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Rayleigh-Taylor Instability in the Presence of a Stratified Shear Layer

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A nonlocal theory of the Rayleigh-Taylor instability which includes the effect of a transverse velocity shear is presented. A two fluid model is used to describe an inhomogeneous plasma under the influence of gravity and sheared equilibrium flow velocity, and to derive a differential equation describing the generalized Rayleigh-Taylor instability. An extensive parametric study is made in the collisionless and collisional regime, and the corresponding dispersion curves are presented. The results are applied to the equatorial F region and to barium releases in the ionosphere.
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RAYLEIGH-TAYLOR INSTABILITY IN THE PRESENCE OF
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I. Introduction

The Rayleigh-Taylor instability (Rayleigh, 1894; Taylor, 1950) arises in a wide range of physical phenomena. This instability is primarily driven by a gravitational force acting on an inverted density gradient (e.g., a heavy fluid supported by a light fluid). In a magnetized plasma these modes can exist in both the collisionless and collisional domains. For example, theoretical models (Hudson and Kennel, 1975; Ossakow, 1979) and numerical simulations (Scannapieco and Ossakow, 1976; Ossakow et al., 1979; Zalesak and Ossakow, 1980; Zalesak et al., 1982) of the collisional Rayleigh-Taylor instability in the earth's ionosphere show that the mode evolves into plasma bubbles that extend upward from the bottomside to the topside of the F-layer. The collisionless interchange type instability (ballooning mode) can exist in the earth's plasmasphere (Vinas, 1980) as well as in laboratory plasmas (Coppi and Rosenbluth, 1966; Coppi et al., 1979). These collisionless modes arise due to an unfavorable curvature in the magnetic field (simulating an effective gravity) in the presence of a pressure gradient. This instability may also arise in the acceleration of a heavy fluid by one of lower density as in targets accelerated by laser ablation (Emery et al., 1982 and references therein) or the deceleration of barium clouds injected in the ionosphere (Pillip, 1971; Rosenberg, 1971; Davis et al., 1974; Fedder, 1980).

In some of the above situations, the equilibrium flow velocity is observed to be inhomogeneous. For example, in the ionosphere, the horizontal plasma velocity during equatorial spread F (ESF) reverses its direction as a function of altitude (Kudeki et al., 1981; Tsunoda, 1981a; Tsunoda and White, 1981; Kelley et al., 1981). In the plasmasphere, steep shear in the flow...
velocity can exist due to the dominating corotating electric field inside the plasmasphere and a convective magnetospheric electric field penetrating across the plasmapause. This can lead to a Kelvin-Helmholtz type erosion of the outer edge of the plasmasphere (Viñas, 1980; Viñas and Madden, 1983). In targets accelerated by laser ablation, the Rayleigh-Taylor instability (Bodner, 1974) non-linearly evolves into a bubble and spike structure (as during ESF) and develops a strong shear in the flow velocity (Harlow and Welsh, 1966; Daly, 1967; Emery et al., 1982). In the absence of any other driving mechanism, the velocity shear gives rise to a transverse Kelvin-Helmholtz instability in fluids (Kelvin, 1910; Chandrasekhar, 1961) and in magnetized plasmas (Mikhailovskii, 1974). All of the above examples suggest a need for a detailed study of configurations in which the two driving mechanisms co-exist, namely, inhomogeneous velocity flows and gravity (or similar forces).

The influence of velocity shear on interchange instabilities has been studied by Drazin (1958) and Chandrasekhar (1961) in the context of fluid models; by Hamieri (1979) in the context of laboratory plasmas; by Viñas (1980) in the context of the plasmasphere; and by Guzdar et al. (1982; 1983) in the context of equatorial spread F. In this paper we report on our detailed study of the influence of velocity shear on the collisionless and collisional Rayleigh-Taylor instability and apply the results to a variety of geophysical phenomena. We find that the velocity shear can have a dramatic effect on the Rayleigh-Taylor instability (Guzdar et al., 1982; 1983). A sufficiently strong velocity shear can stabilize the most unstable modes (i.e., those for which $kL > 1$ where $L$ is the scale length of the inhomogeneity and $k$ is the perpendicular wave number), which leads to maximum growth in the long wavelength regime (for which $kL < 1$). Thus velocity shear, in some
domains, preferentially excites a long wavelength mode, in sharp contrast to the behavior of the mode in the absence of velocity shear.

This paper is divided into five sections. In the next section, we derive the general mode structure equation describing an inhomogeneous collisional plasma which contains a sheared velocity flow and which is under the action of gravity. In the third section we discuss the stability of this plasma in two limits, namely, the Rayleigh-Taylor limit (no velocity shear), and the Kelvin-Helmholtz limit (no gravity). In the fourth section we present the results of the analysis of the generalized Rayleigh-Taylor instability, i.e., when both velocity shear and gravity are present. In the final section, we discuss the results and apply them to geophysical phenomena.
II. Theory

We consider an infinite slab of magnetized plasma. The coordinate system is shown in Fig. 1. The magnetic field is uniform and is in the z direction ($\mathbf{B} = B \hat{z}$), the acceleration due to gravity is in the x direction ($g = -g \hat{x}$), the ambient electric field is inhomogeneous and is in the x direction ($\mathbf{E}_0 = E_0(x) \hat{x}$), and the density is inhomogeneous in the x direction ($n_0 = n_0(x)$). The inhomogeneity in the background electric field leads to a sheared equilibrium flow, $V_0(x) = -c E_0(x)/B \hat{z}$.

The basic assumptions used in the analysis are as follows: (1) the perturbed quantities vary as $\delta p \sim \delta p(x) \exp[i(k_y y - \omega t)]$, where $k_y$ is the wave number along y direction and $\omega = \omega_r + i\gamma$, implying growth for $\gamma > 0$; (2) the ordering in the frequencies is such that $\omega, \nu_{in} \ll \Omega_i$, where $\nu_{in}$ is the ion-neutral collision frequency and $\Omega_i$ is the ion-gyro-frequency; (3) we ignore finite-gyroradius effects by limiting the wavelength domain to $k \rho_i \ll 1$, where $\rho_i$ is the mean ion-Larmor radius; (4) we neglect perturbations along the magnetic field ($k_z = 0$) so that only two dimensional mode structure in the x-y plane is obtained; (5) we retain ion inertia effects, thereby including the ion polarization drift, but ignore electron inertia; and (6) we neglect ion and electron pressure.

A key feature of our analysis is that a nonlocal theory is developed. That is, the mode structure of the potential in the x direction, the direction in which density and the flow velocity are assumed to vary, is determined by a differential equation rather than an algebraic equation obtained by Fourier analysis. This technique allows one to study modes which have wavelengths comparable to the scale size of the inhomogeneities (i.e., $k_y L < 1$, where L represents scale lengths of the boundary layer). In fact, nonlocal theory is
Figure 1. Plasma configuration and slab geometry used in the analysis.
crucial to describe the Kelvin-Helmholtz instability driven by transverse velocity shear (Mikhailovskii, 1974).

Based upon the assumptions discussed above, the fundamental fluid equations used in the analysis are continuity and momentum transfer in the neutral frame of reference:

\[
\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha v_\alpha) = 0, \quad (1)
\]

\[
0 = E + \frac{1}{c} v_e \times B, \quad (2)
\]

\[
m_1 \left( \frac{\partial}{\partial t} + v_\perp \cdot \nabla \right) v_\perp = eE + e \frac{c}{c} (v_\perp \times B) - m_1 v_{in} v_\perp + m_1 g. \quad (3)
\]

where \( \alpha \) denotes species (e for electrons; i for ions).

The equilibrium velocities are, for electrons

\[
v_{e0}(x) = v_0(x) = (c/B) E_0(x) \times \hat{z}, \quad (4)
\]

and for ions (to order \( v_{in}/\Omega_1 \))

\[
v_{i0}(x) = \frac{(c/B)}{[E_0 + (m_1/e)g] \times \hat{z} + (v_{in}/\Omega_1) [E_0 + (m_1/e)g]}
\]

\[
+ \left( \frac{1}{\Omega_1} \right) (v_{i0} \cdot \nabla) E_0 \quad (5)
\]

where \( \Omega_1 = eB/m_1c \). The electrons simply \( E \times B \) drift, while the ions drift with \( v_{i0} \), which incorporates the \( E \times B \) drift, and the effects of gravity and the polarization electric field.

We substitute \( E = -V\phi = -\nabla (\phi_0 + \delta \phi) \), \( V_\alpha = V_{\alpha 0} + \delta V_\alpha \), \( n = n_0 + \delta n \) into Eqs. (1) - (3). To obtain the perturbed velocities we do not Fourier analyze the perturbed quantities in the \( x \) direction since the
equilibrium quantities vary in that direction; we Fourier analyze only in the
y direction. Linearizing Eqs. (2) and (3) and making use of Eqs. (4) and (5),
we find the perturbed velocities to be

\[ \delta V_y = - \frac{c}{B} \left[ ik_y \delta \phi \hat{x} + \frac{3}{3x} \delta \phi \hat{y} \right], \]  
\[ \delta V_y = \frac{\omega}{\Omega_1} \left[ - ik_y \delta \phi + \frac{i \omega - \nu \delta \phi}{\Omega_1} \hat{y} \right], \]  
\[ + \frac{c}{B} \left[ \frac{\partial}{\partial x} \delta \phi - \frac{\nu}{\Omega_1} k_y \delta \phi \right] \hat{y}, \]  

where \( \omega = c_y V_0(x) \) and \( V_0 = V_{0y} = -\frac{c}{B} E_{0x} \).

We substitute Eqs. (6) and (7) into Eq. (1), to obtain

\[ \frac{3}{3t} \delta n_e + i k_y V_0 \delta n_e - i \frac{c}{B} k_y \delta \phi x_0 = 0 \]  

and

\[ \frac{3}{3t} \delta n_i - \frac{c}{B} \Omega_1 n_0 \left[ (- i \omega + \nu \delta \phi + k_y V_0) \left( \frac{3}{3x} - k^2 \right) \delta \phi \right] 
+ i k_y V_0 \delta \phi + \nu \left( \frac{3}{3x} \delta \phi \right) \]

\[ - \frac{c}{B} \left( k_y (1 - V_0/\Omega_1) \right) \delta \phi \]

\[ - \frac{c}{B} \left( - i \omega + \nu \delta \phi + i k_y V_0 \right) n_0 \frac{3}{3x} \delta \phi \]

\[ + (g/\Omega_1 - V_0) i k_y \delta n_i + (\nu \delta \phi) n_0 \frac{3}{3x} \delta n_i \]

\[ + \frac{\delta n_i}{\Omega_1} \left[ (g/\Omega_1 - V_0) \nu \delta \phi + \nu \delta \phi n_0 \right] = 0 \]  

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where we have ignored terms of order $O(\omega/\Omega)$ but have retained terms proportional to $\omega/v_{in}$ and the derivatives of the equilibrium quantities with respect to $x$ (indicated by the primes). We now subtract the electron continuity equation, Eq. (8), from the ion continuity equation, Eq. (9), and impose quasineutrality ($\delta n_e = \delta n_i$). We use Eq. (8) to eliminate $\delta n_i$ and obtain the mode structure equation

$$\frac{3}{3} \frac{\partial^2 \phi}{\partial x^2} + p \frac{\partial}{\partial x} \frac{\partial \phi}{\partial x} + q \frac{\partial \phi}{\partial x} = 0,$$

(10)

where

$$p = \frac{n_i}{n_0} + \frac{iv_{in}}{\tilde{\omega} + iv_{in}} \left( \frac{v_{in}^2}{\tilde{\omega}} - \frac{(n_i/n_0)k_y v_0}{\tilde{\omega}} \right),$$

(11)

$$q = -k_y^2 + \frac{k_y (v_{in}^0 (n_i/n_0) + v_{in}^0)}{(\tilde{\omega} + iv_{in})} + \frac{k_y^2 g (n_i/n_0)}{\tilde{\omega} (\tilde{\omega} + iv_{in})} - \frac{iv_{in} k_y v_0}{\tilde{\omega} (\tilde{\omega} + iv_{in})} \left[ \frac{\omega}{\tilde{\omega}} (n_i/n_0)(v_{in}^0/v_0) + (n_i^n/n_0) + (v_{in}^n/v_{in})(n_i^0/n_0) \right].$$

(12)

Note that the solution of Eq. (10) allows for wavepacket formation instead of plane waves. The properties of the wavepacket are governed by the coefficients $p$ and $q$. From the expression for $q$ we see that the second term involves the free energy associated with the inhomogeneous plasma flow giving rise to Kelvin-Helmholtz instability. The third term has the gravity and the inverted density gradient leading to Rayleigh-Taylor instability. The last term becomes important in the moderately collisional domain.
Now we define the following dimensionless parameters in order to cast Eq. (10) into a dimensionless form

\[ \hat{\omega} = \omega/(g/L)^{1/2}, \hat{\nu} = \nu_1/(g/L)^{1/2}, \hat{\nu}_0 = \nu_0/(gL)^{1/2}, \text{ and } \hat{k} = k_L \]  

(13)

where \( L \) is the characteristic inhomogeneity scale length. We also normalize \( x \) with \( L \) and define a new independent variable \( \chi = x/L \). With these definitions, Eqs. (10)-(12) become

\[ \frac{\partial^2}{\partial \chi^2} \hat{\phi} + \frac{\partial}{\partial \chi} \left( \frac{\partial}{\partial \chi} \hat{\phi} + q \hat{\phi} \right) = 0, \]  

(14)

\[ p = \left( \frac{n_0}{n_0'} \right) + i \frac{\hat{\nu}}{\omega_2} \left[ \frac{\hat{\nu}}{\nu} - \frac{(n_0'/n_0) \hat{k} \hat{\nu}_0}{\omega_1} \right], \]  

(15)

\[ q = -\hat{k}^2 + \hat{k} \left\{ \frac{\hat{\nu}'}{\omega_2} \left( \frac{n_0'/n_0}{\omega} + \frac{n_0'/n_0}{\omega_0} \right) + \frac{\hat{k}^2 (n_0'/n_0)}{\omega_1 \omega_2} \right\} \]  

- \frac{i \hat{\nu}}{\omega_1 \omega_2} \hat{k} \hat{\nu}_0 \left[ (\hat{\nu}_0'/\hat{\nu}_0)\left( \frac{n_0'/n_0}{\omega} \right) + (n_0'/n_0) + (n_0'/n_0) (\hat{\nu}'/\hat{\nu}) \right], \]  

(16)

where \( \hat{\omega}_1 = \hat{\omega} - \hat{k} \hat{\nu}_0(\chi) \) and \( \hat{\omega}_2 = \hat{\omega}_1 + i \hat{\nu} \).

We use the transformation

\[ \delta \phi(\chi) = \psi(\chi) e^{-\int p(\eta) \, d\eta} \]  

\[ \chi \]  

(17)
to write the Eq. (14) as

$$\psi'' + Q(x) \psi = 0,$$

(18)

where

$$Q(x) = q(x) - \frac{p''(x)}{2} - \frac{p(x)^2}{4}.$$  

(19)

In solving the eigenvalue problem, we use WKB boundary conditions on \(\psi\):

$$\psi + \left(1/Q^{1/4}\right) \exp\left(-\int_0^X Q(n) \, dn\right) \quad \text{as } x \pm \infty.$$  

(20)

We refer to \(Q(x)\) as the potential function and we give the plots of \(Q(x)\) and \(\psi\) when we solve Eq. (18) numerically in the next section.

Equations similar to Eq. (18) have been obtained by several authors studying the stability of stratified shear layers in neutral fluids (Drazin, 1958). However, usually a Rayleigh-Taylor stable density profile was chosen, mainly to examine the influence of inertia on the velocity shear induced mixing phenomenon. This situation differs slightly from the one considered in this paper, since we study a situation where inverted density gradients and velocity shear both are sources of free energy. The collisionless case can be compared with the neutral fluid case, and when we set \(\dot{\gamma} = 0\), in Eqs. (14)-(16) we regain the mode structure equation obtained by Drazin (1958). Drazin (1958) considers a weakly inhomogeneous plasma, i.e., by setting \(n_0'/n_0\) to zero everywhere except in the driving term containing the gravity, and obtains a simple equation.
\[
\psi'' + \left[ -k^2 + \frac{R}{(\alpha - \nu_0)^2} + \frac{\nu''}{(\alpha - \nu_0)} \right] \psi = 0
\]  

(21)

where

\[
R \equiv \frac{(g/L)}{(\nu_0/L)^2}
\]  

(22)

and \( \alpha = \omega/k_y \). For this weakly inhomogeneous case, the analysis shows that a neutral stability boundary (where the real and the imaginary parts of \( c \) are zero) can be obtained as

\[
R = \hat{k}^2(1 - \hat{k}^2).
\]  

(23)

This implies that for \( R < 1/4 \) the system is stable. Hamier (1979) considered a more general case applicable to a Tormac machine and arrived at a less stringent condition. A theoretical analysis in the collisionless and collisional cases will be presented in a future paper.
III. Analysis and Results

The generalized mode structure equation [Eq. (14)] can be better understood by first considering two limiting cases: (A) the collisionless and collisional Rayleigh-Taylor instability without velocity shear, and (B) the Kelvin-Helmholtz instability with no collisions or gravity.

A. Rayleigh-Taylor Instability

Setting \( V_0 = 0 \) in Eq. (14) we obtain

\[
\psi'' + \left( \frac{n_0'}{n_0} \right) \psi' - \kappa^2 \left[ 1 - \frac{n_0'/n_0}{\omega} (\omega + iv) \right] \psi = 0
\]  

Eq. (24) can be solved in the local approximation,

\[
\psi(x) = \exp(i k x); \quad k^2 l^2 \gg k^2 l^2 \gg 1
\]  

and we obtain the well known dispersion relation (Haerendel, 1974; Hudson and Kennel, 1975)

\[
\omega^2 + iv_{in} \omega + g/L_n = 0
\]  

which has the solution

\[
\omega = - \frac{iv_{in}}{2} \left[ 1 + \left( 1 - 4g/L_n v_{in}^2 \right) \right]^{1/2},
\]  

where \( L_n^{-1} = \frac{1}{n_0} \frac{3n_0}{\partial x} \). Instability can occur when \( g/L_n < 0 \). The collisionless and collisional solutions are, respectively,
\[ \omega = \pm (g/L_n)^{1/2} \quad \nu \ll |2(g/L_n)^{1/2}|, \]  
\[ \omega = -i \frac{(g/L_n)}{\nu} \quad \nu \gg |2(g/L_n)^{1/2}|. \]  

We now solve Eq. (24) numerically for a density profile

\[ n_0 = \tilde{n}_0 \exp(-\frac{k^2}{2}) + \Delta n. \]  

The results are shown in Fig. 2, which is a plot of normalized growth rate, \( \tilde{\gamma} = \gamma/(g/L)^{1/2} \), vs normalized wave numbers, \( \hat{k} = k/L \). Curve A is for the collisionless Rayleigh-Taylor instability and curve B is for the collisional Rayleigh-Taylor instability with \( \nu = 0.5 \). We use \( \Delta n/n_0 = 0.01 \) for both cases. As expected, the growth rate maximizes in the regime \( \hat{k} \gg 1 \) and the maximum growth rate agrees well with the growth rate predicted by local theory [Eq. (27)] with the growth rate evaluated at the maximum density gradient. The potential function \( Q(\chi) \) given by Eq. (19)) and the wave function \( \psi(\chi) \), corresponding to \( \hat{k} = 1.0 \) are shown in Figs. 3 and 4, respectively. We note that for \( \Delta n/n_0 = 0.01 \), \( n_0/\tilde{n}_0 \) has a maximum at \( \chi = -2.4 \) and a potential well \([-Q(\chi) \text{ is a minimum}] \) is formed around this point as can be seen from Fig. 3. We see from Fig. 4 that the wave function also localizes at \( \chi = -2.4 \). The negative sign implies that the Rayleigh-Taylor instability is active where the density gradient opposes the gravity. We note that the wave function spreads out into the positive region of the \( x \)-axis, where the Rayleigh-Taylor instability is locally stable (gravity acts in the same direction as the density gradient).
Figure 2. Normalized growth rate $\hat{\gamma} \equiv \gamma/\sqrt{g/L}$ vs $\hat{k} \equiv k_y L$ for the Rayleigh-Taylor instability. Curve A represents the collisionless mode ($\hat{\nu} = 0$); curve B represents the collisional mode ($\hat{\nu} = 0.5$), where $\hat{\nu} = \nu/\sqrt{g/L}$. The density profile used is a Gaussian-like profile $n_0 e^{-x^2/2l^2} + \Delta n$, with $\Delta n/n_0 = 0.01$. 
Figure 3. Plot of potential term, $Q(x)$, as a function of $x$ for the collisional case and density profile used in Fig. 2.
Figure 4. Plot of the wavefunction for $\omega_r = 0$, $\gamma = 0.693$, $k = 1.0$, and for the collisional case and density profile used in Fig. 2.
B. Kelvin-Helmholtz Instability

We retain the flow velocity \( V_0 = V_0(x) \) but consider a collisionless, uniform fluid with no gravity. Eq. (14) becomes

\[
\psi'' + \left[ -k_y^2 + \frac{k_y V_0''}{(\omega - k_y V_0)} \right] \psi = 0, \tag{31}
\]

which is well known (Mikhailovskii, 1974). Rayleigh's theorem (Mikhailovskii, 1974) predicts an instability if the velocity profile has a vanishing second derivative between the boundaries, i.e., \( \left[ \frac{\partial^2 V_0}{\partial x^2} \right]_{x_0} = 0 \), where \( x_1 < x_0 < x_2 \) and \( x_1 \) and \( x_2 \) are the boundaries.

Equation (31) is solved for an equilibrium velocity profile

\[
V_0 = \bar{V}_0 \tanh(x/L) \tag{32}
\]

and the results are shown in Fig. 5 (curve A) in which we plot
\( \gamma/(V_0/L) \) versus \( k_y L \). The instability is purely growing and is bounded between \( k_y L = 0 \) and 1 with a maximum growth rate of \( \gamma \sim 0.18 \) \( (V_0/L) \) at \( k_y L = 0.45 \) (Michalke, 1964).

When a density gradient is included, we arrive at the following equation

\[
\psi'' + \left( \frac{n_o}{n_0} \right) \psi' + \left[ -k_y^2 + \frac{[V_0'' + V_0' (n_o/n_0)]}{(\omega - k_y V_0)} \right] \psi = 0. \tag{33}
\]

Using the same procedure as outlined following Eq. (31), we can show that for instability the density and velocity profiles should be such that
\( \left[ \frac{1}{n_0} \frac{\partial}{\partial x} (n_0 V_0') \right]_{x_0} = 0 \) where \( x_0 \) is any point within the boundaries. It is interesting to note from Eq. (33) that no instability exists if the density and velocity profiles are such that \( V_0''/V_0' = -n_o/n_0 \). Equation (33) is
solved numerically for the velocity profile given in Eq. (32) (in this example we have set the density and shear scale lengths to be equal). The properties of this mode are shown in curve B of Fig. 5 for an exponentially decreasing density profile. Two features are to be noted here: (1) that the density gradient has a stabilizing influence which reduces the maximum growth rate from $\gamma = 0.18 \, (V_0/L)$ to $\gamma = 0.074 \, (V_0/L)$, and (2) that the instability exists in the region $0.1 < k_y L < 0.9$ shifting the wave number at which the growth rate maximizes from $k_y L = 0.45$ to $k_y L = 0.55$.

Figure 5. Growth rate $\gamma/(V_0/L)$ vs $k \equiv k_y L$ for the Kelvin-Helmholtz mode. Curve A represents the case $\dot{v} = 0$, $\dot{n} = 0$; Curve B represents the case $\dot{v} = 0$, and $n_0 = \bar{n}_0 \exp(-x/L)$. 
IV. Generalized Rayleigh-Taylor Instability

In the previous section we considered the limiting cases where an inverted density gradient in the presence of gravity and a velocity shear individually give rise to different instabilities. We now consider the general problem where both free energy sources jointly give rise to a generalized Rayleigh-Taylor instability (Hamieri, 1979; Vinas, 1980).

We consider two different cases: (A) a self-consistent equilibrium, and (B) a general equilibrium based on the experimental observations.

A. Self-consistent Equilibrium

We choose the following density profile

\[ n_0(x) = \bar{n}_0 \left(1 + \varepsilon \tanh(x/L)/(1-\varepsilon)\right), \]  

(34)

and the following velocity profile

\[ V_0(x) = \bar{V}_0(\bar{n}_0/n_0(x)) \]  

(35)

such that

\[ \hat{V}_0 = \frac{\bar{V}_0}{(gL)^{1/2}} \frac{\bar{n}_0}{n_0(x)} = \hat{s} \frac{\bar{n}_0}{n_0(x)}, \]

where we have defined a dimensionless parameter \( \hat{s} \equiv \bar{V}_0/(gL)^{1/2} \). Note that the zeroth order continuity equation is satisfied by these profiles for \( V_0 \) constant, i.e.,

\[ \frac{3}{\bar{n}_0} \left[ (n_0(x) V_{10x}(x)) \right] = 0. \]  

(36)
which implies

\[ n_0(x) E_0(x) = \text{constant} \]

or using the definition of \( V_0 \) from Eq. (4),

\[ n_0(x) V_0(x) = \text{constant}. \]

We solve Eq. (14) numerically using these profiles and present the results below.

In Fig. 6 we plot the normalized growth rate \( \gamma \) versus \( k \) for the collisionless and collisional cases. The solid lines represent the collisionless case \( (\gamma = 0) \) and the dashed lines represent the collisional case \( (\gamma = 0.5) \). We set \( \epsilon = 0.8 \) in the density profile. Several points are to be noted in this figure. First, we note that in a shear-free, collisionless Rayleigh-Taylor plasma the growth rate asymptotes to the local growth rate evaluated at the peak density gradient, (solid line), i.e.,

\[ \gamma(k \gg 1) = [g(n'_0 / n_0)]^{1/2} / \chi. \quad (37) \]

For the density profile given in Eq. (36) \( n'_0/n_0 \) maximizes at \( \chi \) obtained from

\[ \tanh \chi = [-1 \pm (1 - \epsilon^2)^{1/2}] / \epsilon. \quad (38) \]
Figure 6. Growth rate $\hat{\gamma}$ vs $\hat{k}$ for the generalized Rayleigh-Taylor instability. Solid lines represent the collisionless case ($\hat{\nu} = 0.0$, $\hat{s} = 0.0$, and 1.0); dashed lines represent the collisional case ($\hat{\nu} = 0.5$, $\hat{s} = 0.0$, and 1.0). The profiles used are the self-consistent profiles, Eqs. (34) and (35).
For $\epsilon = 0.3$ Eq. (38) yields $\chi = -0.55$ and using this in Eq. (37) we find the local growth rate to be $\gamma = 1.0$. This agrees with the growth rate for large $k$ (see Fig. 6, solid line). Second, we find that ion-neutral collisions have a stabilizing influence as seen from the dashed line, which represents the growth rate curve for $\hat{v} = 0.5$ and $\hat{s} = 0$. Third, in a collisionless Rayleigh-Taylor unstable plasma, for $\hat{s} = 1.0$ (shear frequency, $V_0/L$, equal to $\sqrt{g/L}$) velocity shear stabilizes the short wavelength modes (solid line; $\hat{s} = 1.0$); the cut-off mode number, where the growth rate becomes zero, is $k_c \sim 11.0$. As a result the growth rate maximizes at $k \sim 1.5$ and has a maximum value $\gamma_m = 0.675$. Fourth, in a collisional plasma also, with $\hat{v} \sim 0.5$ the short wavelength modes are completely stabilized (dashed line; $\hat{s} = 1.0$). The cut-off mode number in this case is, $k_c \sim 10.0$, which is less than that of the collisionless case. The peak growth rate is also smaller with $\gamma_m = 0.45$ occurring at $k = 1.5$. We see that the ion-neutral collisions not only reduce the growth rate but also reduce the cut-off mode number.

In Fig. 7 we give the plots of the wavefunction for $\hat{s} = 0$, $\hat{k} = 0.5$, $\hat{v} = 0.5$, and $\epsilon = 0.8$. We note that the wavefunction localizes at $\chi_0 = -0.55$ which is the point where the density gradient $(n_0', n_0)$ has an extremum. The wavefunction localizes at this point because $Q(\chi)$ has a local minimum. We refer to this point (for $\hat{s} = 0$) as the Rayleigh-Taylor localization point. The negative sign indicates that the density gradient has to oppose the gravity for instability.

In Fig. 8 we give the wavefunction for the case $\hat{s} = 1$ with the same $\hat{v}$ and $\epsilon$ as in Fig. 7. The solid and the dotted lines represent the real and imaginary parts of the wavefunction, respectively. We note that when
Figure 7. The wave function corresponding to Fig. 6 (solid line; $s = 0$) and $k = 0.5$. 
Figure 8. Real and imaginary parts of the wavefunction, corresponding to Fig. 6 (dotted line, $s = 1.0$) and $k = 0.5$. 
the velocity shear is introduced into the problem, the wavefunction picks up an imaginary part. Furthermore, we find that for $k \geq 1$, the wavefunction localizes at a point closer to the origin in the velocity shear layer.

In comparing our results with Drazin (1958) we note that since we use a density profile whose density gradient is not a constant, and since we have a Raleigh-Taylor unstable plasma, our threshold condition on $R$ is quite different. This aspect will be dealt with in a future paper.

B. General Equilibrium

Recent experimental observations, made during equatorial spread F (ESF) (Kudeki et al., 1981) and in the high latitude ionospheric F region (Kelley et al., 1978), indicate that ionospheric plasmas usually support inhomogeneous equilibrium plasma flows. In the case of ESF it was found that the flow velocity reverses its direction as a function of altitude (x, the direction of the density gradient). Furthermore, the velocity reversal point moves up as the spread F develops. This equilibrium situation, where the flow velocity profile is not related to the density profile in a simple manner, is generated by the coupling of the plasma to the neutral atmosphere, for example, by the neutral winds and the inherent shear in the neutral wind velocity or in the case of ESF due to an incomplete coupling caused by background ionospheric Pedersen conductivity away from the equatorial plane (Zalesak et al., 1982). Our earlier numerical results indicate that the inhomogeneity in $\omega - k_y V_0(x)$, and not necessarily the $V'_0$ and $V''_0$, is primarily responsible for stabilizing the short wavelength interchanger modes (Huba et al., 1983). Therefore, based on the experimental observations (Kudeki et al., 1981; Tsunoda et al., 1981) and our numerical results (Huba et al., 1983), we choose the following density and velocity profiles for a general equilibrium study:
\[ n_0(x) = \overline{n}_0 \frac{(1 + \varepsilon \tanh x)/(1 - \varepsilon)} \] (39)

\[ V_0(x) = \overline{V}_0 \tanh (x - \chi_0) \] (40)

where \( \chi_0 \) is the velocity reversal point (in the ionospheric case, \( \chi_0 \) is the point where the westward flow becomes eastward). Using these profiles we solve Eq. (14) numerically. Also, for simplicity, we choose the density gradient scale length and velocity shear scale length to be equal. The numerical results are given below.

First we study the role of \( \chi_0 \) on stability and determine the optimum \( \chi_0 \) to be used in later calculations. We set \( \nu = 0.5 \) and \( \varepsilon = 0.8 \). In Fig. 9 we plot the normalized growth rate \( \gamma \) versus \( \hat{k} \). Curve A shows the nonlocal collisionless Rayleigh-Taylor instability (\( \hat{s} = 0 \)). Curves B, C, and D correspond to \( \hat{s} = 1 \) for different values of \( \chi_0 \). Curve B gives the growth rates for \( \hat{s} = 1 \) and \( \chi_0 = -2.0 \) which shows a significant reduction in the growth rate. The growth rate maximizes with \( \gamma = 0.52 \) at \( \hat{k} \approx 3.0 \). However, when \( \chi_0 \) is set to -0.55 (the Rayleigh-Taylor localization point) the growth rate is sharply reduced, maximizing at \( \hat{k} \approx 0.5 \) with \( \gamma = 0.256 \) (curve C). For \( \chi_0 = 0 \) (curve D) there is a significant reduction in the growth rate and a severe reduction of the \( \hat{k} \) domain for instability. The instability is bounded between \( \hat{k} = 0.5 \) and 3.6. Here, the growth rate peaks around \( \hat{k} \approx 1.7 \).

For \( \chi_0 = 2.0 \) (not shown) the growth rate curve is similar to that of \( \chi_0 = -2.0 \) (curve C). From this we conclude that the effects of velocity shear are strongest when the velocity reversal point falls in the Rayleigh-Taylor localization region.
Figure 9. Variation of the dispersion curve, $\gamma$ vs $k$, for different transition points ($x_0$), of the flow velocity profile given by $V = \bar{V}_0 \tanh(x - x_0)$ for $x_0 = -0.5$, $-2.0$, and $0.0$. Parameters used are $\hat{s} = 1$, $\hat{\nu} = 0.5$, $\varepsilon = 0.8$, and for the profiles given in Eqs. (39) and (40). Curve A corresponds to shear-free case ($\hat{s} = 0$) where as curves B, C, and D correspond to $\hat{s} = 1.0$ and $x_0 = -2.0$, $-0.55$, and $0.0$ respectively.
In order to throw some light on the variation of the dispersion curves as a function of the velocity shear we plot $\dot{\gamma}$ versus $k$ for various values of $\hat{s}$ keeping $\hat{v}$, $\hat{e}$, and $\chi_0$ fixed at 0.5, 0.3, and -0.55, respectively, in Fig. 10. This figure shows that the general (non self-consistent) profiles yield results similar to those of the self-consistent profiles (sec. IV.A; see fig. 6). As $\hat{s}$ is increased, $k_m$ the mode number at which the growth rate maximizes, moves towards smaller $k$ and the growth rate is substantially reduced. For very large shear, $\hat{s} >> 1$, the mode becomes purely Kelvin-Helmholtz like, preferentially exciting a long wavelength mode ($k \sim 0.45$) with the cut-off $k$ less than 1.0 (to be compared with Fig. 5). This aspect is further illustrated in Fig. 11 where we plot $\dot{\gamma}$ versus $\hat{s}$ for several values of $\hat{s}$. The figure shows that for $\hat{v} = 0.5$, modes with $\hat{k} < 0.8$ are always unstable. No amount of shear (measured in units of $\hat{s}$) stabilizes these modes due to the onset of the Kelvin-Helmholtz instability for these large $\hat{s}$ and small $\hat{k}$. Furthermore, for the parameters used in the figure, moderate to strong velocity shear stabilizes modes with $\hat{k} > 0.8$. An empirical estimate of the shear that stabilizes the smallest $\hat{k}$ mode can be obtained from the $\hat{k} = 0.8$ curve in Fig. 11, i.e., $\hat{s} = 2.5$ or $\frac{\bar{V}_0}{L} \sim 2.5 \sqrt{\hat{g}/L}$; the figure also shows that the critical shear depends on the wavenumber.

From Fig. 10 we see that for $\hat{v} = 0.5$, the mode with $\hat{k} = 0.45$ is the fastest growing mode for large $\hat{s}$. It is interesting to study the behavior of this mode as a function of $\hat{s}$. In Fig. 12 we plot $\dot{\gamma}$ (for $\hat{k} = 0.45$) versus $\hat{s}$. Curves A and B represent the collisionless ($\hat{v} = 0$) and collisional ($\hat{v} = 0.5$) cases respectively. Note that for $\hat{s} = 0$, $\dot{\gamma}$ is purely Rayleigh-Taylor-like; but as $\hat{s}$ increases, $\dot{\gamma}$ initially decreases, which shows that velocity shear is reducing the growth rate of the Rayleigh-Taylor instability. Beyond $\hat{s} > 1$ the velocity shear dominates and the Kelvin-
Figure 10. Study of the growth rate as a function of the mode number in the collisional domain $\hat{\gamma}$ vs $\hat{k}$ and for $\hat{s} = 0, 0.25, 1.0, 2.0, \text{ and } 4.0$. We have used $\chi_0 = -0.55, \hat{\nu} = 0.5, \text{ and } \varepsilon = 0.8$. 
Figure 11. Study of growth rate $\hat{\gamma}$ as a function of $\hat{s}$ for $\hat{k} = 0.4$ thru 2.0. The parameters used are $\hat{\varepsilon} = 0.3$, $\hat{\nu} = 0.5$, and $\chi_0 = -0.55$. 
Figure 12. Study of $\gamma$ versus $s$, for $k = 0.45$ where the Kelvin-Helmholtz instability has maximum growth rate. Curves A and B refer to $\gamma = 0.0$ and 0.5 respectively.
Helmholtz mode sets in. We note that $\gamma$ increases linearly with $s$ for large $s$ since it is normalized to $\sqrt{g/L}$ and not $V_0/L$. For the collisional case we see that the velocity shear has a stronger influence over a broader domain in $s$ and the Kelvin-Helmholtz type instability sets in for larger $(s > 2.5)$.

We see from Fig. 10 that the cut-off mode numbers and the mode numbers where the growth rate maximizes vary significantly as a function of velocity shear. In order to show the values they asymptote to for large velocity shear, we plot the cut off mode numbers $k_c$ (curves A), and the mode numbers of the fastest growing modes, $k_m$ (curves B) as a function of $s$ in Fig. 13. We use $\chi_0 = -0.55$, $\varepsilon = 0.8$, and $\gamma = 0$ and 0.5. Solid lines represent the collisionless case ($\gamma = 0$) and dashed lines represent the collisional case ($\gamma = 0.5$). From the figure, we see that $k_m$ and $k_c$ fall sharply as $s$ is increased and asymptote to smaller $k$ values. For $\gamma = 0$, (solid lines) $k_m$ asymptotes to 0.5 and $k_c$ asymptotes to 1.0. Because for large $s$ the mode is Kelvin-Helmholtz-like, $k_m$ and $k_c$, as expected, attain the values shown in Fig. 5. For $\gamma = 0.5$ (dashed lines) both $k_m$ and $k_c$ are initially smaller than those for the collisionless case. However, as $s$ is increased these maximum and cut-off wavenumbers achieve a minimum value, then rise and again asymptote to similar values as those corresponding to the collisionless case, namely 0.5 and 1.0, respectively.

Finally, in Fig. 14 we show the effects of introducing a spatially dependent collision frequency. In the ionosphere the ion-neutral collision frequency decreases exponentially as a function of the altitude. We use the profile $\varkappa = 0.5\exp(-x/L)$, choosing the scale length to be the same as the density gradient scale length for simplicity. Curve A shows the growth rate curve for constant collision frequency, $\gamma = 0.5$, and for $s = 1.0$, and $\varepsilon = 0.8$. The growth rate maximizes at $\gamma_m = 0.256$ around $k = 0.55$. Curve B,
with $\hat{\nu} = \hat{\nu}(x)$, shows a drastic reduction in the growth rate. The maximum growth rate ($\gamma_m = 0.09$) occurs at $\hat{k} \approx 0.7$. Interestingly, in this case the lower bound of the instability is shifted. The domain of unstable wave numbers is $0.15 < \hat{k} < 1.2$; whereas, for the constant collision frequency case the domain was $0 < \hat{k} < 1.2$ (the lower bound for curve A is not shown in the figure). Curve C for a weaker shear, namely $\hat{s} = 0.5$, shows that the maximum growth rate is comparable to that of curve A. However, in this case modes with wave numbers $\hat{k} > 3.0$ are completely stabilized.
Figure 13. Plots of $\hat{k}_c$ (cut-off mode numbers) and $\hat{k}_m$ (mode numbers where the growth rate maximizes) as a function of $\hat{s}$. The solid and dashed lines correspond to the collisionless and collisional ($\nu = 0.5$) cases respectively. Curves A and B refer to $\hat{k}_c$ and $\hat{k}_m$, respectively.
Figure 14. Normalized growth rate versus the normalized wavenumbers for the case of a spatially dependent collision frequency. The density and velocity profiles are given in Eqs. (39) and (40). The parameters used are $\xi = 0.8$, $\chi_0 = -0.55$, and $\hat{v} = 0.5$. Curve A refers to constant collision frequency, $\hat{v} = 0.5$, and $\hat{s} = 1.0$. Curves B and C refer to $\hat{v} = 0.5 \exp(-\chi)$ and for $\hat{s} = 1.0$ and 0.5 respectively.
V. Discussion and Conclusions

We have investigated the influence of velocity shear on the Rayleigh-Taylor instability. The Rayleigh-Taylor instability is driven by gravity and an inverted density gradient. In general this instability is most unstable in the short wavelength domain, \( k_y L > 1 \), where \( L \) is the density inhomogeneity scale length and \( k_y \) is perpendicular to the density gradient and the magnetic field. We obtain the well known results that the maximum growth rate is given by \( \sqrt{g/L (g/\nu_{\text{in}} L)} \) in the collisionless (collisional) domain. On the other hand, a sheared transverse velocity drives the Kelvin-Helmholtz instability in the long wavelength domain, \( k_y L < 1 \). In the presence of transverse velocity shear, the short wavelength spectrum \( (k_y L > 1) \) of the Rayleigh-Taylor instability is strongly suppressed or stabilized and the growth rate maximizes in the long wavelength domain \( (k_y L < 1) \). Thus, velocity shear causes a long wavelength mode to be preferentially excited; whereas in the absence of velocity shear the dominant wave mode usually has a shorter wavelength determined by initial conditions or non-linear processes. This prominent conclusion had been stated in an earlier paper (Guzdar et. al., 1982, 1983).

We note that the wavepacket generally localizes in the region where the density gradient opposes the gravity, which in our case also happens to be the shear layer. The wave function falls off rapidly away from the localization region of the Raleigh-Taylor instability, but still has some finite amplitude in the stable region (where \( g/7n \) is positive). This is due to the global sampling of the entire density profile.

Another interesting feature of the generalized (including velocity shear) Rayleigh-Taylor instability is its crucial dependence on the velocity reversal point. In the absence of velocity shear, the wave function localizes at a point, say \( x_w \), determined primarily by the background density profile. In the
case of the hyperbolic tangent density profile (Eq. 34), the wave function localizes in the region where the density gradient opposes the gravity. If the velocity reversal point, $x_0$ (where the $y$-component of the equilibrium velocity changes sign), is in the region where the density gradient is parallel to the gravity, the velocity shear has a generally stabilizing influence without the characteristic peak in the growth rate vs wave number curve in the long wavelength domain (Fig. 9). However, when $\lambda_0 = x_0$, velocity shear reduces the growth rate significantly and moves the peak toward longer wavelengths, preferentially exciting longer wavelength modes.

Two possible applications of this theory to ionospheric phenomena have been discussed in a previous paper (Guzdar et al., 1982, 1983). Briefly, the major feature of this theory, viz., preferential excitation of a long wavelength mode, may explain (1) the structuring (1-3 km) of barium releases which are injected across the magnetic field (Linson et al., 1980; Wescott et al., 1980), and (2) the long wavelength (few hundred kms) oscillations of the bottomside $F$ layer during equatorial spread $F$ (Tsunoda and White, 1981; Kelley et al., 1981).

The shaped barium release experiment (Wescott et al., 1980) was conducted at high latitudes in the presence of a pulsating aurora at an altitude of 571 km. Numerical simulations showed that a charge separation induced radial polarization electric field results in an $E \times B$ velocity shear. This shear layer seems to be located in the region where the density gradient is steepest (Wescott et al., 1980). Our analysis does not strictly apply in the auroral environment. However, the basic instability leading to structuring of the barium cloud is possibly a Rayleigh-Taylor type instability (Pillip, 1971; Fedder, 1980) due to the deceleration of the cloud (Scholer, 1970). So our results in the collisionless domain could be applied to this case. For
example, gradient scale sizes of 500 m - 1 km can lead to irregularity scale sizes of 1.5 - 3 kms with growth rates ~ $10^{-2}$ sec$^{-1}$.

Kudeki et al. (1981) have shown, from the observations at Jicamarca using a Radar interferometer technique, that the velocity reversal point moves upward as the spread-F structures evolve. The position of the F-peak was not available at the time of these measurements. However, we conjecture that since the velocity reversal point is at a different location with respect to the F-peak at different times, the velocity shear induced long wavelength modulations of the bottom side F-layer may not be apparent at all times, but may be seen when the velocity reversal point is in the Rayleigh-Taylor localization region (namely, in the bottom side of the F region).

Tsunoda (1983) recently has shown that the background density gradient has a scale length of 25 km when long wavelength fluctuations were observed in the bottom side of the F-layer. No velocity shear measurements, such as the strength of the shear or velocity reversal point, were available. We point out that the measured absense of the velocity shear prior to or immediately after the onset of the wave-like structure is expected because ALTAIR needs the formation of the bubble and spike structures to measure the plasma velocities. Despite the lack of shear data in his paper there is resonable agreement between the data and the theoretical results. However, data on velocity shear, for example by alternate techniques, are crucial to confirm or disprove the theory. The question of short circuiting effects by the E-layer, raised by Tsunoda (1983), needs a closer examination and is not addressed here.

Similar results were obtained by Vinas (1980) in connection with the investigations of the erosion of the plasmapause. We conjectured that strong velocity shear could exist in the plasmapause region and lead to long
wavelength irregularities in competition with the ballooning mode type interchange phenomenon. However, an important difference exists in comparing Vinas' theory with ours, namely, that \( g/L \) is positive in his case, meaning that the heavy fluid supports the lighter fluid. We also find similar results in the topside of the equatorial ionosphere, where gravity acts in the same direction as that of the density gradient (a situation similar to Vinas' case) and the collision frequency is very small. These results indicate that in a Rayleigh-Taylor stable plasma the velocity shear could excite Kelvin-Helmholtz type modes. Thus we can conclude that in the absence of equatorial spread F if the flow velocity in the topside ionosphere is sufficiently strongly inhomogeneous, it can induce some large scale irregularities. Figure 5 (curve B) shows that if sufficient velocity shear exists, irregularities of scale sizes of ~300 km with weak growth rates \( (10^{-3} \text{ s}^{-1}) \) could possibly exist in the weakly collisional topside of the ionosphere.

In conclusion, we have shown that:

(i) Sheared plasma velocity flows can have pronounced effects on the collisional and collisionless Rayleigh-Taylor instabilities. Sufficiently strong velocity shear preferentially excites a long wavelength mode. This result may explain the long wavelength oscillations of the bottomside F layer during equatorial spread F and the prompt structuring of injected barium clouds (Guzdar et al., 1982; 1983).

(ii) Since the wavefunction localizes in the Rayleigh-Taylor unstable region we expect these long wavelength fluctuations to be seen at the bottomside of the F-layer.
(iii) This phenomenon is most likely to occur when the velocity reversal point is within the Rayleigh-Taylor localization region (where gravity opposes the density gradient).

(iv) The generalized Rayleigh-Taylor instability is qualitatively similar but has quantitatively different properties in the collisional and collisionless domains (see fig. 12). The cut-off mode numbers and maximally growing mode numbers are different in these two cases (see fig. 13).

(v) The properties of the Rayleigh-Taylor instability are similar for self-consistent as well as for general equilibrium density and velocity profiles.

(vi) As the velocity shear is increased, the cut-off mode numbers and the maximally growing mode numbers asymptote to values similar to those of the collisionless Kelvin-Helmholtz instability (see fig. 12).

(vii) A spatially dependent collision frequency alters the results drastically by reducing the growth rate, and by restricting the band of unstable wave numbers to a smaller region (see fig. 14).

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