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MECHANICS OF ICE JAM FORMATION IN RIVERS(U) COLD
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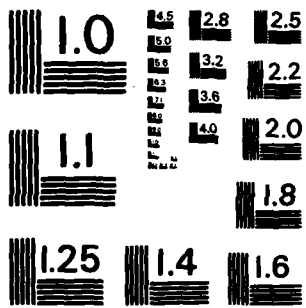
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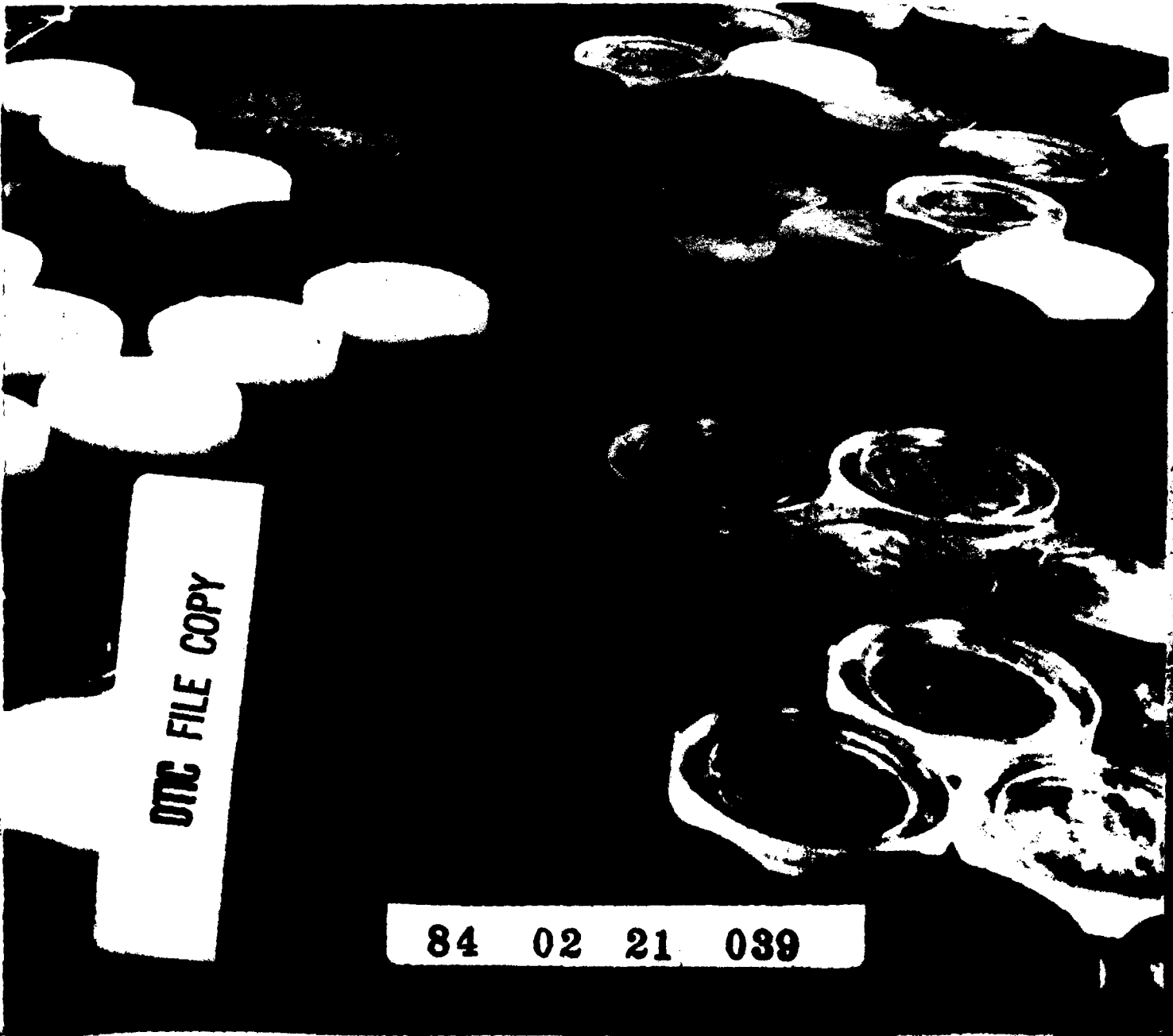


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Mechanics of ice jam formation in rivers



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Cover: Simulating river ice transport with suspended plastic disks in an air table chute.

CRREL Report 83-31

December 1983



Mechanics of ice jam formation in rivers

Norbert L. Ackermann and Hung Tao Shen

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CRREL Report 83-31	2. GOVT ACCESSION NO. AD-A138371	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MECHANICS OF ICE JAM FORMATION IN RIVERS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Norbert L. Ackermann and Hung Tao Shen		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Clarkson College of Technology Potsdam, New York		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS CWIS 31750
11. CONTROLLING OFFICE NAME AND ADDRESS Office of the Chief of Engineers Washington, D.C. 20314		12. REPORT DATE December 1983
		13. NUMBER OF PAGES 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Hydraulics Mathematical model Ice Rivers Ice bridge Surface ice jam Ice jam		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A mathematical model is described that is used to determine the maximum ice conveyance capacity of a river channel. Based upon this model, computer programs were developed that enable the ice discharge to be calculated for steady-state flow conditions. For rivers that have uniform flow, the maximum ice-conveying capacity can be described with a simple function expressed in terms of the size of the ice fragments, channel geometry, and the flow of water in the river. For nonuniform flows, the computer program determines the elevation profile of the surface layer in addition to other flow characteristics, such as the velocity and surface concentration of the ice fragments. The location along this surface profile where the ice conveyance capacity becomes less than the upstream supply is determined and is considered to be the position where a surface ice jam or ice bridge will be formed. ←		

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PREFACE

This report was prepared by Norbert L. Ackermann and Hung Tao Shen of Clarkson College of Technology, Potsdam, New York. Funding for this research was provided by Civil Works Program, *Ice Engineering, CWIS 31750, Prediction of Ice Formation.*

The authors have benefited greatly from discussions with G.E. Frankenstein, G.D. Ashton and D.J. Calkins regarding the mechanics of the formation of river jams.

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NOMENCLATURE

a	empirical constant in stress-strain rate relationship, eq 1
B	width of water surface
b	width of river bottom
C	volumetric or surface concentration of solids
C_D	drag coefficient
C_0	maximum concentration of solids
D	ice fragment diameter
g	acceleration of gravity
I	maximum ice-conveying capacity
k_b	roughness height of river bed
k_i	roughness height of ice cover
n_b	Manning's coefficient
Q	$Q_w + 0.92Q_i$
Q_w	water discharge
Q_i	ice discharge
q	unit width discharge
R	hydraulic radius
S_0	slope of river bed
t	ice fragment thickness
u	velocity
u_i	velocity of ice
x	downstream coordinate direction
x_j	distance from a nearly uniform flow section to ice jam section
y	depth coordinate direction
y_n	normal depth
y_t	total depth of flow
y_{ij}	total depth of flow where $x = x_j$
z	transverse coordinate direction
α	constant; Q_i/I
γ_w	specific weight of water
δ	annular gap width
ϵ	coefficient of restitution
θ	side slope of river bank
λ	linear concentration
μ	coefficient of friction
ρ, ρ_f	density of fluid
ρ_i	density of ice
ρ_s	density of solid
σ	normal stress
τ	shear stress
τ_b	shear stress at river bed
τ_i	shear stress at water/ice cover interface

MECHANICS OF ICE JAM FORMATION IN RIVERS

Norbert L. Ackermann and Hung Tao Shen

INTRODUCTION

The principles that govern the motion of ice fragments on the surface of a river are of interest because of their possible influence on the formation of a river's surface ice cover. When floating in isolation, an ice fragment moves at approximately the local surface velocity of the river or stream. However, if ice fragments occur in sufficient concentration, they collide with one another as well as with the river banks, reducing their downstream velocity. The mutual interference created by adjacent ice fragments can significantly reduce a river's ice-carrying capacity. The extent of this interference depends upon the size distribution and physical characteristics of the ice fragments, their number density, and their mean velocity, as well as the boundary conditions formed by the river banks.

This report presents an analysis of the stresses produced by the collisional transfer of momentum between adjacent ice fragments on a river's surface and describes how stresses influence a river's ice transporting capacity. The maximum ice-conveying capacity of a river can be determined using the theoretical relationships developed in this report. If the upstream supply of ice particles exceeds the maximum value, a rapid increase in the concentration of the surface layer develops and a stationary cover starts to form.

During a river's prefreeze-up period, ice pans commonly form on the river surface and are transported downstream with the river flow. These ice pans may grow in size and occur in sufficient abundance to create a surface ice jam across the river or stream. This surface restriction or barrier causes the upstream progression of an ice cover, since the incoming ice supply cannot be transported farther downstream. Such a surface ice jam or bridge also creates a potential location for a river ice jam in the spring if ice fragments created upstream by spring break-up are arrested by this surface barrier.

It is believed that the motion of the surface layer created by the ice pans in the fall, as well as fragmented ice elements in the stream following the spring break-up, can be described by the analysis presented in this report.

CONSTITUTIVE RELATIONSHIPS

When ice fragments are transported along the surface of a river, the mixture of ice and water that forms the surface layer can be considered as a material or continuum with its own distinctive material properties. At rest, this surface layer forms a rigid boundary that is subject to stresses identical to those that are produced at the river banks and bottom. Stresses within the rigid ice layer are then of interest for determining ice thickening processes, the development of ice pressure ridges, or the conditions for incipient break-up.

Before a stationary, rigid ice cover is formed, the surface layer is formed from a mixture of ice fragments, water, and slush that is transported downstream by gravity. The stresses generated within this moving surface layer have a significant influence upon the rate with which the mixture is transported downstream and, ultimately, upon the conditions required for the formation of a rigid ice cover.

If the stresses within the surface layer result from the collisional transfer of momentum between adjacent particles, the flow of this layer is considered to be inertia-dominated. When a river is at its maximum ice-transporting capacity, the velocities of the ice fragments in the surface layer are of the same order of magnitude as the average water velocities in the river. For these conditions, therefore, the assumption that the stresses within the surface layer are inertia-dominated appears to be reasonable. Although experimental information describing the stresses within a rapidly sheared mixture of ice fragments and water is not available, there are data available on the constitutive (stress-strain rate) relationships of other cohesionless granular solids that can be modified to provide information about the stress state that would exist in the surface layer of a river covered with rapidly flowing ice fragments.

Bagnold (1954) was the first to describe a theoretical framework for analysis and to develop experimental information describing the stresses in a rapidly sheared, neutrally buoyant suspension of granular solids. He used spherical solids of hardened wax and lead that were suspended in water. The mixture of solids and water was contained in the annular region between two concentric cylinders (Fig. 1). The angular velocity and torque required to rotate the outer cylinder was measured to obtain the relationship between the velocity gradient of the mixture within the annular gap, the volumetric concentration of the solids, and the stress required to produce the motion.

The velocity gradient du/dz produced within the annular gap was considered to be constant and equal to u/δ , where u is the velocity of the outer cylinder and δ is equal to the gap width. Bagnold determined that the shear stresses in the inertia-dominated regime could be approximated by the relationship

$$\tau = a \rho_s \left[\frac{D}{(C_0/C)^{1/3} - 1} \right]^2 \left(\frac{du}{dz} \right)^2 \quad (1)$$

where a = an empirical constant

D = diameter of the spherical particles

C = volumetric concentration of solids

C_0 = concentration at the densest packing.

The normal stress σ producing a force perpendicular to the cylinder walls was found to be proportional to the shear stress, as described by

$$\sigma/\tau = \text{constant}. \quad (2)$$

Savage (1978) and Sayed (1981) conducted tests similar to Bagnold's using a variety of materials

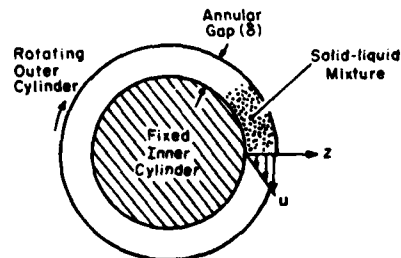


Figure 1. Schematic diagram of Bagnold's laboratory apparatus.

create different solid-liquid mixtures, but the mixture for which experimental data was available that most closely represented the ice fragments and water found on a river surface is the mixture of hard wax spheres in water that was first studied by Bagnold (1954).

Following Bagnold's pioneering study, numerous efforts were made to formulate a theoretical framework that would enable the stresses in a rapidly sheared granular mixture to be predicted. The studies by Ackermann and Shen (1982), Shen and Ackermann (1982), and Shen (1982) were the first to provide a quantitative description of the stresses created in a rapidly sheared mixture of solids and fluids. Their equations described the shear and normal stresses in terms of the density ρ_s and ρ_f of the solid and fluid, the diameter D of the granular solids, the coefficient of kinetic friction μ , the coefficient of restitution ϵ , and the volumetric concentration C within the mixture. An approximation to the complete constitutive equations described by Shen (1982) for the shear and normal stresses of disk-shaped solids in a two-dimensional surface layer is

$$\tau = C_0 \rho_s D^2 \frac{\lambda}{2} \left(\frac{1+\epsilon}{3\pi} \right) \left[\frac{\frac{1+\epsilon}{3\pi} \frac{\lambda}{1+\lambda}}{\frac{4}{\pi} \frac{C_D}{\lambda} \frac{\rho_f}{\rho_s} + \frac{1-\epsilon^2}{4} + \frac{\mu(1+\epsilon)}{\pi} - \frac{\mu^2(1+\epsilon)^2}{4}} \right]^{1/2} \left(\frac{du}{dz} \right)^2 \quad (3)$$

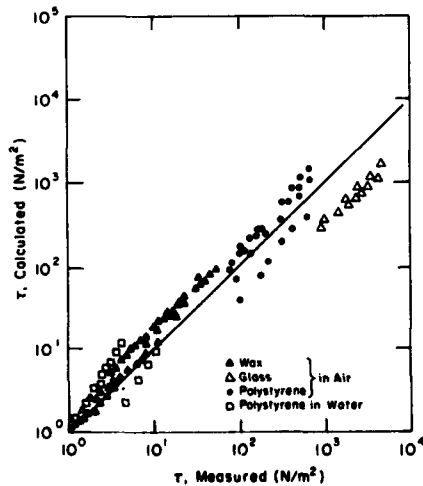
$$\sigma = \frac{-6\sqrt{2}}{\pi} \left[\frac{\frac{1+\epsilon}{3\pi} \frac{\lambda}{1+\lambda}}{\frac{4}{\pi} \frac{C_D}{\lambda} \frac{\rho_f}{\rho_s} + \frac{1-\epsilon^2}{4} + \frac{\mu(1+\epsilon)}{\pi} - \frac{\mu^2(1+\epsilon)^2}{4}} \right]^{1/2} \tau \quad (4)$$

where C_D is the coefficient for fluid drag of the disk and λ is equal to $[(C_0/C)^{1/2} - 1]^{-1}$.

Equation 5 was developed for stresses in which the mixture was formed of sphere-shaped particles:

$$\tau = C_0 \rho_s D^2 \frac{\lambda}{2} \left[\frac{(1+\epsilon)^3 (0.053 + 0.081\mu)^3 \left(\frac{\lambda}{1+\lambda} \right)^3}{\frac{3}{2} \frac{C_D}{\lambda} \frac{\rho_f}{\rho_s} + \frac{1-\epsilon^2}{4} + \frac{\mu(1+\epsilon)^3}{\pi} - \frac{\mu^2(1+\epsilon)^2}{4}} \right]^{1/2} \left(\frac{du}{dz} \right)^2 \quad (5)$$

As reported by Shen and Ackermann (1982), eq 5 accurately describes the variety of laboratory data that are available if a single experimental constant is used as a constant multiplier to eq 5. The agreement between the laboratory data and the theoretical results obtained from eq 5 is shown in Figure 2.



Material	Coef. of restitution ϵ	Coef. of friction μ	Linear concn. λ
Wax in air	0.2	0.2	2.1-7.6
Glass in air	1.0	0.2	6.3-7.5
Polystyrene in air	0.9	0.2	5.5-7.1
Polystyrene in water	0.9	0.2	3.0-6.8

Figure 2. Measured shear stress vs calculated values (eq 5 with a factor of 10).

The data presented consist of all published data presently available and include tests in which the solids were either of hardened wax, polystyrene, or glass and the fluid was either water or air. Laboratory data were not available, however, with which to determine the validity of eq 3 describing the shear stresses in a surface layer of disk-shaped solids. The ratio of eqs 3 and 5 was therefore used as a multiplying factor to convert Bagnold's experimental data for rapidly sheared sphere-shaped solids to an equation appropriate for disk-shaped solids, such as ice fragments on the surface of a stream. Chiou (1982) describes in detail how eqs 3 and 5 and other independent methods were used to modify Bagnold's experimental results to determine the constitutive relationships for rapidly sheared disk-shaped solids in a two-dimensional layer. Each of the methods used to modify eq 1 to represent the motion of disk-shaped solids produced approximately similar results. Chiou determined that the equation that best described the shear stress in a surface layer of ice fragments is

$$\tau = 0.021 \rho_i \lambda (1 + \lambda) D^2 (du/dz)^2 \quad (6)$$

where $\lambda = [(0.91/C)^{1/2} - 1]^{-1}$

ρ_i = density of ice

D = ice fragment diameter

du/dz = velocity gradient of the surface ice layer.

EQUATIONS OF MOTION

Uniform flow

The motion of the surface ice layer in a river is produced by the downstream component of the gravity forces and the shear force produced by the water moving below the ice-covered surface. During steady uniform flow these driving forces are in balance with the resisting forces produced by the river banks. The average motion of the surface layer is described by the velocity profile $u_i = u_i(z)$ shown in Figure 3b. Using eq 6, the internal shear stress τ of the ice layer can be expressed in terms of the velocity gradient, du_i/dz , and concentration C of the surface layer. In this way a link is established between the forces that act on the ice layer and the deformation rates or velocities that these forces create.

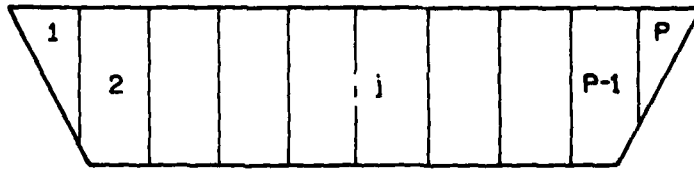
For a system to be in dynamic and kinematic equilibrium there must be a compatibility at the ice/water interface between the forces and deformations of the ice layer and those of the lower layer of ice-free river water. Figure 3b is a pictorial representation of these requirements.

Kinematic constraints

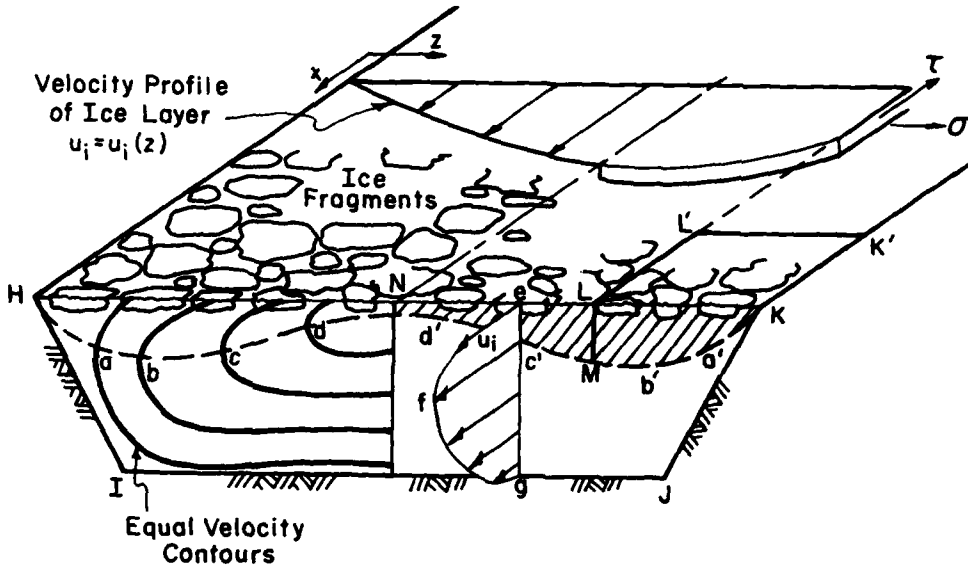
The left-hand portion of Figure 3b illustrates the contours of equal water velocity in the ice-free river water. These contours intersect the ice layer at the point where the surface layer has a velocity equal to that represented by the contour. The vertical velocity profile of the water is shown on the right-hand side of Figure 3b. The maximum velocity occurs at position c' . The locus of points of maximum water velocity over the river width is described by a line through points $H, a, b, c, d, d', c', b', a',$ and K . To maintain kinematic compatibility the vertical velocity profile of the water drawn upward from the river bed at point g must meet, at point f , with the water velocity drawn downward from the moving ice cover at point e . These velocity profiles are described in terms of the boundary shear stress and bed roughness, and must satisfy the relationship

$$u_i + 2.5\sqrt{\tau_i/\rho} \ln(30\bar{ec}'/k_i) = 2.5\sqrt{\tau_b/\rho} \ln(30\bar{c'g}/k_b) \quad (7)$$

where τ_i and k_i refer to the shear and roughness at the lower surface of the moving ice cover, τ_b and k_b refer to the shear and roughness at the river bottom, and \bar{ec}' and $\bar{c'g}$ are the distances between the corresponding points shown in Figure 3b. The integration of the velocity profiles over the channel cross section must also equal the discharge of water that is specified for the river.



a. Schematization for numerical solution.



b. Velocity contours and velocity profile of surface ice layer.

Figure 3. Channel cross section.

Dynamic equilibrium

The shear stress τ within the ice layer and the boundary stresses τ_i and τ_b can all be determined from force balances on free-body diagrams of elements within the flow system. If τ , τ_i , and τ_b are to be the only stresses involved in the free-body analysis, the selection of the free bodies must depend upon the shape of the velocity contours shown in Figure 3b. Since the line through points H, a, b, c, d, d', c', b', a', and K represents a locus of points of maximum velocity, it can be assumed that this is also a line along which the longitudinal shear stress is zero. This can be further illustrated by considering the vertical velocity profile shown on the right-hand side of Figure 3b. Since du/dy at point f on the vertical velocity profile is zero, the shear stress is also equal to zero, since fluid stresses are proportional to some power of the velocity gradient. For a river with uniform flow and no ice cover, the total area $HJKH$ multiplied by the bed slope and specific weight of water would equal the total resistance force per unit length of river exerted by the bed shear τ_b . For an ice-covered channel, the shaded area on the right-hand side of Figure 3b multiplied by the bed slope S_0 and specific weight γ_w of water therefore represents the force per unit of river length acting on the ice-covered surface. In steady uniform flow, the boundary stresses can be described by the relationships

$$(\tau_i)_e = \gamma_w S_0 \overline{ec'} \quad (8)$$

$$(\tau_b)_g = \gamma_w S_0 \overline{c'g} \quad (9)$$

where $(\tau_i)_e$ and $(\tau_b)_g$ refer to the boundary stresses at locations e and g , respectively. The equation for the stress τ within the ice cover $KK'L'L$ is obtained from a force balance on the free body (Fig. 3b). This force balance includes the downstream component of the weight of the ice fragments and water contained in the surface layer as well as the shear forces on the underside of the ice layer represented by the shaded area $Lmb'a'KL$.

These conditions for dynamic equilibrium and kinematic compatibility enable sufficient equations to be written to determine the velocity and volume rate of flow of the ice in the surface layer. The formulation of these equations and their method of solution are described by Ackermann (1979), Free (1979), and Chiou (1982). The solution procedure involves discretizing the channel into P strips or volume elements, as shown in Figure 3a. The equations of motion were written for the water and ice portions of each of the volume elements. These equations were then solved numerically.

Nonuniform flow

Numerous complications are introduced by considering conditions where the river's depth changes in the direction of flow. The stresses τ_i and τ_b can no longer be described by eq 8 and 9, since there is then a variation in the momentum flux in the stream direction. Since none of the channel boundaries then have parallel surfaces, the geometric complexities involved in discretizing the channel into subelements greatly complicate the analysis. For nonuniform flow conditions there are also nonzero values of the transverse mass and momentum flux between each of the P subelements. The development of the equations of motion considering these nonuniform effects is described by Chiou (1982).

ICE TRANSPORT: UNIFORM FLOW

Symmetric channel

The ice-transporting capacity of the river shown in Figure 4 is to be determined when the flow conditions are steady and uniform. Conditions of uniform flow represent an idealized situation in which the depth of flow as well as all other flow conditions remain constant along the length of the river. The channel shown in Figure 4 is symmetric about its centerline, which further simplifies the analysis.

If the number density of the ice fragments on the river surface is sufficiently low, there will be few collisions between the ice particles and the river banks. The gravity component of the ice/water system in the stream direction can then be assumed to be balanced entirely by the shear stresses τ_b between the water and the river banks and bottom. In such a situation, the normal depth y_n can be approximated from the Manning equation expressed in the following form:

$$R(y_n) = \left[\frac{n_b Q}{1.49 S_0^{1/2}} \right]^{3/2} \quad (10)$$

where $R(y_n)$ is the hydraulic radius of the channel cross section that can be expressed in terms of the normal depth y_n , n_b is Manning's coefficient for the river bed, and Q is equal to $Q_w + 0.92 Q_i$, where $0.92 Q_i$ represents the volume of ice transformed to its equivalent volume of water. At low surface concentrations, the volume flow rate of the ice can be expressed as

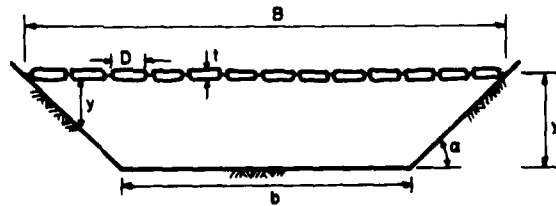


Figure 4. Schematic diagram of channel cross section.

$$Q_i = \int_0^B t \cdot C(z) \cdot u_i(z) dz \quad (11)$$

where

$$u_i = 2.5\sqrt{\tau_b/\rho} \cdot (30y_n/k_b)$$

$$\tau_b = \sqrt{\gamma_w y_n S_0}$$

and t represents the thickness of the ice fragments that occur with the concentration C on the river surface layer. For a given channel geometry and water discharge Q_w , the ice

discharge Q_i can be expressed in terms of the concentration of ice in the surface layer, as shown in Figure 5. The approximately straight line portion of the curve between p and q on Figure 5 can be approximated by eq 11. When the concentration of ice on the river surface increases, the collisions between adjacent particles and the river banks become increasingly effective in producing a mechanism whereby the weight of the ice layer and the underside shear stresses τ_i are transferred to the river banks through the intergranular stresses τ described by eq 6. Under such flow conditions, the complete equations of motion must be solved to determine accurately the relationship between Q_i and the surface ice concentration C shown in Figure 5. Bunte (1980) considered a flow in the trapezoidal channel shown in Figure 4 for which the Manning's n for the ice and river were $n_i = 0.3$ and $n_b = 0.25$. The maximum ice-conveying capacity I , which corresponds to point r in Figure 5, was described by the following dimensionless relationship

$$\phi\left(\frac{I}{Q_w}, \frac{Q_w^2}{gb}, \frac{D}{b}, \frac{D}{t}, \theta, S_0\right) = 0. \quad (12)$$

A quantitative relationship between these variables was obtained from a numerical solution of the equations of motion. These numerical results were obtained for a range of geometries and flows that was considered to represent the range of physically realistic river conditions. A statistical analysis was then performed upon these numerical results to obtain the following equation, which describes the ice-conveying capacity I ,

$$\frac{I}{Q_w} = 2.4 \left(\frac{1}{\theta}\right)^{0.682} \left(\frac{t}{D}\right)^{1.181} \left(\frac{D}{b}\right)^{0.845} (t/D)^{0.083} (1/\theta)^{0.106} \left(\frac{gb^5 S_0}{Q_w^2}\right)^{0.317\theta^{0.135}} \quad (13)$$

A summary of these results was presented by Ackermann et al. (1981). For a side slope of $\theta = 1:1/2$ for example, eq 13 can be approximated as

$$\frac{I}{b} = K \frac{t^{1.16}}{D^{0.33}} b^{0.13} S_0^{0.32} q^{0.36} \quad (14)$$

where I/b is the maximum ice discharge per unit length of bottom width b , q is Q_w/b , and K is a constant. From eq 14 it is apparent that as the characteristic diameter D of the ice fragments increases, the ice-conveying capacity of the river decreases. This relationship is seen in commonly observed field conditions where ice pans are increasingly retarded in their movement downstream as they make contact with adjacent ice pans and grow in size through bonding or freezing at their surfaces of contact. As indicated by eq 14, the unit width ice discharge I/b increases as the bed slope S_0 increases. As the unit discharge q increases, the shear τ_i on the underside of the ice cover also increases, again producing increased values of the ice-conveying capacity. Decreases in the

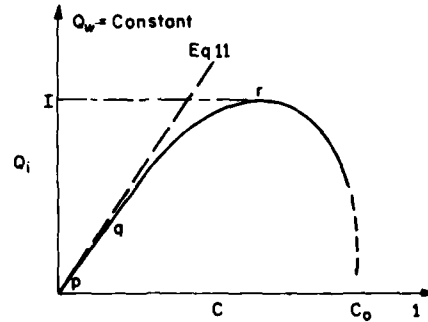


Figure 5. Ice discharge vs surface concentration.

unit discharge l/b are produced by a decrease in the river's base width b . This decrease occurs since, for the same unit discharge, a reduction in the channel width b produces an increase in the velocity gradient du_1/dz , which, as shown in eq 6, increases the intergranular stress within the ice/water mixture in the surface layer.

Asymmetric channel

Figure 6 shows a channel that has no geometric symmetry. The solution of the equations of motion for steady uniform flows in such systems is not significantly complicated. For symmetric channels, the channel centerline provides the location where $du_1/dz = 0$ and hence provides a convenient boundary condition with which to calculate the ice velocity profile. When the channel is asymmetric, the position where $du_1/dz = 0$ is located by a trial-and-error procedure and merely introduces an additional computational step in the numerical solution. Chiou (1982) describes the detailed computational procedure required to determine the ice conveyance capacity of a channel that has no geometric symmetry.

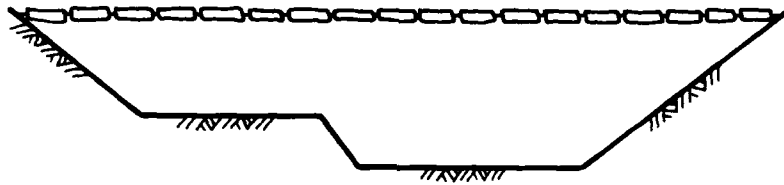


Figure 6. Asymmetric channel.

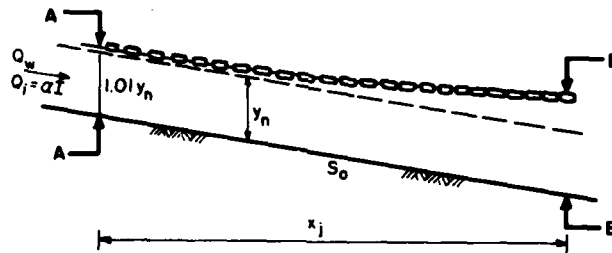


Figure 7. Nonuniform flow in an ice-covered river.

ICE TRANSPORT: NONUNIFORM FLOW

Consider the flow of an ice-covered river in which the discharge of water and ice is Q_w and Q_i , respectively (Fig. 7). If the flow were uniform, the value of Q_i would correspond to a surface ice concentration C , as described in Figure 5. If the concentration of ice upon the river surface were very low, the value of y_n could be approximated by eq 10. For the specified value of Q_w in a uniform flow, there would be a maximum ice-conveying capacity, l , which is also shown in Figure 5. The ice discharge Q_i can be expressed in terms of this maximum conveyance capacity as $Q_i = \alpha l$.

Chiou (1982) considered conditions of nonuniform flow, such as those shown in Figure 7, where the upstream depth y was greater than y_n and increased in the downstream direction. He described the depth of flow at the upstream boundary as equal to $1.01 y_n$. The water surface profile and concentration of ice fragments upon the river surface were computed in a stepwise manner in the downstream direction. For the prescribed values of Q_w and $Q_i = \alpha l$, the river reach of length x_j was determined, within which the equations of motion and conditions for the continuity of flow could be satisfied.

As the depth of flow increased in the direction downstream from section A, the water velocity as well as the slope of the surface profile decreased. The reduction of water velocity and surface

slope correspondingly reduced the driving force on the ice layer, thereby producing an increase in the concentration C of the ice fragments on the surface and a reduction in their downstream velocity u_i . As long as the product of the surface concentration and the velocity of the ice particles could provide for the transport of the specified upstream ice supply Q_i as shown by eq 15, steady-state flow conditions could be maintained and the equations of motion would be satisfied.

$$Q_i = \alpha I = t \int_0^R v_i(z, x) C(z, x) dz. \quad (15)$$

By traveling sufficiently far downstream, however, a location such as section B will eventually be reached where a set of values $u_i(z)$ and $C(z)$ cannot be found that satisfy the equations of motion. The volume rate of flow Q_i established at the upstream boundary can therefore not be maintained below that section, and unsteady-state conditions are created. The concentration of ice at section B will increase with time, the ice fragments will be compressed by forces from the upstream flow, and a solid ice cover will start to form. This situation creates the conditions required to initiate an ice jam or obstruction of the surface layer at location B.

Chiou (1982) simulated numerous flow situations by solving the equations of motion for non-uniform flows in a river with a trapezoidal cross section. He developed the digital computer program that was used to find the numerical solutions; it was restricted to nonuniform flows in symmetric channels in which the depth increased in the downstream direction. Figures 8 through 12 provide some of Chiou's (1982) results.

It was found that, even when the channel was conveying ice at its maximum capacity, the depth of flow described by the water surface profile was essentially the same as when the water surface was ice-free. The small effect of the ice cover upon the deviation of the water surface profile resulted from the fact that when the river transported ice at its maximum conveyance capacity the entire ice cover was moving. The ice cover therefore did not present a stationary boundary that would produce interfacial stresses of the magnitude that existed between the water and the river bottom. Hence, when applying these results to rivers in the pre-freezeup period, it can be assumed that the presence of ice pans on the river's surface does not significantly influence the water surface elevation. Figure 8 demonstrates this condition by describing conditions when the flow in a river is $30,000 \text{ ft}^3/\text{s}$ ($850 \text{ m}^3/\text{s}$). The river has a bed slope of 0.0001 with a bottom width of 250 ft (77.2 m) and side slopes of $2:1$. The maximum ice-conveying capacity of the river is $I = 1950 \text{ ft}^3/\text{s}$ (55.2 m^3). Three conditions are given, with the ice entering the upstream boundary at flow rates of $Q_i = 0.25I, 0.5I$, and $0.97I$ (i.e. $\alpha = 0.25, 0.5$ and 0.97). For these conditions the value of $1.01y_n$

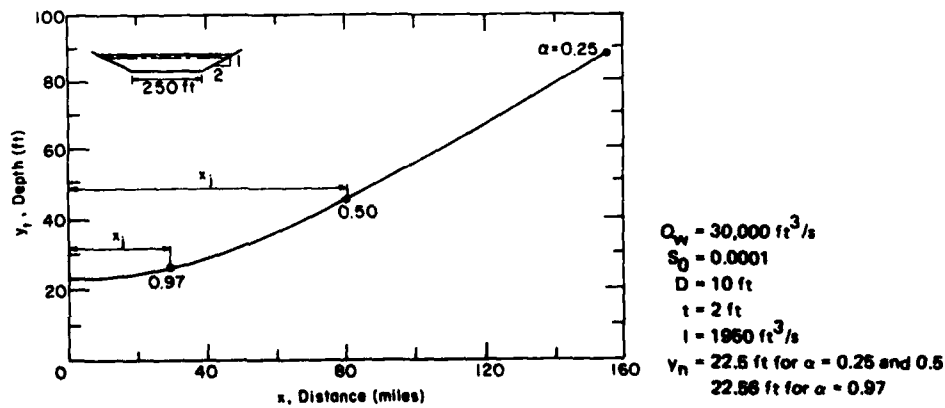


Figure 8. Water surface profiles for three different ice discharges ($\alpha = 0.25, 0.5$ and 0.97).

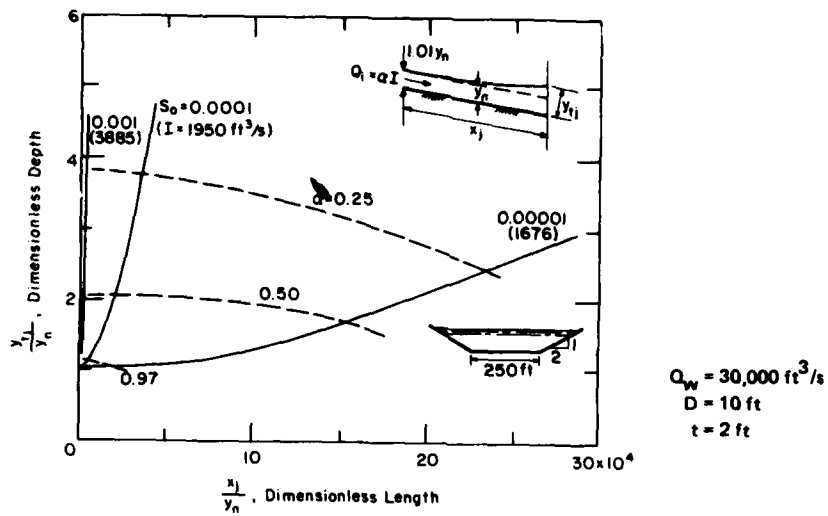


Figure 9. Effect of bed slope S_0 on location of surface ice jam.

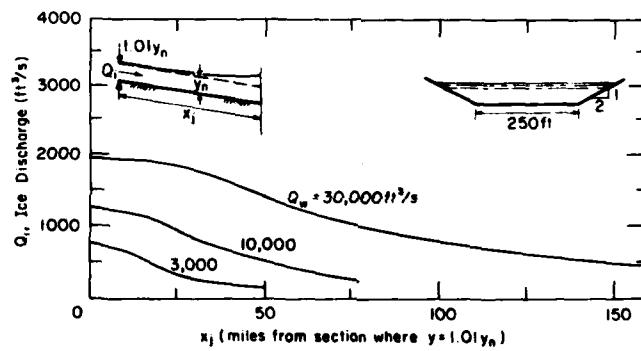


Figure 10. Ice discharge vs distance to surface ice jam for different river water discharges.

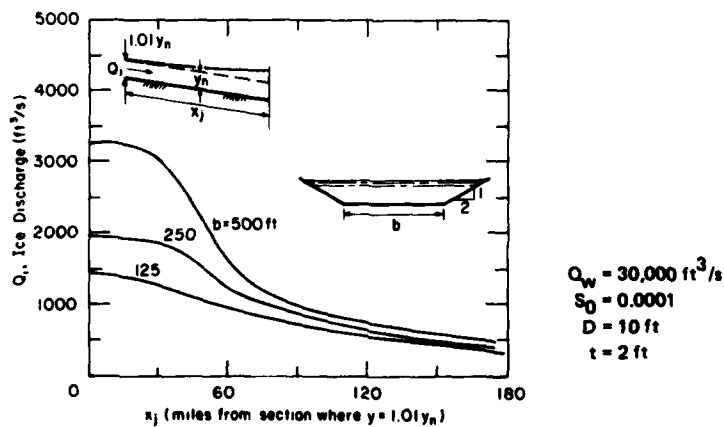


Figure 11. Ice discharge vs distance to surface ice jam for different bottom widths.

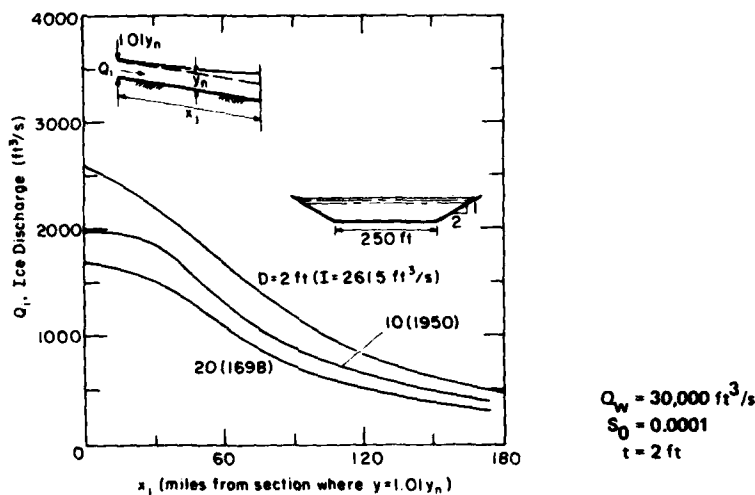


Figure 12. Ice discharge vs distance to surface ice jam for different diameters of ice fragments.

at the upstream boundary is 22.5 ft, 22.5 ft, and 22.56 ft, respectively. Figure 8 shows the relationship between the centerline or total depth of flow y_t and the distance x downstream from the $x = 0$ location where $y_t = 1.01y_n$. The river reach of length x_j extending downstream from section A (Fig. 7), within which the discharge Q_i could be maintained, is also described in Figure 8. For example, Figure 8 shows that the river can maintain an ice discharge of $Q_i = 0.5(1950 \text{ ft}^3/\text{s})$ for an 80-mile reach downstream from the section where the flow has a depth 1% greater than the normal depth. As seen from the figure, the depth at this limiting downstream location is approximately twice the normal depth.

Figure 9 shows how the slope of the channel bottom and the ice discharge influence the reach x_j when the river discharge is $Q_w = 30,000 \text{ ft}^3/\text{s}$. The depth and distance downstream to the location of the surface jam is presented in dimensionless form, utilizing the depth y_n as the normalizing parameter. The figure applies to ice fragment diameters $D = 10 \text{ ft}$ and ice thicknesses $t = 2 \text{ ft}$. The figure enables the maximum downstream depth for which the ice discharge Q_i could be transported to be determined.

Figure 10 shows how discharges Q_w and Q_i influence the distance x_j between the upstream boundary, where $y = 1.01y_n$, and the location of the surface ice jam. Another interpretation is that the downstream section located at x_j represents the location of the channel cross section where the ice-conveying capacity becomes less than the upstream supply $Q_i = \alpha I$. For a given ice discharge Q_i , the distance x_j increases with increasing values of Q_w . Variations in the bed width of the trapezoidal channel influence the relationship between x_j and the ice discharge Q_i (Fig. 11). The ice-conveying capacity is significantly increased if the diameter of the surface ice fragments is reduced (Fig. 12). This relationship results from the fact that the shear stress in the surface ice layer varies in direct proportion to the square of the diameter of the surface particles.

Chiou's (1982) analysis shows that the water surface profile for steady nonuniform flows is not significantly influenced by the movement of ice on the surface layer. A greatly simplified method of analysis could therefore be developed where the river depth and surface slope were first established by assuming the flow to be ice-free. With the water surface slope thus fixed, the river could then be considered to have a surface layer of moving ice fragments. The predetermined slope would enable the gravity forces on the ice layer to be calculated, and the ice-conveying capacity could be determined by establishing a force balance between the streamwise gravity component acting on the surface layer, the intergranular stresses, and the boundary shear from the river banks.

In such an analysis, the shear stresses τ_i from the water would be ignored, and the results would thus underestimate the ice-conveying capacity. Such an analysis, however, would be very simple to perform and would provide a reasonable approximate solution.

FURTHER CONSIDERATIONS

Several assumptions were made in this investigation that could not be readily substantiated but that were expedient for purposes of analysis. The first assumptions were that no slip conditions existed at the river banks (Fig. 13a) and that the concentration of ice fragments on the river surface was constant over the full river width (Fig. 13b). A third assumption was that eq 6 accurately describes the constitutive equations for the surface ice layer. These are believed to be reasonable assumptions of conditions that would exist before the formation of a surface ice bridge. One contradiction, however, was introduced by eq 2, which indicated that the normal stresses σ were proportional to the shear stresses. The inconsistency that this introduces is illustrated in Figure 13. Figure 13c shows the distribution of shear stresses in the ice-covered surface layer. If the forces from the water underneath the ice layer are ignored, the shear stress in the surface ice layer varies linearly from zero at the channel centerline to a maximum at the river bank. Since eq 2 indicates that normal stresses are proportional to the shear stresses, it is then consistent to draw a distribution of normal stresses as in Figure 13d. However, there are no uniformly distributed transverse forces in the river, acting normal to the river banks, that would produce such a linear variation in the normal stresses over the river's cross section. From considerations of a transverse force balance, it would appear that the normal stress σ should be constant across the entire river width. This conclusion, however, is inconsistent with the linearly varying normal stress distribution. This inconsistency was found to exist in other theories that describe the flow of granular materials in chutes and conduits (Jenkins and Cowin 1979). The authors believe, however, that they have identified the causes of this paradoxical situation and are developing a complete solution to the problem. The solution will enable the variation in the concentration over the river cross section to be determined and will allow the slip conditions that exist at the river banks to be accurately modeled. With these modifications, the analytical solutions that have been presented can be improved.

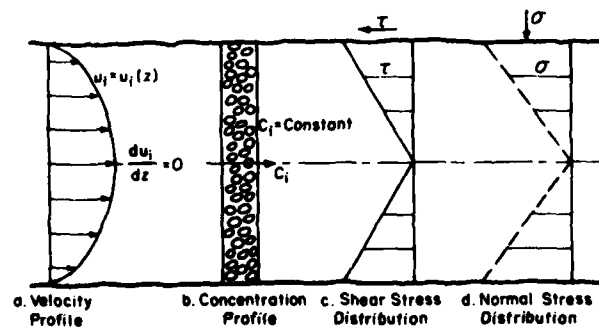


Figure 13. Schematic of the assumptions upon which the analysis is based.

BASIS FOR MODEL IMPROVEMENT

In Ackermann and Shen (1982), the authors developed a theory that describes the mechanism by which both normal and shear stresses are developed in a rapidly sheared granular material. By including a single empirical constant, their theory has been shown to agree with experimental results where granular mixtures have been sheared in the opening created between two closely spaced boundaries having a relative velocity y (Fig. 1). Within such small gaps the velocity distribution is considered to be linear. If the granular flow occurs within boundaries that are not closely spaced, such as between two river banks, the mechanism for stress generation within the granular mixture is believed to be more complicated than that previously reported by Ackermann and Shen (1982), who reported that Reynolds-type stresses were produced by the transfer of momentum between colliding particles. Unlike existing theories for describing the turbulence of homogeneous fluids, these Reynolds-type stresses could be accurately determined from theoretical considerations. The lateral diffusion of the turbulent energy created by the collision of the ice fragments was ignored, however, when eq 6 was derived to describe the shear stresses in the mixture of ice and water on the river's surface layer. To describe completely the stress state within the surface layer of a river, an energy balance equation that includes the generation of turbulent energy within the mixture, its dissipation into heat, and its lateral transfer must be considered. Only by introducing these modifications will it be possible to obtain the complete solution to the problem of the transportation of ice on a river surface. The concentration of ice is not constant over the river's width, and the normal stresses between the ice fragments and the river banks are an important quantity in the solution. The identification of these normal stresses will permit evaluation of the slip of ice fragments at the boundary.

By using the changes suggested above to modify the constitutive relationships and by including the simplifications that were made obvious from the work by Chiou (1982), a more accurate model of the transportation of ice on a river surface can be developed. The authors are currently conducting laboratory tests in which disk-shaped solids are transported by gravity forces down a smooth and evenly inclined chute. Numerous holes on the chute surface provide a vertical air supply that greatly reduces the frictional forces between the disks and the surface of the chute. The correct interpretation of the results from these tests is believed to depend upon obtaining an analytical solution to the problem described above, which considers the dispersion of turbulent energy through the granular flow.

CONCLUSIONS

The maximum ice-conveying capacity of a river depends upon the amount of river water discharge, the channel geometry, the size of the ice fragments, the depth of flow, and the slope of the water surface profile. A single equation has been found that describes the ice conveyance capacity of the channel when the flow in a river is uniform so that the depth of the flow remains constant. When the flow is nonuniform, however, the velocity and concentration of ice fragments upon the river surface are obtained from a numerical solution of the equations of motion. If the volume rate of the flow of ice supplied by the upstream reach of a river is specified, the depth of flow can be determined at the point where the incoming supply exceeds the local ice-conveying capacity of the river. At such a location a surface ice jam is going to occur.

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Ackermann, Norbert L.

Mechanics of ice jam formation in rivers / by Norbert L. Ackermann and Hung Tao Shen. Hanover, N.H.: U.S. Cold Regions Research and Engineering Laboratory; Springfield, Va.: available from National Technical Information Service, 1983.

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