
\]

(19)

We require that the tangential component of the electric field be continuous at \( x = 0 \) (Hasegawa, 1971) which means that \( \phi \) is continuous at \( x = 0 \). This is equivalent to requiring that the interface velocity and the fluid velocity perpendicular to the interface be equal (Chandrasekhar, 1961), i.e., \( \delta \mathbf{V}_x \) is continuous at the discontinuity. Thus, \( \phi_1 = \phi_2 \) in Eq. (19) so that

\[
\phi(x) = \begin{cases} 
-k_y \Gamma x & x > 0 \\
-k_y \Gamma x & x < 0 
\end{cases}
\]

(20)

To obtain the dispersion equation we integrate Eq. (13) across the discontinuity at \( x = 0 \). Thus, we have

\[
\int_{-\varepsilon}^{\varepsilon} \left( \frac{n_0 \varepsilon}{\gamma} \right) dx = \int_{-\varepsilon}^{\varepsilon} \left[ n_0 k_y^2 \Gamma^2 - \frac{n_1 k_y}{\omega} \left( k_y \frac{\varepsilon_1}{\varepsilon_2} - k_y \frac{\varepsilon_2}{\varepsilon_1} \right) \frac{c_0^2}{\varepsilon_2} - k_y \mathbf{V}_d \frac{n_0}{\varepsilon_1} \right] dx = 0
\]

(21)

Since \( \phi \) is continuous across the boundary at \( x = 0 \), it is found that Eq. (21) leads to (letting \( \varepsilon \to 0 \))
\[(n_0 \phi^-)_1 - (n_0 \phi^-)_2 = -\frac{\Omega_1}{\omega} k_y \left( k_y \frac{E_y}{n_1} - k_y \frac{v_{in}}{n_1} \frac{cE_0}{B_0} - k_z v_d \right) (n_1 - n_2) \phi_0 \]  

(22)

where \((1, 2)\) indicate the regions \(x > 0 (\pm C)\) and \(x < 0 (-C)\), respectively.

Substituting Eq. (20) into Eq. (22)

\[\omega \tau = (k_y \frac{E_y}{n_1} - k_y \frac{v_{in}}{n_1} \frac{cE_0}{B_0} - k_z v_d \tau \frac{n_1 - n_2}{n_1 + n_2}) \]  

(23)

which is the general dispersion equation which describes long wavelength interchange modes.

It is interesting to compare the short wavelength (i.e., local) and long wavelength (i.e., nonlocal) dispersion equations, that is, Eqs. (14) and (23), respectively. These equations are, in fact, quite similar. We define an effective wavenumber \(\hat{k}\) given by \(\hat{k} = k_y \tau = (k_y^2 + i k_z^2 \frac{n_1}{\omega})^{1/2}\) and use the definition of the density gradient scale length \((L = (n_0/n_0)^{-1})\) to note the following. The transition from the short wavelength limit to the long wavelength limit can be made by substituting \((n_1 - n_2)/(n_1 + n_2)\) for \((kL)^{-1}\) in Eq. (14). By doing this Eq. (23) is recovered. A significant feature of this substitution is that the wavenumber dependence of the growth rates in the long and short wavelength limits will be different. It will be shown that the short wavelength growth rate is independent of \(\hat{k}\) while the long wavelength growth rate is proportional to \(\hat{k}^{-p}\) where \(p < 1\) and the actual value of \(p\) depends upon the plasma parameters.

Finally, we close this section by rewriting Eq. (23) in a form which is in line with ionospheric structuring terminology. That is, we define \(M = n_1/n_2\) which can be thought of as the ratio of Pedersen conductivity
inside the plasma cloud to that outside the plasma cloud. With this definition, Eq. (23) takes the form

$$\omega \sim \Gamma = (k_y \frac{\delta}{\Omega_i} - k_y \frac{\nu_{\text{in}}}{\Omega_i} \frac{cE_0}{B} - k_z V_d) \frac{M - 1}{M + 1}. \quad (24)$$

IV. ANALYTICAL RESULTS

In this section we present analytical expressions for the growth rates of the interchange instabilities under study in both the collisional ($\nu_{\text{in}} \gg \omega$) and collisionless limits ($\omega \gg \nu_{\text{in}}$). Simplifying assumptions will be made in order to distill essential parameter scalings of the growth rates.

A. Collisional Limit ($\nu_{\text{in}} \gg \omega$)

In this limit $\Gamma = \nu_{\text{in}}$ and $\Gamma^2 = 1 + (k_y^2/k_z^2)(\Omega_e/\nu_{\text{el}})(\Omega_i/\nu_{\text{in}})$. The general nonlocal dispersion equation is then given by

$$\gamma = - \frac{\Omega_i}{\Gamma^2 \nu_{\text{in}} L} \left( \frac{\delta}{\Omega_i} - \frac{\nu_{\text{in}} cE_0}{\Omega_i} \frac{k_z}{k_y} V_d \right) \Gamma \frac{k_y L \frac{M - 1}{M + 1}}{\gamma} \quad (25)$$

where the subscript $\text{nl}$ denotes nonlocal. Instability (i.e., $\gamma > 0$) can occur for $M > 1$ when $g < 0$ (Rayleigh-Taylor), $E_0 > 0$ ($E \times B$ gradient drift), and $V_d > 0$ (current convective). We note that the local (short wavelength) growth rate is given by

$$\gamma_i = - \frac{\Omega_i}{\Gamma^2 \nu_{\text{in}} L} \left( \frac{\delta}{\Omega_i} - \frac{\nu_{\text{in}} cE_0}{\Omega_i} \frac{k_z}{k_y} V_d \right) \quad (26)$$

which is independent of $k_y$. The nonlocal (long wavelength) growth rate can then be expressed as
Thus, the growth rates of the collisional interchange instabilities relevant to ionospheric structuring in the long wavelength limit are directly proportional to $k_y(M-1)/(M+1)$. This limit applies to the low altitude F region ($\lesssim 400$ km).

B. Collisionless Limit ($\omega \gg \nu_{in}$)

In this limit $\tilde{\omega} = \omega$ so that $\Gamma^2 = 1 + i(k_z^2/k_y^2)[(\nu_e/\nu_{ei})(\tilde{\omega}/\omega)]$. Since $\Gamma^2$ can be a function of $\omega$ in this case (i.e., for $k_z$ finite), the relationship between the local and nonlocal growth rates is not, in general, as simple as in the collisional case. We consider then two limits: $k_z = 0$ and $k_z \neq 0$. For the case $k_z = 0$ the relevant instabilities are the Rayleigh-Taylor and the $E_x B_y$ gradient drift. For $k_z \neq 0$ the relevant instability is the current convective. These results apply to the high altitude F region ($\gtrsim 500$ km).

1. $k_z = 0$

In the collisionless flute mode limit ($k \cdot B = 0$), we find that $\Gamma = 1$ so that the nonlocal, long wavelength growth rate is simply (from Eq. (24))

$$\gamma_{ni} = [\tilde{E}_0/L - \nu_{in} \delta_0/L^2]^{1/2}[k_y L (M - 1)/(M + 1)]^{1/2}$$

(29)

Instability can occur for $M > 1$ when $g/L < 0$ (Rayleigh-Taylor) and $E_0/L > 0$ ($E_x B_y$ gradient drift). Again, we note that the local, short wavelength growth rate is given by
so that

\[ \gamma_{nl} = \left[ k_y L \left( \frac{M-1}{M+1} \right) \right]^{1/2} \gamma_z \]

Thus, in contrast to the collisional Rayleigh-Taylor and \( \xi \times B \) gradient drift instabilities, the growth rate of the collisionless long wavelength mode is proportional to \( [k_y L (M-1)/(M+1)]^{1/2} \).

2. \( k_z \neq 0 \)

In the collisionless limit with \( k_z \) finite we consider only the current convective instability (i.e., we take \( g = 0 \) and \( E_0 = 0 \)). In this limit \( r^2 = 1 + i (k_z^2/k_y^2) (\Omega_e/\nu_{ei}) (\nu_d/\omega) \) so that the long wavelength, nonlocal dispersion equation is given by

\[
\omega^2 (1 + i \frac{k_z^2}{k_y^2} \frac{\Omega_i}{\nu_{ei}})^{1/2} = -k_v \Omega_i \frac{M-1}{M+1}
\]

We obtain a simple expression for the growth rate by assuming that \( \omega/\Omega_i \ll (k_y^2/k_z^2) (\Omega_e/\nu_{ei}) \). In this limit Eq. (32) becomes

\[
\omega^3 = +i k_y^2 \frac{\nu_{ei}}{\Omega_e} \left( \frac{M-1}{M+1} \right)^2
\]

We find that

\[ \gamma_{nl} = \left( \frac{\nu_{ei}}{\Omega_e} \frac{\Omega_i L}{\nu_d} \right)^{1/3} (k_y L)^{2/3} \left( \frac{M-1}{M+1} \right)^{2/3} \frac{\nu_d}{L} \]

12
We can relate this nonlocal growth rate to the \textit{maximum} local growth rate $\gamma_{m\ell}$ which is given by (Chaturvedi and Ossakow, 1981)

$$\gamma_{m\ell} = \frac{\left(\frac{\nu_e}{\Omega}\right)^{1/3} V_d}{L}$$

Thus, we find that

$$\gamma_{n\ell} = a[k_{yL}(M-1)/(M+1)]^{2/3} \gamma_{m\ell}$$

where $a$ is a factor of order unity. Thus, the growth rate of the long wavelength, collisionless current convective instability is proportional to $[k_{yL}(M-1)/(M+1)]^{2/3}$.

V. \textbf{CONCLUSION}

We have derived a general, nonlocal dispersion equation (Eq. (13)) which describes the long wavelength limit ($kL \ll 1$) of several interchange instabilities (i.e., Rayleigh-Taylor, $\vec{B} \times \vec{E}$ gradient drift, and current convective). Analytical expressions for the growth rates of these instabilities have been presented in both the collisional ($\nu_{in} \gg \omega$) and collisionless ($\omega \gg \nu_{in}$) regimes. An important result is the scaling of $\gamma_{n\ell}$ with $k_{yL}$ and $M$. These scalings are summarized in Table I. The implications of this work in regard to ionospheric structuring are discussed by Zalesak and Hubs, and by Wittwer in these proceedings.

\textbf{Acknowledgments}

We thank Le\textsuperscript{er} for his interest and encouragement in this research. This work has been supported by the Defense Nuclear Agency.
Fig. 1  Plasma configuration and slab geometry used in the analysis.  (a) Plasma equilibrium quantities and geometry.  (b) Plasma density profile with a discontinuity at $x = 0$. 
Table 1—Scaling of growth rates with respect to $k$ and $M$

<table>
<thead>
<tr>
<th>Instability</th>
<th>Collisional ($v_{in} &gt;&gt; \omega$)</th>
<th>Collisionless ($\omega &gt;&gt; v_{in}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh-Taylor</td>
<td>$k_y L \left( \frac{M-1}{M+1} \right)$</td>
<td>$\left[ k_y L \left( \frac{M-1}{M+1} \right) \right]^{1/2}$</td>
</tr>
<tr>
<td>$\mathbf{E} \times \mathbf{B}$ gradient drift</td>
<td>$k_y L \left( \frac{M-1}{M+1} \right)$</td>
<td>$\left[ k_y L \left( \frac{M-1}{M+1} \right) \right]^{1/2}$</td>
</tr>
<tr>
<td>Current convective</td>
<td>$k_y L \left( \frac{M-1}{M+1} \right)$</td>
<td>$\left[ k_y L \left( \frac{M-1}{M+1} \right) \right]^{2/3}$</td>
</tr>
</tbody>
</table>
References


